(6.1)
$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 0 \\ 2 & 1 & -1 & 1 & -2 \\ 1 & 1 & -3 & 1 & 4 \end{array}\right) \to \left(\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 1 & -1 & -5 & 2 \\ 0 & 1 & -5 & 1 & 10 \end{array}\right) \to$$

$$\to \left(\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 1 & -1 & -5 & 2 \\ 0 & 0 & -4 & 6 & 8 \end{array}\right)$$

$x_4 = c$

$4x_3 = 6x_4 - 8;\ x_3 = \frac{3}{2}x_4 - 2$

$x_2 = x_3 + 5x_4 + 2 = \frac{3}{2}x_4 - 2 + 5x_4 + 2 = \frac{13}{2}x_4$

$2x_1 = -x_2 + x_3 - x_4 - 2 = -6x_4 - 4$

$x_1 = -3x_4 - 2$

$\left(-3x_4 - 2;\ \frac{13}{2}x_4;\ \frac{3}{2}x_4 - 2,\ x_4\right)$

(6.2) а) $\left(\begin{array}{ccc|c} 3 & -1 & 1 & 4 \\ 2 & -5 & -3 & -17 \\ 1 & 1 & -1 & 0 \end{array}\right)$ $\operatorname{rank} A = 3$, $\operatorname{rank} \bar{A} = 3$ $\Rightarrow$ одно реш.

б) $\left(\begin{array}{ccc|c} 2 & -4 & 6 & 1 \\ 1 & -2 & 5 & -2 \\ 3 & -6 & 9 & 5 \end{array}\right)$ $\operatorname{rank} A = 1$, $\operatorname{rank} \bar{A} = 2$ $\Rightarrow$ нет реш.

в) $\left(\begin{array}{ccc|c} 1 & 2 & 5 & 4 \\ 3 & 1 & -8 & -2 \end{array}\right)$ $\operatorname{rank} A = 2$, $\operatorname{rank} \bar{A} = 2$ $\Rightarrow$ бесч. мн. реш.

(6.3)
$$\left(\begin{array}{cccc|c} 1 & 3 & -2 & 4 & 3 \\ 0 & 5 & 0 & 1 & 2 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 0 & 2 & 1 \end{array}\right)$$

$\operatorname{rank} A = \operatorname{rank} \bar{A} = 4$

одно решение

(6.4)
$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 4 & 5 & 6 & b \\ 7 & 8 & 9 & c \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & -3 & -6 & b - 4a \\ 0 & -6 & -12 & c - 7a \end{array}\right)$$

$\operatorname{rank} A = 2$

или $\operatorname{rank} \bar{A} = 3$ (нет решений)

$\operatorname{rank} \bar{A} = 3$, если $c - 7a \neq 2(b - 4a)$

$c - 7a \neq 2b - 8a$

$c + a \neq 2b$

(7.1) а) $\left(\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & -4 & 7 \end{array}\right)$

$\det A = -4 + 6 = 2 \neq 0$

$\det A_1 = \begin{vmatrix} 1 & -2 \\ 7 & -4 \end{vmatrix} = -4 + 14 = 10$

$\det A_2 = \begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix} = 7 - 3 = 4$

$x_1 = \frac{10}{2} = 5$

$x_2 = \frac{4}{2} = 2$

$$\left(\begin{array}{ccc|c} 2 & -1 & 5 & 10 \\ 1 & 1 & -3 & -2 \\ 2 & 4 & 1 & 1 \end{array}\right)$$

$\det A = \begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & -3 \\ 2 & 4 & 1 \end{vmatrix} = 43$ $\quad \det A_1 = \begin{vmatrix} 10 & -1 & 5 \\ -2 & 1 & 3 \\ 1 & 4 & 1 \end{vmatrix} = 146$

$\det A_2 = \begin{vmatrix} 2 & 10 & 5 \\ 1 & -2 & -3 \\ 2 & 1 & 1 \end{vmatrix} = 216$ $\quad \det A_3 = \begin{vmatrix} 2 & -1 & 10 \\ 1 & 1 & -2 \\ 2 & 4 & 1 \end{vmatrix} = -40$

$x_1 = \frac{146}{43}$; $x_2 = \frac{216}{43}$; $x_3 = -\frac{40}{43}$

7.2 a) $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 9 & 12 \\ 3 & 26 & 30 \end{pmatrix}$ (2) (3)

$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 0 \end{pmatrix}$

(4) $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 20 & 18 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix} = U$

б) $\begin{pmatrix} 1 & 1 & 2 & 4 \\ 2 & 5 & 8 & 9 \\ 3 & 18 & 29 & 18 \\ 4 & 22 & 53 & 33 \end{pmatrix}$ (2), (3), (4) $\to \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 15 & 23 & 6 \\ 0 & 18 & 45 & 17 \end{pmatrix}$ (5), (6)

$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 5 & 1 & 0 \\ 4 & 6 & 7 & 1 \end{pmatrix}$

(7) $\begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 21 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix} = U$

7.3 $\begin{pmatrix} 2 & 1 & 3 \\ 4 & 7 & 5 \\ 9 & 8 & 4 \end{pmatrix} \to \begin{matrix} x_2 & x_3 & x_1 \\ 1 & 3 & 2 \\ 7 & 5 & 11 \\ 8 & 4 & 9 \end{matrix}$ (7/8) $\to$ (5/4) $\begin{pmatrix} 1 & 3 & 2 \\ 0 & -16 & -3 \\ 0 & -20 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 2 \\ 0 & -16 & -3 \\ 0 & 0 & -13/4 \end{pmatrix} = U$

$L = \begin{pmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \\ 8 & 5/4 & 1 \end{pmatrix}$

$Ly = B$ $\quad y_2 = 1$; $y_3 = -6 - 7y_2 = -13$

$y_1 = -8y_2 - 5/4\, y_3 = -8 + \frac{5 \cdot 13}{4} = \frac{13}{4}$

$Ux = (1, -13, 13/4)$

$x_1 = -1$; $x_3 = (-13 - 3)/-16 = 1$

$x_2 = 1 - 2x_1 - 3x_3 = 1 + 2 - 3 = 0$ $\quad (-1; 0; 1)$

7,4

$$\left(\begin{array}{ccc|c} 81 & -45 & 45 & 531 \\ -45 & 50 & -15 & -460 \\ 45 & -15 & 38 & 193 \end{array}\right)$$

$$l_{11} = \sqrt{a_{11} - \sum_{k=1}^{1-1} l_{1k}^2} = \sqrt{a_{11}} = 9$$

$$l_{21} = \frac{a_{21}}{l_{11}} = -\frac{45}{9} = -5$$

$$l_{31} = \frac{a_{31}}{l_{11}} = \frac{45}{9} = 5$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2} = \sqrt{50 - 25} = 5$$

$$l_{32} = \frac{1}{l_{22}}\left(a_{32} - l_{21} l_{31}\right) = \frac{1}{5}\left(-15 - (-5)\cdot(5)\right) = \frac{-15+25}{5} = 2$$

$$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2} = \sqrt{38 - 25 - 4} = 3$$

$$L = \begin{pmatrix} 9 & 0 & 0 \\ -5 & 5 & 0 \\ 5 & 2 & 3 \end{pmatrix} \qquad L^T = \begin{pmatrix} 9 & -5 & 5 \\ 0 & 5 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$Ly = b$

$9y_1 = 531$, $y_1 = 59$

$-5y_1 + 5y_2 = -460$; $5y_2 = -460 + 295$; $y_2 = -33$

$5y_1 + 2y_2 + 3y_3 = 193$; $3y_3 = 193 + 2\cdot 33 - 5\cdot 59 = -36$, $y_3 = -12$

$L^T x = y$

$3x_3 = -12$, $x_3 = -4$

$5x_2 + 2x_3 = -33$; $5x_2 = -33 + 8 = -25$; $x_2 = -5$

$9x_1 = 5x_2 - 5x_3 + 59 = -25 + 20 + 59 = 54$, $x_1 = 6$ $(6; -5; -4)$