Control Systems

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Contents

Abstract—**This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Compensators

1.1. Why do we use compensators.
Solution: In order to obtain the desired performance of the system, we use compensating networks.

1.2. What is the use of a Phase lead compensator.
Solution: Phase lead compensator is used to increase the stability or speed of response of a system.

1.3. What is the use of phase lag compensator.
Solution: Phase lag compensator lag compensator is used to reduce (but not eliminate) the steady-state error.

1.4. Write the general expression for the transfer function of a phase lead compensator.
Solution:

$$H(s) = \frac{(1 + \alpha s T)}{(1 + T s)} \tag{1.4.1}$$

α is generally less than 1 for Phase lead compensator.

1.5. Find the frequency range in which phase introduced by the phase lead compensator reaches maximum
Solution: Transfer function can be rewritten as

$$H(j\omega) = \frac{(1 + j\alpha\omega T)}{(1 + j\omega T)} \tag{1.5.1}$$

Then phase introduced by transfer function is given by

$$\phi = tan^{-1}(\alpha\omega T) - tan^{-1}(\omega T) \tag{1.5.2}$$

Upon differentiating

$$\frac{d\phi}{d\omega} = \frac{\alpha T}{1 + (\alpha\omega T)^2} - \frac{T}{1 + (\omega T)^2} \tag{1.5.3}$$

Equating it to zero to find the maximum

$$\frac{\alpha T}{1 + (\alpha\omega T)^2} = \frac{T}{1 + (\omega T)^2} \tag{1.5.4}$$

$$\alpha(1 + (\omega T)^2) = 1 + (\omega\alpha T)^2 \tag{1.5.5}$$

$$\alpha + \alpha(\omega T)^2 = 1 + (\omega\alpha T)^2 \tag{1.5.6}$$

$$(\alpha - 1) = (\omega T)^2\alpha(\alpha - 1) \tag{1.5.7}$$

$$(\omega)^2 = \frac{1}{(T^2)\alpha} \tag{1.5.8}$$

$$\omega = \frac{1}{T\sqrt{\alpha}} \tag{1.5.9}$$

Clearly

$$\frac{1}{\alpha T} < \frac{1}{T\sqrt{\alpha}} < \frac{1}{T} \tag{1.5.10}$$

So the frequency range in which phase lead introduced by this compensator reaches maximum in

$$\omega\epsilon(\frac{1}{\alpha T}, \frac{1}{T}) \tag{1.5.11}$$

1.6. Write the general expression for the transfer function of phase lag compensator.
Solution:

$$H(s) = \frac{1}{\beta}\frac{(1 + \beta s T)}{(1 + T s)} \tag{1.6.1}$$

β is generally greater than 1 for Phase lead compensator.

1.7. Write the general expression for the transfer function of a Phase lag-lead compensator.
Solution: The Transfer Function of the Phase

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lag-lead compensator is

$$H(s) = \frac{(1+\alpha s T_1)(1+\beta s T_2)}{(1+T_1 s)(1+T_2 s)} \tag{1.7.1}$$

α and β are generally chosen in such a way that

$$\alpha\beta = 1 \tag{1.7.2}$$

It is to be noted that phase introduced by lag part is close to zero in the frequency range where lead part reaches maximum and vice versa.

1.8. Write the expression for phase introduced by phase lag-lead compensator.
Solution:

$$\phi = tan^{-1}(\frac{\omega T_1}{\beta}) + tan^{-1}(\beta\omega T_2) - tan^{-1}(\omega T_1) - tan^{-1}(\omega T_2) \tag{1.8.1}$$

1.9. Consider the Transfer function of a phase lag-lead compensator

$$C(s) = \frac{(1+\frac{s}{0.1})(1+\frac{s}{100})}{(1+\frac{s}{1})(1+\frac{s}{10})} \tag{1.9.1}$$

Find the frequency range in which the phase (lead) introduced by the compensator reaches the maximum
Solution: Comparing the given Transfer function with with equation(1.7.1), we get $\alpha = 10$, $T_1 = 1$, $\beta = 0.1$, $T_2 = 0.1$. Substituting these values in equation (1.8.1)

$$\phi = tan^{-1}(10\omega) + tan^{-1}(0.01\omega) - tan^{-1}(\omega) - tan^{-1}(0.1\omega) \tag{1.9.2}$$

As we are trying to find the range of frequencies in which Phase lead introduced by the compensator is maximum, the phase introduced by the lag part of compensator will be close to zero.

$$\phi = tan^{-1}(10\omega) - tan^{-1}(\omega) \tag{1.9.3}$$

This is similar to equation 1.5.2 and we already know that in this case ϕ is maximum for

$$\omega\epsilon(0.1, 1) \tag{1.9.4}$$

1.10. Verify using a python plot
Solution:

codes/EE18BTECH11044.py