Wein-bridge oscillator

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For the Wein-bridge oscillator of Fig 0, use the expression for loop gain in (3.5) to find the poles of the closed-loop system. Give the expression for the pole, Q and use it to show that to locate the poles in the right half of s plane, $\frac{R_2}{R_1}$ must be selected to be greater than 2.

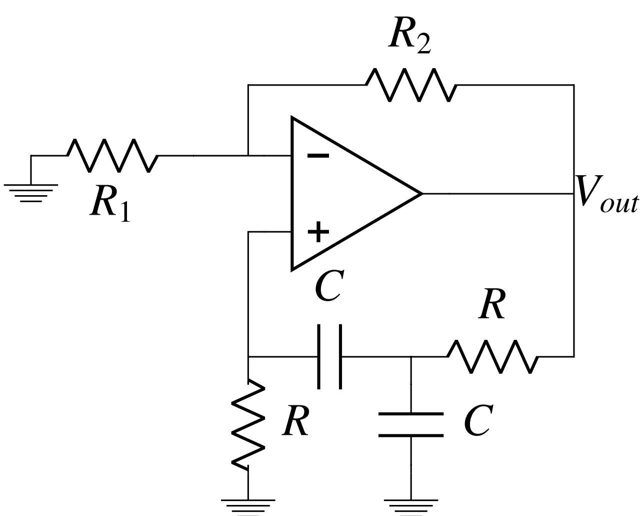

Fig. 0

1.
2. Compare the basic structure for a sinusoidal oscillator with Wein-bridge oscillator and give expressions for G and H.
   Solution:
   - Comparring Fig 0 and Fig 2, we get

$$G = 1 + \frac{R_2}{R_1} \tag{2.1}$$

$$H = \frac{Z_p}{Z_p + Z_s} \tag{2.2}$$

where,

$$Z_p = \frac{R}{RSC + 1} \tag{2.3}$$

$$Z_s = \frac{RSC + 1}{SC} \tag{2.4}$$

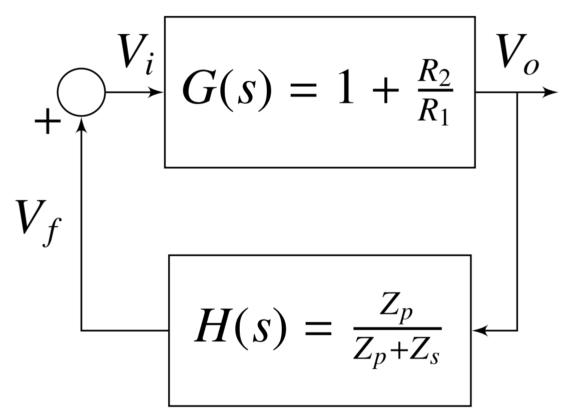

Fig. 2

3. Give the expression for loop gain for Wein-bridge oscillator.
   Solution:

$$T(s) = A(s)\beta(s) \tag{3.1}$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + Z_s Y_p} \tag{3.2}$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + (\frac{sRC+1}{sC})(\frac{sRC+1}{R})} \tag{3.3}$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{s^2R^2C^2+sRC+sRC+1}{sRC}} \tag{3.4}$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} \tag{3.5}$$

4. Write the characteristic equation for Wein-bridge oscillator.

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Solution:

$$1 - T(s) = 0 \tag{4.1}$$

$$1 - \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} = 0 \tag{4.2}$$

$$3 + sRC + \frac{1}{sCR} = 1 + \frac{R_2}{R_1} \tag{4.3}$$

$$3 - 1 + sRC + \frac{1}{sRC} - \frac{R_2}{R_1} = 0 \tag{4.4}$$

$$2s + s^2RC + \frac{1}{RC} - s\frac{R_2}{R_1} = 0 \tag{4.5}$$

$$s^2RC + s(2 - \frac{R_2}{R_1}) + \frac{1}{RC} = 0 \tag{4.6}$$

$$s^2 + s\frac{1}{RC}(2 - \frac{R_2}{R_1}) + \frac{1}{R^2C^2} = 0 \tag{4.7}$$

5. Write the general expression for the characteristic equation.
Solution:

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \tag{5.1}$$

6. State the **Barkhausen criterion** for sustained oscillations with frequency ω_0.
Solution:

$$T(j\omega_0) = G(j\omega_0)H(j\omega_0) = 1 \tag{6.1}$$

- That is, at ω_0 the phase of the loop gain should be zero and the magnitude of loop gain should be 1.
- Only for a ∞ gain,system will produce a finite output for zero input.

7. Give the definition of **Quality factor**(Q) and explain its significance.
Solution:
- It is a parameter of an oscillatory system expressing the relationship between stored energy and energy dissipation.
- The "purity" of output sine waves will be a function of the selectivity feedback network.
- That is, higher the value of Q for frequency selective network, the less the harmonic content of sine wave produced.

8. Compare the equations 4.7 and 5.1 and give expressions for Q and ω_0
Solution:

$$\omega_0^2 = \frac{1}{R^2C^2} \tag{8.1}$$

$$\omega_0 = \frac{1}{RC} \tag{8.2}$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}(2 - \frac{R_2}{R_1}) \tag{8.3}$$

$$Q = \frac{1}{(2 - \frac{R_2}{R_1})} \tag{8.4}$$

$$\tag{8.5}$$

9. Using Eq 8.4 calculate the value of $\frac{R_2}{R_1}$ for which poles lie on right hand of s-plane.
Solution:
Poles lie on imaginary axis for $Q = \infty$

$$2 - \frac{R_2}{R_1} = 0 \tag{9.1}$$

$$\frac{R_2}{R_1} = 2 \tag{9.2}$$

$\therefore$ For poles to lie on right hand side of s-plane

$$\frac{R_2}{R_1} > 2 \tag{9.3}$$

10. Verify the above calculations using a Python code.
Solution:

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codes/ee18btech11044_3_1.py
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- This figure shows how the location of poles vary if $\frac{R_2}{R_1}$ is varied for a fixed ω_0.
- I have varied $\frac{R_2}{R_1}$ from -10 to 10.

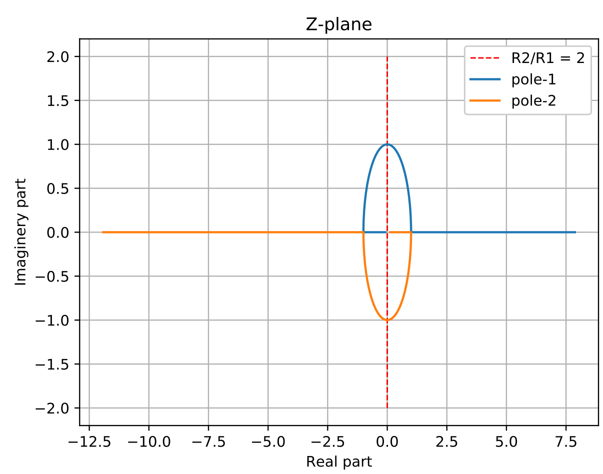

Fig. 10

11. Simulate the circuit shown in Fig 0 using spice simulators. Plot the output generated using python.
Solution:
You can find the netlist for the simulated circuit here:

spice/ee18btech11044/ee18btech11044.net

You can find the python script used to generate the output here:

spice/ee18btech11044/ee18btech11044_spice.py

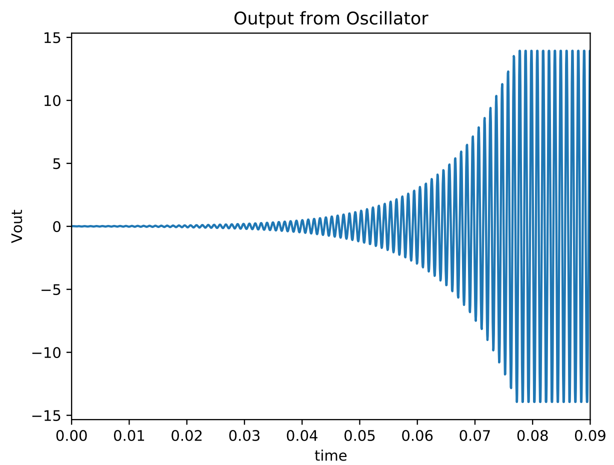

Fig. 11

12. Tabulate the values of Resistors and Capacitors you have chosen for the simulation.
Solution:

Parameter	Value
R_1	$10k\Omega$
R_2	$20.3k\Omega$
R_p	$10k\Omega$
R_s	$10k\Omega$
C_s	$16nF$
R_p	$10k\Omega$
C_p	$16nF$

TABLE 12

Where, according to Fig 0

$$R_p = R_s = R \tag{12.1}$$
$$C_p = C_s = C \tag{12.2}$$

13. Calculate the frequency of sinusoidal generated for the combination of R and C chosen using Eq 8.2
Solution:
Frequency generated is given by

$$\omega_0 = \frac{1}{RC} \tag{13.1}$$
$$\omega_0 = 6250rad/sec \tag{13.2}$$
$$f_0 = 995.22Hz. \tag{13.3}$$

14. Calculate the frequency of sinusoidal wave using plot generated from simulation.
Solution:
- Consider a part of plot generated from simulation shown in the Fig 14.
- Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 14.

$$T_0 = 0.0856452 - 0.0846361 \tag{14.1}$$
$$f_0 = 1/T_0 \tag{14.2}$$
$$f_0 = 990.98Hz. \tag{14.3}$$

- We get the frequencies calculated from the formulae and the plot to be approximately same.
- Use this script to generate Fig 14

spice/ee18btech11044/ee18btech11044_spice_2.py

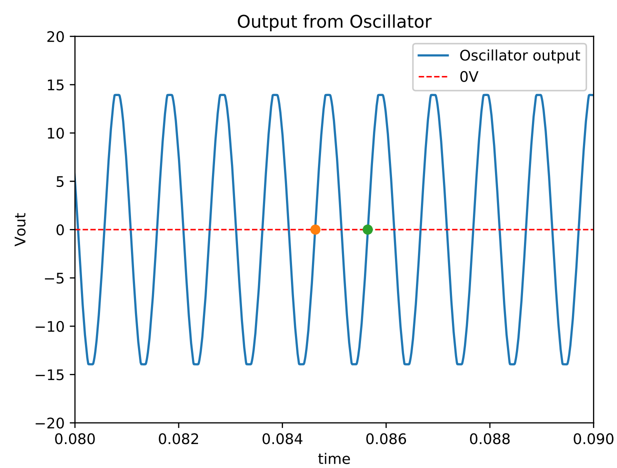

Fig. 14