Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

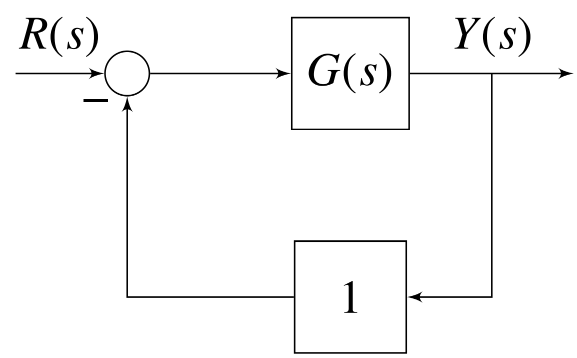

Fig. 1.2

1 Compensators

1.1. For a unity feedback system shown in 1.2, $\frac{10}{s(s+1)}$. Design a lead compensator such that the phase margin of the system is 45° and appropriate steady state error is less than or equal to $\frac{1}{15}$ units of the final output value. Further the gain crossover frequency of the system must be less than 7.5rad/sec.

1.2. For the control system shown in 1.2 write the steady state output for step input.
Solution:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{1.2.1}$$

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{sR(s)G(s)}{1+G(s)} \tag{1.2.2}$$

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{G(s)}{1+G(s)} \tag{1.2.3}$$

1.3. What do you mean by steady state error and write the expression for steady state error for control system shown in 1.2 considering step input.
Solution: Steady-state error is the difference between the input and the output for a prescribed test input as time tends to infinity.

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+G(s)} \tag{1.3.1}$$

1.4. Write the general expression for the transfer function of a phase lead compensator.
Solution:

$$G_c(s) = K_{comp} \frac{(1+\alpha T s)}{(1+T s)} \tag{1.4.1}$$

1.4 shows the compensated control system.

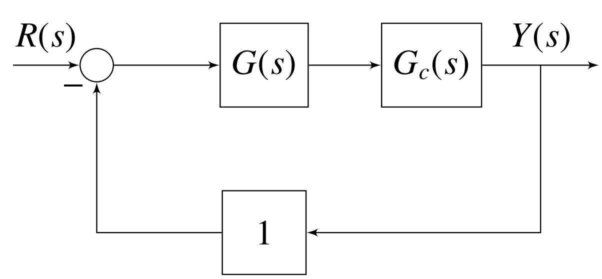

Fig. 1.4

1.5. Calculate the steady state output value and steady state error for the control system shown in 1.2, where G(s) = $\frac{10}{s(s+1)}$. Consider the input to be unit step.
Solution:

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Steady state value:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} \frac{10}{10 + s(s+1)} \tag{1.5.1}$$
$$\lim_{t\to\infty} y(t) = 1 \tag{1.5.2}$$

Steady state error:

$$e_{ss} = \lim_{s\to 0} \frac{s(s+1)}{10 + s(s+1)} \tag{1.5.3}$$
$$e_{ss} = 0 \tag{1.5.4}$$

1.6. Choose a value of K_{comp} which satisfies the steady state error condition.
Solution: As the steady state error for unit step response is always zero, any value of K_{comp} satisfies the steady state error condition in compensated system. For simplicity let us choose $K_{comp} = 1$.

1.7. Calculate phase margin and gain cross over frequency of open-loop transfer function G(S).
Solution:
Gain cross over frequency:

$$G(j\omega) = \frac{10}{j\omega(j\omega + 1)} \tag{1.7.1}$$
$$|G(j\omega)| = \frac{10}{\sqrt{\omega^4 + \omega^2}} \tag{1.7.2}$$
$$\frac{10}{\sqrt{\omega^4 + \omega^2}} = 1 \tag{1.7.3}$$
$$\omega_{gc} = 3.084 \tag{1.7.4}$$

Phase Margin:

$$\phi = -90^\circ - tan^{-1}(\omega) \tag{1.7.5}$$
$$pm = 180^\circ - 90^\circ - tan^{-1}(\omega_{gc}) \tag{1.7.6}$$
$$pm = 17.966^\circ \tag{1.7.7}$$
$$\tag{1.7.8}$$

1.8. Write the expression for maximum phase of a lead compensator and the frequency where it occurs
Solution:

$$\phi_{max} = sin^{-1}(\frac{\alpha - 1}{\alpha + 1}) \tag{1.8.1}$$
$$\omega_m = \frac{1}{T\alpha} \tag{1.8.2}$$

1.9. Calculate the value of ϕ_{max} required to meet desired phase margin.
Solution:

$$\phi_{max} = 45^\circ - pm + 15^\circ \tag{1.9.1}$$
$$\phi_{max} = 45^\circ - 17.966^\circ + 15^\circ \tag{1.9.2}$$
$$\phi_{max} = 42.034^\circ \tag{1.9.3}$$

Here the extra 15^o has been added to compensate for the shift in ω_{gc}.

1.10. Using (1.8.1) calculate the value of α
Solution:

$$sin(42.034^\circ) = \frac{\alpha - 1}{\alpha + 1} \tag{1.10.1}$$
$$0.669 = \frac{\alpha - 1}{\alpha + 1} \tag{1.10.2}$$
$$0.331\alpha = 1.669 \tag{1.10.3}$$
$$\alpha = 5.04 \tag{1.10.4}$$

1.11. Choose appropriate value for ω_m.
Solution:
- For maximum increase in phase margin we have to ensure that ϕ_{max} occurs at frequency close to ω_{gc} of G(s).

$$\omega_m = 3.084 rad/sec \tag{1.11.1}$$

- We know that ω_{gc} gets shifted slightly when we cascade a compensator to original transfer function, to compensate for the shift we have already added an extra 15° to ϕ_{max}.

1.12. Using (1.8.2) calculate the value of T.
Solution:

$$T = \frac{1}{\omega_m \alpha} \tag{1.12.1}$$
$$T = \frac{1}{15.54} \tag{1.12.2}$$
$$T = 0.064 \tag{1.12.3}$$

1.13. Write the final expression of the Lead compensator designed.
Solution:

$$G_c(s) = \frac{(1 + 0.322s)}{(1 + 0.064s)} \tag{1.13.1}$$

- Zero at s = -3.084
- Pole at s = -15.54

1.14. Verify using a python plot.
Solution:

```
codes/ee18btech11044_2.py
```

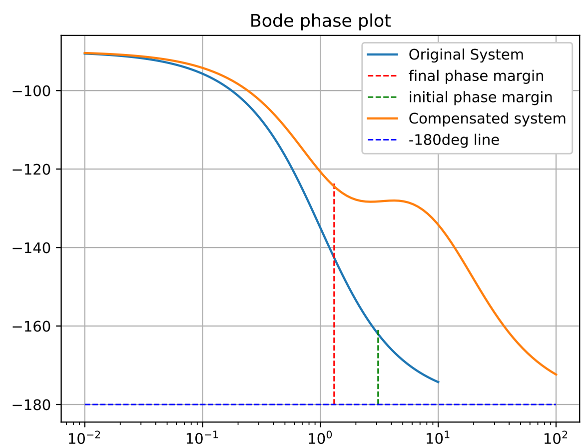
Bode phase plot
Original System
final phase margin
initial phase margin
Compensated system
-180deg line
−100
−120
−140
−160
−180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Fig. 1.14