b) $f(x_2) \geq f(x_1) + \nabla f^T(x_1)(x_2 - x_1) + \frac{1}{2L} \|\nabla f(x_2) - \nabla f(x_1)\|_2^2$

Let define $g_{x_1}(x_2) \triangleq f(x_2) - f(x_1) - \nabla f^T(x_1)(x_2 - x_1)$

Since f is convex therefor: $f(x_2) \geq f(x_1) + \nabla f^T(x_1)(x_2 - x_1)$

$$\Rightarrow f(x_2) - f(x_1) - \nabla f^T(x_1)(x_2 - x_1) \geq 0$$

$$\Rightarrow g_{x_1}(x_2) \geq 0$$

In particular $g_{x_1}(x_1) = 0 \Rightarrow g_{x_1}(x_1) = \min_x g_{x_1}(x_2)$

and $\nabla g_{x_1}(x_1) = -\nabla f(x_1) + \nabla f(x_1) = 0$

from the optimality of x_1, it then follows that

$$g_{x_1}(x_1) \leq \min_\eta g_{x_1}(x_2 - \eta \nabla g_{x_1}(x_2))$$

(*)

$$= \min_\eta f(x_2 - \eta \nabla g_{x_1}(x_2)) - f(x_1) - \nabla f^T(x_1)(x_2 - \eta \nabla g_{x_1}(x_2) - x_1)$$

By deffinition of L-smooth we have:

$$f(x_2 - \eta \nabla g_{x_1}(x_2)) \leq f(x_2) + \nabla f^T(x_2)(-\eta \nabla g_{x_1}(x_2)) + \frac{L}{2} \|\eta \nabla g_{x_1}(x_2)\|_2^2$$

In the follows from (*) we have:

$$g_{x_1}(x_1) \leq \min_\eta f(x_2) + \nabla f^T(x_2)(-\eta \nabla g_{x_1}(x_2)) + \frac{L}{2} \|\eta \nabla g_{x_1}(x_2)\|_2^2$$

$$- f(x_1) - \nabla f^T(x_1)(x_2 - x_1 - \eta \nabla g_{x_1}(x_2))$$