

5.1 $A = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}$

$$\begin{vmatrix} -1-\lambda & -6 \\ 2 & 6-\lambda \end{vmatrix} = 0$$

$$(1+\lambda)(\lambda-6) + 12 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda_1 = 2 \quad \lambda_2 = 3$$

$$\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{cases} -x_1 - 6x_2 = 2x_1 \\ 2x_1 + 6x_2 = 2x_2 \end{cases}$$

$$\begin{cases} -6x_2 = 3x_1 \\ 4x_2 = -2x_1 \end{cases} \quad x_1 = -2x_2$$

например, $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 3 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{cases} -x_1 - 6x_2 = 3x_1 \\ 2x_1 + 6x_2 = 3x_2 \end{cases}$$

$$\begin{cases} 6x_2 = -4x_1 \\ 3x_2 = -2x_1 \end{cases}$$

например, $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$$x_1 = -\frac{3}{2}x_2$$

5.2 $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{vmatrix} -1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0; \quad (\lambda+1)^2 = 0; \quad \lambda_{1,2} = -1$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{cases} -x_1 = -x_1 \\ -x_2 = -x_2 \end{cases} \Rightarrow x_{1,2} \text{ - любое}$$

5.3 $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \quad x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} 2 = \lambda \\ -1 + 3 = \lambda \end{cases} \Rightarrow \lambda = 2, \text{ год}$$

5.4 $A_2 = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad x = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$

$$\begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$$

$$\begin{cases} -9 = 3\lambda \\ 9 = -3\lambda \\ -12 = -4\lambda \end{cases} \Rightarrow \begin{cases} \lambda = -3 \\ \lambda = 3 \\ \lambda = 3 \end{cases} \quad \text{нет}$$