Wein-bridge oscillator

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1. For the Wein-bridge oscillator of Fig 1.1, use the expression for loop gain to find the poles of the closed-loop system. Give the expression for the pole, Q and use it to show that to locate the poles in the right half of s plane, $\frac{R_2}{R_1}$ must be selected to be greater than 2.

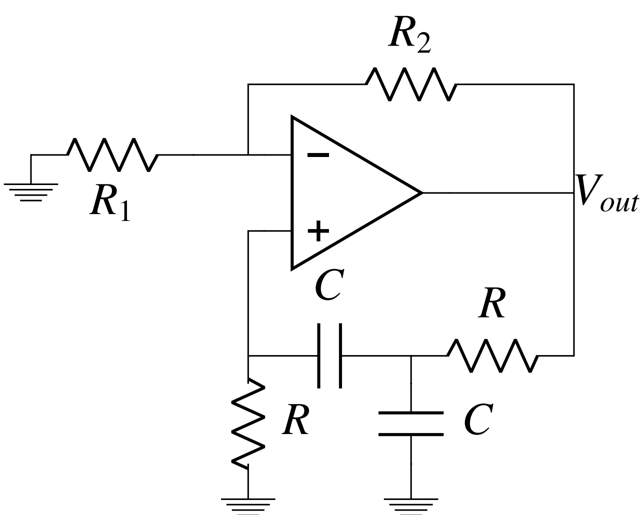

Fig. 1.1

2. Compare the basic structure for a sinusoidal oscillator with Wein-bridge oscillator and give expressions for G and H.
Solution:
Comparring Fig 1.1 and Fig 2.2, we get

$$G = 1 + \frac{R_2}{R_1} \tag{2.1}$$

$$H = \frac{Z_p}{Z_p + Z_s} \tag{2.2}$$

where,

$$Z_p = \frac{R}{RSC + 1} \tag{2.3}$$

$$Z_s = \frac{RSC + 1}{SC} \tag{2.4}$$

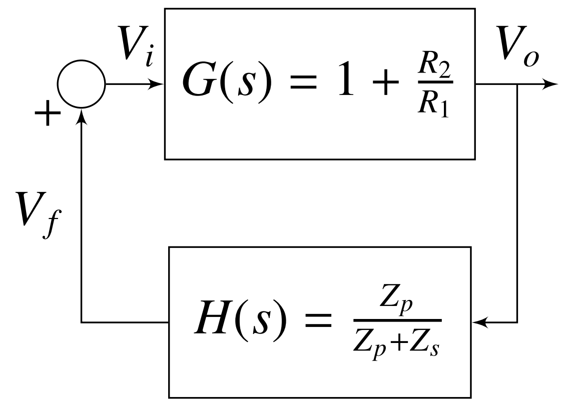

Fig. 2.2

3. Write the closed loop Transfer function T(s) for wein-bridge oscillator.
Solution:

$$T(s) = \frac{(1 + \frac{R_2}{R_1})R^2C^2s^2 + 3RCs + 1}{R^{C2}s^2 + (2 - \frac{R_2}{R_1})RCs + 1} \tag{3.1}$$

4. Write the characteristic equation for Wein-bridge oscillator.
Solution:

$$1 - L(s) = 0 \tag{4.1}$$

$$1 - \frac{1 + \frac{R_2}{R_1}}{3 + sCR + \frac{1}{sCR}} = 0 \tag{4.2}$$

$$3 + sRC + \frac{1}{sCR} = 1 + \frac{R_2}{R_1} \tag{4.3}$$

$$3 - 1 + sRC + \frac{1}{sRC} - \frac{R_2}{R_1} = 0 \tag{4.4}$$

$$2s + s^2RC + \frac{1}{RC} - s\frac{R_2}{R_1} = 0 \tag{4.5}$$

$$s^2RC + s(2 - \frac{R_2}{R_1}) + \frac{1}{RC} = 0 \tag{4.6}$$

$$s^2 + s\frac{1}{RC}(2 - \frac{R_2}{R_1}) + \frac{1}{R^2C^2} = 0 \tag{4.7}$$

5. Write the general expression for the characteristic equation.

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Solution:

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = 0 \tag{5.1}$$

6. State the **Barkhausen criterion** for sustained oscillations with frequency ω_0.
Solution:

$$L(j\omega_0) = G(j\omega_0)H(j\omega_0) = 1 \tag{6.1}$$

- That is, at ω_0 the phase of the loop gain should be zero and the magnitude of loop gain should be 1.
- Only for a ∞ gain,system will produce a finite output for zero input.

7. Give the definition of **Quality factor**(Q) and explain its significance.
Solution:
- It is a parameter of an oscillatory system expressing the relationship between stored energy and energy dissipation.
- The "purity" of output sine waves will be a function of the selectivity feedback network.
- That is, higher the value of Q for frequency selective network, the less the harmonic content of sine wave produced.

8. Compare the equations 4.7 and 5.1 and give expressions for Q and ω_0
Solution:

$$\omega_0^2 = \frac{1}{R^2C^2} \tag{8.1}$$

$$\omega_0 = \frac{1}{RC} \tag{8.2}$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}(2 - \frac{R_2}{R_1}) \tag{8.3}$$

$$Q = \frac{1}{(2 - \frac{R_2}{R_1})} \tag{8.4}$$

$$\tag{8.5}$$

9. Using Eq 8.4 calculate the value of $\frac{R_2}{R_1}$ for which poles lie on right hand of s-plane.
Solution:
Poles lie on imaginary axis for $Q = \infty$

$$2 - \frac{R_2}{R_1} = 0 \tag{9.1}$$

$$\frac{R_2}{R_1} = 2 \tag{9.2}$$

$\therefore$ For poles to lie on right hand side of s-plane

$$\frac{R_2}{R_1} > 2 \tag{9.3}$$

10. Verify the above theory using a Python code.
Solution:

```
codes/ee18btech11044/ee18btech11044_3_1.
    py
```

- This figure shows how the location of poles vary if $\frac{R_2}{R_1}$ is varied for a fixed ω_0.
- I have varied $\frac{R_2}{R_1}$ from -10 to 10.

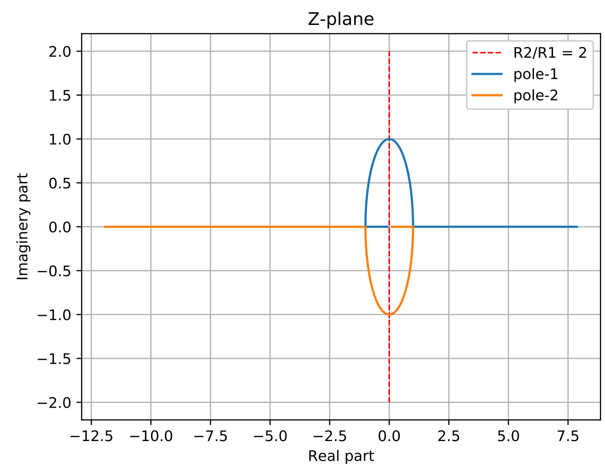

Fig. 10.3

11. Choose appropriate values of Resistances and Capacitors to simulate the circuit.
Solution:

Parameter	Value
R_1	$10k\Omega$
R_2	$20.3k\Omega$
R_p	$10k\Omega$
R_s	$10k\Omega$
C_s	$16nF$
R_p	$10k\Omega$
C_p	$16nF$

TABLE 11

Where, according to Fig 1.1

$$R_p = R_s = R \tag{11.1}$$

$$C_p = C_s = C \tag{11.2}$$

12. Calculate the frequency of sinusoid generated for the values given in table 11.
Solution:
   - Calculating poles of transfer function for $\frac{R_2}{R_1}$= 2.03 using a python script.

```
codes/ee18btech11044/
    ee18btech11044_3_1.py
```

   - We get the poles as 93.75 + j 6249.2968 and 93.75 - j 6249.2968, which correspond to an exponentially increasing sinusoid of frequency 995.11Hz.
13. Substituting the values shown in table 11 in the Eq 3.1, Plot the impulse response and step response using a python code.
Solution:
   - Refer Fig 13.4 for impulse response of transfer function.
   - Code for generating impulse response

```
codes/ee18btech11044/
    ee18btech11044_3_2.py
```

   - Refer D=Fig 13.5 for step response of transfer function.
   - Code for generating step response

```
codes/ee18btech11044/
    ee18btech11044_3_3.py
```

   - As expected from the poles we are getting an exponentially increasing sinusoid as both impulse response and step response.

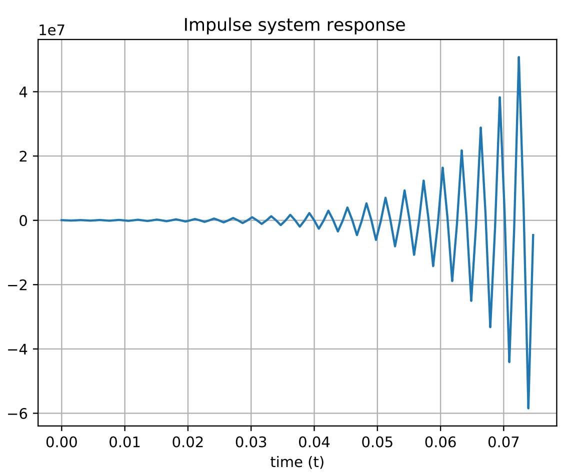

Fig. 13.4

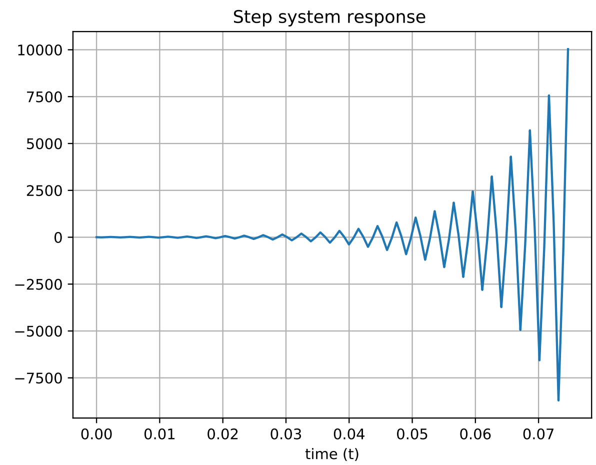

Fig. 13.5

14. Simulate the circuit shown in Fig 1.1 using spice simulators. Substitute the values shown in table 11 and plot the output using a python script.
Solution:
   - Refer Fig **??** for the spice simulation output.
   - You can find the netlist for the simulated circuit here:

```
spice/ee18btech11044/
    ee18btech11044_3_1.net
```

   - You can find the python script used to generate the output here:

```
spice/ee18btech11044/
    ee18btech11044_spice_3_1.py
```

15. Calculate the frequency of sinusoidal wave using plot generated from simulation.
Solution:
   - Consider a part of plot generated from simulation shown in the Fig 15.7.
   - Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 15.7.

$$T_0 = 0.0856452 - 0.0846361 \tag{15.1}$$
$$f_0 = 1/T_0 \tag{15.2}$$
$$f_0 = 990.98Hz. \tag{15.3}$$

   - We get the frequencies calculated from the formulae and the plot to be approximately same.
   - Use this script to generate Fig 15.7

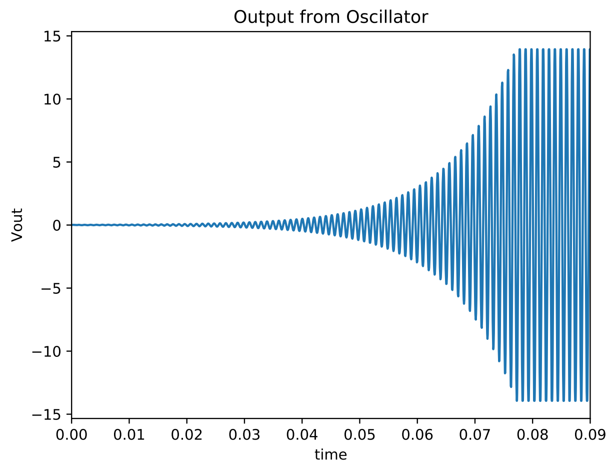

Fig. 14.6

```
spice/ee18btech11044/
    ee18btech11044_spice_3_2.py
```

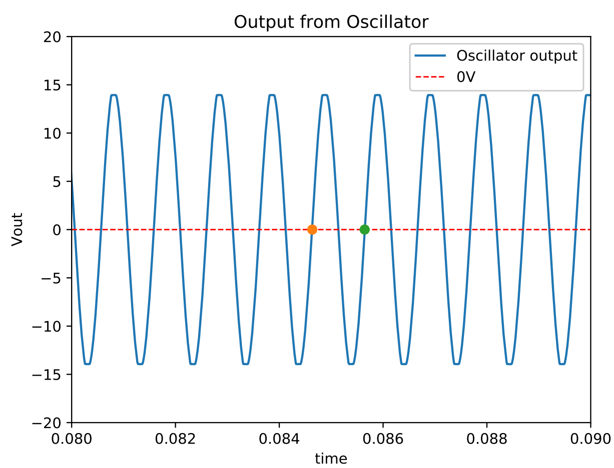

Fig. 15.7

16. Calculate the poles of transfer function when the ratio $\frac{R_2}{R_1}$ is taken to be 2.
Solution:
    - Calculating poles of transfer function for $\frac{R_2}{R_1}$= 2 using a python script.

    ```
    codes/ee18btech11044/
        ee18btech11044_3_4.py
    ```

    - We get the poles to be purely imaginery +6250j and -6250j, which correspond to a sinusoid of frequency 995.22Hz.

17. Substituting the values shown in table 11 in the Eq 3.1, Consider the ratio of $\frac{R_2}{R_{1¢}}$ = 2 and plot the impulse response and step response using a python code.
Solution:
    - Refer Fig 17.8 for impulse response of transfer function.
    - Code for generating impulse response

    ```
    codes/ee18btech11044/
        ee18btech11044_3_5.py
    ```

    - Refer D=Fig 17.9 for step response of transfer function.
    - Code for generating step response

    ```
    codes/ee18btech11044/
        ee18btech11044_3_6.py
    ```

    - As expected from the poles we are getting pure sinusoid as both impulse response and step response.

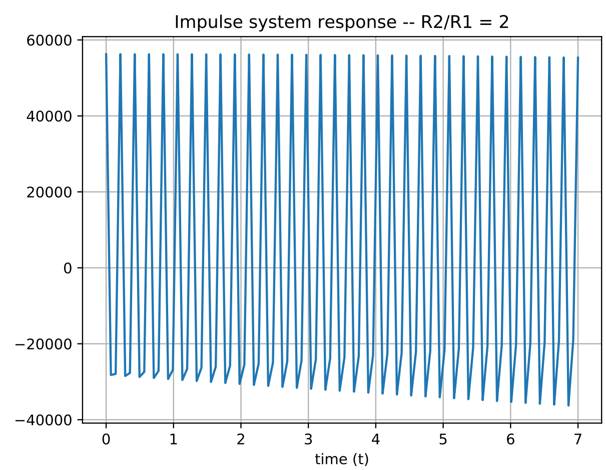

Fig. 17.8

18. Simulate the circuit shown in Fig 1.1 using spice simulators. Consider the ratio of $\frac{R_2}{R_{1¢}}$ = 2 and plot the output using a python script.
Solution:
    - Refer Fig **??** for the spice simulation output.
    - You can find the netlist for the simulated circuit here:

    ```
    spice/ee18btech11044/
        ee18btech11044_3_2.net
    ```

    - You can find the python script used to generate the output here:

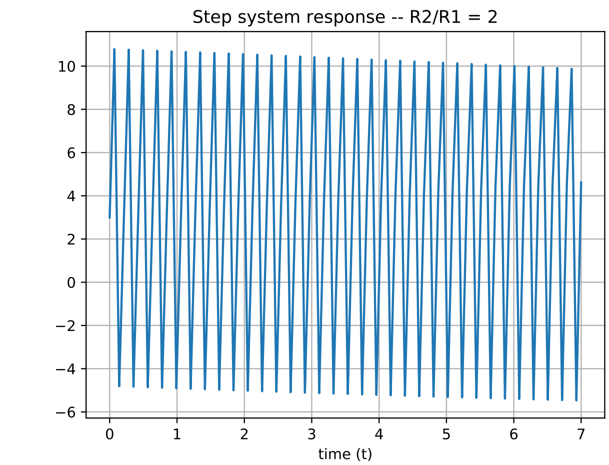

Fig. 17.9

```
spice/ee18btech11044/
    ee18btech11044_spice_3_3.py
```

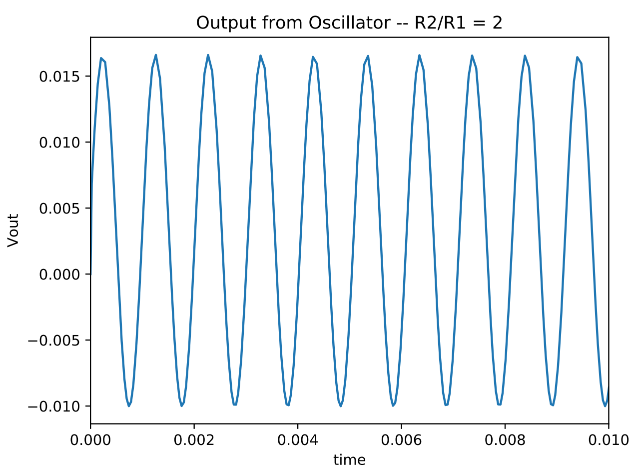

Fig. 18.10

19. Calculate the frequency of sinusoidal wave using plot generated from simulationfor the case where $\frac{R_2}{R_1} = 2$.
Solution:
  - Consider a part of plot generated from simulation shown in the Fig 19.11.
  - Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 19.11.

$$T_0 = 0.00227823 - 0.00127016 \tag{19.1}$$
$$f_0 = 1/T_0 \tag{19.2}$$
$$f_0 = 991.9944Hz. \tag{19.3}$$

  - We get the frequencies calculated from the formulae and the plot to be approximately same.
  - Use this script to generate Fig 19.11

```
spice/ee18btech11044/
    ee18btech11044_spice_3_4.py
```

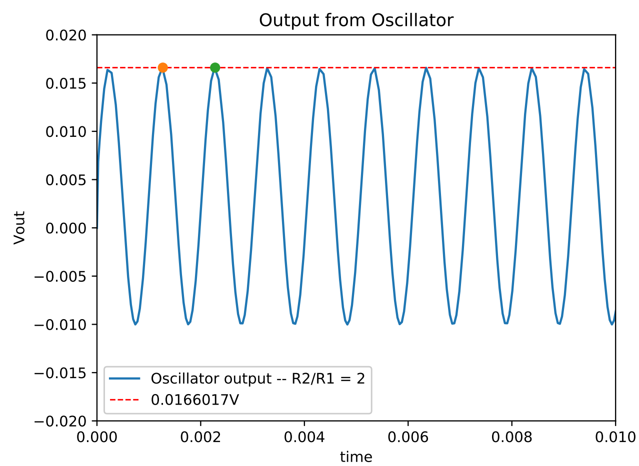

Fig. 19.11

20. Calculate the poles of transfer function when the ratio $\frac{R_2}{R_1}$ is taken to be 9.
Solution:
  - Calculating poles of transfer function for $\frac{R_2}{R_1}$= 9 using a python script.

```
codes/ee18btech11044/
    ee18btech11044_3_7.py
```

  - We get the poles two real and distinct poles at 42838.13728906053 and 911.8627109394729, which correspond to an exponentially increasing function.

21. Substituting the values shown in table 11 in the Eq 3.1. Consider the ratio of $\frac{R_2}{R_1¢} = 9$ and plot the impulse response and step response using a python code.
Solution:
  - Refer Fig 21.12 for impulse response of transfer function.
  - Code for generating impulse response

```
codes/ee18btech11044/
    ee18btech11044_3_8.py
```

- Refer D=Fig 21.13 for step response of transfer function.
- Code for generating step response

```
codes/ee18btech11044/
    ee18btech11044_3_9.py
```

- As expected from the poles we are getting an exponentially increasing function as both impulse response and step response.

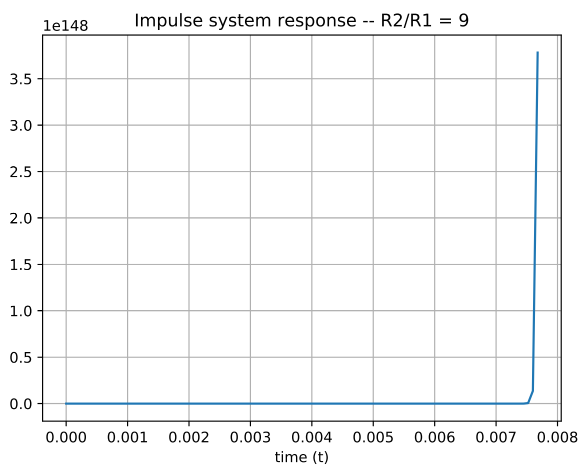

Fig. 21.12

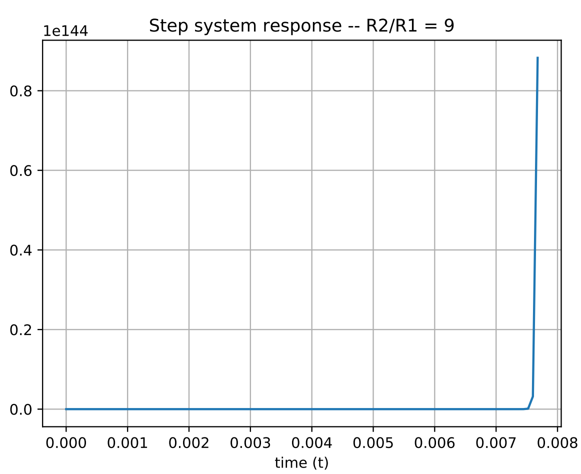

Fig. 21.13

22. Simulate the circuit shown in Fig 1.1 using spice simulators. Consider the ratio of $\frac{R_2}{R_1¢} = 9$ and plot the output using a python script.

Solution:

- Refer Fig **??** for the spice simulation output.
- You can find the netlist for the simulated circuit here:

```
spice/ee18btech11044/
    ee18btech11044_3_3.net
```

- You can find the python script used to generate the output here:

```
spice/ee18btech11044/
    ee18btech11044_spice_3_5.py
```

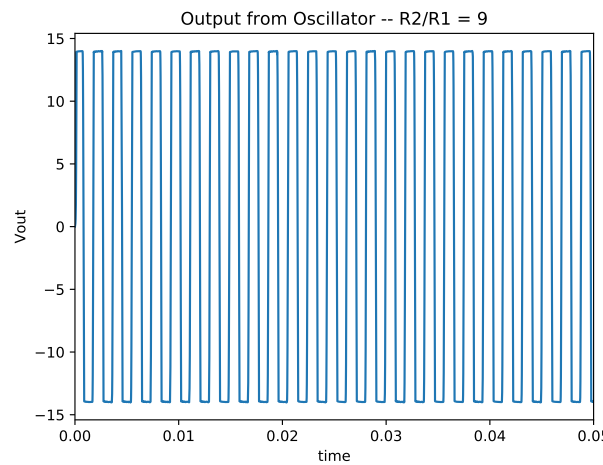

Fig. 22.14

23. Calculate the frequency of sinusoidal wave using plot generated from simulationfor the case where $\frac{R_2}{R_1} = 9$.

Solution:

- Consider a part of plot generated from simulation shown in the Fig 23.15.
- Calculating the Time-period of the sinusoidal wave generated using the two points marked in the Fig 23.15.

$$T_0 = 0.00453629 - 0.00270161 \tag{23.1}$$
$$f_0 = 1/T_0 \tag{23.2}$$
$$f_0 = 545.054 Hz. \tag{23.3}$$

- We get the frequencies calculated from the formulae and the plot to be approximately same.
- Use this script to generate Fig 23.15

```
spice/ee18btech11044/
    ee18btech11044_spice_3_6.py
```

- In this case because of the saturation voltage in op-amp we are getting a curve similar to square wave.

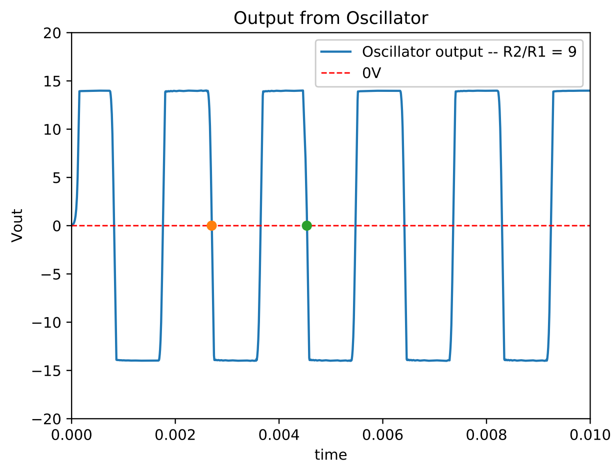

Fig. 23.15

24. Describe the output depending upon the nature of poles.
Solution:
   - For purely imaginery poles, we expect a sinusoidal output (As in the case $\frac{R_2}{R_1} = 2$).
   - For complex conjugate poles, we expect an exponentially increasing sinusoid as the output.(As in the case $2 < \frac{R_2}{R_1} < 4$).
   - For real and distinct poles, we expect an exponential output.(As in the case $\frac{R_2}{R_1} > 4$).
   - As you can observe behaviour of output generated by python scripts can be explained from the location of poles.
   - In the case of spice simulation, the output is always sinusoidal irrespective of location of poles.
   - Only in the case of marginal stability the output of ngspice seems to be valid.
   - Please note that python plots are of expected shape but frequency from python plots are absurd.