Завдання 1

$R = 5$ см

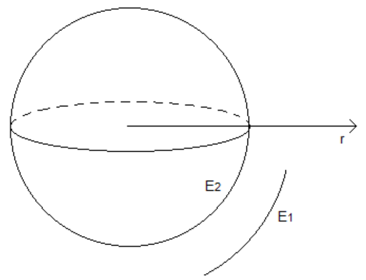
$\rho(r) = \rho_0 \dfrac{r}{R}$

$\rho_0 = 50 \dfrac{\text{нКл}}{\text{м}^3}$

$\varphi(r = \infty) = 0$

$E(r) - ?$

$\varphi(r) - ?$



$$\oint \vec{E} d\vec{S} = \frac{1}{\varepsilon_0} q$$

$$\oint \vec{E} d\vec{S} = \int E dS \cos 0 = E \int dS = ES$$

$$q = \int \rho dV$$

$$dV = S dr = 4\pi r^2 dr$$

$$q = \int_V \rho_0 \frac{r}{R} dV = \int_0^r \rho_0 \frac{r}{R} 4\pi r^2 dr = \frac{4\pi\rho_0}{R} \int_0^r r^3 dr = \frac{\pi r^4 \rho_0}{R}\bigg|_0^r = \frac{\pi\rho_0}{R} r^4$$

$$E_2 4\pi r^2 = \frac{\pi r^4 \rho_0}{\varepsilon_0 R} \Rightarrow E_2 = \frac{\rho_0}{4\varepsilon_0 R} r^2; \; E_1 4\pi r^2 = \frac{\pi R^3 \rho_0}{\varepsilon_0} \Rightarrow E_1 = \frac{R^3 \rho_0}{4\varepsilon_0 r^2}$$

$$E(r) = \begin{cases} E_1 = \dfrac{R^3\rho_0}{4\varepsilon_0 r^2}, & r \geq R \\ E_2 = \dfrac{\rho_0}{4\varepsilon_0 R} r^2, & r < R \end{cases} \Rightarrow E(r) = \begin{cases} E_1 = \dfrac{0.177}{r^2} \text{ (В/м)}, & r \geq R \\ E_2 = 28248.6\, r^2 \text{ (В/м)}, & r < R \end{cases}$$

$$\vec{E} = -\operatorname{grad} \varphi \Rightarrow d\vec{r}\vec{E} = -d\varphi \Rightarrow drE = -d\varphi \Rightarrow \int_1^2 drE = \int_1^2 -d\varphi \Rightarrow \int_1^2 drE = -(\varphi_2 - \varphi_1)$$

$$1)\ \varphi_{\text{зовні}} = \int_r^\infty E_1 dr = \int_r^\infty \frac{R^3\rho_0}{4\varepsilon_0 r^2} dr = \frac{R^3\rho_0}{4\varepsilon_0 r} = \frac{0.177}{r} \text{ (В)}$$

$$2)\ \varphi_{\text{вс}} = \int_R^\infty E_1 dr + \int_r^R E_2 dr = \int_R^\infty \frac{R^3\rho_0}{4\varepsilon_0 r^2} dr + \int_r^R \frac{\rho_0}{4\varepsilon_0 R} r^2 dr = \frac{R^2\rho_0}{4\varepsilon_0} + \frac{\rho_0(R^3 - r^3)}{12\varepsilon_0 R} = 4.7 - 9416.2 r^3 \text{ (В)}$$

$$\varphi = \begin{cases} \dfrac{0.177}{r} \text{ (В)}, & r \geq R \\ 4.7 - 9416.2 r^3 \text{ (В)}, & r < R \end{cases}$$

Завдання 3

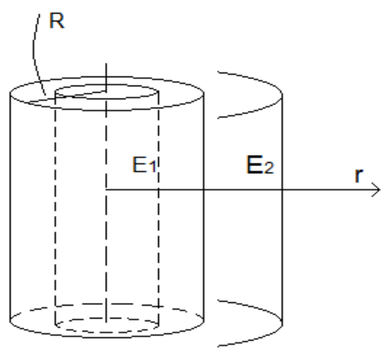
$R = 10$ см

$\rho(r) = \rho_0 \dfrac{r}{R}$

$\rho_0 = 10 \dfrac{\text{нКл}}{\text{м}^3}$

$E(r) - ?$

$\Delta\varphi - ?$



$$\oint \vec{E} d\vec{S} = \frac{1}{\varepsilon_0} q$$

$$\oint \vec{E} d\vec{S} = \int E dS \cos 0 = E \int dS = ES$$

$$q = \int \rho dV$$

$$dV = S dr = 2\pi r h dr$$

$$q = \int_V \rho_0 \frac{r}{R} dV = \int_0^r \rho_0 \frac{r}{R} 2\pi r h dr = \frac{2\pi h \rho_0}{3R} r^3$$

$$E_1 2\pi r h = \frac{2\pi h r^3 \rho_0}{3\varepsilon_0 R} \Rightarrow E_1 = \frac{\rho_0}{3\varepsilon_0 R} r^2; \; E_2 2\pi h r = \frac{2\pi h \rho_0 R^3}{3R\varepsilon_0} \Rightarrow E_2 = \frac{R^2 \rho_0}{3\varepsilon_0 r}$$

$$E(r) = \begin{cases} E_1 = \dfrac{\rho_0}{3\varepsilon_0 R} r^2, & r < R \\ E_2 = \dfrac{R^2\rho_0}{3\varepsilon_0 r}, & r \geq R \end{cases} \Rightarrow E(r) = \begin{cases} E_1 = 3766.5\, r^2 \text{(В/м)}, & r < R \\ E_2 = \dfrac{3.767}{r} \text{ (В/м)}, & r \geq R \end{cases}$$

$$\vec{E} = -\operatorname{grad} \varphi \Rightarrow d\vec{r}\vec{E} = -d\varphi \Rightarrow drE = -d\varphi \Rightarrow \int_1^2 drE = \int_1^2 -d\varphi \Rightarrow \int_1^2 drE = -(\varphi_2 - \varphi_1)$$

$$\Delta\varphi = \int_0^R E_1 dr = \int_0^R \frac{\rho_0}{3\varepsilon_0 R} r^2 dr = \frac{\rho_0 R^2}{9\varepsilon_0} = 1.255 \text{ (В)}$$

Завдання 2

$R = 10$ см

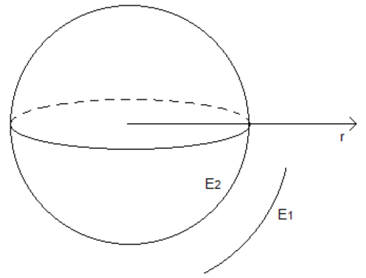
$\rho(r) = \rho_0 \dfrac{r^2}{R^2}$

$\rho_0 = 500 \dfrac{\text{нКл}}{\text{м}^3}$

$\varphi(r=\infty) = 0$

$E(r)-?$

$\varphi(r)-?$



$$\oint \vec{E}\vec{dS} = \frac{1}{\varepsilon_0} q$$

$$\oint \vec{E}\vec{dS} = \int EdS\cos 0 = E\int dS = ES$$

$$q = \int \rho dV$$

$$dV = Sdr = 4\pi r^2 dr$$

$$q = \int_V \rho_0 \frac{r^2}{R^2} dV = \int_0^r \rho_0 \frac{r^2}{R^2} 4\pi r^2 dr = \frac{4\pi\rho_0}{R^2}\int_0^r r^4 dr = \frac{4\pi r^5 \rho_0}{5R^2}\bigg|_0^r = \frac{4\pi\rho_0}{5R^2} r^5$$

$$E_2 4\pi r^2 = \frac{4\pi r^5 \rho_0}{5\varepsilon_0 R^2} \Rightarrow E_2 = \frac{\rho_0}{5\varepsilon_0 R^2} r^3;\; E_1 4\pi r^2 = \frac{4\pi R^3 \rho_0}{5\varepsilon_0} \Rightarrow E_1 = \frac{R^3\rho_0}{5\varepsilon_0 r^2}$$

$$E(r) = \begin{cases} E_1 = \dfrac{R^3\rho_0}{5\varepsilon_0 r^2}, & r \geq R \\ E_2 = \dfrac{\rho_0}{5\varepsilon_0 R^2} r^3, & r < R \end{cases} \Rightarrow E(r) = \begin{cases} E_1 = \dfrac{11.3}{r^2}\ (\text{В/м}), & r \geq R \\ E_2 = 1129943.5\, r^2\ (\text{В/м}), & r < R \end{cases}$$

$$\vec{E} = -\operatorname{grad}\varphi \Rightarrow d\vec{r}\vec{E} = -d\varphi \Rightarrow drE = -d\varphi \Rightarrow \int_1^2 drE = \int_1^2 -d\varphi \Rightarrow \int_1^2 drE = -(\varphi_2 - \varphi_1)$$

$$1)\ \varphi_{\text{зовні}} = \int_r^\infty E_1 dr = \int_r^\infty \frac{R^3\rho_0}{5\varepsilon_0 r^2} dr = \frac{R^3\rho_0}{5\varepsilon_0 r} = \frac{11.3}{r}\ (\text{В})$$

$$2)\ \varphi_{\text{вс}} = \int_R^\infty E_1 dr + \int_r^R E_2 dr = \int_R^\infty \frac{R^3\rho_0}{5\varepsilon_0 r^2} dr + \int_r^R \frac{\rho_0}{5\varepsilon_0 R^2} r^3 dr = \frac{R^2\rho_0}{5\varepsilon_0} + \frac{\rho_0(R^4 - r^4)}{20\varepsilon_0 R} = 115.8 - 28248.6r^3\ (\text{В})$$

$$\varphi = \begin{cases} \dfrac{11.3}{r}\ (\text{В}), & r \geq R \\ 115.8 - 28248.6r^3\ (\text{В}), & r < R \end{cases}$$

Завдання 4

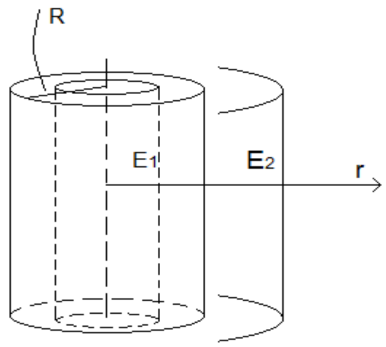
$R = 100$ см

$\rho(r) = \rho_0 \dfrac{r^2}{R^2}$

$\rho_0 = 100 \dfrac{\text{нКл}}{\text{м}^3}$

$E(r)-?$

$\Delta\varphi-?$



$$\oint \vec{E}d\vec{S} = \frac{1}{\varepsilon_0} q$$

$$\oint \vec{E}d\vec{S} = \int EdS\cos 0 = E\int dS = ES$$

$$q = \int \rho dV$$

$$dV = Sdr = 2\pi rhdr$$

$$q = \int_V \rho_0 \frac{r^2}{R^2} dV = \int_0^r \rho_0 \frac{r^2}{R^2} 2\pi rhdr = \frac{\pi h\rho_0}{2R^2} r^4$$

$$E_1 2\pi rh = \frac{\pi h r^4 \rho_0}{2\varepsilon_0 R^2} \Rightarrow E_1 = \frac{\rho_0}{4\varepsilon_0 R^2} r^3;\; E_2 2\pi hr = \frac{\pi h\rho_0 R^4}{2R^2\varepsilon_0} \Rightarrow E_2 = \frac{R^2\rho_0}{4\varepsilon_0 r}$$

$$E(r) = \begin{cases} E_1 = \dfrac{\rho_0}{4\varepsilon_0 R^2} r^3, & r < R \\ E_2 = \dfrac{R^2\rho_0}{4\varepsilon_0 r}, & r \geq R \end{cases} \Rightarrow E(r) = \begin{cases} E_1 = 2824.9\, r^3 (\text{В/м}), & r < R \\ E_2 = \dfrac{2824.9}{r}\ (\text{В/м}), & r \geq R \end{cases}$$

$$\vec{E} = -\operatorname{grad}\varphi \Rightarrow d\vec{r}\vec{E} = -d\varphi \Rightarrow drE = -d\varphi \Rightarrow \int_1^2 drE = \int_1^2 -d\varphi \Rightarrow \int_1^2 drE = -(\varphi_2 - \varphi_1)$$

$$\Delta\varphi = \int_0^R E_1 dr = \int_0^R \frac{\rho_0}{4\varepsilon_0 R^2} r^3 dr = \frac{\rho_0 R^2}{16\varepsilon_0} = 706.2\ (\text{В})$$