

$$(1.1) \quad f_1(x) = e^x; \quad f_2(x) = 1; \quad f_3(x) = x+1; \quad f_4(x) = x \cdot e^x$$

$$f_4(x) = f_3(x) \cdot f_2(x) \cdot f_1(x)$$

$$(1.2) \quad f_1(x) = 2; \quad f_2(x) = x; \quad f_3(x) = x^2; \quad f_4(x) = (x+1)^2$$

$$f_4(x) = x^2 + 2x + 1 = f_3(x) + 2f_2(x) + \frac{1}{2}f_1(x)$$

$$(1.3) \quad x = (2, 3, 5) \\ b_1 = (2, 0, 0); \quad b_2 = (0, 0, 10); \quad b_3 = (0, 1, 0)$$

$$(2, 3, 5) = \underbrace{(2, 0, 0)}_{b_1} + \underbrace{(0, 3, 0)}_{3b_3} + \underbrace{(0, 0, 5)}_{1/2 b_2} = \frac{1}{2}b_1 + b_2 + 3b_3 \\ \left(\frac{1}{2}; 1; 3\right)$$

$$(1.4) \quad 3x^2 - 2x + 2$$

$$a) \quad 1, x, x^2$$

$$e_1, e_2, e_3$$

$$3x^2 - 2x + 2 = \text{---} = 2e_1 - 2e_2 + 3e_3; \quad (2, -2, 3)$$

$$b) \quad x^2, x+1, 1 \\ e_1, e_2, e_3$$

$$3x^2 - 2x + 2 = 3x^2 - 2(x+1) = 3e_1 - 2e_2; \quad (3, -2, 0)$$

$$(1.5) \quad a) L: (0, b, c) \cup (a, 0, c) \cup (0, 0, c)$$

$$u+v = (0, b_1, c_1) + (a_2, 0, c_2) = (a_2, b_1, c_1+c_2) \notin L$$

$$b) \quad \text{linear combination } u_1, \dots, u_n$$

$$u = \alpha_1 u_1 + \dots + \alpha_n u_n$$

$$v = \beta_1 u_1 + \dots + \beta_n u_n$$

$$u+v = (\alpha_1 + \beta_1)u_1 + \dots + (\alpha_n + \beta_n)u_n$$

$$\alpha u = \alpha \alpha_1 u_1 + \dots + \alpha \alpha_n u_n$$

$$y \in L$$



2.1 a)  $(0, 3, 6) \quad (-4, 7, 9)$   
 $(x, y) = 0 - 21 + 54 = 33$

b)  $(7, -4, 0, 1) \quad (-3, 1, 1, 2)$   
 $(x, y) = -21 - 4 + 0 + 2 = -23$

2.2 a)  $(4, 2, 4) \quad b = (12, 3, 4)$

$\|a\|_1 = 4 + 2 + 4 = 10$

$\|a\|_2 = \sqrt{16 + 4 + 16} = 6$

$\|b\|_1 = 12 + 3 + 4 = 19$

$\|b\|_2 = \sqrt{144 + 9 + 16} = 13$

$\cos \varphi = \frac{48 + 6 + 16}{6 \cdot 13} = \frac{70}{6 \cdot 13} = \frac{35}{39}$

2.4 a)  $(1, 0, 0) \quad (0, 0, 1)$

$(e_1, e_2) = 0$

$(e_1, e_1) = 1 \quad (e_2, e_2) = 1$  — ga

$(e_1, e_1) = 0$   
 $(e_1, e_2) = 1$

b)  $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \quad (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0) \quad (0, 0, 1)$

$e_1 e_2 = \frac{1}{2} - \frac{1}{2} + 0 = 0 \quad e_1 e_3 = 0 \quad e_2 e_3 = 0$  — ga

$e_1 e_1 = \frac{1}{2} + \frac{1}{2} + 0 = 1 \quad e_2 e_2 = \frac{1}{2} + \frac{1}{2} + 0 = 1 \quad e_3 e_3 = 0 + 0 + 1 = 1$

c)  $(\frac{1}{2}, -\frac{1}{2}, 0) \quad (0, \frac{1}{2}, \frac{1}{2}) \quad (0, 0, 1)$

$e_1 e_2 = 0 - \frac{1}{4} + 0 \neq 0$  — HET

d)  $(1, 0, 0) \quad (0, 1, 0) \quad (0, 0, 1)$

$e_1 e_2 = 0 \quad e_1 e_3 = 0 \quad e_2 e_3 = 0 \quad e_1 e_1 = 1 \quad e_2 e_2 = 1 \quad e_3 e_3 = 1$  — ga

2.3 a)  $|x| |y|$

✓ 1.  $|x| \cdot |y| = |y| \cdot |x|$

✓ 2.  $|\lambda x| |y| = \lambda |x| |y|$

— 3.  $|x_1 + x_2| |y| \leq |x_1| |y| + |x_2| |y|$

✓ 4.  $|x| \cdot |x| \geq 0; \quad |x| \cdot |x| = 0 \Leftrightarrow x = 0$  — HET

b)  $3|x| |y| \cos \varphi$

✓ 1.  $3|x| |y| \cos \varphi = 3|y| |x| \cos \varphi$

✓ 2.  $3|\lambda x| |y| \cos \varphi = \lambda 3|x| |y| \cos \varphi$

✓ 3.  $3|x_1 + x_2| |y| \cos \varphi = 3(|x_1| |y| \cos \varphi + |x_2| |y| \cos \varphi) = 3|x_1| |y| \cos \varphi + 3|x_2| |y| \cos \varphi$  — ga

✓ 4.  $3|x| |x| \cos 0 \geq 0; \quad 3|x| |x| = 0 \Leftrightarrow x = 0$