(3.1)

a) A 4×2, B 4×2 — AB N/a, BA N/a
b) A 2×5, B 5×3 — AB 2×5, BA N/a
c) A 8×3, B 3×8 — AB 8×8, BA 3×3
d) A 4×4, B 4×4 — AB 4×4, BA 4×4

(3.2) $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 5 & -3 \\ 3 & 5 \end{pmatrix}$

$\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 4+0 & -1-10 \\ 12+0 & -3+0 \end{pmatrix} = \begin{pmatrix} 4 & -11 \\ 12 & -3 \end{pmatrix}$

$\begin{pmatrix} 4 & -1 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 4-3 & -8 \\ 15 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -8 \\ 15 & 0 \end{pmatrix}$

(3.3)

$A = \begin{pmatrix} 1 & 7 \\ 3 & -6 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 5 \\ 2 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & -4 \\ 1 & 1 \end{pmatrix}$

$3A - 2B + 4C = \begin{pmatrix} 3 & 21 \\ 9 & -18 \end{pmatrix} - \begin{pmatrix} 0 & 10 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 8 & -16 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 11 & -5 \\ 9 & -12 \end{pmatrix}$

(3.4)

$A = \begin{pmatrix} 4 & 1 \\ 5 & -2 \\ 2 & 3 \end{pmatrix} \quad A^T = \begin{pmatrix} 4 & 5 & 2 \\ 1 & -2 & 3 \end{pmatrix}$

$AA^T = \begin{pmatrix} 16+1 & 20-2 & 8+3 \\ 20-2 & 25+4 & 10-6 \\ 8+3 & 10-6 & 4+9 \end{pmatrix} = \begin{pmatrix} 17 & 18 & 11 \\ 18 & 29 & 4 \\ 11 & 4 & 17 \end{pmatrix}$

$A^T A = \begin{pmatrix} 16+25+4 & 4-10+6 \\ 4-10+6 & 1+4+9 \end{pmatrix} = \begin{pmatrix} 45 & 0 \\ 0 & 14 \end{pmatrix}$

(4.1)

a) $\begin{vmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{vmatrix} = \sin^2 x + \cos^2 x = 1$

b) $\begin{vmatrix} 4 & 2 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 9 \end{vmatrix} = 45 \cdot 9 = 180$

c) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix} = 1\cdot5\cdot9 + 2\cdot6\cdot7 + 3\cdot4\cdot8 - 7\cdot5\cdot3 - 8\cdot6\cdot1 - 9\cdot4\cdot2 =$
$= 45 + 84 + 96 - 105 - 48 - 72 = 0$

(4.2) $\det(A) = 4$

$\det(A^2) = \det(A) \cdot \det(A) = 16$

$\det(A^T) = \det(A) = 4$

$\det(2A) = 2^n \det(A) = 2^n \cdot 4$, gde n - red. matrice A

4.3

$$A = \begin{pmatrix} -2 & 7 & -3 \\ 4 & -14 & 6 \\ -3 & 7 & 13 \end{pmatrix} \cdot -2 \begin{pmatrix} -2 & 7 & -3 \\ -2 & 7 & -3 \\ -3 & 7 & 13 \end{pmatrix} \Rightarrow \det A = 0 \Rightarrow$$

A - вырожд.

4.4

а) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, rank A = 2

б) $\begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 3 \\ 2 & 3 & 5 & 6 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 2 & 3 & 5 & 6 \end{pmatrix}$, rank A = 3