HW #4 Due: 5/01/2020

1. If we want to use ICA for dimensionality reduction, can we directly pick independent components with larger energy? Explain.
2. Use the FA approach to reduce the feature dimension from 4 to 3 for Iris data set. As usual, take 70% of the samples as training set and perform FA. Use 5-NN to classify the test set and then report the average accuracy after 10 trials. For simplicity, you may assume $\Psi = 0$ and use the pseudo inverse solution.
3. Suppose we have two classes of data. The samples in the first class are from $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_1 and x_2 are jointly Gaussian with $\mu_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\Sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The samples in the second class are from $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ where y_1 and y_2 are jointly Gaussian with $\mu_y = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $\Sigma_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find the $\boldsymbol{w}_{opt}$ (with normalization) and optimal decision value α_{opt}.
4. We have a dataset $S = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Follow the k-means algorithm to complete the assignment step and the update step for one run. Use k = 2, initial $\boldsymbol{\mu}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ and $\boldsymbol{\mu}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ in the computation.
5. In this problem we want to use the SOFM algorithm to reproduce the picture of the "BMU + Error Map" in the PPT file. To begin with, we need to determine the size of the feature map. By counting the number of colored dots in the picture, we guess the size of the map is (39×39). Next, we choose a reasonable number of iteration, such as 5,000, in the algorithm. Third, we determine λ by $\lambda = \frac{\text{iteration \#}}{\text{map radius}}$. (a) What is the map radius in this case? Let $\eta(0) = 0.1$ and $\gamma_{i,j}(0) = 0.05$ for farthest locations if the BMU is at the center location of the map. (b) Find η_0 and σ_0 accordingly. (c) Complete the program and present your results. Note: Because different initial weights and different parameter settings, your map likely looks differently from the one on the PPT file.