Chapter 4
Preliminary Design

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Updated: 4/7/2015

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4.1. Introduction

The purpose of this chapter is to describe the preliminary design phase of an aircraft. Based on the Systems Engineering approach, an aircraft will be designed during three phases: 1. Conceptual design phase, 2. Preliminary design phase, and 3. Detail design phase. In the conceptual design phase, the aircraft will be designed in concept without the precise calculations. In another word, almost all parameters are determined based on a decision making process and a selection technique. On the other hand, the preliminary design phase tends to employ the outcomes of a calculation procedure. As the name implies, in the preliminary design phase, the parameters that are determined are not final and will be altered later. In addition, in this phase, parameters are essential and will directly influence the entire detail design phase. Therefore the ultimate care must be taken to insure the accuracy of the results of the preliminary design phase.

Three fundamental aircraft parameters that are determined during the preliminary design phase are: 1. Aircraft maximum take-off weight (W_{TO}), 2. Wing reference area (S_W or S_{ref} or S), and 3. Engine thrust (T_E or T) or engine power (P_E or P). Hence, three primary aircraft parameters of W_{TO}, S and T (or P) are the output of the preliminary design phase. These three parameters will govern the aircraft size, the manufacturing cost, and the complexity of calculation. If during the conceptual design phase, a jet engine is selected, the engine thrust is calculated during this phase. But, if during the conceptual design phase, a prop-driven engine is selected, the engine power is calculated during this phase. A few other non-important aircraft parameters such as aircraft zero-lift drag coefficient and aircraft maximum lift coefficient are estimated in this phase too.

The preliminary design phase is performed in two steps:

Step 1: Estimate aircraft maximum take-off weight
Step 2: Determine wing area and engine thrust (or power) simultaneously

In this chapter, two design techniques are developed. First a technique based on the statistics is developed to determine wing reference area and engine thrust (or power). Second, another technique is developed based on the aircraft performance requirements (such as maximum speed, range, and take-off run) to determine the wing area and the engine thrust (or power). This technique is sometime referred to as the matching plot or matching chart, due to its graphical nature. In some references, this process and this design phase is referred to as "*initial sizing*". This is due to the nature of the process which literally determines the size of three fundamental features of the aircraft.

Figure 4.1 illustrates a summary of the preliminary design process. In general, the first technique is not accurate (in fact, it is an estimation) and the approach may carry some inaccuracies, while the second technique is very accurate and the results are reliable.

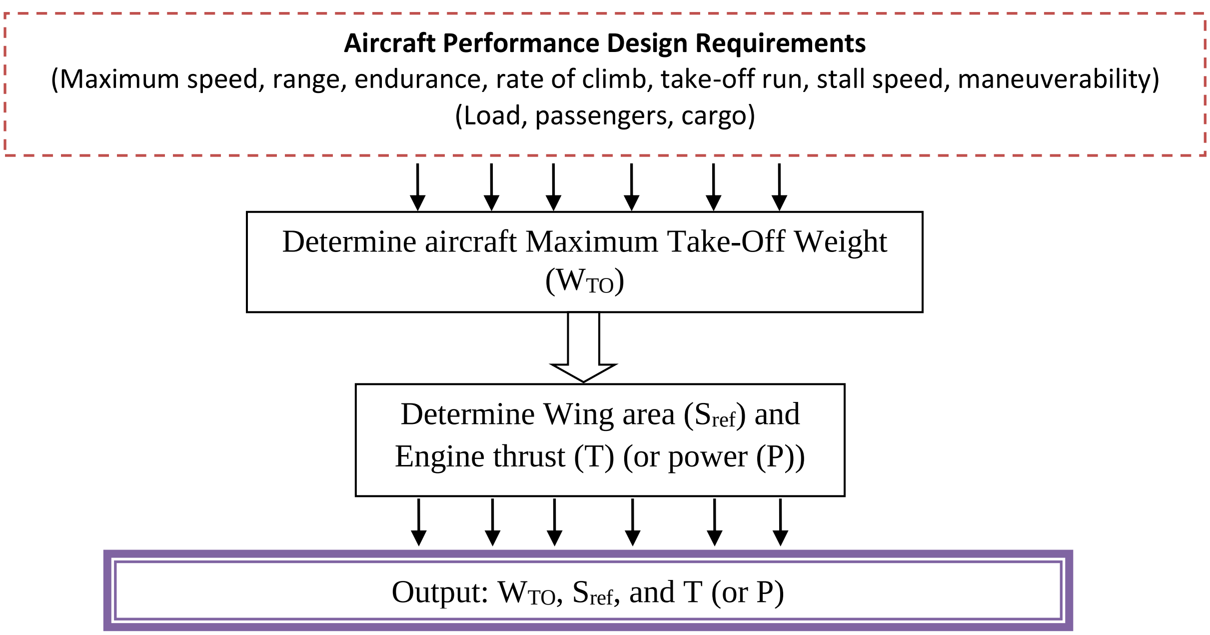

Figure 4.1. Preliminary design procedure

4.2. Maximum Take-Off Weight Estimation

4.2.1. The General Technique

The purpose of this section is to introduce a technique to obtain the first estimate of the maximum take-off weight (or all-up weight) for an aircraft before it is designed and built. The word estimation is intentionally selected to indicate the degree of the accuracy and reliability of the output. Hence, the value for the maximum take-off weight is not final and must be revised in the later design phases. The result of this step may have up to about 20% inaccuracies, since it is not based on its own aircraft data. But the calculation relies on the other aircraft data with similar configuration and mission. Thus, we are adopting the past history as the major source of the information for the calculation in this step. At the end of the detail design phase, the take-off weight estimation is repeated by using another more accurate technique which will be introduced in Chapter 10. As described in Chapter 1, the aircraft design nature in iterative, thus, a new data for the maximum take-off weight requires a new round of calculations and new designs for all aircraft components such as wing, tail and fuselage.

Since the accuracy of the result of this design step largely depends on the past history, one must be careful to utilize only the aircraft data that are current, and the aircraft are similar in configuration and mission. The currency of data and similarity play a vital role as there are many aspects to compare. As the years pass, the science of materials and also the manufacturing technologies are changing and improving. For instance, every year, new engineering materials are introduced to the market; that are lighter and stronger. New materials such as composite materials have caused a revolution in the aircraft industry. In addition, new power transmission technologies such as fly-by-wire allowed the aircraft to be much lighter than expected. The trend is continuing, therefore, the more current data results in a more reliable estimation.

Due to the fact that various aircraft manufacturing industries are employing different approaches in their products, more than one aircraft data must be obtained. The suggestion is to use at least five different aircraft data to estimate the take-off weight of your aircraft. Aircraft manufacturing companies such as Boeing, Airbus, Lockheed, Grumman, Cessna, Raytheon, Bombardier, Dassult, Emberaer, Learjet, and Jetstream, each have different management systems, design techniques, and market approaches. Thus, their aircraft productions have several differences including maximum take-off weight. When you are selecting several aircraft for data application; select aircraft from different companies and even from different regions of the world. Another recommendation is to choose aircraft data from recent years. For example, a comparison between fighters in World War I era (e.g. Avro 504), World War II era (such as Mustang and Spitfire) as and the current modern advance fighters such F-16 Fighting Falcon demonstrates how lighter are the current aircraft compared with older ones.

4.2.2. Weight Buildup

An aircraft has a range of weights from minimum to maximum depending upon the number of pilots and crew, fuel, and payloads (passengers, loads, luggage, and cargo). As the aircraft flies, the fuel is burning and the aircraft weight is decreasing. The most important weight in the design of an aircraft is the maximum allowable weight of the aircraft during take-off operation. It is also referred to as all up weight. The design maximum take-off weight (MTOW or W_{TO}) is the total weight of an aircraft when it begins the mission for which it is designed. The maximum design take-off weight is not necessarily the same as the maximum nominal take-off weight, since some aircraft can be overloaded beyond design weight in an emergency situation, but will suffer a reduced performance and reduced stability. Unless specifically stated, maximum take-off weight is the design weight. It means every aircraft component (e.g. wing, tail) is designed to support this weight.

The general technique to estimate the maximum take-off weight is as follows: the aircraft weight is broken into several parts. Some parts are determined based on statistics, but some are calculated from performance equations.

Maximum take-off weight is broken into four elements:

1. Payload weight (W_{PL})
2. Crew weight (W_C)
3. Fuel weight (W_f)
4. Empty weight (W_E)

$$W_{TO} = W_{PL} + W_C + W_F + W_E \tag{4.1}$$

The payload weight and crew weight are almost known and determined from the given data (by customer and standards) and are not depending on the aircraft take-off weight. On the other hand, the empty weight and fuel weight are both functions of the maximum take-off weight. Hence, to simplify the calculation, both fuel weight and empty weight are expressed as fractions of the maximum take-off weight. Hence:

$$W_{TO} = W_{PL} + W_C + \left(\frac{W_f}{W_{TO}}\right)W_{TO} + \left(\frac{W_E}{W_{TO}}\right)W_{TO} \tag{4.2}$$

This can be solved for W_{TO} as follows:

$$W_{TO} - \left(\frac{W_f}{W_{TO}}\right)W_{TO} - \left(\frac{W_E}{W_{TO}}\right)W_{TO} = W_{PL} + W_C \tag{4.3}$$

The take-off weight can be factored out:

$$W_{TO}\left[1 - \left(\frac{W_f}{W_{TO}}\right) - \left(\frac{W_E}{W_{TO}}\right)\right] = W_{PL} + W_C \tag{4.4}$$

Thus:

$$W_{TO} = \frac{W_{PL} + W_C}{1 - \left(\frac{W_f}{W_{TO}}\right) - \left(\frac{W_E}{W_{TO}}\right)} \quad (4.5)$$

In order to find W_{TO}, one needs to determine four variables of W_{PL}, W_C, W_f/W_{TO} and W_E/W_{TO}. The first three parameters, namely payload, crew, and fuel fraction are determined fairly accurately, but the last parameter (i.e. empty weight fraction) is estimated from statistics.

4.2.3. Payload Weight

The payload is the net carrying capacity of an aircraft. An aircraft is originally required and designed to carry the payload or useful load. The payload includes luggage, cargo, passenger, baggage, store, military equipment, and other intended loads. Thus, the name payload has a broad meaning. For instance, sometimes the Space Shuttle cannot successfully land on Kennedy Space Center in Florida due to the poor weather conditions. So, Shuttle will first land at another runway such as one in Edward Air Force Base in California, and then it will be carried out by a Boeing 747 (Figures 3.7, 3.12, and 9.4) to Florida. Thus, the Space Shuttle is called the payload for Boeing 747 in this mission.

In case of the passenger aircraft, the passengers' weight is to be determined. Actual passenger weights must be used in computing the weight of an aircraft with a limited seating capacity. Allowance must be made for heavy winter clothing when such is worn. There is no standard human, since every kinds of passenger (such as infant young, senior) may get in the plane. To make the calculation easy, one might assume a number as the tentative weight for a typical passenger and then multiply this value by the number of passengers. There are several references in human factor and ergonomic engineering areas that have these numbers. Federal Aviation Administration [1] has regulated this topic and the reader is encouraged to consult with its publications; Federal Aviation Regulations (FAR). For example, FAR part 25 which regulates airworthiness standards for transport aircraft asks the aircraft designers to consider the reasonable numbers for an average passenger. The following is a suggested value for the passenger weight based on published data.

$$W_{pass} = 180 \text{ lb} \quad (4.6)$$

Note that this number is updated every year (due to obesity and other issues), so it is recommended to consult with FAA publications for an accurate data. For instance, FAA in 2005 issued an Advisory Circular [2] and had several recommendations for airlines. One example is illustrated in Table 4.1. In this table, the standard average passenger weight includes 5 pounds for summer clothing, 10 pounds for winter clothing, and a 16-pound allowance for personal items and carry-on bags. Where no gender is given, the standard average passenger weights are based on the assumption that 50 percent of passengers are male and 50 percent of passengers are female. The weight of children under the age of 2 has been factored into the standard average and segmented adult passenger weights.

In determining the total weight of passengers, it is wise to consider the worst case scenario which is the heaviest possible case. It means that all passengers are considered to be adult and male. Although this is a rare case, but it guarantees the flight safety. In a passenger aircraft, the water and food supply must be carried in long trips. But these are included in the empty weight.

No	Passenger	Weight Per Passenger (*lb*)	
		Summer	Winter
1	Average adult	190	195
2	Average adult male	200	205
3	Average adult female	179	184
4	Child weight (2 years to less than 13 years of age)	82	87

Table 4.1. Standard average passenger weights [2]

The weight of luggage and carry-on bag is another item that must be decided. FAA has some recommendations about the weight of bag and luggage in a passenger aircraft. But due to high rising fuel cost, airlines have regulated the weight themselves. For instance, majority of airlines are currently accepting two bags of 70 lbs for international flight and one bag of 50 lbs in domestic flight. There is some news that these numbers are going to drop in near future.

4.2.4. Crew Weight

Another part of the aircraft weight is the weight of the people who are responsible to conduct the flight operations and serving passengers and payload. A human piloted aircraft needs at least one human to conduct the flight. In case of a large passenger aircraft, more staff (e.g. copilot, flight engineer, navigation pilot) may be needed. Moreover, one or more crew is necessary to serve the passengers. In case of a large cargo aircraft, several officers are needed to locate the loads and secure them in the right place.

In a large transport aircraft, this weight count almost nothing compared with the aircraft all-up weight. In a hang glider, however, the weight of the pilot count for more than 70% of the aircraft weight. Therefore, in the smaller aircraft, more attention must be paid in determining the weight of the pilot. Two parameters must be determined in this part: 1. Number of pilots and crew members, 2. Weight of each crew.

In a small GA or a fighter aircraft, number of pilots is given to the designer, but in a large passenger and cargo aircraft, more pilots and more crew are needed to conduct the flight operation safely. In the 1960s, a large transport aircraft was required to have two pilots plus one flight engineer and one navigation engineer. Due to the advance in avionic systems, the last two jobs are cancelled, and left to pilot and copilot to take care of them this is due to the fact that more and more measurement devices are becoming electronic and integrated and illustrated in one large display. In 1950s, a large transport aircraft such as Boeing 727 had about 200 gauges, instruments, knobs, switches, lights, display, and handles that must be monitored and controlled throughout the flight operation. However, thanks to digital electronics and modern computers, at the moment, one pilot not only can conduct the flight safely, but is also able to monitor tens of flight variables and aircraft motions through a display and a control platform simultaneously.

If the aircraft is under commercial flight operations, it would be operating under Parts 119 and 125. Flight attendant's weight is designated in 119.3. In subpart I of part 125, there are Pilot-in-command and second-in-command qualifications. There may be space on the aircraft for more crew members, but based on the language of the document, two flight crew members is the minimum allowed.

FAA [1] has regulated the number of crew for transport aircraft. Based on FAR Part 125, Section 125.269, for airplanes having more than 100 passengers, two flight attendants plus one additional flight attendant for each unit of 50 passengers above 100 passengers are required:

Each certificate holder shall provide at least the following flight attendants on each passenger-carrying airplane used:

(1) For airplanes having more than 19 but less than 51 passengers—one flight attendant.

(2) For airplanes having more than 50 but less than 101 passengers—two flight attendants.

(3) For airplanes having more than 100 passengers—two flight attendants plus one additional flight attendant for each unit (or part of a unit) of 50 passengers above 100 passengers.

No	Aircraft	W_C/W_{TO} (%)
1	Hang glider/Kite/Paraglider	70-80
2	Single-seat Glider/Sail plane	10-20
3	Two-seat Motor glider	10-30
4	Ultra-light	30-50
5	Micro-light	20-40
6	Very light aircraft (VLA)	15-25
7	GA single-seat piston engine	10-20
8	GA multi-seat	10-30
9	Agriculture	2-3
10	Business jet	1.5-3
11	Jet trainer	4-8
12	Large transport aircraft	0.04-0.8
13	Fighter	0.2-0.4
14	Bomber	0.1-0.5

Table 4.2. Typical values for the crew weight fraction [3]

Therefore, for instance, a large passenger aircraft is required to have two pilots plus eight flight attendants. The followings regulations are reproduced [1] from FAR Part 119, section 119.3:

Crew--for each crewmember required by the Federal Aviation Regulations--
(A) For male flight crewmembers--180 pounds.
(B) For female flight crewmembers--140 pounds.
(C) For male flight attendants--180 pounds.
(D) For female flight attendants--130 pounds.
(E) For flight attendants not identified by gender--140 pounds.

The following sentence is also reproduced [1] from FAR Part 125, Section 125.9:

Crew -- 200 pounds for each crewmember required under this chapter

The reader is encouraged to observe the particular FAA standards which apply to the case.

For military aircraft, particularly fighters, pilots are usually equipped with helmet, goggle, g-suite, and other special equipment (such as pressure system). Not only the fighter pilot is often heavier than a civil pilot, but also each equipment weight must be added to the pilot's weight. For more information the reader is encouraged to consult with military standards. Ref. [4] has some useful information and standards. The general rule to determine the weight of each pilot, flight attendant, or crew is similar to what is introduced in Section 8.2.3 (i.e. equation 4.6). In order to obtain the certificate, the designer must follow FAA regulations [1].

In case of a home-built or special mission aircraft (such as the non-stop globe circling aircraft; Voyager, or the aircraft to carry another aircraft to space for the first time; Space Ship One), the weight of each pilot is exactly obtained by weighting the specified pilot on scale. Table 4.2 demonstrates typical values of the crew weight fraction for several aircraft.

4.2.5. Fuel Weight

Another part of the aircraft maximum take-off weight is the fuel weight. The required amount of the total fuel weight necessary for a complete flight operation depends upon the mission to be followed, the aerodynamic characteristics of the aircraft, and the engine specific fuel consumption. The mission specification is normally given to the designer and must be known. The aircraft aerodynamic model and the specific fuel consumption may be estimated from the aircraft configuration that is designed in the conceptual design phase. Recall from equation 4.5 that we are looking for fuel fraction (W_f/W_{TO}).

The first step to determine the total fuel weight is to define the flight mission segments. Three typical mission profiles are demonstrated in Figure 4.2 for three typical aircraft; i.e., transport, fighter, and reconnaissance. A typical flight mission for a General Aviation aircraft is often very similar to a flight mission of a transport aircraft but the duration is shorter. For other types of aircraft such as trainer, agriculture, bomber, the designer can build the mission profile based on the given information from the customer.

Each flight mission consists of several segments, but usually one of them takes the longest time. The main feature of the flight of a transport aircraft is "cruise" that makes the longest segment of the flight. The main feature of the flight of a reconnaissance/patrol/monitor/relay aircraft is loitering that makes the longest segment of the flight. The main feature of the flight of a fighter aircraft is "dash" that makes the longest segment of the flight. In terms of flight mechanics, the cruising flight is measured by "*range*", a loitering flight is measured by "*endurance*", and dash is measured by "*radius of action*".

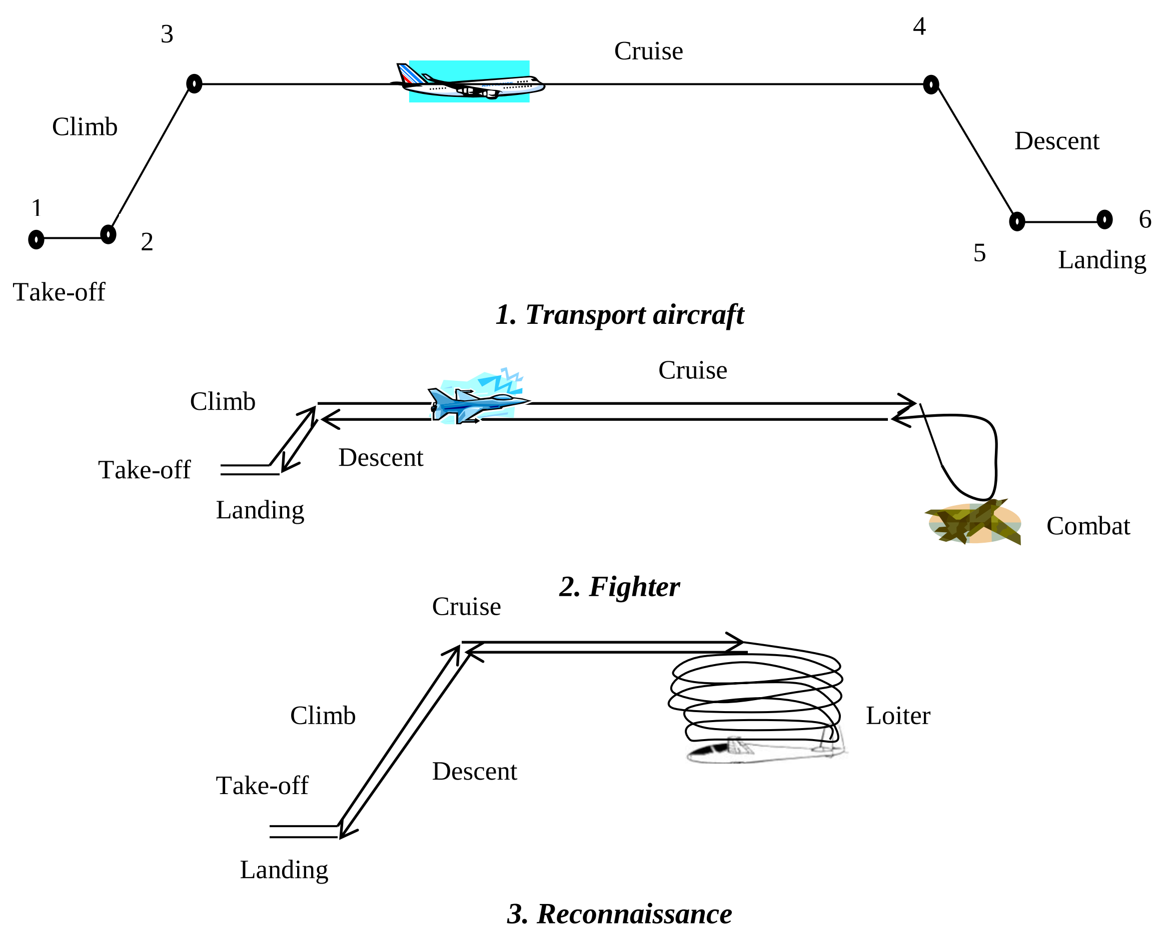

Figure 4.2. Typical mission profiles for three typical aircraft

For analysis, each mission segment is numbered; with 1 denote the beginning of take-off and 2 is the end of take-off. For example, in the case of a regular flight of a transport aircraft, segments could be numbered as follows: 1. taxi/take-off, 2. climb, 3. cruise, 4. descent, 5. landing. In a similar fashion, the aircraft weight at each phase of the flight mission can be numbered. Hence, W_1 is the aircraft weight at the beginning of take-off (i.e. maximum take-off weight). W_2 is the aircraft weight at the end of take-off which is the beginning of climb phase. W_3 is the aircraft weight at the end of climb phase which is the beginning of cruising phase. W_4 is the aircraft weight at the end of cruising phase which is the beginning of descending phase. W_5 is the aircraft weight at the end of descending phase which is the beginning of landing phase. Finally, W_6 is the aircraft weight at the end of landing phase. Thus, for any mission segment "*i*", the mission segment weight fraction is expressed as (W_{i+1}/W_i). If these weight fraction can be estimated for all of the segments, they can be multiplied together to find the ratio of the aircraft weight at the end of flight operation, divided by the initial weight; i.e. maximum take-off weight. This ratio would then be employed to determine the total fuel fraction.

During each segment, the fuel is burnt and the aircraft loses weight. If an aircraft has a mission to drop load or parachute, the technique must be applied with a slight correction. The aircraft weight at the end of a segment divided by its weight at the beginning of that segment is called the segment weight fraction. For instance, W_4/W_3 in the flight mission of figure 4.2-1 is the fuel fraction during cruise segment. The will make a base for estimating the required fuel weight and fuel fraction during a flight operation. The difference between the aircraft weight at the end of flight (i.e. landing) and the aircraft weight at the beginning of flight (i.e. take-off) is exactly equal to the fuel weight:

$$W_{TO} - W_{Land} = W_f \tag{4.7}$$

Thus, in a regular flight mission, the ratio between the aircraft weight at the end of flight to the aircraft weight at the beginning of flight is:

$$\frac{W_{Landing}}{W_{TO}} = \frac{W_{TO} - W_f}{W_{TO}} \tag{4.8}$$

Therefore, for the case of a mission with 5 segments as shown in figure 4.2-1, the fuel weight fraction is obtained as follows:

$$\frac{W_f}{W_{TO}} = 1 - \frac{W_6}{W_1} \tag{4.9}$$

where $\frac{W_6}{W_1}$ can be written as:

$$\frac{W_6}{W_1} = \frac{W_2}{W_1}\frac{W_3}{W_2}\frac{W_4}{W_3}\frac{W_5}{W_4}\frac{W_6}{W_5} \tag{4.10}$$

For other flight missions, the reader is required to identify the segments and to build a similar numbering system to derive a similar equation. For the sake of flight safety, it is recommended to carry a reserve fuel in case that the intended airport is closed, so the aircraft has to land on another nearby airport. FAA regulation requires the transport aircraft to carry 20% more fuel than needed or a flight of 45 minutes to observe the airworthiness standards. The extra fuel required for safety purposes is almost 5 percent of aircraft total weight, so it is applied as follows:

$$\frac{W_f}{W_{TO}} = 1.05\left(1 - \frac{W_6}{W_1}\right) \tag{4.11}$$

Therefore, in order to find the fuel weight fraction, one must first determine these weight fractions for all of the mission segments (e.g. $\frac{W_2}{W_1}$, $\frac{W_3}{W_2}$, $\frac{W_4}{W_3}$, $\frac{W_5}{W_4}$, $\frac{W_6}{W_5}$). There are primarily six flight segments as take-off, climb, cruise, loiter, descent, and landing. These flight phases or segments can be divided into two groups:

No	**Mission segment**	W_{i+1}/W_i
1	Taxi and take-off	0.98
2	Climb	0.97
3	Descent	0.99
4	Approach and landing	0.997

Table 4.3. Typical average segment weight fractions

1. The segments during which the fuel weight that is burnt is almost nothing and negligible compared with maximum take-of weight. These include, taxi, take-off, climb, descent, approach, and landing. The fuel weight fractions for these mission segments are estimated based on the statistics. Table 4.3 illustrates typical average values for fuel fractions of take-off, climb, descent, and landing.
2. The segments during which the fuel weight that is burnt is considerable. These include cruise and loiter and are determined through mathematical calculations.

Table 4.4 shows the fuel weight fractions for several aircraft.

4.2.5.1. Cruise Weight Fraction for Jet Aircraft

The fuel weight fraction for cruise segment is determined by employing the Breguet range equation. By definition, range is the total distance that an aircraft can fly with full fuel tank and without refueling. This consists of take-off, climb, cruise, descent, and landing and does not include the wind effect (either positive or negative). Since this definition is not applicable for our case, we resort to Gross Still Air Range which does not include any segment other than cruising flight. In order to cruise, basically there are three flight programs that satisfy trim requirements. They are:

Flight program 1. Constant altitude-constant lift coefficient flight

Flight program 2. Constant airspeed-constant lift coefficient flight

Flight program 3. Constant altitude-constant airspeed flight

No	**Aircraft**	**Type**	**Range (km)**	**S (m^2)**	**m_{TO} (kg)**	**m_f (kg)**	$\frac{m_f}{m_{TO}}$
1	MIT Daedalus 88	Man-powered	N/C[1]	29.98	104	0	0
2	Volmer VJ-25 Sunfun	Hang glider/Kite	N/C	15.14	140.5	50	0
3	Manta Fledge III	Sailplane/Glider	N/C	14.95	133	0	0
4	Merlin E-Z	Ultra-light	-	15.33	476	163	0.342
5	Pilatus PC-12	Turboprop transport	3,378	25.81	4,100	1,200	0.293
6	C-130J Hercules	Military transport	5,250	162.12	70,305	17075	0.243
7	Beech super king air B200	Light transport	2,204	28.18	5,670	1,653	0.292
8	Hawkeye E-2C	Early warning	2,854	65.03	24,687	5,624	0.228
9	MD-95 ER	Jet transport	3,705	92.97	54,885	10433	0.19
10	Airbus 380-841	Wide bodied airliner	15,200	845	590,000	247,502	0.419
11	Boeing 777	Airliner	10,556	427.8	229,520	94,210	0.41
12	Beechcraft 390	Light business jet	1,457	22.95	5,670	1,758	0.31
13	F-16C	Fighter	2,742	27.87	19,187	3,104	0.16
14	Voyager	Circumnavigation	39,000	30.1	4398	3168	0.72
15	Global hawk	Unmanned reconnaissance	24,985	50.2	10,387	6536	0.629

Table 4.4. Fuel weight fraction for several aircraft

Each flight program has a unique range equation, but for simplicity, we use the second flight program, since its equation is easiest to apply in our preliminary design phase. The range equation for a jet aircraft is slightly different than that of a prop-driven aircraft. The origin of the difference is that jet engine is generating thrust (T), while a prop-driven engine produces power (P). Thus, they are covered separately.

For an aircraft with the jet engine (i.e. turbojet and turbofan), the optimum range equation [5] with the specified speed of $V_{(L/D)max}$ is:

$$R_{\max} = \frac{0.866 V_{R_{\max}}}{C} \left(\frac{L}{D}\right)_{\max} \ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.12}$$

[1] Not Constant

where W_i denotes the aircraft weight at the beginning of cruise, and W_{i+1} is the aircraft weight at the end of cruising flight. Thus, the term $\frac{W_i}{W_{i+1}}$ indicates the fuel weight fraction for cruise segment. Also, the parameter C is the engine specific fuel consumption and L/D is the lift-to-drag ratio. The cruising speed is usually a performance requirement and is given. But, two parameters of C and $(L/D)_{max}$ are unknown at this moment, since we are in the preliminary design phase and the aerodynamic aspect of the aircraft and also propulsion system are not determined. Again, we resort to historical value and employ the data for similar aircraft. Table 4.5 shows typical values for maximum lift-to-drag ratio of several aircraft. Supersonic transport aircraft Concorde (Figures 7.24, 11.15) tend to have a lift-to-drag ratio of 7.1 at a speed of Mach 2.

No	Aircraft type	$(L/D)_{max}$
1	Sailplane (glider)	20-35
2	Jet transport	12-20
3	GA	10-15
4	Subsonic military	8-11
5	Supersonic fighter	5-8
6	Helicopter	2-4
7	Homebuilt	6-14
8	Ultralight	8-15

Table 4.5. The typical maximum lift-to-drag ratio for several aircraft

From "*Flight Mechanics*", the reader may recall that there are three differences between an economic cruising flight and a flight to maximize range.

1. Almost no aircraft is cruising to maximize the range, since it ends up having a longer trip and have some operational difficulties. Most transport aircraft are recommended to fly with the Carson's speed that is 32% higher than the speed for maximizing range.

$$V_C = 1.32 \quad V_{(L/D)_{\max}} \tag{4.13}$$

2. On the other hand, in a cruising flight with the Carson's speed, the lift-to-drag ratio slightly less than the maximum lift-to-drag ratio; i.e.:

$$\left(\frac{L}{D}\right)_{cruise} = \frac{\sqrt{3}}{2}\left(\frac{L}{D}\right)_{\max} = 0.866\left(\frac{L}{D}\right)_{\max} \tag{4.14}$$

3. In a cruising flight, the maximum engine thrust is not normally employed. This is to reduce the cost and the engine specific fuel consumption (C).

For more details the reader is referred to [5]. By taking into account these above-mentioned economic and operational considerations, the equation 4.12 is modified as follows:

$$R = 0.866\frac{V_C}{C}\left(\frac{L}{D}\right)_{\max}\ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.15}$$

Therefore, the cruise fuel weight ratio is determined as:

$$\frac{W_{i+1}}{W_i} = e^{\frac{-R \cdot C}{0.866V(L/D)_{\max}}} \tag{4.16}$$

The definition and typical values for the variable C is presented in Section 4.2.5.5.

4.2.5.2. Cruise Weight Fraction for Prop-Driven Aircraft

The definition and flight approaches to satisfy a trimmed operation for a specified range are discussed in Section 4.5.2.1. Since the type of propulsion system is a prop-driven, the engine is generating power, and the propeller efficiency influences the overall thrust. The same as the case for a jet aircraft, there are three flight approaches to hold aircraft trim despite the loss of weight due to fuel burn. For the sake of simplicity and due to the expected accuracy in the preliminary design phase, we select only one of them. If the design requirements specify that the aircraft must have a different approach, one need to employ the relevant equation.

For an aircraft with the prop-driven engine (i.e. piston-prop or turboprop), the optimum range will be achieved, when the aircraft is flying with the minimum drag speed. Thus the range equation [5] is:

$$R_{\max} = \frac{\eta_P (L/D)_{\max}}{C}\ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.17}$$

This is for the case where lift coefficient (C_L) or angle of attack (α) is held constant. In another word, either flight speed is decreasing, or flight altitude is increasing (air density is decreased) to compensate for the loss of aircraft weight. This is referred to as *Breguet* range equation for prop-driven aircraft. Therefore, the cruise fuel weight ratio is determined as:

$$\frac{W_{i+1}}{W_i} = e^{\frac{-R \cdot C}{\eta_P (L/D)_{\max}}} \tag{4.18}$$

The definition and typical values for the variable C is presented in Section 4.2.5.5. In this equation, all parameters except C are without unit. Since the unit of range is in terms of length (such as m, km, ft, and nm), the unit of C must be converted into the reciprocal of length (such as 1/m, 1/km, 1/ft, and 1/nm). Recall that the unit of C is initially "lb/(hr.hp)" or "N/(hr.W)".

4.2.5.3. Loiter Weight Fraction for Jet Aircraft

The aircraft performance criterion loiter is measured with a parameter called endurance. In order to determine the fuel fraction for loitering flight, the equation for endurance is used. Endurance (E) is the length of time that an aircraft can remain airborne for a given expenditure of fuel and for a specified set of flight condition. For some aircraft (such as reconnaissance, surveillance, and border monitoring), the most important performance parameter of their mission is to be airborne as much as possible. Several technical aspects of endurance and range are similar. The only difference is to consider how long (time) the aircraft can fly rather than how far (distance) it can travel. The objective for this flight is to minimize the fuel consumption, because the aircraft has limited fuel. A *loiter* is a flight condition that the endurance is its primary objective. For more information and the derivation, the reader is encouraged to consult with [5]. The endurance equation [5] for a jet aircraft is:

$$E_{max} = \frac{(L/D)_{max}}{C} \ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.19}$$

Therefore, fuel weight ratio for a loitering flight is determined as:

$$\frac{W_{i+1}}{W_i} = e^{\frac{-E \cdot C}{(L/D)_{max}}} \tag{4.20}$$

The definition and typical values for the variable C is presented in Section 4.2.5.5. Since the unit of E is in terms of time (such as second and hour), the unit of C must be converted into the reciprocal of time (such as 1/sec and 1/hr). Recall that the unit of C is initially "lb/(hr.lb)" or "N/(hr.N)".

4.2.5.4. Loiter Weight Fraction for Prop-Driven Aircraft

The definition and flight approaches to satisfy a trimmed operation for a specified loiter are discussed in Section 4.5.2.3. Since the type of propulsion system is a prop-driven, the engine is generating power, and the propeller efficiency influences the overall thrust. The same as the case for a jet aircraft, there are three flight approaches to hold aircraft trim despite the loss of weight due to fuel burn. For the sake of simplicity and due to the expected accuracy in the preliminary design phase, we select only the case where flight speed is decreasing (i.e. Constant altitude-constant lift coefficient flight). If the design requirements specify that the aircraft must have a different approach, one need to employ the relevant endurance equation.

For an aircraft with the prop-driven engine (i.e. piston-prop or turboprop), the optimum endurance will be achieved, when the aircraft is flying with the minimum drag speed. Thus the range equation [5] is:

$$E_{max} = \frac{(L/D)_{E\max} \eta_P}{C V_{E_{max}}} \ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.21}$$

For a prop-driven aircraft, the endurance will be maximized when $\left(C_L^{3/2}/C_D\right)$ ratio is at its maximum value. In another word:

$$(L/D)_{E\max} = 0.866(L/D)_{\max} \tag{4.22}$$

Then:

$$E_{\max} = \frac{0.866(L/D)_{\max}\,\eta_P}{CV_{E_{\max}}}\ln\left(\frac{W_i}{W_{i+1}}\right) \tag{4.23}$$

Therefore, for a prop-driven aircraft, the fuel weight fraction for a loitering flight is determined as:

$$\frac{W_{i+1}}{W_i} = e^{\frac{-E\cdot C\cdot V_{E_{\max}}}{0.866\eta_P (L/D)_{\max}}} \tag{4.24}$$

The speed for maximum endurance (V_{Emax}) for a prop-driven aircraft [5] happens when the aircraft is flying with the minimum power speed (i.e. V_{Pmin}). Since the aircraft has not yet been fully designed at preliminary design phase, the calculation of minimum power speed cannot be implemented. Hence, the recommendation is to use a reasonable approximation. The minimum power speed for most prop-driven aircraft is about 20% to 40% higher than the stall speed. Then:

$$V_{E_{\max}} = V_{P_{\min}} \approx 1.2V_S - 1.4V_S \tag{4.25}$$

The definition and typical values for the variable C is presented in Section 4.2.5.5. Since the unit of E is in terms of time (such as second and hour), and the unit of speed in distance per time, the unit of C must be converted into the reciprocal of distance (such as 1/m and 1/ft). Recall that the unit of C is initially "lb/(hr.lb)" or "N/(hr.N)".

4.2.5.5. Specific Fuel Consumption

The remaining unknown in the range and endurance relationships (Equations 4.16, 4.18, 4.20, and 4.24) is the specific fuel consumption (C or SFC). The specific fuel consumption is a technical figure of merit for an engine that indicates how efficiently the engine is burning fuel and converting it to thrust. SFC depends on the type and the design technology of the engine and also the type of fuel. Specific fuel consumption is used to describe the fuel efficiency of an engine with respect to its mechanical output.

Various grades of fuel have evolved during the development of jet engines in an effort to ensure both satisfactory performance and adequate supply. JP-8 is the most commonly used fuel for US Air Force jet aircraft. The US Navy uses JP-5, a denser, less volatile fuel than JP-8, which allows it to be safely stored in the skin tanks of ships. The most common commercial aircraft fuel is Jet A and Jet A-1. In general, piston engine fuels are about 10% lighter than jet fuels.

The SFC for jet engines (turbojet and turbofan) is defined as the weight (sometimes mass) of fuel needed to provide a given thrust for a given period (e.g. lb/hr/lb or g/sec/N in SI units). In propeller driven engines (piston, turboprop and turboshaft), SFC measures the mass of fuel needed to provide a given thrust or power for a given period. The common unit of measure in British unit is lb/hp/hr (i.e. lb/(hp.hr)); that is, pounds of fuel consumed for every horsepower generated during one hour of operation, (or kg/kW/hr in SI units). Therefore a lower number indicates better efficiency.

The unit of C can be converted readily between SI and British units. For instance, a typical piston engine has a SFC of about 0.5 lb/hp/hr or (0.3 kg/kW/hr or 83 g/MJ), regardless of the design of any particular engine. As an example, if a piston engine consumes 400 lb of fuel to produce 200 hp for four hours, its SFC will be as follows:

$$SFC = \frac{400 \quad lb}{4hr \times 200hp} = 0.5\frac{lb}{hr.hp} = 2.98\frac{N}{hr.kW}$$

Table 4.6 shows typical values of SFC for various engines. It is very important to use consistent unit in the range and endurance equations. In general, the unit of C in the range equation must be 1 over time unit (e.g. 1/sec). If the SI unit is used (e.g. km/hr for cruising speed), the unit of C must be 1/hr. If the British Unit is utilized (e.g. ft/sec for the cruising speed), the unit of C must be 1/sec. moreover, the unit of C in the endurance equation must be 1 over unit of distance (e.g. 1/m or 1/ft). The following is two examples to demonstrate how to convert the unit from lb/hp.hr to 1/ft, and to convert the unit of lb/hr.lb to 1/sec. Recall that 1 hp is equivalent to 550 lb.ft/sec, and one hour contains 3,600 seconds.

No	Engine type	SFC in cruise	SFC in loiter	Unit (British Unit)
1	Turbojet	0.9	0.8	lb/hr/lb
2	Low bypass ratio Turbofan	0.7	0.8	lb/hr/lb
3	High bypass ratio Turbofan	0.4	0.5	lb/hr/lb
4	Turboprop	0.5-0.8	0.6 – 0.8	lb/hr/hp
5	Piston (fixed pitch)	0.4 – 0.8	0.5 – 0.7	lb/hr/hp
6	Piston (variable pitch)	0.4 – 0.8	0.4 – 0.7	lb/hr/hp

Table 4.6. Typical values of SFC for various engines

$$SFC = 0.5\frac{lb}{hr.hp} = 0.5\frac{lb}{(3600\sec)\cdot\left(550\frac{lb \cdot ft}{\sec}\right)} = \frac{0.5}{3600 \times 550}\frac{1}{ft} = \frac{0.5}{1980000}\frac{1}{ft} = 2.52 \times 10^{-7}\frac{1}{ft}$$

$$SFC = 0.7 \frac{lb}{hr.lb} = 0.7 \frac{1}{3600 \sec} = 0.000194 \frac{1}{\sec}$$

No	Aircraft	Type	Engine	S (m^2)	m_{TO} (kg)	m_E (kg)	$\frac{W_E}{W_{TO}}$
1	Voyager	Circumnavigation	piston	30.1	4398	1020	0.23
2	Questair Spirit	Sport homebuilt	Piston	6.74	771	465	0.6
3	Skystar Kitfox V	Kit-built	Piston	12.16	544	216	0.397
4	Beech Bonanza A36	Utility	Piston	16.8	1,655	1,047	0.63
5	Air & Space 20A	Autogyro	Piston	11.33[2]	907	615	0.68
6	Stemme S10	Motor glider	Piston	18.7	850	640	0.75
7	BN2B Islander	Multirole transport	Turboprop	30.19	2,993	1866	0.62
8	C-130H Hercules	Tactical transport	Turboprop	162.12	70,305	34,686	0.493
9	Saab 2000	Regional transport	Turboprop	55.74	22,800	13,800	0.605
10	ATR 42	Regional transport	Turboprop	54.5	16,700	10,285	0.616
11	Air Tractor AT-602	Agricultural	Turboprop	31.22	5,443	2,471	0.454
12	Cessna 750	Business jet	Turbofan	48.96	16,011	8,341	0.52
13	Gulfstream V	Business jet	Turbofan	105.63	40,370	21,228	0.523
14	Falcon 2000	Business transport	Turbofan	49.02	16,238	9,405	0.58
15	Airbus A340	Wide bodied airliner	Turbofan	363.1	257,000	123,085	0.48
16	MD-90	Airliner	Turbofan	112.3	70,760	39,916	0.564
17	Beechjet	Military trainer	Turbofan	22.43	7,303	4,819	0.66
18	Boeing 777-300	Wide bodied airliner	Turbofan	427.8	299,370	157,215	0.525
19	Airbus 380-841	Wide bodied airliner	Turbofan	845	590,000	270,015	0.485
20	BAe Sea Harrier	Fighter and attack	Turbofan	18.68	11,880	6,374	0.536
21	F-16C Falcon	Fighter	Turbofan	27.87	12,331	8,273	0.67
22	Eurofighter 2000	Fighter	Turbofan	50	21,000	9,750	0.46
23	Volmer VJ-25 Sunfun	Hang glider/Kite	No engine	15.14	140.5	50	0.35
24	Manta Fledge III	Sailplane/Glider	No engine	14.95	133	33	0.25
25	MIT Daedalus 88	Man-powered	Prop-human	29.98	104	32	0.307
26	Global hawk	Unmanned	Turbofan	50.2	10,387	3,851	0.371
27	Edge 540T	Remote controlled	Piston	0.774	6.76	5.58	0.825

Table 4.7. Empty weight fraction for several aircraft [3]

[2] The value is for the area of rotor disk.

4.2.6. Empty Weight

The last term in determining maximum take-off weight in equation 4.5 is the empty weight fraction ($\frac{W_E}{W_{TO}}$). At this moment (preliminary design phase), the aircraft has been design only conceptually, hence, there is no geometry or sizing. Therefore, the empty weight fraction cannot be calculated analytically. The only way is to past history and statistics. Table 4.7 shows the empty weight fraction for several aircraft. The only known information about the aircraft is the configuration and aircraft type based on the mission. According to this data, the author has developed a series of empirical equations to determine the empty weight fraction. The equations are based on the published data taken from Ref. [3] and other sources. In general, the empty weight fraction varies from about 0.2 to about 0.75. Figure 4.3 shows the human powered aircraft Daedalus with an empty-weight-to-take-off-weight ratio of 0.3.

Figure 4.3. Human powered aircraft Daedalus (Courtesy of NASA)

$$\frac{W_E}{W_{TO}} = aW_{TO} + b \qquad (4.26)$$

where a and b are found in Table 4.8. Note that the equation 4.26 is curve fitted in British units system. Thus the unit for maximum take-off weight and empty weight is lb. Table 4.8 illustrates statistical curve-fit values for the trends demonstrated in aircraft data as shown in Table 4.7. Note that the unit of W_{TO} in Table 4.8 is lb. This is included due to the fact that all data in FAR publications are in British units.

In Table 4.8, the assumption is that the either the entire aircraft structure or majority of aircraft components are made up of aluminum. The preceding take-off weight calculations have thus implicitly assumed that the new aircraft would also be constructed of aluminum. In case that the aircraft is expected to be made up of composite material, the value of $\frac{W_E}{W_{TO}}$ must be multiplied by 0.9. The values for GA aircraft in Table 4.8 are for Normal aircraft. If a GA aircraft is of utility type, the value of $\frac{W_E}{W_{TO}}$ must be multiplied by 1.03. If a GA aircraft is of acrobatic type, the value of $\frac{W_E}{W_{TO}}$ must be multiplied by 1.06.

No	Aircraft	a	b
1	Hang glider	-1.58×10^{-4}	0.29
2	Man-powered	-1.05×10^{-5}	0.31
3	Glider/Sailplane	-2.3×10^{-4}	0.59
4	Motor-glider	1.21×10^{-4}	0.55
5	Micro-light	-7.22×10^{-5}	0.481
6	Homebuilt	-4.6×10^{-5}	0.68
7	Agricultural	-7.62×10^{-6}	0.6
8	GA-single engine	1.543×10^{-5}	0.57
9	GA-twin engine	5.74×10^{-6}	0.59
10	Twin turboprop	-8.2×10^{-7}	0.65
11	Jet trainer	1.39×10^{-6}	0.64
12	Jet transport	-7.754×10^{-8}	0.576
13	Business jet	1.13×10^{-6}	0.48
14	Fighter	-1.1×10^{-5}	0.97
15	Long-range, long-endurance	1.07×10^{-5}	0.126
16	Small remote controlled (RC)	-0.00296	0.87

Table 4.8. The coefficients "a" and "b" for the empirical equation of 4.26

Figure 4.19 illustrates the British fighter aircraft Aerospace Harrier GR9 (Figure 4.19) with a thrust-to-weight ratio of 1.13, and the transport aircraft Antonov An-124 (Figure 4.19) with a thrust-to-weight ratio of 0.231. Figure 4.20 shows the nonconventional composite aircraft Voyager aircraft with an empty-weight-to-take-off-weight ratio of 0.23, while Figure 4.21 demonstrates the fighter aircraft General Dynamics F-16C Fighting Falcon (Figure 3.12) with an empty-weight-to-take-off-weight ratio of 0.69.

4.2.7. Practical Steps of the Technique

The technique to determine the aircraft maximum take-off weight has *eleven* steps as follows:

Step 1. Establish the flight mission profile and identify the mission segments (similar to figure 4.2).

Step 2. Determine number of flight crew members

Step 3. Determine number of flight attendants

Step 4. Determine the overall weight of flight crew and flight attendants and also flight crew and attendants weight ratio

Step 5. Determine the overall weight of payloads (i.e. passengers, luggage, bag, cargo, store, loads, etc.)

Step 6. Determine fuel weight ratios for the segments of taxi, take-off, climb, descent, approach, and landing (use Table 4.3).

Step 7. Determine fuel weight ratios for the segments of range and loiter using equations introduced in Section 4.2.5.

Step 8. Find the overall fuel weight ratio using equations similar to equations 4.10 and 4.11.

Step 9. Substitute the value of overall fuel weight ratio into the equation 4.5.

Step 10. Establish the empty weight ratio by using equation 4.26 and Table 4.8.

Step 11. Finally, two equations of 4.5 (derived in step 9) and 4.26 (derived in step 10) must be solved simultaneously and find two unknowns of W_{TO} and $\frac{W_E}{W_{TO}}$. The primary unknown that we are looking for is W_{TO} which is the aircraft maximum take-off weight. These two equations form a set of nonlinear algebraic equations and may be solved by employing an engineering software package such as MathCad[3] and MATLAB[4]. If you do not have access to such software packages, a trial and error technique can be employed to solve the equations.

A fully solved example in Section 4.4 demonstrates the application of the technique to estimate the aircraft maximum take-off weight.

4.3. Wing Area and Engine Sizing

4.3.1. A Summary of the Technique

In the first step of the aircraft preliminary design phase, the aircraft most fundamental parameter (i.e. aircraft maximum take-off weight; W_{TO}) is determined. The technique was introduced in Section 4.2. The second crucial step in the aircraft preliminary design phase is to determine wing reference area (S_{ref}) plus engine thrust (T). However, if the aircraft propulsion system has been chosen to be prop-driven, the engine power will be determined. Hence, two major outputs of this design step are:

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[3] Mathcad is a registered trademark of Mathsoft, Inc.
[4] MATLAB is a registered trademark of Mathworks, Inc.

1. Wing reference area (S or S_{ref})
2. Engine Thrust (T), or Engine Power (P)

Unlike the first step in preliminary design phase at which the main reference was statistics, this phase is solely depending upon the aircraft performance requirements and employs flight mechanics theories. Hence, the technique is an analytical approach and the results are highly reliable without inaccuracy. The aircraft performance requirements that are utilized to size the aircraft in this step are:

- Stall speed (V_s)
- Maximum speed (V_{max})
- Maximum rate of climb (ROC_{max})
- Take-off run (S_{TO})
- Ceiling (h_c)
- Turn requirements (turn radius and turn rate)

Recall that the following aircraft performance requirements have been used to determine aircraft maximum take-off weight (Section 4.2):

- Range (R)
- Endurance (E)

Hence, they will not be utilized again in this technique. There are a few aircraft parameters (such as aircraft maximum lift coefficient) which may be needed throughout the technique, but they have not been analytically calculated prior to this preliminary design phase. These parameters will currently be estimated based on the statistics that will be provided in this Section. However, in the later design phase, where their exact values are determined, these calculations will be repeated to correct the inaccuracies. References [5], [6] and [7] introduce techniques of aircraft performance analysis.

In this section, three new parameters are appeared in almost all equations. So, we need to define them first:

1. **Wing loading:** The ratio between aircraft weight and the wing area is referred to as wing loading and represented by W/S. This parameter indicates that how much load (aircraft i.e. weight weight) is held by each unit area of the wing.
2. **Thrust-to-weight ratio:** The ratio between aircraft engine thrust and the aircraft weight is referred to as thrust loading and is represented by T/W. This parameter indicates that how heavy is the aircraft with respect to engine thrust. The term thrust-to-weight ratio is associated with jet aircraft (e.g. turbofan or turbojet engines). Ref. [8] refers to this term as the thrust loading. Although this designated name seems convenient to use, but it does not seem to fit very well to the concept related to thrust and weight. However, W/T seems to be a

more convenient symbol to be referred to as the thrust loading; which means how much weight is carried by each unit of thrust.

3. **Power loading:** The ratio between aircraft weight and the engine power is referred to as power loading and is represented by W/P. This parameter indicates that how heavy is the aircraft with respect to engine power. A better name for this parameter is weight-to-power ratio. This term is associated with propeller-driven aircraft (turboprop or piston engines).

Table 4.9 illustrates wing loading, power loading and thrust loading for several aircraft. In general, two desired parameters (S and T (or P)) are determined in ***six*** steps. The following is the steps to determine wing area and engine power for a prop-driven aircraft. If the aircraft is jet-driven, substitute the word thrust loading instead of power loading. The principles and steps of the technique are similar for both types of aircraft.

No	Aircraft	Type	W_{TO} (lb)	S (ft^2)	P (hp)	W/S (lb/ft^2)	W/P (lb/hp)
1	C-130 Hercules	Large Transport	155,000	1754	4×4508	88.37	8.59
2	Beech bonanza	Utility-Piston prop	2,725	178	285	15.3	9.5
3	Gomolzig RF-9	Motor glider	1642	193.7	80	8.5	20.5
4	Piaggio P180 Avanti	Transport	10,510	172.2	2×800	61	6.5
5	Canadair CL-215T	Amphibian	43,500	1080	2×2100	40.3	10.3
6	Socata TB30 Epsilon	Military trainer	2,756	97	300	28.4	9.2
7	DHC-8 Dash 8-100	Short range Transport	34,500	585	2×2000	59	8.6
8	Beechcraft King Air 350	Utility twin turboprop	15,000	310	2×1,050	48.4	7.14

1. Prop-driven aircraft

No	Aircraft	Type	W_{TO} (lb)	S (ft^2)	T (lb)	W/S (lb/ft^2)	T/W (lb/lb)
1	Paragon spirit	Business jet	5,500	140	1,900	39.3	0.345
2	Cessna 650 Citation VII	Business jet	22,450	312	2 × 4,080	71.9	0.36
3	F-15 Eagle	Fighter	81,000	608	2 × 23,450	133.2	0.58
4	Lockheed C-5 Galaxy	Transport	840,000	6,200	4 × 43,000	135.5	0.205
5	Boeing 747-400	Airliner	800,000	5,825	4 × 56,750	137.3	0.28
6	F-5A Freedom Fighter	Fighter	24,700	186	2 × 3500	132.3	0.283
7	AV-8B Harrier II	VTOL Fighter	20,750	243.4	23,500	85.2	1.133
8	F-16C Falcon	Fighter	27,185	300	29,588	90.6	1.09
9	B-2 Spirit	Bomber	336,500	5,000	4 × 17,300	67.3	0.206
10	Eurofighter	Fighter	46,297	538	2 × 16,000	86	0.691
11	Embraer EMB 190	Regional jet	105,359	996	2×14,200	195.8	0.27

2. Jet aircraft

Table 4.9. Wing loading, power loading and thrust loading (in British Unit) of several aircraft

1. Derive one equation for each aircraft performance requirement (e.g. V_s, V_{max}, ROC, S_{TO}, h_c, R_{turn}, ω_{turn}). If the aircraft is prop-driven, the equations are in the form of W/P as functions of W/S as follows:

$$\left(\frac{W}{P}\right)_{V_s} = f_1\left(\frac{W}{S}, V_S\right) \tag{4.27a}$$

$$\left(\frac{W}{P}\right)_{V_{\max}} = f_2\left(\frac{W}{S}, V_{\max}\right) \tag{4.27b}$$

$$\left(\frac{W}{P}\right)_{S_{TO}} = f_3\left(\frac{W}{S}, S_{TO}\right) \tag{4.27c}$$

$$\left(\frac{W}{P}\right)_{ROC} = f_4\left(\frac{W}{S}, ROC\right) \tag{4.27d}$$

$$\left(\frac{W}{P}\right)_{ceiling} = f_5\left(\frac{W}{S}, h_c\right) \tag{4.27e}$$

$$\left(\frac{W}{P}\right)_{turn} = f_6\left(\frac{W}{S}, R_{turn}, \omega_{turn}\right) \tag{4.27f}$$

However, if the aircraft is jet-driven, the equations are in the form of T/W as functions of W/S. The details of the derivation are presented in the next Sections.

2. Sketch all derived equations in one plot. The vertical axis is power loading (W/P) and the horizontal axis is the wing loading (W/S). Thus, the plot illustrates the variations of power loading with respect to wing loading. These graphs will intersect each other in several pints and may produce several regions.

3. Identify the acceptable region inside the regions that are produced by the axes and the graphs. The acceptable region is the region that meets all aircraft performance requirements. A typical diagram is shown in figure 4.4. The acceptable region is recognized by the fact that as a performance variable (say V_{max}) is varied inside the permissible limits; the power loading must behave to confirm that trend. For instance, consider the graph of power loading versus wing loading for maximum speed. Assume that the power loading is inversely proportional to the maximum speed. Now, if the maximum speed is increased; which is inside the permissible limits, the power loading is decreased. So the reduction in power loading is acceptable. Hence, the lower region of this particular graph is acceptable.

4. **4.** Determine the design point (i.e. the optimum selection). The design point on the plot is only one point that yields the smallest engine in terms of power (i.e. the lowest cost). For the case of the jet aircraft, the design point yields an engine with the smallest thrust.
5. **5.** From the design point, obtain two numbers; corresponding wing loading; $(W/S)_d$ and corresponding power loading; $(W/P)_d$. A typical graphical technique is illustrated in figure 4.4 (for prop-driven aircraft) and figure 4.5 (for jet aircraft). For the case of the jet aircraft, read corresponding thrust loading; $(T/W)_d$.
6. **6.** Calculate the wing area and engine power from these two values, since the aircraft maximum take-off weight (W_{TO}) has been previously determined. The wing area is calculated by dividing the aircraft take-off weight by wing loading. The engine power is also calculated by dividing the aircraft take-off weight by power loading.

$$S = W_{TO} \Big/ \left(\frac{W}{S}\right)_d \tag{4.28}$$

$$P = W_{TO} \Big/ \left(\frac{W}{P}\right)_d \tag{4.29}$$

While, in the case of a jet aircraft, the engine thrust is calculated by multiplying the aircraft take-off weight by thrust loading:

$$T = W_{TO} \times \left(\frac{T}{W}\right)_d \tag{4.28a}$$

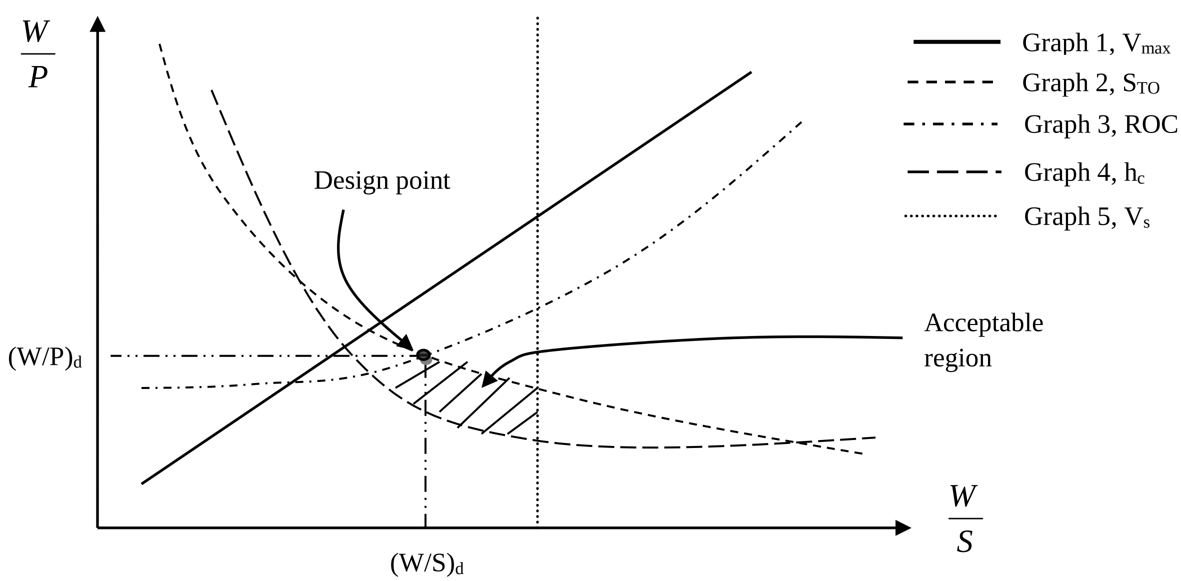

Figure 4.4. Matching plot for a prop-driven aircraft

The principles of the technique are originally introduced in a NASA technical report [9] and they were later developed by [8]. The technique is further developed by the author in this Section. This graph that contains several performance charts is sometimes referred to as the matching plot, matching chart, or matching diagram.

It must be mentioned that there is an analytical solution to this set of equations. One can write a computer program and apply all limits and inequalities. The results will be the values for two required unknowns (s and T (or P)). Extreme caution must be taken to use consistent units in the application process. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient. Since in some of the equation, W/S is on the denominator, do not begin the horizontal axis of the plot from zero. This is to avoid the value of W/P to go toward infinity. So, it is suggested to have values of W/S from, say 5 lb/ft^2 to say 100 lb/ft^2 (in British unit).

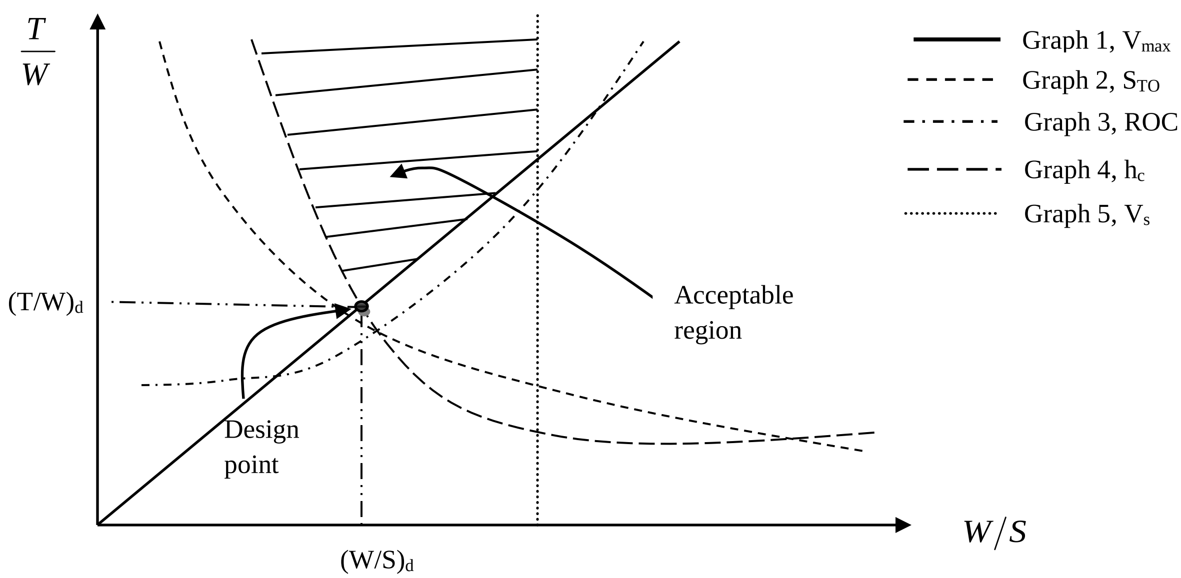

Figure 4.5. Matching plot for a jet aircraft

If the performance requirements are completely consistent, the acceptable region would be only one point; that is the design point. However, as the performance requirements are more scattered, the acceptable region becomes wider. For instance, if the aircraft is required to have a rate of climb of 10,000 fpm, but the absolute ceiling is required to be only 15,000 ft, this is assumed to be a group of non-consistent design requirements. The reason is that a 10,000 fpm rate of climb requires a powerful engine, but an 15,000 ft absolute ceiling requires a low thrust engine. It is clear that a powerful engine is easily capable of satisfying a low altitude absolute ceiling. This type of performance requirements makes the acceptable region in the matching chart a wide one. An example application is presented in Section 4.4. Now, the derivations of equations for each performance requirement are carried out using the mathematical methods and practical methods.

4.3.2. Stall Speed

One of the aircraft performance requirements is a limit to the minimum allowable speed. Only helicopters (or rotary wing aircraft) are able to fly (i.e. hover) with a zero forward airspeed. The other conventional (i.e. fixed-wing) aircraft need to have a minimum airspeed in order to be airborne. For most aircraft the mission demands a stall speed not higher than some minimum value. In such a case, the mission specification includes a requirement for a minimum speed. From the lift equation, as the aircraft speed is decreased, the aircraft lift coefficient must be increased, until the aircraft stalls. Hence, the minimum speed that an aircraft can fly with is referred to as the stall speed (V_s).

An aircraft must be longitudinally trimmed at any cruising flight condition including at any flight speed. The range of acceptable speeds is between the stall speed and the maximum speed. In a cruising flight with the stall speed, the aircraft weight must be balanced with the lift (L).

$$L = W = \frac{1}{2}\rho V_s^{\,2} S C_{L_{max}} \tag{4.30}$$

where ρ denotes the air density at the specified altitude, and C_{Lmax} is the aircraft maximum lift coefficient. From, equation 4.30, we can derive the following when dividing both sides by “S”.

$$\left(\frac{W}{S}\right)_{V_s} = \frac{1}{2}\rho V_s^{\,2} C_{L_{max}} \tag{4.31}$$

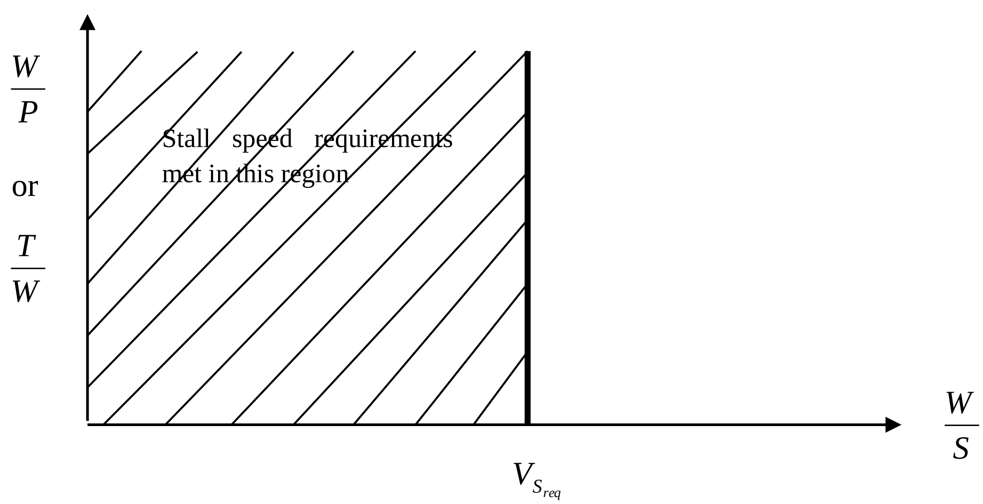

Figure 4.6. Stall speed contribution in constructing matching plot

This provides the first graph in the matching plot. The wing sizing based on stall speed requirements is represented by equation 4.31 as the variations of wing loading versus stall speed. As can be seen from equation 4.31; neither power loading (W/P) nor thrust loading (T/W) have contribution to wing loading in this case. In another word, the wing loading to satisfy the stall speed requirements is not a function of power loading no thrust loading. Therefore, the graph of power loading or thrust loading versus wing loading is always a vertical line in matching plot as sketched in figure 4.6.

In general, a low stall speed is desirable, since the lower stall speed results in a safer aircraft. When an unfortunate aircraft crash happens, a lower stall speed normally causes lower damages and fewer casualties. On the other hand, a lower stall speed results in a safer take-off and a safer landing, since an aircraft in a lower take-off and landing speed is more controllable. This is due to the fact that the take-off speed and the landing speed are often slightly higher than the stall speed (normally 1.1 to 1.3 times stall speed). Hence, in theory, any stall speed less than the stall speed specified by the mission requirements is acceptable. Therefore, the left side of the graph in figure 4.5 or figure 4.6 is an acceptable region and in the right side, the stall requirement is not met. So, by specifying a maximum allowable stall speed, equation 4.31 provides a maximum allowable wing loading for a given value of C_{Lmax}.

No	Aircraft	Type	m_{TO} (kg)	S (m^2)	V_s (knot)	C_{Lmax}
1	Volmer VJ-25 Sunfun	Hang glider/Kite	140.5	15.14	13	3.3
2	Manta Fledge III	Sailplane/Glider	133	14.95	15	2.4
3	Euro Wing Zephyr II	Microlight	340	15.33	25	2.15
4	Campana AN4	Very Light	540	14.31	34	1.97
5	Jurca MJ5 Sirocco	GA two seat	760	10	59	1.32
6	Piper Cherokee	GA single engine	975	15.14	47.3	1.74
7	Cessna 208-L	GA single turboprop	3,629	25.96	61	2.27
8	Short Skyvan 3	Twin turboprop	5,670	35.12	60	2.71
9	Gulfstream II	Business twin jet	29,700	75.2	115	1.8
10	Learjet 25	Business twin jet	6,800	21.5	104	1.77
11	Hawkeye E-2C	Early warning	24,687	65.03	92	2.7
12	DC-9-50	Jet Airliner	54,900	86.8	126	2.4
13	Boeing 727-200	Jet Airliner	95,000	153.3	117	2.75
14	Airbus 300	Jet Airliner	165,000	260	113	3
15	F-14 Tomcat	Fighter	33,720	54.5	110	3.1

Table 4.10. Maximum lift coefficient for several aircraft [3]

Based on FAR Part 23, a single engine aircraft and also multiengine aircraft with a maximum take-off weight of less than 6,000 lb may not have a stall speed greater than 61 knot.

A very light aircraft (VLA) that is certified with EASA[5] may not have a stall speed greater than 45 knot.

$$V_s \le 61 \quad knot \qquad \text{(FAR 23)} \qquad (4.32)$$

$$V_s \le 45 \quad knot \qquad \text{(EASA CS-VLA)} \qquad (4.33)$$

There are no maximum stall speed requirements for transport aircraft that are certified by FAR Part 25. It is clear that the stall speed requirements can be met with flap up configuration, since flap deflection allows for a higher lift coefficient and thus lower stall speed. An example application is presented in Section 4.4.

The equation 4.31 has two unknowns (ρ and C_{Lmax}) which often are not provided by the customer, so must be determined by the aircraft designer. The air density must be chosen to be at sea level (ρ = 1.225 kg/m^3, or 0.2378 slug/ft^3), since it provides the highest air density, which results in the lowest stall speed. This selection helps to more satisfy the stall speed requirement.

No	Aircraft type	C_{Lmax}	V_s (knot)
1	Hang glider/Kite	2.5-3.5	10-15
2	Sailplane/Glider	1.8-2.5	12-25
3	Microlight	1.8-2.4	20-30
4	Very light	1.6-2.2	30-45
5	GA-light	1.6-2.2	40-61
6	Agricultural	1.5-2	45-61
7	Homebuilt	1.2-1.8	40-70
8	Business jet	1.6-2.6	70-120
9	Jet transport	2.2-3.2	95-130
10	Supersonic fighter	1.8-3.2	100-120

Table 4.11. Typical values of maximum lift coefficient and stall speed for different types of aircraft

The aircraft maximum lift coefficient is mainly functions of wing and airfoil design, and also high lift device. The wing and airfoil design, and also high lift device selection are discussed in Chapter 5. At this moment of design phase (preliminary design); where the wing has not been designed yet; and high lift device has not yet been finalized; it is recommended to select a reasonable value for the maximum lift coefficient. Table 4.10 presents the maximum lift coefficient for several aircraft. This table also provides the aircraft stall speed for your information. If the stall speed is not given by the aircraft customer, use this table as a useful reference. This selection must be honored in the wing design (Chapter 5), hence, select a

[5] Joint Aviation Requirements (the aviation standards for several European countries)

reasonable maximum lift coefficient. Table 4.11 presents typical values of maximum lift coefficient and stall speed for different types of aircraft.

Employ extreme caution in using the units for variables. In SI system, the unit of W is N, the unit of S is m^2, the unit of V_s is m/sec, and the unit of ρ is kg/m^3. However, In British system, the unit of W is lb, the unit of S is ft^2, the unit of V_s is ft/sec, and the unit of ρ is $slug/ft^3$.

4.3.3. Maximum Speed

Another very important performance requirement; particularly for fighter aircraft; is the maximum speed. Two major contributors; other than aircraft weight; to the satisfaction of this requirement is the wing area and engine thrust (or power). It this section, the relevant equations are derived for the application in the matching plot. The derivations are presented in two separate sub-sections; one sub-section for jet aircraft (4.3.3.1), and another sub-section for prop-driven aircraft (4.3.3.2).

4.3.3.1. Jet Aircraft

Consider a jet aircraft which is flying with the maximum constant speed at a specified altitude (ρ_{alt}). The aircraft is in longitudinal trim; hence, the maximum engine thrust (T_{max}) must be equal to the maximum aircraft drag (D_{max}) and aircraft weight (W) must be equal to the lift (L).

$$T_{\max} = D_{\max} \tag{4.34}$$

$$W = L \tag{4.35}$$

where lift and drag are two aerodynamic forces and are defined as:

$$D = \frac{1}{2}\rho V_{\max}^2 S C_D \tag{4.36}$$

$$L = \frac{1}{2}\rho V_{\max}^2 S C_L \tag{4.37}$$

On the other hand, the engine thrust is decreasing with increasing aircraft altitude. This requires knowledge of how the engine thrust of an aircraft varies with airspeed and altitude. A general relationship between engine thrust and the altitude; which is represented by air density (ρ) is:

$$T_{alt} = T_{SL}\left(\frac{\rho}{\rho_o}\right) = T_{SL}\sigma \tag{4.38}$$

where ρ_o is the sea level air density, T_{alt} is the engine thrust at altitude, and T_{SL} is the engine thrust at sea level. By substituting the equations 4.36 and 4.38 into equation 4.34, we will have:

$$T_{SL}\sigma = \frac{1}{2}\rho V_{\max}^2 S C_D \tag{4.39}$$

The aircraft drag coefficient has two contributors; zero-lift drag coefficient (C_{Do}) and induced drag coefficient (C_{Di}):

$$C_D = C_{D_o} + C_{D_i} = C_{D_o} + K \cdot C_L^2 \tag{4.40}$$

where K is referred to as induced drag factor and is determined by:

$$K = \frac{1}{\pi \cdot e \cdot AR} \tag{4.41}$$

Typical values for e (Oswald span efficiency factor) are between 0.7 and 0.95. The typical values for wing aspect ratio (AR) are given in Table 5.8 of Chapter 5. Substitution of equation 4.40 into equation 4.39 yields:

$$T_{SL}\sigma = \frac{1}{2}\rho V_{\max}^2 S\left(C_{D_o} + K \cdot C_L^2\right) \tag{4.42}$$

From equation 4.35 and 4.37, the aircraft lift coefficient can be derived as:

$$C_L = \frac{2W}{\rho V_{\max}^2 S} \tag{4.43}$$

Substituting this lift coefficient into equation 4.42 provides:

$$T_{SL}\sigma = \frac{1}{2}\rho V_{\max}^2 S\left(C_{D_o} + K \cdot \left[\frac{2W}{\rho V_{\max}^2 S}\right]^2 \right) \tag{4.44}$$

Now, we can simplify this equation as:

$$T_{SL}\sigma = \frac{1}{2}\rho V_{\max}^2 S C_{D_o} + \frac{1}{2}\rho V_{\max}^2 S \frac{K(2W)^2}{\left(\rho V_{\max}^2 S\right)^2} = \frac{1}{2}\rho V_{\max}^2 S C_{D_o} + \frac{2KW^2}{\rho V_{\max}^2 S} \tag{4.45}$$

Both sides of this equation can be divided by aircraft weight (W) as:

$$\frac{T_{SL}}{W}\sigma = \frac{1}{2}\rho V_{\max}^2 \frac{S}{W} C_{D_o} + \frac{2KW^2}{\rho V_{\max}^2 SW} \tag{4.46}$$

This can also be written as:

$$\left(\frac{T_{SL}}{W}\right)_{V_{\max}} = \rho_o V_{\max}^2 C_{D_o} \frac{1}{2\left(\frac{W}{S}\right)} + \frac{2K}{\rho\sigma V_{\max}^2}\left(\frac{W}{S}\right) \tag{4.47}$$

Thus, thrust loading (T/W) is a nonlinear function of wing loading (W/S) in terms of maximum speed, and may be simplified as:

$$\left(\frac{T}{W}\right) = \frac{aV_{\max}^2}{\left(\frac{W}{S}\right)} + \frac{b}{V_{\max}^2}\left(\frac{W}{S}\right) \tag{4.48}$$

The wing and engine sizing based on maximum speed requirements is represented by equation 4.47 as the variations of thrust loading versus wing loading. This variation of T/W as a function of W/S based on V_{max} can be sketched by using equation 4.47 in constructing the matching plot as shown in figure 4.7. In order to determine the acceptable region, we need to find what side of this graph is satisfying the maximum speed requirement. As the value of V_{max} in equation 4.47 is increased, the value of thrust-to-weight ratio (T/W) is increased too. Since, any value of V_{max} greater than the specified maximum speed is satisfying the maximum speed requirement, so the region above the graph is acceptable.

Employ extreme caution to use a consistent unit when applying the equation 4.47 (either in SI system or British system). In SI system, the unit of V_{max} is m/sec, the unit of W is N, the unit of T is N, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of V_{max} is ft/sec, the unit of W is lb, the unit of T is lb, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. An example application is presented in Section 4.4.

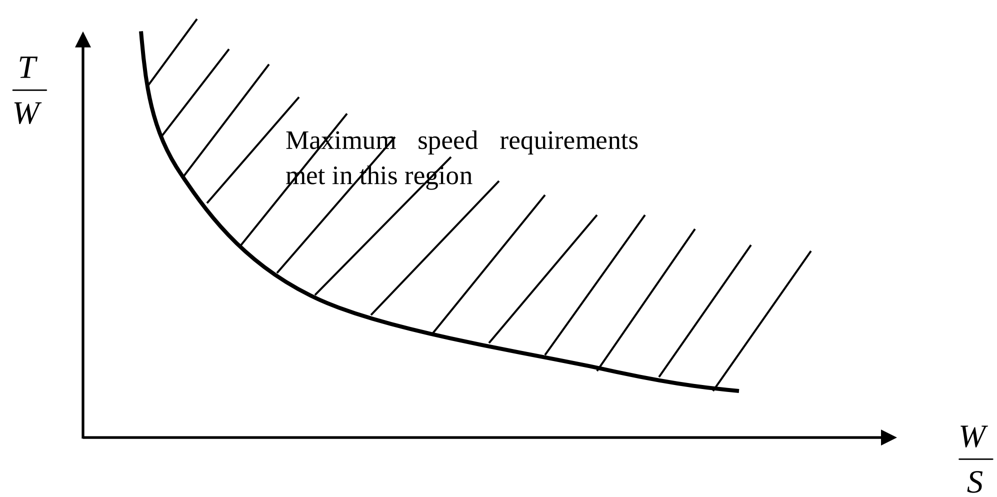

Figure 4.7. Maximum speed contribution in constructing matching plot for a jet aircraft

If instead of the maximum speed, the cruising speed is given as a design requirement, assume that the maximum speed is about 20 to 30 percent greater than cruise speed. This is due to the fact that cruise speeds for jet aircraft are usually calculated at 75 to 90 percent thrust.

$$V_{\max} = 1.2V_C \quad to \quad 1.3V_C \tag{4.49}$$

Section 4.3.3.3 provides a technique to estimate the aircraft zero-lift drag coefficient (C_{Do}).

4.3.3.2. Prop-driven Aircraft

Consider a prop-driven aircraft which is flying with the maximum constant speed at a specified altitude (ρ_{alt} or simply ρ). The aircraft is in longitudinal trim; hence, the maximum available engine power (P_{max}) must be equal to the maximum required power (P_{req}); which is thrust multiplied by maximum speed.

$$P_{avl} = P_{req} \Rightarrow \eta_P P_{\max} = TV_{\max} \tag{4.50}$$

where engine thrust (T) must be equal to the aircraft drag (D); equation 4.36. On the other hand, the engine power is decreasing with increasing aircraft altitude. This requires knowledge of how the engine power of an aircraft varies with airspeed and altitude. A general relationship between engine power and the altitude; which is represented by air density (ρ) is:

$$P_{alt} = P_{SL}\left(\frac{\rho}{\rho_o}\right) = P_{SL}\sigma \tag{4.51}$$

where P_{alt} is the engine power at altitude, and P_{SL} is the engine power at sea level. By substituting the equations 4.36 and 4.51 into equation 4.50, we will obtain:

$$\eta_P P_{SL}\sigma = \frac{1}{2}\eta_P \rho V_{\max}^2 SC_D \cdot V_{\max} = \frac{1}{2}\rho V_{\max}^3 SC_D \tag{4.52}$$

Aircraft drag coefficient is (C_D) defined by equation 4.40, and aircraft lift coefficient (C_L) is provided by equation 4.43. Substitution of C_D (equation 4.40) and C_L (equation 4.43) into equation 4.52 yields:

$$\eta_P P_{SL}\sigma = \frac{1}{2}\rho V_{\max}^3 S\left(C_{D_o} + K \cdot \left[\frac{2W}{\rho V_{\max}^2 S}\right]^2\right) \tag{4.53}$$

or:

$$\eta_P P_{SL}\sigma = \frac{1}{2}\rho V_{\max}^3 SC_{D_o} + \frac{1}{2}\rho V_{\max}^3 S\frac{K(2W)^2}{\left(\rho V_{\max}^2 S\right)^2} = \frac{1}{2}\rho V_{\max}^3 SC_{D_o} + \frac{2KW^2}{\rho SV_{\max}} \tag{4.54}$$

Both sides of this equation can be divided by aircraft weight (W) as:

$$\frac{P_{SL}}{W}\eta_P\sigma = \frac{1}{2}\rho V_{\max}^3 \frac{S}{W}C_{D_o} + \frac{2KW^2}{\rho V_{\max} SW} = \frac{1}{2}\rho C_{D_o} V_{\max}^3 \frac{1}{\left(\frac{W}{S}\right)} + \frac{2K}{\rho V_{\max}}\left(\frac{W}{S}\right) \tag{4.55}$$

This equation can be inverted as follows:

$$\left(\frac{W}{P_{SL}}\right) = \frac{\eta_P\sigma}{\frac{1}{2}\rho V_{\max}^3 C_{D_o}\frac{1}{\left(\frac{W}{S}\right)} + \frac{2K}{\rho V_{\max}}\left(\frac{W}{S}\right)} \Rightarrow$$

$$\left(\frac{W}{P_{SL}}\right)_{V_{\max}} = \frac{\eta_P}{\frac{1}{2}\rho_o V_{\max}^3 C_{D_o}\frac{1}{\left(\frac{W}{S}\right)} + \frac{2K}{\rho\sigma V_{\max}}\left(\frac{W}{S}\right)} \tag{4.56}$$

Thus, power loading (W/P) is a nonlinear function of wing loading (W/S) in terms of maximum speed, and may be simplified as:

$$\left(\frac{W}{P}\right) = \frac{\eta_P}{\frac{aV_{\max}^3}{\left(\frac{W}{S}\right)} + \frac{b}{V_{\max}}\left(\frac{W}{S}\right)} \tag{4.57}$$

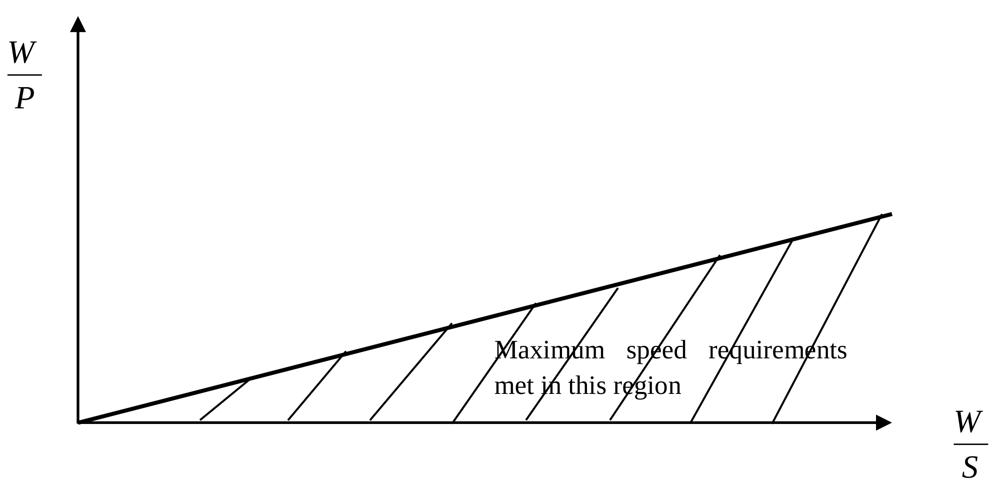

Figure 4.8. Maximum speed contribution in constructing matching plot for a prop-driven aircraft

The wing and engine sizing based on maximum speed requirements is represented by equation 4.57 as the variations of power loading versus wing loading. The variation of W/P as a function of W/S based on V_{max} for a prop-driven aircraft can be sketched by using equation 4.56 in constructing the matching plot as shown in figure 4.8. In order to determine the acceptable region, we need to find what side of this graph is satisfying the maximum speed requirement. As the value of V_{max} in equation 4.56 is increased, the value of power loading (P/W) is decreased. This is due to the fact that the first term in the denominator of equation 4.56 is V_{max}^3. Since, any value of V_{max} greater than the specified maximum speed is satisfying the maximum speed requirement, so the region below the graph is acceptable. Extreme caution must be taken to use consistent units in the application process. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient.

Employ extreme caution to use a consistent unit when applying the equation 4.56 (either in SI system or British system). In SI system, the unit of V_{max} is m/sec, the unit of W is N, the unit of P is Watt, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of V_{max} is ft/sec, the unit of W is lb, the unit of P is lb.ft/sec, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient. Recall that each horse power (hp) is equivalent to 550 lb.ft/sec. An example application is presented in Section 4.4.

If instead of the maximum speed, the cruising speed is given as a design requirement, assume that the maximum speed is about 20 to 30 percent greater than cruise speed.

$$V_{max} = 1.2V_C \quad to \quad 1.3V_C \tag{4.58}$$

This is due to the fact that cruise speeds for prop-driven aircraft are usually calculated at 75 to 90 percent power.

4.3.3.2. Aircraft C_{Do} Estimation

An important aircraft parameter that must be known and is necessary in constructing the matching plot is the aircraft zero-lift drag coefficient (C_{Do}). Although the aircraft is not aerodynamically designed yet at this phase of design, but there is a reliable way to estimate this parameter. The technique is primarily based on a statistics. However, in most references; such as [3]; the aircraft C_{Do} is not given, but it can be readily determined based on aircraft performance which is often given.

Consider a jet aircraft that is flying with its maximum speed at a specified altitude. The governing trim equations are introduced in Section 4.3.3.1 and the relationships are expanded until we obtain the following equation:

$$\left(\frac{T_{SL}}{W}\right) = \rho_o V_{\max}^2 C_{D_o} \frac{1}{2\left(\frac{W}{S}\right)} + \frac{2K}{\rho\sigma V_{\max}^2}\left(\frac{W}{S}\right) \tag{4.47}$$

The aircraft C_{Do} can be obtained from this equation as follows:

$$\left(\frac{T_{SL}}{W}\right) - \frac{2KW}{\rho\sigma V_{\max}^2 S} = \rho_o V_{\max}^2 C_{D_o} \frac{S}{2W} \Rightarrow C_{D_o} = \frac{\frac{T_{SL}}{W} - \frac{2KW}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 \frac{S}{2W}} \tag{4.59}$$

or

$$C_{D_o} = \frac{2T_{SL_{\max}} - \frac{4KW^2}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 S} \tag{4.60}$$

If the aircraft is prop-driven, the engine thrust is a function of engine power, airspeed, and propeller efficiency (η_P), so:

$$T_{\max} = \frac{P_{\max} \cdot \eta_P}{V_{\max}} \tag{4.61}$$

where prop efficiency is about 0.7 to 0.85 when an aircraft is cruising with its maximum speed.

No	Aircraft type	C_{Do}
1	Jet transport	0.015 − 0.02
2	Turboprop transport	0.018 − 0.024
3	Twin-engine piton prop	0.022 − 0.028
4	Small GA with retractable landing gear	0.02 − 0.03
5	Small GA with fixed landing gear	0.025 − 0.04
6	Agricultural	0.04 − 0.07
7	Sailplane/Glider	0.012 − 0.015
8	Supersonic fighter	0.018 − 0.035
9	Homebuilt	0.025 − 0.04
10	Microlight	0.02 − 0.035

Table 4.12. Typical values of C_{Do} for different types of aircraft

The equation 4.61 can be substituted into the equation 4.60:

$$C_{D_o} = \frac{2\frac{P_{SL_{\max}} \cdot \eta_P}{V_{\max}} - \frac{4KW^2}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 S} \qquad (4.62)$$

The equations 4.60 and 4.62 are employed to determine the aircraft C_{Do} for jet and prop-driven aircraft respectively. In these equations, $T_{SL_{\max}}$ is the maximum engine thrust at sea level, and $P_{SL_{\max}}$ is the maximum engine power at sea level, ρ is the air density at flight altitude, and σ is the relative air density at flight altitude. Make sure to use a consistent unit for all parameters (either in metric unit or British unit).

In order to estimate the C_{Do} for the aircraft which is under preliminary design, calculate the C_{Do} of several aircraft which have similar performance characteristics and similar configuration. Then find the average C_{Do} of those aircraft. If you have selected five similar aircraft, then the C_{Do} of the aircraft under preliminary design is determined as follows:

$$C_{D_o} = \frac{C_{D_{o1}} + C_{D_{o2}} + C_{D_{o3}} + C_{D_{o4}} + C_{D_{o5}}}{5} \qquad (4.63)$$

where $C_{D_{oi}}$ is the C_{Do} of ith aircraft. Table 4.12 presents typical values of C_{Do} for different types of aircraft. References [5] and [10] present details of the technique to calculate complete C_{Do} of an aircraft.

Example 4.1. C_{Do} Calculation

Determine the zero-lift drag coefficient (C_{Do}) of the fighter aircraft F/A-18 Hornet (Figures 2.11, 6.12, and 12.27) which is flying with a maximum speed of Mach 1.8 at 30,000 ft. This fighter has the following characteristics:

T_{SLmax} = 2 × 71,170 N, m_{TO} = 16,651 kg, S = 37.16 m^2, AR = 3.5, e = 0.7

Solution:

We need to first find out maximum speed in terms of m/sec. The air density at 30,000 ft is 0.000892 slug/ft^3 or 0.46 kg/m^3, and the air temperature is 229 K. From Physics, we know that speed of sound in a function of air temperature. Thus:

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 229} = 303.3 \ \frac{m}{\sec} \qquad (4.64)$$

From Aerodynamics, we know that Mach number is the ratio between airspeed to the speed of sound. Hence, the aircraft maximum speed is:

$$M = \frac{V}{a} \Rightarrow V_{maax} = M_{\max} \cdot a = 1.8 \times 303.3 = 546 \quad \frac{m}{\sec} \tag{4.65}$$

The induced drag factor is:

$$K = \frac{1}{\pi \cdot e \cdot AR} = \frac{1}{3.14 \times 0.7 \times 3.5} \Rightarrow K = 0.13 \tag{4.41}$$

Then:

$$C_{D_o} = \frac{2T_{SL_{\max}} - \dfrac{4KW^2}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 S} = \frac{2 \times 2 \times 71{,}170 - \dfrac{4 \times 0.13 \times (16{,}651 \times 9.81)}{0.46 \times \left(\dfrac{0.46}{1.225}\right) \times (546)^2 \times (37.16)}}{1.225 \times (546)^2 \times (37.16)} = 0.02 \tag{4.60}$$

Thus, the zero-lift drag coefficient (C_{Do}) of the fighter aircraft F/A-18 Hornet at 30,000 ft is 0.02.

Example 4.2. Aircraft C_{Do} Estimation

You are a member of a team that is designing a transport aircraft which is required to carry 45 passengers with the following performance features:

1. Max speed: at least 300 knots at sea level
2. Max range: at least 1,500 km
3. Max rate of climb: at least 2,500 fpm
4. Absolute ceiling: at least 28,000 ft
5. Take-off run: less than 4,000 ft

In the preliminary design phase, you are required to estimate the zero-lift drag coefficient (C_{Do}) of such aircraft. Identify five current similar aircraft and based on their statistics, estimate the C_{Do} of the aircraft being designed.

Solution:

Ref. [3] is a reliable source to look for similar aircraft in terms of performance characteristics. Shown below is Table 4.13 illustrating five aircraft with similar performance requirements as the aircraft that is being designed. There are 3 turboprops and 2 jets, so either engine configuration may be satisfactory. All of them are twin engines, and have retractable gear. There are no mid-wing aircraft listed here. The wing areas are very similar, ranging from 450 ft^2 - 605 ft^2. Except for the Bombardier Challenger 604 which can carry 19 passengers (the minimum requirement for the aircraft being designed) the other four listed aircraft can accommodate approximately 50 passengers. The weights of the aircraft vary, with the Challenger 800 weighing the most. The power of the prop-driven aircraft are all around 2000 hp/engine, and then thrust for the jet aircraft is around 8000 lb/engine.

In order to calculate the C_{Do} of each aircraft, equation 4.60 is employed for the jet aircraft and equation 4.62 is used for the prop-driven aircraft.

No	Name	Pax	V_{max} (knot)	Range (km)	ROC (fpm)	S_{TO} (ft)	Ceiling (ft)
1	DHC-8 Dash 8-300B	50	287	1,711	1,800	3,600	25,000
2	Antonov 140 (Figure 3.15)	46	310 @ 23,620 ft	1,721	1,345	2,890	25,000
3	Embraer 145MP	50	410@ 37,000 ft	3,000	1,750	6,465	37,000
4	Bombardier Challenger 604	19	471 @ 17,000 ft	4,274	3,395	2,910	41,000
5	Saab 340 (Figure 8.21)	35	280 @ 20,000 ft	1,750	2,000	4,325	25,000

Table 4.13. Characteristics of five aircraft with similar performance

$$C_{D_o} = \frac{2T_{SL_{\max}} - \dfrac{4KW^2}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 S} \tag{4.60}$$

$$C_{D_o} = \frac{2\dfrac{P_{SL_{\max}} \cdot \eta_P}{V_{\max}} - \dfrac{4KW^2}{\rho\sigma V_{\max}^2 S}}{\rho_o V_{\max}^2 S} \tag{4.62}$$

The Oswald span efficiency factor was assumed to be 0.85, and the prop efficiencies for the propeller aircraft were assumed to be 0.82. Example 4.1 shows the application of the equation 4.60 for a jet aircraft, the following is the application of equation 4.62 for Saab 340, a turboprop-driven airliner. The cruise altitude for Saab 340 is 20,000 ft, so the air density at 20,000 ft is 0.001267 slug/ft^3 and the relative air density at 20,000 ft is 0.533.

No	Aircraft	Type	W_o (lb)	P (hp)/T (lb)	S (ft^2)	AR	C_{Do}
1	DHC-8 Dash 8-300B	Twin-turboprop	41,100	2×2500 hp	605	13.4	0.02
2	Antonov An-140	Twin-turboprop	42,220	2×2,466 hp	549	11.5	0.016
3	Embraer EMB-145	Regional jet	42,328	2×7040 lb	551	7.9	0.034
4	Bombardier Challenger 604	Business jet	47,600	2×9,220 lb	520	8	0.042
5	Saab 340	Twin-turboprop	29,000	2×1750 hp	450	11	0.021

Table 4.14. C_{Do} of five similar aircraft

$$K = \frac{1}{\pi \cdot e \cdot AR} = \frac{1}{3.14 \times 0.85 \times 11} = 0.034 \tag{4.41}$$

$$C_{D_o} = \frac{2\frac{2\times1750\times550\times0.82}{280\times1.688} - \frac{4\times0.034\times(29{,}000)^2}{0.001267\times0.533\times(280\times1.688)^2\times450}}{0.002378\times(280\times1.688)^2\times450} \Rightarrow C_{D_o} = 0.021 \quad (4.62)$$

where 1 knot is equivalent to 1.688 ft/sec and 1 hp is equivalent with 550 lb.ft/sec.

The aircraft geometries, engine powers, and C_{Do} and also the results of the calculation are shown in Table 4.14. The zero lift drag coefficient for two turboprop aircraft is very similar, 0.02 or 0.021 and one is only 0.016. This coefficient for jet aircraft is higher, 0.034 and 0.042. It seems these three numbers (0.016, 0.034, and 0.042) are unrealistic; therefore, some of the published data are not reliable. The estimation of C_{Do} of the aircraft being design is determined by taking the average of five C_{Do}.

$$C_{D_o} = \frac{C_{D_{o1}} + C_{D_{o2}} + C_{D_{o3}} + C_{D_{o4}} + C_{D_{o5}}}{5} = \frac{0.02+0.016+0.034+0.042+0.021}{5} \quad (4.63)$$
$$\Rightarrow C_{D_o} = 0.027$$

Therefore, the C_{Do} for the aircraft under preliminary design will be assumed to be 0.027.

4.3.4. Take-Off Run

The take-off run (S_{TO}) is another significant factor in aircraft performance and will be employed in constructing matching chart to determine wing area and engine thrust (or power). The take-off requirements are frequently spelled out in terms of minimum ground run requirements, since every airport has a limited runway. Take-off run is defined as the distance between take-off starting point to the location of standard obstacle that the aircraft must clear (Figure 4.9). The aircraft is required to clear an imaginary obstacle at the end of airborne section, so take-off run includes ground section plus airborne section. The obstacle height is determined by airworthiness standard. Based on FAR Part 25, obstacle height (h_o) is 35 ft for passenger aircraft, and based on FAR Part 23 Section 23.53, obstacle height is 50 ft for GA aircraft. There is no FAR requirement for take-off run; instead FAR has a number of regulations on the balanced field length.

4.3.4.1. Jet Aircraft

Based on [5], take-off run for a jet aircraft is determined by the following equation:

$$S_{TO} = \frac{1.65W}{\rho g S C_{D_G}} \ln\left[\frac{\frac{T}{W} - \mu}{\frac{T}{W} - \mu - \frac{C_{D_G}}{C_{L_R}}}\right] \quad (4.66)$$

where μ is the friction coefficient of the runway (see Table 4.15) and C_{D_G} is defined as:

$$C_{D_G} = \left(C_{D_{TO}} - \mu C_{L_{TO}}\right) \quad (4.67a)$$

The parameter C_{L_R} is the aircraft lift coefficient at take-off rotation and is obtained from:

$$C_{L_R} = \frac{2mg}{\rho S V_R^2} \quad (4.67b)$$

where V_R is the aircraft speed at rotation which is about $1.1V_s$ to $1.2V_s$. The aircraft drag coefficient at take-off configuration ($C_{D_{TO}}$) is:

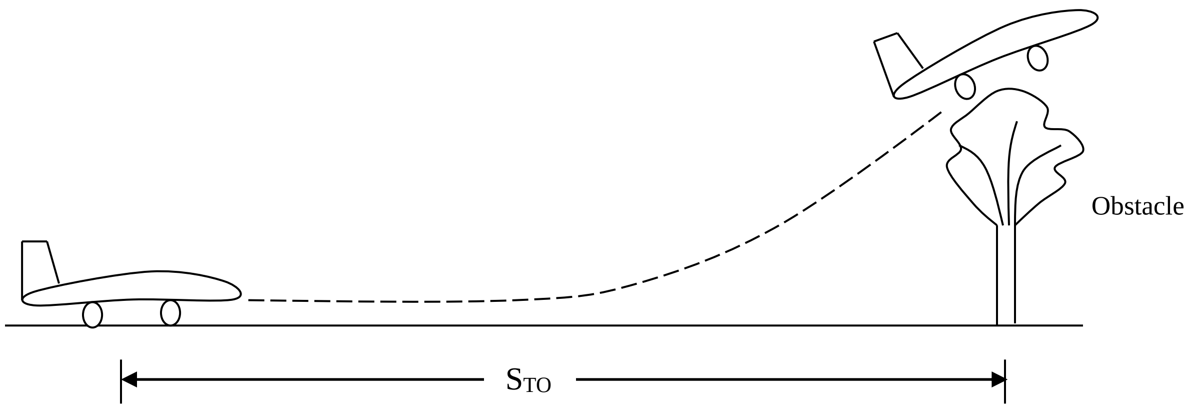

Figure 4.9. The definition of take-off run

$$C_{D_{TO}} = C_{D_{oTO}} + KC_{L\,TO}^{2} \quad (4.68)$$

where aircraft zero-lift drag coefficient at take-off configuration ($C_{D_{oTO}}$) is:

$$C_{D_{oTO}} = C_{D_o} + C_{D_{oLG}} + C_{D_{oHLD\_TO}} \quad (4.69a)$$

where C_{Do} is the clean-aircraft zero-lift drag coefficient (see Table 4.12), $C_{D_{oLG}}$ is the landing gear drag coefficient, and $C_{D_{oHLD\_TO}}$ is the high lift device (e.g. flap) drag coefficient at take-off configuration. The typical values for $C_{D_{oLG}}$ and $C_{D_{oHLD\_TO}}$ are as follows:

$$C_{D_{oLG}} = 0.006 \ \ to \ \ 0.012$$
$$C_{D_{oHLD\_TO}} = 0.003 \ \ to \ \ 0.008 \quad (4.69b)$$

where the take-off lift coefficient is determined as:

$$C_{L_{TO}} = C_{L_C} + \Delta C_{L_{flapTO}} \tag{4.69c}$$

where the C_{L_C} is the aircraft cruise lift coefficient and $\Delta C_{L_{flapTO}}$ is the additional lift coefficient that is generated by flap at take-off configuration. The typical value for aircraft cruise lift coefficient is about 0.3 for a subsonic aircraft and 0.05 for a supersonic aircraft. The typical value for take-off flap lift coefficient ($\Delta C_{L_{flapTO}}$) is about 0.3 to 0.8. The equation 4.66 can be manipulated to be formatted as the thrust loading (T/W) in terms of wing loading (W/S) and take-off run. The derivation is as follows:

$$\frac{\rho g S C_{D_G} S_{TO}}{1.65W} = \ln\left[\frac{\frac{T}{W} - \mu}{\frac{T}{W} - \mu - \frac{C_{D_G}}{C_{L_R}}}\right] \Rightarrow \frac{\frac{T}{W} - \mu}{\frac{T}{W} - \mu - \frac{C_{D_G}}{C_{L_R}}} = \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right) \Rightarrow \tag{4.70}$$

$$\frac{T}{W} - \mu = \left(\frac{T}{W} - \mu - \frac{C_{D_G}}{C_{L_{TR}}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] \Rightarrow$$

$$\frac{T}{W} - \mu = \left(\frac{T}{W}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] \Rightarrow$$

$$\frac{T}{W} - \left(\frac{T}{W}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] = \mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] \Rightarrow$$

$$\frac{T}{W}\left[1 - \left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right]\right] = \mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{S}{W}\right)\right] \Rightarrow$$

Finally:

No	**Surface**	**Friction coefficient (μ)**
1	Dry concrete/asphalt	0.03-0.05
2	Wet concrete/asphalt	0.05
3	Icy concrete/asphalt	0.02
4	Turf	0.04-0.07
5	Grass	0.05-0.1
6	Soft ground	0.1-0.3

Table 4.15. Friction coefficients for various runway surfaces

$$\left(\frac{T}{W}\right)_{S_{TO}} = \frac{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]}{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)} \tag{4.71}$$

The wing and engine sizing based on take-off run requirements are represented by equation 4.71 as the variations of thrust loading versus wing loading. The variation of T/W as a function of W/S based on S_{TO} for a jet aircraft can be sketched by using equation 4.71 in constructing the matching plot as shown in Figure 4.10. In order to determine the acceptable region, we need to find what side of this graph is satisfying the take-off run requirement. Both the numerator and the denominator of equation 4.71 contain an exponential term with a positive power that includes the parameter S_{TO}.

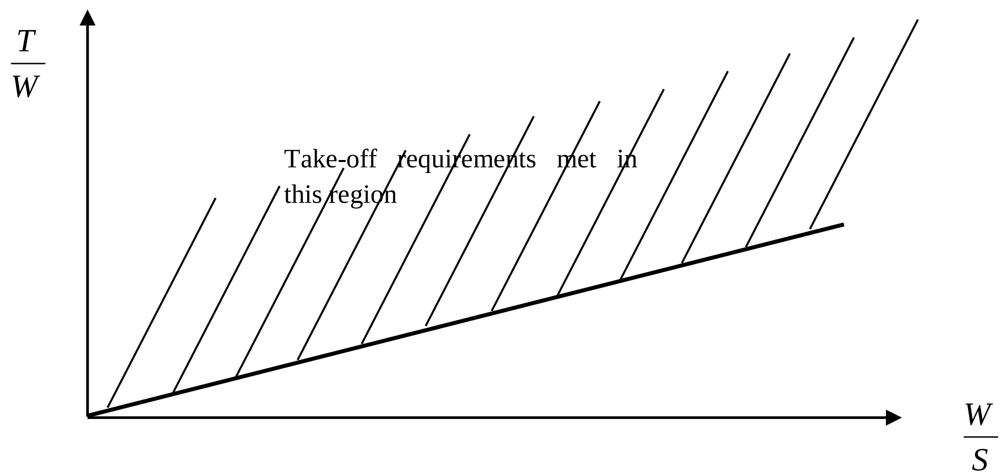

Figure 4.10. Take-off run contribution in constructing matching plot for a jet aircraft

As the value of take-off run (S_{TO}) in equation 4.71 is *increased*, the value of thrust-t-weight ratio (T/W) would *drop*. Since, any value of S_{TO} greater than the specified take-off run is not satisfying the take-off run requirement, so the region *below* the graph (Figure 4.10) is not acceptable. Employ extreme caution to use a consistent unit when applying the equation 4.71 (either in SI system or British system). In SI system, the unit of S_{TO} is m, the unit of W is N, the unit of T is N, the g is 9.81 m/s^2, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of S_{TO} is ft, the unit of W is lb, the unit of T is lb, the unit of g is 32.17 ft/s^2, the unit of S is ft^2, and the unit of ρ is slug/ft^3. An example application is presented in Section 4.4.

4.3.4.2. Prop-driven Aircraft

In a prop-driven aircraft, the engine thrust is a function of propeller efficiency and the aircraft speed. However, take-off operation is considered as an accelerating motion, so the aircraft speed is not constant. The aircraft speed varies quickly from zero to rotation speed and then to take-off speed. The take-off speed (V_{TO}) is normally slightly greater than the stall speed (V_s).

$$V_{TO} = 1.1V_s \quad to \quad 1.3V_s \tag{4.72}$$

The following is reproduced directly from FAR 23.51:

For normal, utility, and acrobatic category airplanes, the speed at 50 feet above the takeoff surface level must not be less than:

(1) or multiengine airplanes, the highest of—

(i) A speed that is shown to be safe for continued flight (or emergency landing, if applicable) under all reasonably expected conditions, including turbulence and complete failure of the critical engine;

(ii) 1.10 V_{MC}; or

(iii) 1.20 V_{S1}.

(2) For single-engine airplanes, the higher of—

(i) A speed that is shown to be safe under all reasonably expected conditions, including turbulence and complete engine failure; or

(ii) 1.20 V_{S1}.

Furthermore, the prop efficiency is not constant and is much lower than its maximum attainable efficiency. If the prop is of fixed pitch type, its efficiency is considerably higher than that of a variable pitch. To include the above mentioned variations in the aircraft speed and prop efficiency, the engine thrust is suggested to be estimated by the following equations:

$$T_{TO} = \frac{0.5P_{\max}}{V_{TO}} \qquad \text{(Fixed-pitch propeller)} \tag{4.73a}$$

$$T_{TO} = \frac{0.6P_{\max}}{V_{TO}} \qquad \text{(Variable-pitch propeller)} \tag{4.73b}$$

This demonstrates that the prop efficiency for a fixed-pitch propeller is 0.5, and for a variable-pitch propeller is 0.6. The above thrust estimation works for majority of aero-engines. A better thrust model might be found through engine manufacturer. By substituting equation 4.73 into equation 4.71, we obtain:

$$\left(\frac{\frac{\eta_P P_{max}}{V_{TO}}}{W}\right)_{S_{TO}} = \frac{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]}{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)} \tag{4.74}$$

or

$$\left(\frac{P}{W}\right)_{S_{TO}} = \frac{V_{TO}}{\eta_P}\frac{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]}{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)} \tag{4.75}$$

This equation can be inverted and is written as follows:

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)}{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]}\frac{\eta_P}{V_{TO}} \tag{4.76}$$

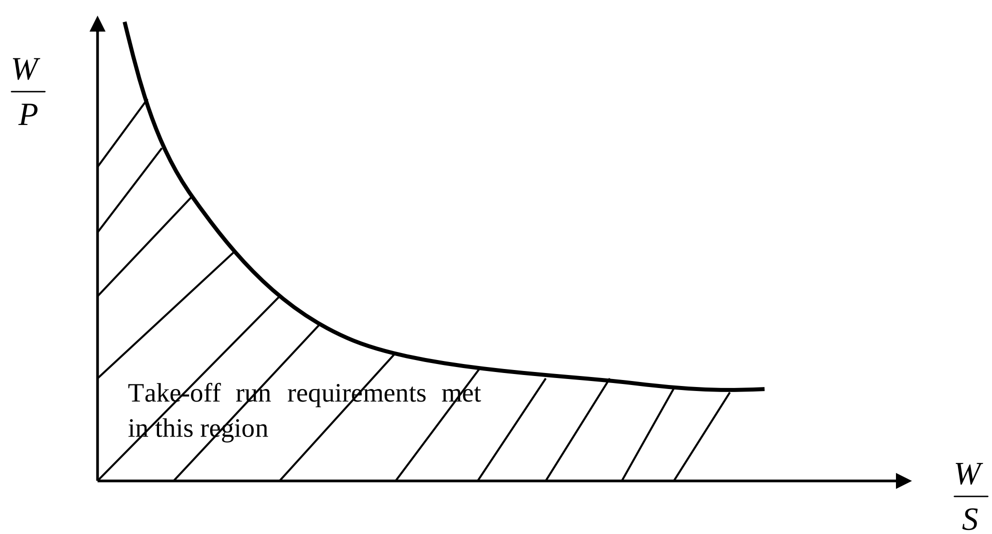

Figure 4.11. Take-off run contribution in constructing matching plot for a prop-driven aircraft

The wing and engine sizing based on take-off run requirement is represented by equation 4.76 as the variations of power loading versus wing loading. Remember, that prop efficiency is 0.5 for a fixed-pitch propeller and is 0.6 for a variable-pitch propeller. The variation of W/P as a function of W/S based on S_{TO} for a prop-driven aircraft can be sketched by using equation 4.76 in constructing the matching plot as shown in figure 4.11. In order to determine the acceptable region, we need to find what side of this graph is satisfying the take-off run requirement. Both the numerator and the denominator of equation 4.71 contain an exponential term with a positive power that includes the parameter S_{TO}. As the take-off run is increased, the magnitude of the exponential term will increase.

As the value of S_{TO} in equation 4.76 is *increased*, the value of power loading (W/P) is going *up*. Since, any value of S_{TO} greater than the specified take-off run is not satisfying the take-off run requirement, so the region *above* the graph is not acceptable. Employ extreme caution to use a consistent unit when applying the equation 4.76 (either in SI system or British system).

In SI system, the unit of S_{TO} is m, the unit of W is N, the unit of P is Watt, the unit of S is m^2, the unit of V_{TO} is m/sec, the variable g is 9.81 m/s^2, and the unit of ρ is kg/m^3. However, In British system, the unit of S_{TO} is ft, the unit of W is lb, the unit of P is lb.ft/sec, the unit of S is ft^2, the unit of V_{TO} is ft/sec, the variable g is 32.17 ft/s^2, and the unit of ρ is slug/ft^3. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient. Recall that each horse power (hp) is equivalent to 550 lb.ft/sec. An example application is presented in Section 4.4.

4.3.5. Rate of Climb

Every type of aircraft must meet certain rate of climb requirements. For civil aircraft, the climb requirements of FAR[6] Part 23 (for GA aircraft), or FAR part 25 (for transport aircraft) must be met. For military aircraft, the requirements specified by military standards, handbooks, and specifications[7] must be met. In some instances, climb requirements are spelled out in terms of time-to-climb, but this can be readily translated into rate of climb requirements. Rate of climb is defined as the aircraft speed in the vertical axis or the vertical component of the aircraft airspeed. Hence rate of climb is about how fast an aircraft gains height.

Based on FAR Part 23 Section 23.65, there are requirements for gradient of climb as follows:

(a) Each normal, utility, and acrobatic category reciprocating engine-powered airplane of 6,000 pounds or less maximum weight must have a steady climb gradient at sea level of at least 8.3 percent for landplanes or 6.7 percent for seaplanes and amphibians.

(b) Each normal, utility, and acrobatic category reciprocating engine-powered airplane of more than 6,000 pounds maximum weight and turbine engine-powered airplanes in the normal, utility, and acrobatic category must have a steady gradient of climb after takeoff of at least 4 percent.

---

[6] Ref. [1]
[7] For instance, see: MIL-C-005011B (USAF), Military specification charts: Standard aircraft characteristics and performance, piloted aircraft, 1977

The derivation of an expression for wing and engine sizing based upon rate of climb requirements for jet and prop-driven aircraft are examined separately. Since the maximum rate of climb is obtained at sea level, the air density in equations in this section implies the sea level air density.

4.3.5.1. Jet Aircraft

In general, the Rate Of Climb (ROC) is defined as the ratio between excess power and the aircraft weight:

$$ROC = \frac{P_{avl} - P_{req}}{W} = \frac{(TV - DV)}{W} \tag{4.77}$$

This can be written as:

$$ROC = V\left[\frac{T}{W} - \frac{D}{W}\right] = V\left[\frac{T}{W} - \frac{D}{L}\right] = V\left[\frac{T}{W} - \frac{1}{L/D}\right] \tag{4.78}$$

In order to maximize rate of climb, both engine thrust and lift-to-drag ratio must be maximized. This is to maximize the magnitude of the term inside the bracket in equation 4.78.

$$ROC_{\max} = V_{ROC_{\max}}\left[\frac{T_{\max}}{W} - \frac{1}{(L/D)_{\max}}\right] \tag{4.79}$$

In order to maximize lift-to-drag ratio, the climb speed must be such the aircraft drag is minimized, as outlined by [5] as follows:

$$V_{ROC_{\max}} = V_{\min_D} = \sqrt{\frac{2W}{\rho S\sqrt{\frac{C_{D_o}}{K}}}} \tag{4.80}$$

Substituting equation 4.80 into equation 4.79 yields:

$$ROC_{\max} = \sqrt{\frac{2W}{\rho S\sqrt{\frac{C_{D_o}}{K}}}}\left[\frac{T_{\max}}{W} - \frac{1}{(L/D)_{\max}}\right] \tag{4.81}$$

This equation can be further manipulated to be in the form of thrust loading as a function of wing loading. Hence:

$$\left[\frac{T_{\max}}{W} - \frac{1}{\left(L/D\right)_{\max}}\right] = \frac{ROC_{\max}}{\sqrt{\frac{2W}{\rho S \sqrt{\frac{C_{D_o}}{K}}}}} \Rightarrow \frac{T_{\max}}{W} = \frac{ROC_{\max}}{\sqrt{\frac{2W}{\rho S \sqrt{\frac{C_{D_o}}{K}}}}} + \frac{1}{\left(L/D\right)_{\max}} \tag{4.82}$$

Thus:

$$\left(\frac{T}{W}\right)_{ROC} = \frac{ROC}{\sqrt{\frac{2}{\rho \sqrt{\frac{C_{D_o}}{K}}}\left(\frac{W}{S}\right)}} + \frac{1}{\left(L/D\right)_{\max}} \tag{4.83}$$

The wing and engine sizing based on rate of climb requirements is represented in equation 4.83 as the variations of thrust loading versus wing loading. Since, the fastest climb is obtained at sea level; where the engine thrust is at its maximum value; the air density must be considered at sea level.

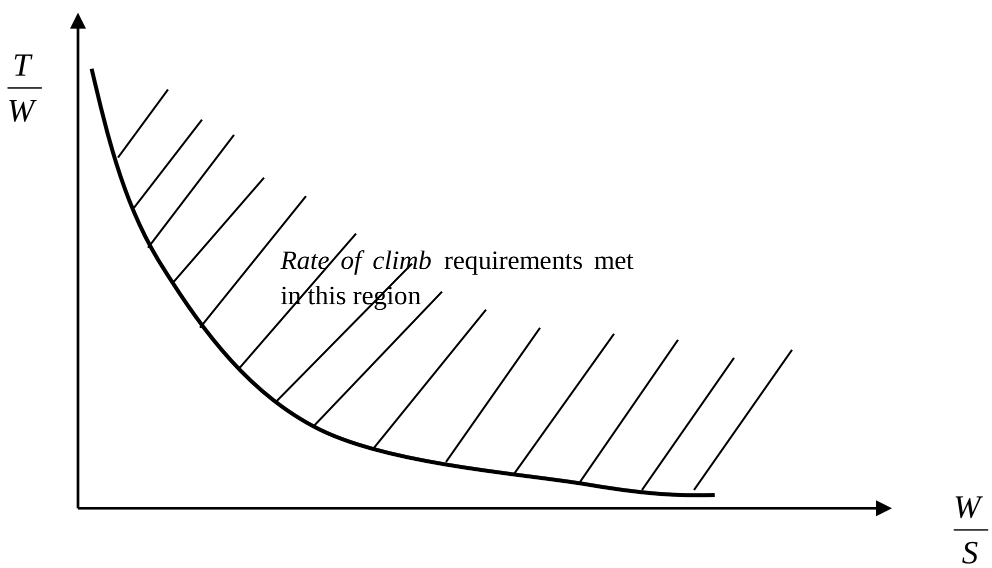

Figure 4.12. Rate of climb contribution in constructing matching plot for a jet aircraft

The variation of T/W as a function of W/S based on ROC for a jet aircraft can be sketched by using equation 4.83 in constructing the matching plot as shown in figure 4.12. In order to determine the acceptable region, we need to find what side of this graph is satisfying the take-off run requirement. Since the rate of clime (ROC) is in the denominator, as the rate of clime is in equation 4.83 is *increased*, the value of thrust loading (T/W) is going *up*. Since, any value of ROC greater than the specified ROC is satisfying the rate of climb requirement, so the region *above* the graph is acceptable. Employ extreme care to use a consistent unit when applying the equation 4.83 (either in SI system or British system). The typical value of maximum lift-to-drag ratio for several types of aircraft is given in Table 4.5.

Employ extreme caution to use a consistent unit when applying the equation 4.83 (either in SI system or British system). In SI system, the unit of ROC is m/sec, the unit of W is N, the unit of T is N, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of ROC is ft/sec, the unit of W is lb, the unit of T is lb, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. An example application is presented in Section 4.4.

4.3.5.2. Prop-driven Aircraft

Returning to the definition of rate of climb in Section 4.3.5.1, and noting that available power is the engine power times the prop efficiency, we have:

$$ROC = \frac{P_{avl} - P_{req}}{W} = \frac{\eta_P P - DV}{W} \tag{4.84}$$

where the speed to obtain maximum rate of climb for a prop-driven aircraft [5] is:

$$V_{ROC_{\max}} = \sqrt{\frac{2W}{\rho S \sqrt{\frac{3C_{D_o}}{K}}}} \tag{4.85}$$

By substituting equation 4.85 into equation 4.84, we obtain:

$$ROC_{\max} = \frac{\eta_P P_{\max}}{W} - \frac{D}{W} \sqrt{\frac{2W}{\rho S \sqrt{\frac{3C_{D_o}}{K}}}} \tag{4.86}$$

However, aircraft drag is a function of aircraft speed, wing area as follows:

$$D = \frac{1}{2} \rho V^2 S C_D \tag{4.36}$$

An expression for wing loading is obtained by inserting equation 4.36 into equation 4.86, as follows:

$$ROC_{\max} = \frac{\eta_P P_{\max}}{W} - \frac{\frac{1}{2} \rho V^2 S C_D}{W} \sqrt{\frac{2W}{\rho S \sqrt{\frac{3C_{D_o}}{K}}}} \tag{4.87}$$

This equation can be further simplified[8] as follows:

$$ROC_{\max} = \frac{\eta_P P_{\max}}{W} - \sqrt{\frac{2}{\rho\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}}\right) \tag{4.88}$$

This equation may be manipulated and inverted to obtain the power loading as follows:

$$\frac{P_{\max}}{W} = \frac{ROC_{\max}}{\eta_P} + \sqrt{\frac{2}{\rho\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right) \Rightarrow$$

$$\left(\frac{W}{P}\right)_{ROC} = \frac{1}{\frac{ROC}{\eta_P} + \sqrt{\frac{2}{\rho\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \tag{4.89}$$

where, the prop efficiency (η_P) in climbing flight is about 0.7. The typical value of maximum lift-to-drag ratio for several types of aircraft is given in Table 4.5.

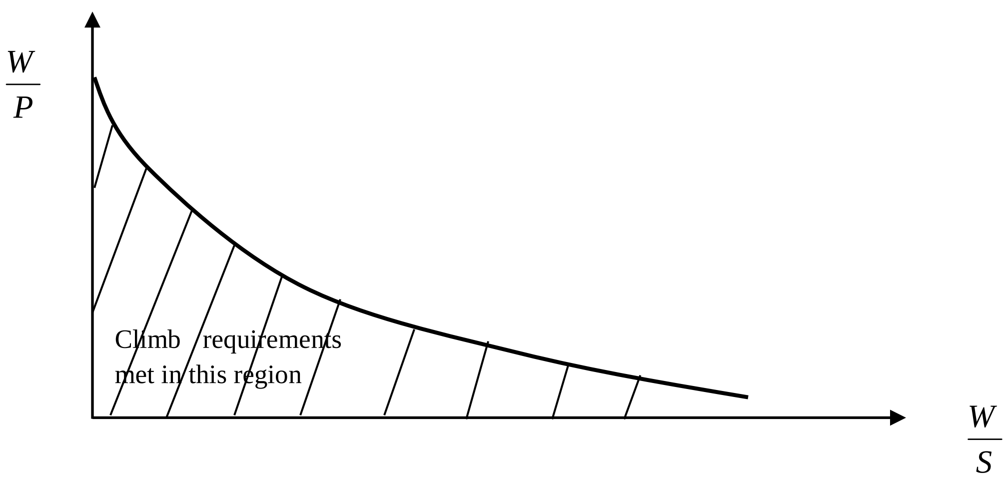

Figure 4.13. Rate of climb contribution in constructing matching plot for a prop-driven aircraft

The wing and engine sizing based on rate of climb requirements is represented in equation 4.89 as the variations of power loading versus wing loading. The prop efficiency in climbing flight is about 0.5 to 0.6. The variation of W/P as a function of W/S based on ROC for a prop-driven aircraft can be sketched by using equation 4.89 in constructing the matching plot

[8] The simplification is given in Ref. [7].

as shown in figure 4.13. In order to determine the acceptable region, we need to find what side of this graph is satisfying the climb requirements.

We note that the rate of climb (ROC) is in the denominator; hence, as the value of ROC in equation 4.89 is *increased*, the value of thrust loading (W/P) will *drop*. Since, any value of ROC greater than the specified rate of climb is satisfying the climb requirement, so the region *below* the graph is acceptable.

Employ extreme caution to use a consistent unit when applying the equation 4.89 (either in SI system or British system). In SI system, the unit of ROC is m/sec, the unit of W is N, the unit of P is Watt, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of ROC is ft/sec, the unit of W is lb, the unit of P is lb.ft/sec, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient. Recall that each horse power (hp) is equivalent to 550 lb.ft/sec. An example application is presented in Section 4.4.

4.3.6. Ceiling

Another performance requirement that influences the wing and engine sizing is the ceiling. Ceiling is defined as the highest altitude that an aircraft can safely have a straight level flight. Another definition is the highest altitude that an aircraft can reach by its own engine and have sustained flight. For many aircraft, ceiling is not a crucial requirement, but for others such as reconnaissance aircraft SR-71 Black Bird, ceiling of about 65,000 ft was the most difficult performance requirement to meet. This design requirement made the designers to design and invent a special engine for this mission. In general, there are four types of ceiling:

1. Absolute Ceiling (h_{ac}): As the name implies, absolute ceiling is absolute maximum altitude that an aircraft can ever maintain level flight. In another term, the ceiling is the altitude at which the rate of climb[9] is zero.
2. Service Ceiling (h_{sc}): Service ceiling is defined as the highest altitude at which the aircraft can climb with the rate of 100 ft per minute (i.e. 0.5 m/sec). Service ceiling is lower than absolute ceiling.
3. Cruise Ceiling (h_{cc}): Cruise ceiling is defined as the altitude at which the aircraft can climb with the rate of 300 ft per minute (i.e. 1.5 m/sec). Cruise ceiling is lower than service ceiling.
4. Combat Ceiling (h_{cc}): Combat ceiling is defined as the altitude at which a fighter can climb with the rate of 500 ft per minute (i.e. 5 m/sec). Combat ceiling is lower than cruise ceiling. This ceiling is defined only for fighter aircraft.

These four definitions are summarized as follows:

[9] Rate of climb is covered is Ref 7.

$$
\begin{aligned}
&ROC_{AC} = 0 \\
&ROC_{SC} = 100 \quad fpm \\
&ROC_{CrC} = 300 \quad fpm \\
&ROC_{CoC} = 500 \quad fpm
\end{aligned}
\tag{4.90}
$$

In this section, an expression for wing and engine sizing based on ceiling requirements are derived in two sections: 1. Jet aircraft, 2. Prop-driven aircraft. Since the ceiling requirements are defined based on the rate of climb requirements, the equations developed in the Section 4.3.5 are employed.

4.3.6.1. Jet Aircraft

An expression for the thrust loading (T/W); as a function of wing loading (W/S) and rate of climb; was derived in equation 4.83. It can also be applied to ceiling altitude as follows:

$$
\left(\frac{T_C}{W}\right) = \frac{ROC_C}{\sqrt{\dfrac{2}{\rho_C \sqrt{\dfrac{C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}} + \frac{1}{(L/D)_{\max}} \tag{4.91}
$$

where ROC_C is the rate of climb at ceiling, and T_C is the engine maximum thrust at ceiling. On the other hand, the engine thrust is a function of altitude, or air density. The exact relationship depends upon the engine type, engine technology, engine installation, and airspeed. At this moment of design phase, which the aircraft is not completely designed, the following approximate relationship (as introduced in Section 4.3) is utilized:

$$
T_C = T_{SL}\left(\frac{\rho_C}{\rho_o}\right) = T_{SL}\sigma_C \tag{4.92}
$$

Inserting this equation into equation 4.91 yields the following:

$$
\left(\frac{T_{SL}\sigma_C}{W}\right) = \frac{ROC_C}{\sqrt{\dfrac{2}{\rho_C \sqrt{\dfrac{C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}} + \frac{1}{(L/D)_{\max}} \tag{4.93}
$$

By modeling the atmosphere, one can derive an expression for the relative air density (σ) as a function of altitude (h). The followings are reproduced from [11].

$$
\sigma = \left(1 - 6.873 \times 10^{-6} h\right)^{4.26} \qquad \text{(from 0 to 36,000 ft)} \tag{4.94a}
$$

$$\sigma = 0.2967 \exp\left(1.7355 - 4.8075 \times 10^{-5} h\right) \qquad \text{(from 36,000 to 65,000 ft)} \qquad (4.94b)$$

This pair of equations is in British units, i.e. the unit of h is ft. For the atmospheric model in SI units, refer to the references such as [5] and [7]. Appendices A and B illustrate the pressure, temperature, and air density at various altitudes in SI and British units respectively. By moving σ in equation 4.93 to the right hand side, the following is obtained:

$$\left(\frac{T}{W}\right)_{h_C} = \frac{ROC_C}{\sigma_C \sqrt{\dfrac{2}{\rho_C \sqrt{\dfrac{C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}} + \frac{1}{\sigma_C (L/D)_{\max}} \qquad (4.95)$$

Since at the absolute ceiling (h_{AC}), the rate of climb is zero ($ROC_{AC} = 0$), the corresponding expression for the thrust loading will be obtained by eliminating the first term of the equation 4.95:

$$\left(\frac{T}{W}\right)_{h_{AC}} = \frac{1}{\sigma_{AC} (L/D)_{\max}} \qquad (4.96)$$

where σ_C is the relative air density at the required ceiling, σ_{AC} is the relative air density at the required absolute ceiling, and ROC_C is the rate of climb at the required ceiling (h_C). The wing and engine sizing based on ceiling requirements (h_C or h_{AC}) are represented in equations 4.95 and 4.96 as relative air density (σ and σ_{AC}) and can be obtained by equation 4.94. The rate of climb at different ceilings is defined at the beginning of this section (equation 4.90). The typical value of maximum lift-to-drag ratio for several types of aircraft is given in Table 4.5.

Equation 4.95 represents the contribution of cruise, service, or combat ceiling (h_C) to size engine and wing. However, equation 4.96 represents the contribution of absolute ceiling (h_{AC}) to size engine and wing. Equations 4.95 and 4.96 represent the nonlinear variations of thrust loading versus wing loading as a function of ceiling. The variations of T/W as a function of W/S based on h_C or h_{AC} for a jet aircraft can be sketched by using equations 4.95 and 4.96 in constructing the matching plot as shown in figure 4.14.

In order to determine the acceptable region, we need to find out what side of this graph is satisfying the ceiling run requirement. Equation 4.95 has two positive terms; one includes ROC_C and σ_C, while another one includes only σ_C. The ceiling rate of climb (ROC_C) is in the numerator of the first term, and in the denominator of both terms; so, as the rate of clime is in equation 4.95 is *decreased*, the value of thrust loading (T/W) *drops*. Since, any value of ROC greater than the specified ROC_C, or any altitude higher than the required ceiling is satisfying the ceiling requirement, so the region *above* the graph is acceptable.

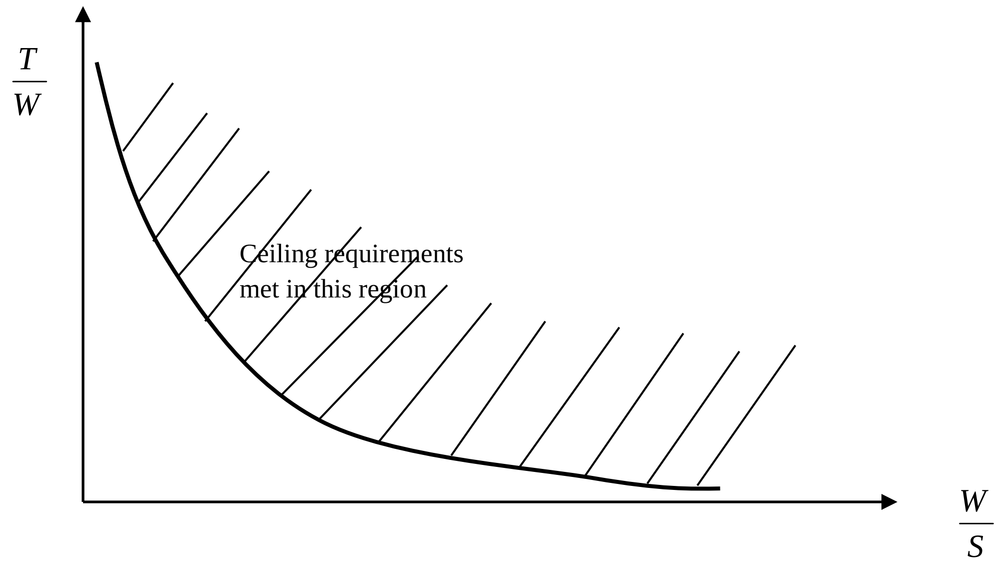

Figure 4.14. Ceiling contribution in constructing matching plot for a jet aircraft

Employ extreme care to use a consistent unit when applying the equation 4.95 and 4.96 (either in SI system or British system). In SI system, the unit of ROC is m/sec, the unit of W is N, the unit of T is N, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of ROC is ft/sec, the unit of W is lb, the unit of T is lb, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. An example application is presented in Section 4.4.

4.3.6.2. Prop-driven Aircraft

An expression for the power loading (W/P); as a function of wing loading (W/S) and rate of climb; was derived in equation 4.89. It can also be applied to ceiling altitude as follows:

$$\left(\frac{W}{P_C}\right) = \frac{1}{\dfrac{ROC_C}{\eta_P} + \sqrt{\dfrac{2}{\rho_C\sqrt{\dfrac{3C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}\left(\dfrac{1.155}{(L/D)_{\max}\eta_P}\right)} \tag{4.97}$$

where ROC_C is the rate of climb at ceiling, ρ_C is the air density at the ceiling, and P_C is the engine maximum thrust at ceiling. On the other hand, the engine power is a function of altitude, or air density. The exact relationship depends upon the engine type, engine technology, engine installation, and airspeed. At this moment of design phase, which the aircraft is not completely designed, the following approximate relationship (as introduced in Section 4.3) is utilized:

$$P_C = P_{SL}\left(\frac{\rho_C}{\rho_o}\right) = P_{SL}\sigma_C \tag{4.98}$$

Inserting this equation into equation 4.97 yields the following:

$$\left(\frac{W}{P_{SL}\sigma_C}\right)=\frac{1}{\frac{ROC_C}{\eta_P}+\sqrt{\frac{2}{\rho_C\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \tag{4.99}$$

By moving σ_C in equation 4.99 to right hand side, the following is obtained:

$$\left(\frac{W}{P_{SL}}\right)_C=\frac{\sigma_C}{\frac{ROC_C}{\eta_P}+\sqrt{\frac{2}{\rho_C\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \tag{4.100}$$

Since at the absolute ceiling (h_{AC}), the rate of climb is zero (ROC_{AC} = 0), the corresponding expression for the power loading will be obtained by eliminating the first term of the denominator of the equation 4.100:

$$\left(\frac{W}{P_{SL}}\right)_{AC}=\frac{\sigma_{AC}}{\sqrt{\frac{2}{\rho_{AC}\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \tag{4.101}$$

where σ_C is the relative air density at the required ceiling, σ_{AC} is the relative air density at the required absolute ceiling, and ROC_C is the rate of climb at the required ceiling (h_C). The wing and engine sizing based on ceiling requirements (h_C or h_{AC}) are represented in equations 4.100 and 4.101. The relative air density (σ and σ_{AC}); as a function of ceiling; can be obtained through equation 4.94. The rate of climb at different types of ceilings is defined at the beginning of this section (equation 4.90). The typical value of maximum lift-to-drag ratio for several types of aircraft is given in Table 4.5.

The variation of W/P as a function of W/S based on h_C or h_{AC} for a prop-driven aircraft can be sketched by using equation 4.100 or 4.101 in constructing the matching plot as shown in figure 4.15. In order to determine the acceptable region, we need to find what side of this graph is satisfying the climb requirements.

Equation 4.101 has σ_C in the numerator, while it has ρ_C in the numerator of the numerator. As the altitude is increased, the air density (ρ) and relative air density (σ) are decreased. Hence, by increasing the altitude, the magnitude of the right hand side in equation 4.101 is *decreased*, and the value of thrust loading (T/W) *drops*. Since, any value of h greater than the specified h_C, or any altitude higher than the required ceiling is satisfying the ceiling requirement, so the region *below* the graph is acceptable.

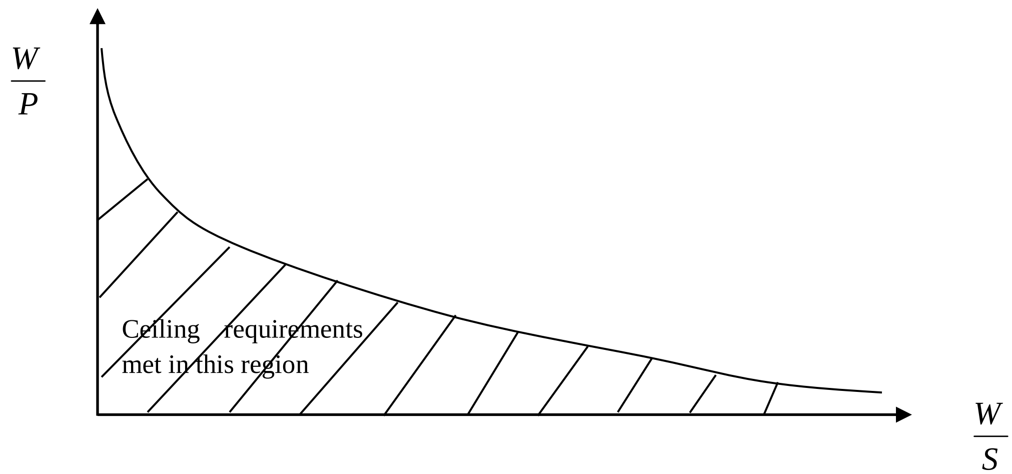

Figure 4.15. Ceiling contribution in constructing matching plot for a prop-driven aircraft

Employ extreme caution to use a consistent unit when applying the equation 4.100 and 4.101 (either in SI system or British system). In SI units system, the unit of ROC is m/sec, the unit of W is N, the unit of P is Watt, the unit of S is m^2, and the unit of ρ is kg/m^3. However, In British system, the unit of ROC is ft/sec, the unit of W is lb, the unit of P is lb.ft/sec or hp, the unit of S is ft^2, and the unit of ρ is $slug/ft^3$. If the British units are used, convert the unit of W/P to lb/hp to make the comparison more convenient. Recall that each horse power (hp) is equivalent to 550 lb.ft/sec. An example application is presented in Section 4.4.

4.4. Design Examples

In this section, two fully solved design examples are provided; one to estimate maximum take-off weight (W_{TO}), and another one to determine wing reference area (S) and engine power (P).

Example 4.3. Maximum Take-Off Weight

Problem statement: You are to design a conventional civil transport aircraft that can carry 700 passengers plus their luggage. The aircraft must be able to fly with a cruise speed of Mach 0.8, and have a range of 9500 km. At this point, you are only required to estimate the aircraft maximum take-off weight. You need to follow FAA regulations and standards. Assume that the aircraft equipped with two high bypass ratio turbofan engines and is cruising at 35,000 ft altitude.

Solution:

Hint: Since FAR values are in British unit, we convert all units to British unit.

Step 1. The aircraft is a stated to be civil transport and to carry 700 passengers. Hence, the aircraft must follow FAR Part 25. Therefore all selections must be based on Federal Aviation Regulations. The regular mission profile for this aircraft consists of taxi and take-off, climb, cruise, descent, loiter, and landing (see Figure 4.16).

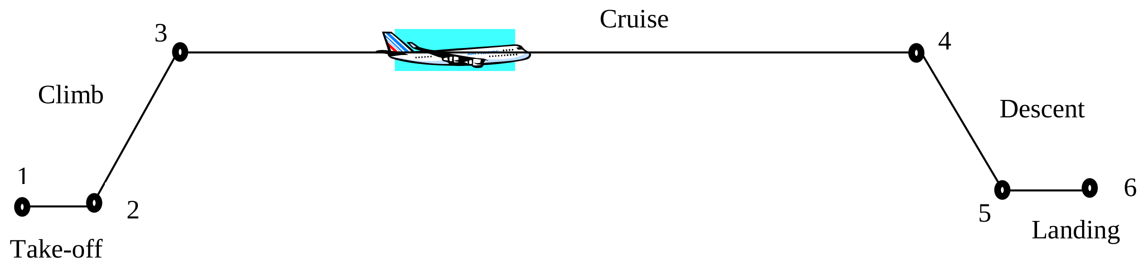

Figure 4.16. Mission profile for the transport aircraft in Example 4.3

Step 2. Flight crew

The aircraft is under commercial flight operations, so it would be operating under Parts 119 and 125. Flight attendant's weight is designated in 119.3. In subpart I of part 125, there are Pilot-in-command and second-in-command qualifications. There may be space on the aircraft for more crew members, but based on the language of the document, two flight crew members is the minimum allowed. Also, the criteria for determining minimum flight crew could be found from Appendix D of FAR part 25. In order to have flight crew to perform the basic workload functions (which listed in Appendix D of FAR part 25 and in section 119.3) safely and comfortably, we designate **two crew** members as one pilot and one copilot.

Step 3. Flight attendants

The number of flight attendants is regulated by FAR Part 125, Section 125.269:
For airplanes having more than 100 passengers -- two flight attendants plus one additional flight attendant for each unit (or part of a unit) of 50 passengers above 100 passengers.

Since there are 700 passengers, number of flight attendants must be 14.

700 = 100 + (12 × 50) => 2 + (12 × 1) =14

Step 4. Weight of flight crew and attendants:

As defined in section 125.9 Definitions, flight crew members are assumed to have a weight of 200 lbs. On the other hand, flight attendant's weight is designated in 119.3 and requires that 140 lb be allocated for a flight attendant whose sex in unknown. Thus, the total weight of flight crew members and flight attendants is:

200 + 200 + (14 × 140) => W_C = 2,360 lb

Step 5. The weight of payloads

The payload for a passenger aircraft primarily includes passengers and their luggage and baggage. In reality, passengers could be a combination of adult males, adult females, children, and infants. Table 4.1 shows the nominal weight for each category. To observe the reality and to be on the safe side, an average weight of 180 lb id selected. This weight includes the allowance for personal items and carry-on bags. On the other hand, 100 lbs of luggage is considered for each passenger. So the total payload would be:

(700 × 180) + (700 × 100) => W_{PL} = 196,000 lb

Step 6. Fuel weight ratios for the segments of taxi, take-off, climb, descent, approach and landing

Using Table 4.3 and the numbering system shown in Figure 4.2, we will have the following fuel weight ratios:

Taxi, take-off: $\frac{W_2}{W_1} = 0.98$

Climb: $\frac{W_3}{W_2} = 0.97$

Descent: $\frac{W_5}{W_4} = 0.99$

Approach and landing: $\frac{W_6}{W_5} = 0.997$

Step 7. Fuel weight ratio for the segment of range

The aircraft has jet (turbofan) engine, so equation 4.16 must be employed. In this flight mission, cruise is the third phase of flight.

$$\frac{W_4}{W_3} = e^{\frac{-R \cdot C}{0.866 V (L/D)_{\max}}} \tag{4.16}$$

where range (R) is 9500 km, C is 0.4 lb/hr/lb (from Table 4.6) or 0.4/3600 1/sec, and $(L/D)_{max}$ is 17 (chosen from Table 4.5). The aircraft speed (V) would be the Mach number times the speed of sound [5].

$$V = M \cdot a = 0.8 \times 296.6 = 237.3 \ \frac{m}{\sec} = 778.5 \ \frac{ft}{\sec} \tag{4.65}$$

where the speed of sound at 35,000 ft altitude is 296.6 m/sec. Thus,

$$\frac{W_4}{W_3} = e^{\frac{-R \cdot C}{0.866 V (L/D)_{\max}}} = e^{\frac{-9,500,000 \times 3.28 \times \frac{0.4}{3600}}{0.866 \times 778.5 \times 17}} = e^{-0.302} \Rightarrow \frac{W_4}{W_3} = 0.739 \tag{4.16}$$

Step 8. Overall fuel weight ratio

By using equations similar to equations 4.10 and 4.11, we obtain:

$$\frac{W_6}{W_1} = \frac{W_2}{W_1}\frac{W_3}{W_2}\frac{W_4}{W_3}\frac{W_5}{W_4}\frac{W_6}{W_5} = 0.98 \times 0.97 \times 0.739 \times 0.99 \times 0.997 \Rightarrow \frac{W_6}{W_1} = 0.694 \tag{4.10}$$

$$\frac{W_f}{W_{TO}} = 1.05\left(1 - \frac{W_6}{W_1}\right) = 1.05(1 - 0.694) \Rightarrow \frac{W_f}{W_{TO}} = 0.322 \tag{4.11}$$

Step 9. Substitution

The known values are substituted into the equation 4.5.

$$W_{TO} = \frac{W_{PL} + W_C}{1 - \left(\frac{W_f}{W_{TO}}\right) - \left(\frac{W_E}{W_{TO}}\right)} = \frac{196,000 + 2,360}{1 - 0.322 - \left(\frac{W_E}{W_{TO}}\right)} = \frac{198,360}{0.678 - \left(\frac{W_E}{W_{TO}}\right)} \tag{4.5}$$

Step 10. Empty weight ratio

The empty weight ratio is established the by using equation 4.26, where the coefficients “a” and “b” are taken from Table 4.8.

$a = -7.754 \times 10^{-8}$, b = 0.576 (Table 4.8)

Thus:

$$\frac{W_E}{W_{TO}} = aW_{TO} + b \Rightarrow \frac{W_E}{W_{TO}} = -7.754 \times 10^{-8} W_{TO} + 0.576 \tag{4.26}$$

Step 11. Final step

The following two equations (one from step 9 and one from step 10) must be solved simultaneously.

$$W_{TO} = \frac{198{,}360}{0.678 - \left(\frac{W_E}{W_{TO}}\right)}$$ (Equation 1) (Step 9)

$$\frac{W_E}{W_{TO}} = -7.754 \times 10^{-8} W_{TO} + 0.576$$ (Equation 2) (Step 10)

MathCad software may be used to solve this set of two nonlinear algebraic equations as follows:

assumption: $x := 0.6$ $y := 100000$

Given

$$y = \frac{198360}{0.678 - x} \qquad x = -7.75410^{-8} \cdot y + 0.576$$

$$\mathrm{Find}(x, y) = \begin{pmatrix} 0.493 \\ 1071658.013 \end{pmatrix}$$

Thus, the empty weight ratio is 0.493 and the maximum take-off weight is:

W_{TO} = 1,071,658 lb = 4,766,972 N

So, the maximum take-off mass is:

m_{TO} = 486,095 kg

An alternative way to find the W_{TO} is the trial and error technique, as shown in Table 4.16. It is observed that after seven trials, the error reduces to only 0.4% which is acceptable. This technique produces a similar result (**W_{TO} = 1,074,201**). Due to the rounding, this technique does not yield as accurate solution as the previous technique.

The third alternative way is to solve the equations analytically. We first manipulate Equation 1 as follows:

$$W_{TO} = \frac{198{,}360}{0.677 - \left(\frac{W_E}{W_{TO}}\right)} \Rightarrow 0.677 - \left(\frac{W_E}{W_{TO}}\right) = \frac{198{,}360}{W_{TO}} \Rightarrow \left(\frac{W_E}{W_{TO}}\right) = 0.677 - \frac{198{,}360}{W_{TO}}$$

Then, we need to substitute the right-hand-side it into Equation 2, and simplify:

$$0.677-\frac{198{,}360}{W_{TO}}=-7.754\times10^{-8}W_{TO}+0.576\Rightarrow 7.754\times10^{-8}W_{TO}+0.576-0.677+\frac{198{,}360}{W_{TO}}=0$$

$$\Rightarrow -7.754\times10^{-8}W_{TO}+\frac{198{,}360}{W_{TO}}-0.101=0$$

Iteration	**Step 1**	**Step 2**	**Step 3**	Error (%)
	Guess W_{TO} (lb)	Substitute W_{TO} of Step 1 into the first equation: $\frac{W_E}{W_{TO}}=-7.754\times10^{-8}W_{TO}+0.576$	Substitute W_E/W_{TO} of Step 2 into the second equation: $W_{TO}=\frac{198{,}360}{0.677-\left(\frac{W_E}{W_{TO}}\right)}$	
1	1,500,000	0.4597	912,797 lb	39.1
2	912,797	0.505	1,154,744 lb	-26.5
3	1,154,744	0.486	1,041,047 lb	9.8
4	1,041,047	0.495	1,091,552 lb	-4.8
5	1,091,552	0.491	1,068,525 lb	2.1
6	1,068,525	0.493	1,078,902 lb	-0.96
7	1,078,902	0.4923	**1,074,201 lb**	0.4

Table 4.16. Trial and error technique to determine maximum take-off weight of the aircraft in Example 4.3

This nonlinear algebraic equation has one unknown (W_{TO}); and only one acceptable (reasonable) solution. This alternative technique is also producing a similar result. For comparison, it is interesting to note that the maximum take-off weight of the giant transport aircraft Airbus 380 with 853 passengers is 1,300,700 lb. Thus, the aircraft maximum aircraft weight would be:

$$W_{TO}=1{,}071{,}658 \quad lb\Rightarrow m_{TO}=486{,}095 \quad kg$$

Example 4.4. Wing and Engine Sizing

Problem Statement: In the preliminary design phase of a turboprop transport aircraft, the maximum take-off weight is determined to be 20,000 lb and the aircraft C_{Do} is determined to be 0.025. The hob airport is located at a city with the elevation of 3,000 ft. By using the matching plot technique, determine wing area (S) and engine power (P) of the aircraft that is required to have the following performance capabilities:

a. Maximum speed: 350 KTAS at 30,000 ft
b. Stall speed: less than 70 KEAS
c. Rate of climb: more than 2700 fpm at sea level
d. Take-off run: less than 1200 ft (on a dry concrete runway)

e. Service ceiling: more than 35,000 ft
f. Range: 4,000 nm
g. Endurance: 2 hours

Assume any other parameters that you may need for this aircraft.

Solution:

First, it must be noted that range and endurance requirements do not have any effect on the engine power or wing area, so we ignore them at this design phase. The air density at 3,000 ft is 0.002175 slug/ft^3 and at 30,000 ft is 0.00089 slug/ft^3.

The matching plot is constructed by deriving five equations:

1. **Stall speed**

The stall speed is required to be greater than 70 KEAS. The wing sizing based on stall speed requirements is represented by equation 4.31. From Table 4.11, the aircraft maximum lift coefficient is selected to be 2.7.

$$\left(\frac{W}{S}\right)_{V_s} = \frac{1}{2}\rho V_s^{\,2} C_{L_{\max}} = \frac{1}{2}\times 0.002378\times(70\times 1.688)^2\times 2.7 = 44.8 \ \frac{lb}{ft^2} \qquad \text{(4.31) or } \textbf{(E-1)}$$

where 1 knot is equivalent to 1.688 ft/sec.

2. **Maximum speed**

The maximum speed is required to be greater than 350 KTAS at 30,000 ft. The wing and engine sizing based on maximum speed requirements for a prop-driven aircraft are represented by equation 4.41.

$$\left(\frac{W}{P_{SL}}\right)_{V_{\max}} = \frac{\eta_P}{\frac{1}{2}\rho_o V_{\max}^3 C_{D_o}\frac{1}{\left(\frac{W}{S}\right)} + \frac{2K}{\rho\sigma V_{\max}}\left(\frac{W}{S}\right)} \qquad (4.56)$$

From Table 5.8 (Chapter 5), the wing aspect ratio (AR) is selected to be 12. From Section 4.3.3, the Oswald span efficiency factor is considered to be 0.85. Thus:

$$K = \frac{1}{\pi\cdot e\cdot AR} = \frac{1}{3.14\times 0.85\times 12} = 0.031 \qquad (4.41)$$

The air relative density (σ) at 30,000 ft is 0.00089/0.002378 or 0.374. The substitution yields:

$$\left(\frac{W}{P_{SL}}\right)_{V_{\max}} = \frac{0.7\times 550}{0.5\times 0.002378\times(350\times 1.688)^3\times 0.025\frac{1}{\left(\frac{W}{S}\right)}+\frac{2\times 0.031}{0.00089\times 0.374\times(350\times 1.688)}\left(\frac{W}{S}\right)}$$

or

$$\left(\frac{W}{P_{SL}}\right)_{V_{\max}} = \frac{385}{6129.7\frac{1}{\left(\frac{W}{S}\right)}+0.317\left(\frac{W}{S}\right)} \quad \left(\frac{lb}{hp}\right) \tag{E-2}$$

where the whole term is multiplied by 550 to convert lb/(lb.ft/sec) to lb/hp, and the prop efficiency is assumed to be 0.7.

3. Take-off run

The take-off run is required to be greater than 1,200 ft at the elevation of 3,000 ft. The wing and engine sizing based on take-off run requirements for a prop-driven aircraft are represented by equation 4.76. Recall that the air density at 3,000 ft is 0.002175 slug/ft^3.

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{1-\exp\left(0.6\rho g C_{D_G} S_{TO}\frac{1}{W/S}\right)}{\mu-\left(\mu+\frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO}\frac{1}{W/S}\right)\right]}\frac{\eta_P}{V_{TO}} \tag{4.76}$$

where based on Table 4.15, μ is 0.04. The take-off speed is assumed to be:

$$V_{TO} = 1.1V_s = 1.1\times 70 = 77 \quad KEAS \tag{4.72}$$

Take-off lift and drag coefficients are:

$$C_{L_{TO}} = C_{L_C} + \Delta C_{L_{flapTO}} \tag{4.69c}$$

where the aircraft lift coefficient C_{L_C} is assumed to be 0.3 and $\Delta C_{L_{flapTO}}$ to be 0.6. Thus:

$$C_{L_{TO}} = 0.3+0.6 = 0.9 \tag{4.69c}$$

$$\begin{aligned} C_{D_{o_{LG}}} &= 0.009 \\ C_{D_{o_{HLD\_TO}}} &= 0.005 \end{aligned} \tag{4.69a}$$

$$C_{D_{oTO}} = C_{D_o} + C_{D_{oLG}} + C_{D_{oHLD\_TO}} = 0.025 + 0.009 + 0.005 = 0.039 \tag{4.69}$$

$$C_{D_{TO}} = C_{D_{oTO}} + KC_{L\,TO}^{2} = 0.039 + 0.031(0.9)^2 = 0.064 \tag{4.68}$$

The take-off rotation lift coefficients is:

$$C_{L_R} = \frac{C_{L_{\max}}}{(1.1)^2} = \frac{C_{L_{\max}}}{1.21} = \frac{2.7}{1.21} = 2.231 \tag{4.69b}$$

The variable C_{D_G} is:

$$C_{D_G} = \left(C_{D_{TO}} - \mu C_{L_{TO}}\right) = 0.064 - 0.04 \times 0.9 = 0.028 \tag{4.67}$$

It is assumed that the propeller is of variable pitch type, so based on equation 4.73b, the prop efficiency is 0.6. The substitution yields:

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)}{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]} \frac{\eta_P}{V_{TO}} \tag{4.76}$$

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{\left[1 - \exp\left(0.6 \times 0.002175 \times 32.2 \times 0.028 \times 1{,}200 \frac{1}{(W/S)}\right)\right]}{0.04 - \left(0.04 + \frac{0.028}{2.231}\right)\left[\exp\left(0.6 \times 0.002175 \times 32.2 \times 0.028 \times 1{,}200 \frac{1}{(W/S)}\right)\right]} \left(\frac{0.6}{77 \times 1.688}\right) \times 550$$

or

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{\left[1 - \exp\left(\frac{1.426}{(W/S)}\right)\right]}{0.04 - (0.053)\left[\exp\left(\frac{1.426}{(W/S)}\right)\right]} (0.0046 \times 550) \qquad \frac{lb}{hp} \qquad \textbf{(E-3)}$$

Again, the whole term is multiplied by 550 to convert lb/(lb.ft/sec) to lb/hp.

4. Rate of climb

The rate of climb run is required to be greater than 2,700 fpm (or 45 ft/sec) at sea level. The wing and engine sizing based on rate of climb requirements for a prop-driven aircraft are represented by equation 4.89. Based on Table 4.5, the maximum lift-to-drag ratio is selected to be 18.

$$\left(\frac{W}{P}\right)_{ROC} = \frac{1}{\frac{ROC}{\eta_P} + \sqrt{\frac{2}{\rho\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \quad (4.89)$$

The substitution yields:

$$\left(\frac{W}{P}\right)_{ROC} = \frac{1\times 550}{\frac{2{,}700}{60\times 0.7} + \sqrt{\frac{2}{0.002378\sqrt{\frac{3\times 0.025}{0.031}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{18\times 0.7}\right)} \quad (4.89)$$

$$\left(\frac{W}{P}\right)_{ROC} = \frac{1\times 550}{64.3+\left(\sqrt{540.7\left(\frac{W}{S}\right)}\right)(0.092)} \quad \textbf{(E-4)}$$

And again, the whole term is multiplied by 550 to convert lb/(lb.ft/sec) to lb/hp.

5. Service ceiling

The service ceiling is required to be greater than 35,000 ft. The wing and engine sizing based on service ceiling requirements for a prop-driven aircraft are represented by equation 4.100. At service ceiling, the rate of climb is required to be 100 ft/min (or 1.667 ft/sec). At 35,000 ft altitude, the air density is 0.000738 slug/ft^3 (Appendix B); so the relative air density is 0.31. The substitution yields:

$$\left(\frac{W}{P_{SL}}\right)_{C} = \frac{\sigma_C}{\frac{ROC_C}{\eta_P} + \sqrt{\frac{2}{\rho_C\sqrt{\frac{3C_{D_o}}{K}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{(L/D)_{\max}\eta_P}\right)} \quad (4.100)$$

$$\left(\frac{W}{P}\right)_{C} = \frac{0.31\times 550}{\frac{100}{60\times 0.7} + \sqrt{\frac{2}{0.000738\sqrt{\frac{3\times 0.025}{0.031}}}\left(\frac{W}{S}\right)}\left(\frac{1.155}{18\times 0.7}\right)} \quad (4.100)$$

or

$$\left(\frac{W}{P}\right)_C = \frac{170.5}{2.38 + \left(\sqrt{1742.3\left(\frac{W}{S}\right)}\right)(0.092)} \qquad \textbf{(E-5)}$$

And again, the whole term is multiplied by 550 to convert lb/(lb.ft/sec) to lb/hp.

1. Construction of matching plot

Now, we have five equations of E-1, E-2, E-3, E-4, and E-5. In all of them, power loading is defined as functions of wing loading. When we plot all of them in one graph, the figure 4.17 will be produced. Recall in this example, that the unit of W/S in lb/ft^2, and the unit of W/P is lb/hp.

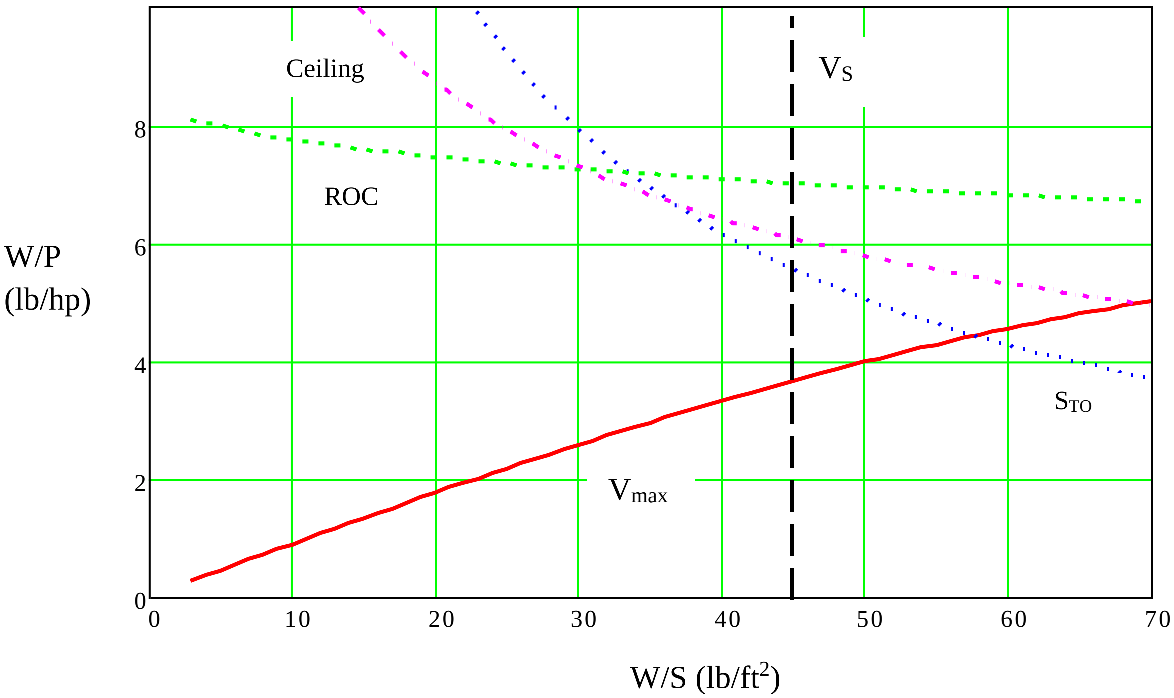

Figure 4.17. Matching plot for example problem 4.4

Now, we need to recognize the acceptable regions. As we discussed in Section 4.3, the region below each graph is satisfying the performance requirements. In another word, the region above each graph is not satisfying the performance requirements. For the case of stall speed, the region in the left side of the graph is satisfying stall speed requirements (see figure 4.18). Hence, the region between the graphs of maximum speed, take-off run and stall speed is the target area.

In this region, we are looking for the smallest engine (lowest power) that has the lowest operating cost. Thus the highest point (figure 4.18) of this region is the design point. Therefore the wing loading and power loading will be extracted from figure 4.18 as:

$$\left(\frac{W}{P}\right)_d = 3.64$$

$$\left(\frac{W}{S}\right)_d = 44.8$$

Then, the wing area and engine power will be calculated as follows:

$$S = W_{TO} \Big/ \left(\frac{W}{S}\right)_d = \frac{20{,}000 lb}{44.8 \frac{lb}{ft^2}} = 446.4 \quad ft^2 = 41.47 \quad m^2 \tag{4.27}$$

$$P = W_{TO} \Big/ \left(\frac{W}{P}\right)_d = \frac{20{,}000 lb}{3.64 \frac{lb}{hp}} = 5{,}495.5 \quad hp = 4{,}097.2 \quad kW \tag{4.28}$$

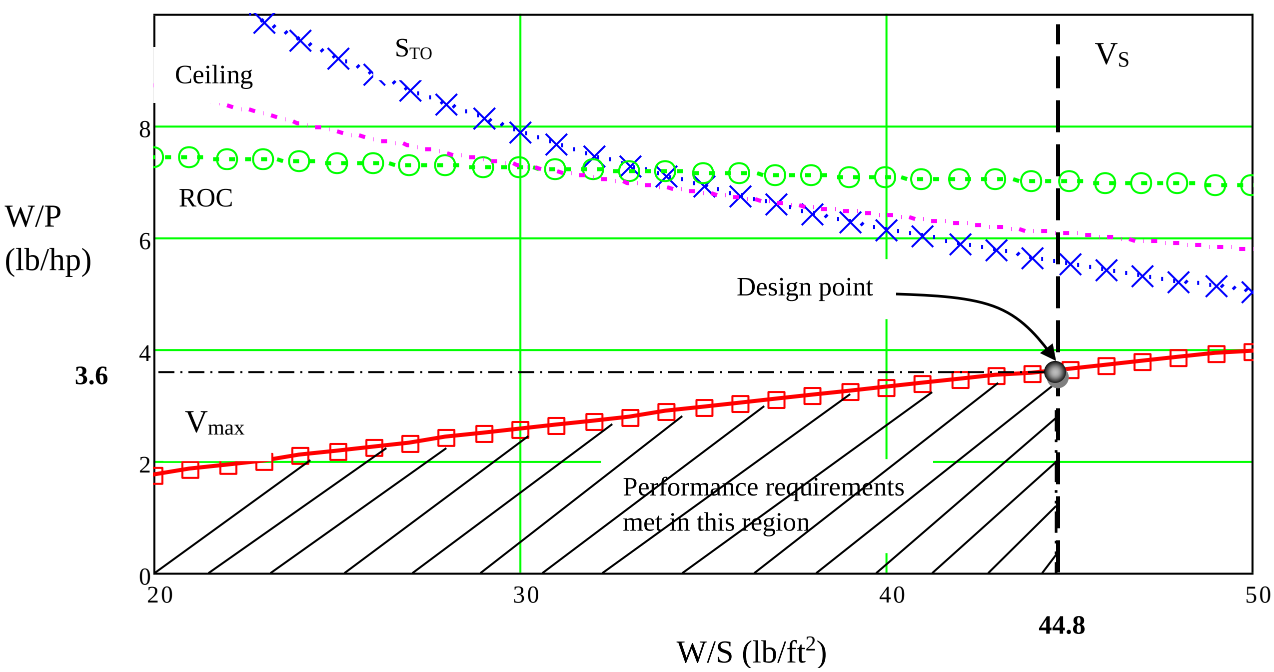

Figure 4.18. Acceptable regions in the matching plot for example problem 4.4.

Therefore, the wing area and engine power will be:

S = 446.4 ft^2, P = 5,495.5 hp

1. Harrier GR9

2. Antonov An-124

Figure 4.19. British Aerospace Harrier GR9 with a thrust-to-weight ratio of 1.13, and Antonov An-124 with a thrust-to-weight ratio of 0.231 (Courtesy of Antony Osborne)

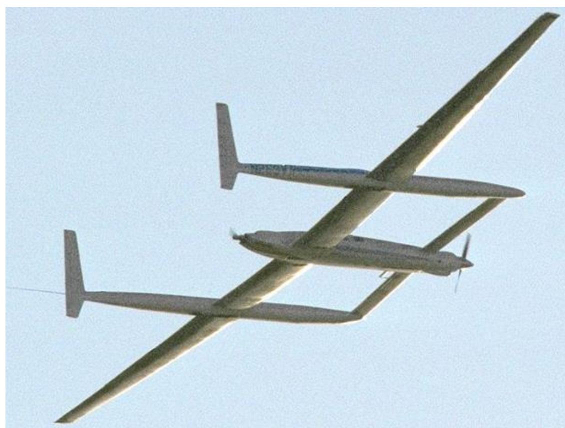

Figure 4.20. Voyager aircraft with an empty-weight-to-take-off-weight ratio of 0.23 (Courtesy of NASA)

Figure 4.21. F-16 falcon with an empty-weight-to-take-off-weight ratio of 0.69 (Courtesy of Antony Osborne)

Problems

1. Determine the zero-lift drag coefficient (C_{Do}) of the two-seat ultra-light aircraft Scheibe SF 40 which is flying with a maximum cruising speed of 81 knot at sea level. This aircraft has one piston engine and the following characteristics:

$$P_{SLmax} = 44.7 \text{ kW}, m_{TO} = 400 \text{ lb}, S = 13.4 \text{ m}^2, AR = 8.7, e = 0.88, \eta_P = 0.75$$

2. Determine the zero-lift drag coefficient (C_{Do}) of the fighter aircraft F-16C Falcon which is flying with a maximum speed of Mach 2.2 at 40,000 ft. This fighter has a turbofan engine and the following characteristics:

$$T_{SLmax} = 29{,}588 \text{ lb}, W_{TO} = 27{,}185 \text{ lb}, S = 300 \text{ ft}^2, AR = 3.2, e = 0.76$$

3. Determine the zero-lift drag coefficient (C_{Do}) of the jet fighter aircraft F-15 Eagle which is flying with a maximum speed of Mach 2.5 at 35,000 ft. This fighter has two turbofan engines and the following characteristics:

$$T_{SLmax} = 2 \times 23{,}450 \text{ lb}, W_{TO} = 81{,}000 \text{ lb}, S = 608 \text{ ft}^2, AR = 3, e = 0.78$$

4. Determine the zero-lift drag coefficient (C_{Do}) of the transport aircraft Boeing 747-400 which is flying with a maximum speed of Mach 0.92 at 35,000 ft. This aircraft has four turbofan engines and the following characteristics:

$$T_{SLmax} = 4 \times 56{,}750, W_{TO} = 800{,}000 \text{ lb}, S = 5{,}825 \text{ ft}^2, AR = 10.2, e = 0.85$$

5. Determine the zero-lift drag coefficient (C_{Do}) of the fighter aircraft Eurofighter which is flying with a maximum speed of Mach 2 at 35,000 ft. This fighter has two turbofan engines and the following characteristics:

$$T_{SLmax} = 2 \times 16{,}000, W_{TO} = 46{,}297 \text{ lb}, S = 538 \text{ ft}^2, AR = 2.2, e = 0.75$$

6. Determine the zero-lift drag coefficient (C_{Do}) of the bomber aircraft B-2 Spirit which is flying with a maximum speed of Mach 0.95 at 20,000 ft. This aircraft has four turbofan engines and the following characteristics:

$$T_{SLmax} = 4 \times 17{,}300 \text{ lb}, W_{TO} = 336{,}500 \text{ lb}, S = 5{,}000 \text{ ft}^2, AR = 6.7, e = 0.73$$

7. Determine the zero-lift drag coefficient (C_{Do}) of the military transport aircraft C-130 Hercules which is flying with a maximum speed of 315 knot at 23,000 ft. This aircraft has four turboprop engines and the following characteristics:

$$P_{SLmax} = 4 \times 4508 \text{ hp}, W_{TO} = 155{,}000 \text{ lb}, S = 1{,}754 \text{ ft}^2, AR = 10.1, e = 0.92, \eta_P = 0.81$$

8. Determine the zero-lift drag coefficient (C_{Do}) of the transport aircraft Piaggio P180 Avanti which is flying with a maximum speed of 395 knot at 20,000 ft. This aircraft has two turboprop engines and the following characteristics:

$$P_{SLmax} = 2 \times 800 \text{ hp}, W_{TO} = 10{,}510 \text{ lb}, S = 172.2 \text{ ft}^2, AR = 12.1, e = 0.88, \eta_P = 0.84$$

9. Determine the zero-lift drag coefficient (C_{Do}) of the small Utility aircraft Beech Bonanza which is flying with a maximum speed of 166 knot at sea level. This aircraft has one piston engine and the following characteristics:

$$P_{SLmax} = 285 \text{ hp}, W_{TO} = 2{,}725 \text{ lb}, S = 178 \text{ ft}^2, AR = 6, e = 0.87, \eta_P = 0.76$$

10. Determine the zero-lift drag coefficient (C_{Do}) of the multi-mission aircraft Cessna 208 Caravan which is flying with a maximum cruising speed of 184 knot at 10,000 ft. This aircraft has one turboprop engine and the following characteristics:

$$P_{SLmax} = 505 \text{ kW}, m_{TO} = 3{,}970 \text{ kg}, S = 26 \text{ m}^2, AR = 9.7, e = 0.91, \eta_P = 0.75$$

11. You are a member of a team that is designing a GA aircraft which is required to have 4 seats and the following performance features:

1. Max speed: at least 150 knots at sea level
2. Max range: at least 700 km
3. Max rate of climb: at least 1,800 fpm
4. Absolute ceiling: at least 25,000 ft
5. Take-off run: less than 1,200 ft

At the preliminary design phase, you are required to estimate the zero-lift drag coefficient (C_{Do}) of such aircraft. Identify five current similar aircraft and based on their statistics, estimate the C_{Do} of the aircraft being designed.

12. You are a member of a team that is designing a business jet aircraft which is required to carry 12 passengers and the following performance features:

1. Max speed: at least 280 knots at sea level
2. Max range: at least 1,000 km
3. Max rate of climb: at least 3,000 fpm
4. Absolute ceiling: at least 35,000 ft
5. Take-off run: less than 2,000 ft

At the preliminary design phase, you are required to estimate the zero-lift drag coefficient (C_{Do}) of such aircraft. Identify five current similar aircraft and based on their statistics, estimate the C_{Do} of the aircraft being designed.

13. You are a member of a team that is designing a fighter aircraft which is required to carry two pilots and the following performance features:

1. Max speed: at least Mach 1.8 at 30,000 ft
2. Max range: at least 1,500 km
3. Max rate of climb: at least 10,000 fpm
4. Absolute ceiling: at least 45,000 ft
5. Take-off run: less than 2,800 ft

At the preliminary design phase, you are required to estimate the zero-lift drag coefficient (C_{Do}) of such aircraft. Identify five current similar aircraft and based on their statistics, estimate the C_{Do} of the aircraft being designed.

14. You are involved in the design a civil transport aircraft which can carry 200 passengers plus their luggage. The aircraft must be able to fly with a cruise speed of Mach 0.8, and have a range of 10,000 km. At this point, you are only required to estimate the aircraft maximum take-off weight. You need to follow FAA regulations and standards. Assume that the aircraft equipped with two high bypass ratio turbofan engines and is required to cruise at 37,000 ft altitude.

15. You are to design a surveillance/observation aircraft which can carry four crew members. The aircraft must be able to fly with a cruise speed of Mach 0.3, and have a range of 2,000 km and an endurance of 15 hours. At this point, you are only required to estimate the aircraft maximum take-off weight. Assume that the aircraft equipped with two turboprop engines and is required to cruise at 8,000 m altitude.

16. You are involved in the design a jet trainer aircraft with that can carry one pilot and one student. The aircraft must be able to fly with a cruise speed of Mach 0.4, and have a range of 1,500 km. At this point, you are only required to estimate the aircraft maximum take-off weight. Assume that the aircraft equipped with one turboprop engine and is required to cruise at 20,000 ft altitude.

17. In the preliminary design phase of a GA (normal) aircraft, the maximum take-off weight is determined to be 2,000 lb and the aircraft C_{Do} is determined to be 0.027 and the engine is selected to be one piston-prop. By using the matching plot technique, determine wing area (S) and engine power (P) of the aircraft that is required to have the following performance capabilities:

a. Maximum speed: 180 KTAS at 20,000 ft
b. Stall speed: less than 50 KEAS
c. Rate of climb: more than 1,200 fpm at sea level
d. Take-off run: less than 800 ft (on a dry concrete runway)
e. Service ceiling: more than 25,000 ft
f. Range: 1,000 nm
g. Endurance: 1 hours

Assume any other parameters that you may need for this aircraft.

18. At the preliminary design phase of a jet transport aircraft, the maximum take-off weight is determined to be 120,000 lb and the aircraft C_{Do} is determined to be 0.022. The hob airport is located at a city with the elevation of 5,000 ft. By using the matching plot technique, determine wing area (S) and engine thrust (T) of the aircraft that is required to have the following performance capabilities:

a. Maximum speed: 370 KTAS at 27,000 ft
b. Stall speed: less than 90 KEAS
c. Rate of climb: more than 3,200 fpm at sea level
d. Take-off run: less than 3,000 ft (on a dry concrete runway)
e. Service ceiling: more than 40,000 ft
f. Range: 8,000 nm
g. Endurance: 5 hours

Assume any other parameters that you may need for this aircraft.

19. In the preliminary design phase of a fighter aircraft, the maximum take-off mass is determined to be 12,000 kg and the aircraft C_{Do} is determined to be 0.028. By using the matching plot technique, determine wing area (S) and engine thrust (T) of the aircraft that is required to have the following performance capabilities:

a. Maximum speed: Mach 1.9 at 10,000 m
b. Stall speed: less than 50 m/sec
c. Rate of climb: more than 50 m/sec at sea level
d. Take-off run: less than 1,000 m (on a dry concrete runway)
e. Service ceiling: more than 15,000 m
f. Radius of action: 4,000 km

Assume any other parameters that you may need for this aircraft.

20. At the preliminary design phase of a twin-turboprop regional transport aircraft, the maximum take-off mass is determined to be 16,000 kg and the aircraft C_{Do} is determined to be 0.019. By using the matching plot technique, determine wing area (S) and engine power (P) of the aircraft that is required to have the following performance capabilities:

a. Maximum speed: Mach 0.6 at 2,500 m
b. Stall speed: less than 190 km/hr
c. Rate of climb: more than 640 m/min at sea level
d. Take-off run: less than 1,100 m (on a dry concrete runway)
e. Service ceiling: more than 9,000 m
f. Range: 7,000 km

Assume any other parameters that you may need for this aircraft.

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