Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

1 Feedback Circuits

1.0.1. For the Wein-bridge oscillator of Fig 1.0.1, use the expression for loop gain in Eq 1.0.3.5 to find the poles of the closed-loop system. Give the expression for the pole , Q and use it to show that to locate the poles in the right half of s plane, $\frac{R_2}{R_1}$ must be selected to be greater than 2.

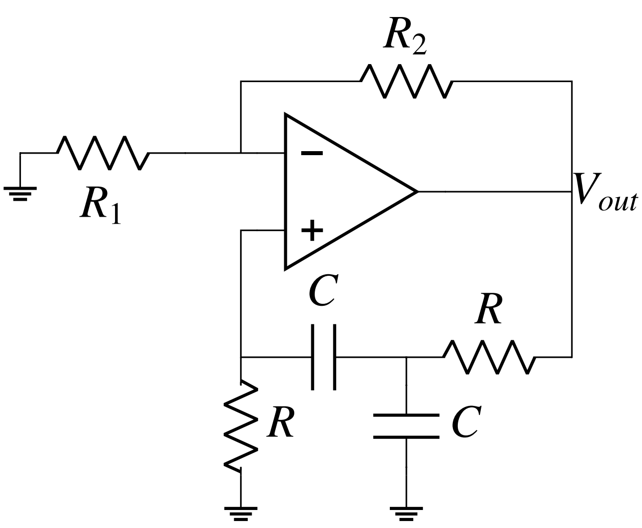

Fig. 1.0.1

1.0.2. Compare the basic structure for a sinusoidal oscillator with Wein-bridge oscillator and give expressions for A and β.
Solution:

- Comparring Fig 1.0.1 and Fig 1.0.2, we get

$$A = 1 + \frac{R_2}{R_1} \tag{1.0.2.1}$$

$$\beta = \frac{Z_p}{Z_p + Z_s} \tag{1.0.2.2}$$

where,

$$Z_p = \frac{R}{RSC + 1} \tag{1.0.2.3}$$

$$Z_s = \frac{RSC + 1}{SC} \tag{1.0.2.4}$$

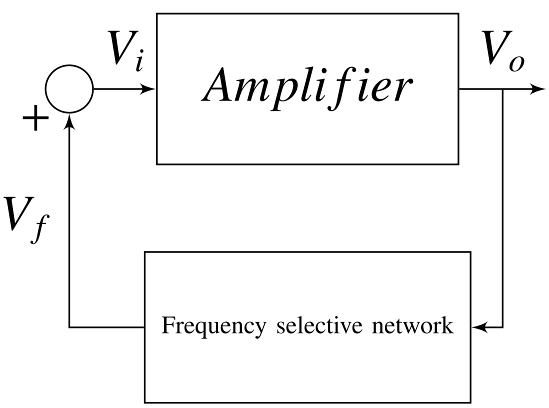

Fig. 1.0.2

1.0.3. Give the expression for loop gain for Wein-bridge oscillator.
Solution:

$$L(s) = A(s)\beta(s) \tag{1.0.3.1}$$

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{1 + Z_s Y_p} \tag{1.0.3.2}$$

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{1 + (\frac{sRC+1}{sC})(\frac{sRC+1}{R})} \tag{1.0.3.3}$$

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{s^2R^2C^2+sRC+sRC+1}{sRC}} \tag{1.0.3.4}$$

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} \tag{1.0.3.5}$$

1.0.4. Write the characteristic equation for Wein-

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bridge oscillator.
Solution:

$$1 - L(s) = 0 \tag{1.0.4.1}$$

$$1 - \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} = 0 \tag{1.0.4.2}$$

$$3 + sRC + \frac{1}{sCR} = 1 + \frac{R_2}{R_1} \tag{1.0.4.3}$$

$$3 - 1 + sRC + \frac{1}{sRC} - \frac{R_2}{R_1} = 0 \tag{1.0.4.4}$$

$$2s + s^2RC + \frac{1}{RC} - s\frac{R_2}{R_1} = 0 \tag{1.0.4.5}$$

$$s^2RC + s(2 - \frac{R_2}{R_1}) + \frac{1}{RC} = 0 \tag{1.0.4.6}$$

$$s^2 + s\frac{1}{RC}(2 - \frac{R_2}{R_1}) + \frac{1}{R^2C^2} = 0 \tag{1.0.4.7}$$

1.0.5. Write the general expression for the characteristic equation.
Solution:

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \tag{1.0.5.1}$$

1.0.6. State the **Barkhausen criterion** for sustained oscillations with frequency ω_0.
Solution:

$$L(j\omega_0) = A(j\omega_0)\beta(j\omega_0) = 1 \tag{1.0.6.1}$$

- That is, at ω_0 the phase of the loop gain should be zero and the magnitude of loop gain should be 1.
- Only for a ∞ gain,system will produce a finite output for zero input.

1.0.7. Give the definition of **Quality factor**(Q) and explain its significance.
Solution:

- It is a parameter of an oscillatory system expressing the relationship between stored energy and energy dissipation.
- The "purity" of output sine waves will be a function of the selectivity feedback network.
- That is, higher the value of Q for frequency selective network, the less the harmonic content of sine wave produced.

1.0.8. Compare the equations 1.0.4.7 and 1.0.5.1 and give expressions for Q and ω_0
Solution:

$$\omega_0^2 = \frac{1}{R^2C^2} \tag{1.0.8.1}$$

$$\omega_0 = \frac{1}{RC}\frac{\omega_0}{Q} = \frac{1}{RC}(2 - \frac{R_2}{R_1}) \tag{1.0.8.2}$$

$$Q = \frac{1}{(2 - \frac{R_2}{R_1})} \tag{1.0.8.3}$$

$$\tag{1.0.8.4}$$

1.0.9. Using Eq 1.0.8.3 calculate the value of $\frac{R_2}{R_1}$ for which poles lie on right hand of s-plane.
Solution:
Poles lie on imaginary axis for $Q = \infty$

$$2 - \frac{R_2}{R_1} = 0 \tag{1.0.9.1}$$

$$\frac{R_2}{R_1} = 2 \tag{1.0.9.2}$$

$\therefore$ For poles to lie on right hand side of s-plane

$$\frac{R_2}{R_1} > 2 \tag{1.0.9.3}$$

1.0.10. Verify the above calculations using a Python code.
Solution:

```
codes/ee18btech11044_3_1.py
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- This figure shows how the location of poles vary if $\frac{R_2}{R_1}$ is varied for a fixed ω_0.
- I have varied $\frac{R_2}{R_1}$ from -10 to 10.

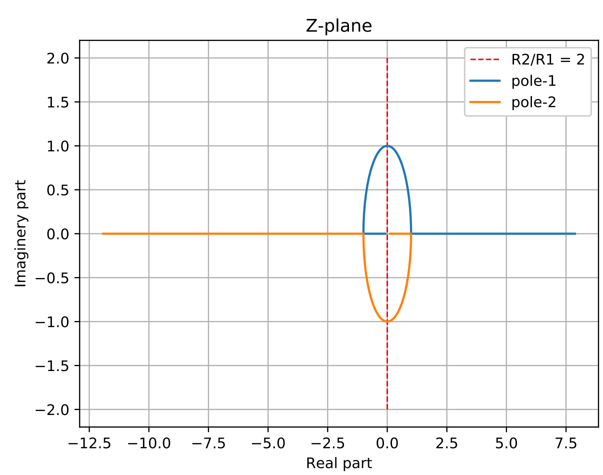

Fig. 1.0.10