CONTROL SYSTEMS

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Problem Statement

Q) The transfer function C(s) of a compensator is given below.

$$C(s) = \frac{(1 + \frac{s}{0.1})(1 + \frac{s}{100})}{(1 + \frac{s}{1})(1 + \frac{s}{10})}$$

The frequency range in which the phase (lead) introduced by the compensator reaches the maximum is
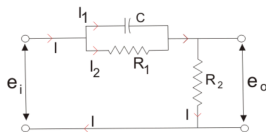
Theory
Introduction

- In order to obtain the desired performance of the system, we use compensating networks.
- Compensating networks are applied to the system in the form of feed forward path gain adjustment.
- Compensate a unstable system to make it stable.
- Compensating networks can also increase the steady state accuracy of the system.
- The three types of compensators — lag, lead and lag-lead are most commonly used electrical compensators.

Now let us see what lag, lead and lag-lead compensators briefly.

Theory

Phase lead compensator

- A system which has one pole and one dominating zero (the zero which is closer to the origin than all over zeros is known as dominating zero.) is known as lead network.

- The phase-lead compensator adds positive phase to the system over the frequency range $1/\alpha T$ to $1/T$.

The transfer function of phase lead compensator is given by

$$TransferFunction = (\frac{1 + \alpha s T}{1 + s T})$$

Theory
Phase lead compensator

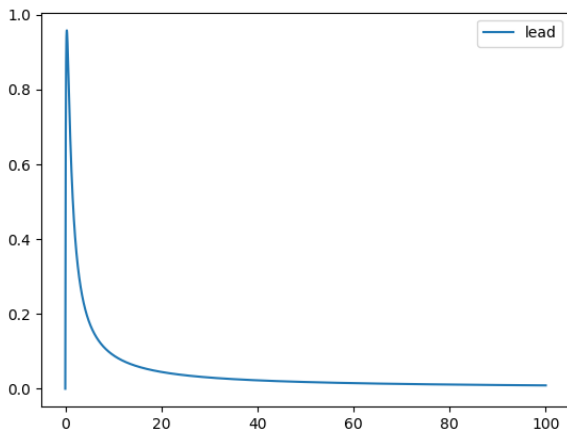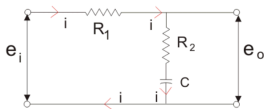
Figure: Phase plot for a phase lead compensator

Theory
Phase lag compensator

- A system which has one zero and one dominating pole (the pole which is closer to origin that all other poles is known as dominating pole) is known as lag network.
- The phase-lag compensator adds negative phase to the system over the frequency range $1/T$ to $1/\beta T$

The transfer function of phase lag compensator is given by

$$TransferFunction = \frac{1}{\beta}\left(\frac{1 + \beta sT}{1 + sT}\right)$$
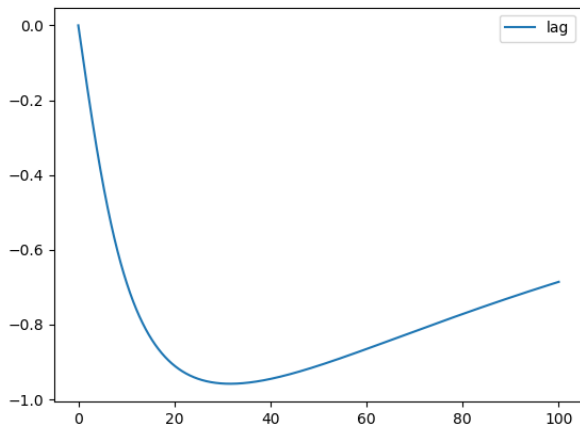
Theory
Phase lag compensator


Figure: Phase plot for a phase lag compensator

Theory

Phase lag-lead compensator

With single lag or lead compensation design specifications may not
satisfied. For an unstable uncompensated system, lead compensation
provides fast response whereas lag compensation stabilize the system. So
we need multiple compensators in cascade.
Given below is the circuit diagram for the phase lag- lead compensation
network.
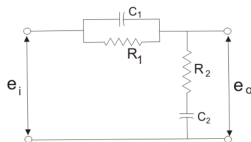
The transfer function of phase lag-lead compensator is given by

$$TransferFunction = \frac{(1 + \alpha s T_1)(1 + \beta s T_2)}{(1 + s T_1)(1 + s T_2)}$$

Theory

- Lead and lag compensators are used quite extensively in control. A lead compensator can increase the stability or speed of response of a system.
- lag compensator can reduce (but not eliminate) the steady-state error.
- Generally $\alpha > 1$ and $\beta < 1$.
- In a phase lead network pole is farther from origin than zero.
- In a phase lag network zero is farther from origin than pole.
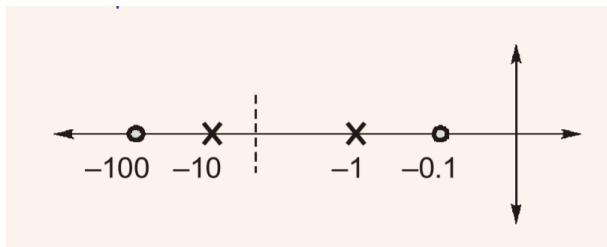
Solution

The given transfer function is :

$$C(s) = \frac{(1 + \frac{s}{0.1})(1 + \frac{s}{100})}{(1 + \frac{s}{1})(1 + \frac{s}{10})}$$

$$C(s) = \frac{(1 + 0.01s)(1 + 10s)}{(1 + 0.1s)(1 + s)}$$

- Clearly the given transfer function is of a Phase lag-lead compensator.
- Comparing the equations, we get $\alpha = 10$, $T_1 = 1$, $\beta = 0.1$ and $T_2 = 0.1$.
- The pole zero diagram for the given compensator transfer function would be :

Solution

- In the above pole zero plot the pole-zero pair closer to the origin corresponds to phase lead part and pole-zero pair away from origin corresponds to the phase lag part.
- The frequency range in which the phase(lead) introduced by the compensator reaches maximum will be when phase lead part is dominating over the phase lag part.
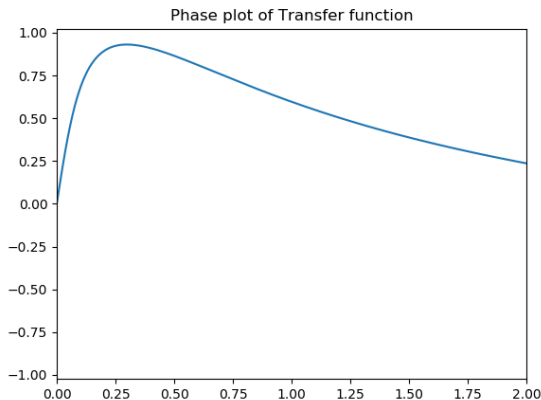- This will occur when ω ranges between 0.1 and 1.

Verification

Plotting the phase of transfer function as a function of ω.

Phase plot of Transfer function
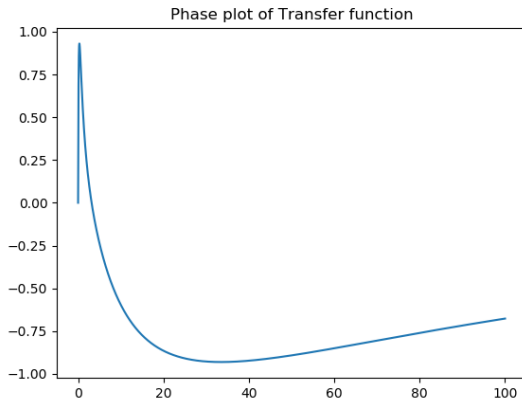
Verification

Limiting the axis from 0 to 2, we can clearly observe the maxima between 0.1 and 1.

Phase plot of Transfer function

The End