

# Control Systems

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## CONTENTS

### 1 Feedback Circuits 1

**Abstract**—The objective of this manual is to introduce control system design at an elementary level.

#### 1 FEEDBACK CIRCUITS

1.0.1. For the Wein-bridge oscillator of Fig 1.0.1, use the expression for loop gain in Eq 1.0.3.5 to find the poles of the closed-loop system. Give the expression for the pole,  $Q$  and use it to show that to locate the poles in the right half of  $s$  plane,  $\frac{R_2}{R_1}$  must be selected to be greater than 2.

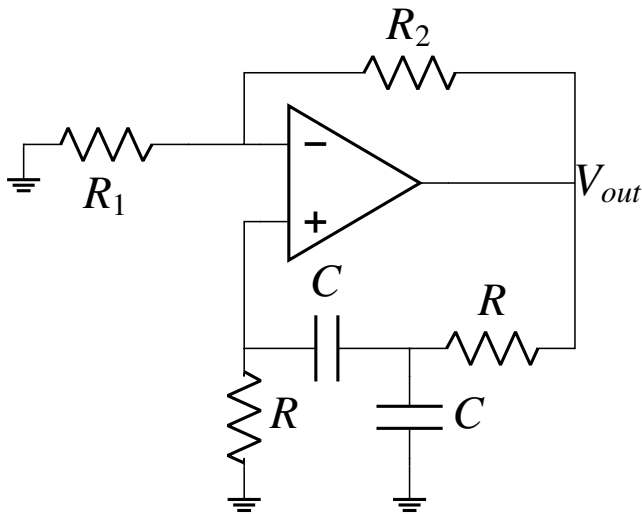


Fig. 1.0.1

1.0.2. Compare the basic structure for a sinusoidal oscillator with Wein-bridge oscillator and give expressions for  $G$  and  $H$ .

**Solution:**

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• Comparing Fig 1.0.1 and Fig 1.0.2, we get

$$G = 1 + \frac{R_2}{R_1} \quad (1.0.2.1)$$

$$H = \frac{Z_p}{Z_p + Z_s} \quad (1.0.2.2)$$

where,

$$Z_p = \frac{R}{RSC + 1} \quad (1.0.2.3)$$

$$Z_s = \frac{RSC + 1}{SC} \quad (1.0.2.4)$$

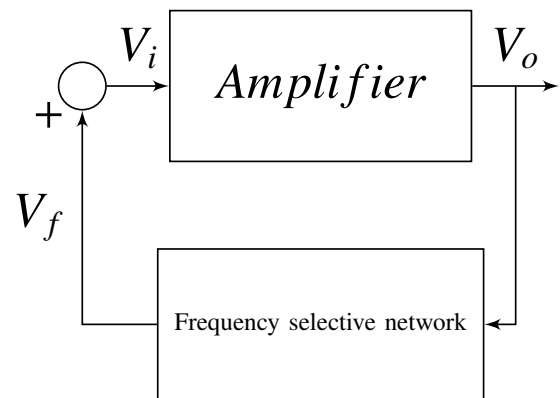


Fig. 1.0.2

1.0.3. Give the expression for loop gain for Wein-bridge oscillator.

**Solution:**

$$T(s) = A(s)\beta(s) \quad (1.0.3.1)$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + Z_s Y_p} \quad (1.0.3.2)$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \left(\frac{sRC+1}{sC}\right)\left(\frac{sRC+1}{R}\right)} \quad (1.0.3.3)$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{s^2 R^2 C^2 + sRC + sRC + 1}{sRC}} \quad (1.0.3.4)$$

$$T(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} \quad (1.0.3.5)$$

1.0.4. Write the characteristic equation for Wein-

bridge oscillator.

**Solution:**

$$1 - T(s) = 0 \quad (1.0.4.1)$$

$$1 - \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} = 0 \quad (1.0.4.2)$$

$$3 + sRC + \frac{1}{sCR} = 1 + \frac{R_2}{R_1} \quad (1.0.4.3)$$

$$3 - 1 + sRC + \frac{1}{sRC} - \frac{R_2}{R_1} = 0 \quad (1.0.4.4)$$

$$2s + s^2RC + \frac{1}{RC} - s\frac{R_2}{R_1} = 0 \quad (1.0.4.5)$$

$$s^2RC + s(2 - \frac{R_2}{R_1}) + \frac{1}{RC} = 0 \quad (1.0.4.6)$$

$$s^2 + s\frac{1}{RC}(2 - \frac{R_2}{R_1}) + \frac{1}{R^2C^2} = 0 \quad (1.0.4.7)$$

1.0.5. Write the general expression for the characteristic equation.

**Solution:**

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \quad (1.0.5.1)$$

1.0.6. State the **Barkhausen criterion** for sustained oscillations with frequency  $\omega_0$ .

**Solution:**

$$T(j\omega_0) = G(j\omega_0)H(j\omega_0) = 1 \quad (1.0.6.1)$$

- That is, at  $\omega_0$  the phase of the loop gain should be zero and the magnitude of loop gain should be 1.
- Only for a  $\infty$  gain, system will produce a finite output for zero input.

1.0.7. Give the definition of **Quality factor(Q)** and explain its significance.

**Solution:**

- It is a parameter of an oscillatory system expressing the relationship between stored energy and energy dissipation.
- The "purity" of output sine waves will be a function of the selectivity feedback network.
- That is, higher the value of Q for frequency selective network, the less the harmonic content of sine wave produced.

1.0.8. Compare the equations 1.0.4.7 and 1.0.5.1 and give expressions for Q and  $\omega_0$

**Solution:**

$$\omega_0^2 = \frac{1}{R^2C^2} \quad (1.0.8.1)$$

$$\omega_0 = \frac{1}{RC} \frac{\omega_0}{Q} = \frac{1}{RC} (2 - \frac{R_2}{R_1}) \quad (1.0.8.2)$$

$$Q = \frac{1}{(2 - \frac{R_2}{R_1})} \quad (1.0.8.3)$$

$$(1.0.8.4)$$

1.0.9. Using Eq 1.0.8.3 calculate the value of  $\frac{R_2}{R_1}$  for which poles lie on right hand of s-plane.

**Solution:**

Poles lie on imaginary axis for  $Q = \infty$

$$2 - \frac{R_2}{R_1} = 0 \quad (1.0.9.1)$$

$$\frac{R_2}{R_1} = 2 \quad (1.0.9.2)$$

$\therefore$  For poles to lie on right hand side of s-plane

$$\frac{R_2}{R_1} > 2 \quad (1.0.9.3)$$

1.0.10. Verify the above calculations using a Python code.

**Solution:**

codes/ee18btech11044\_3\_1.py

- This figure shows how the location of poles vary if  $\frac{R_2}{R_1}$  is varied for a fixed  $\omega_0$ .
- I have varied  $\frac{R_2}{R_1}$  from -10 to 10.

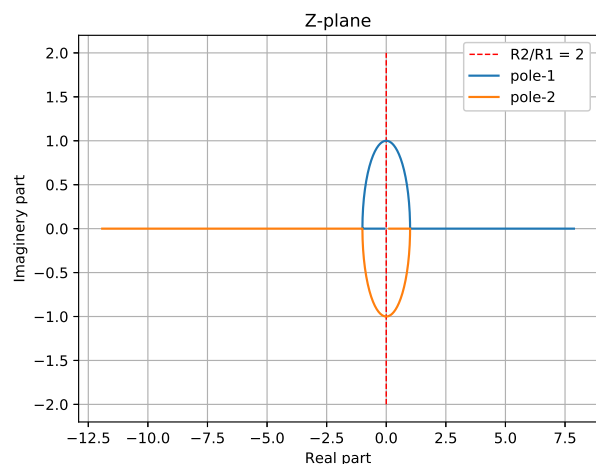


Fig. 1.0.10

1.0.11. Simulate the circuit shown in Fig 1.0.1 using spice simulators. Plot the output generated us-

ing python.

**Solution:**

You can find the netlist for the simulated circuit here:

`spice/ee18btech11044.net`

You can find the python script used to generate the output here:

`codes/ee18btech11044_spice.py`

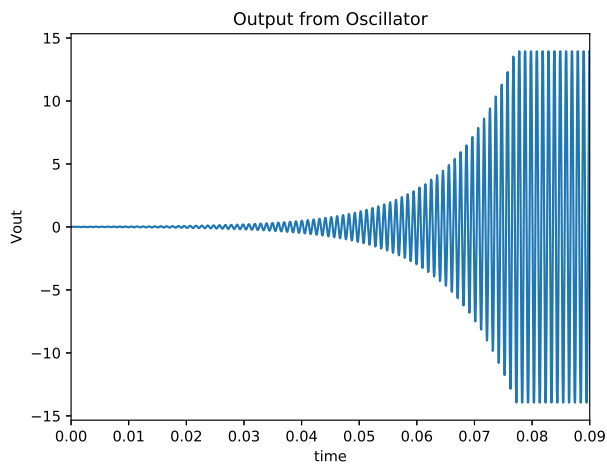


Fig. 1.0.11