f has L-Lipschitz continuous gradient

we then have:

$$\Rightarrow f(x_2) - f(x_1) - \nabla f^T(x_1)(x_2 - x_1) \leq \int_0^1 t L \|x_2 - x_1\|_2 \cdot \|x_2 - x_1\|_2 \, dt$$

$$\leq \int_0^1 t L \|x_2 - x_1\|_2^2 \, dt$$

$$\leq L \|x_2 - x_1\|_2^2 \int_0^1 t \, dt$$

$$\leq \frac{L}{2} \|x_2 - x_1\|_2^2$$

$$\Rightarrow f(x_2) - f(x_1) - \nabla f^T(x_1)(x_2 - x_1) \leq \frac{L}{2} \|x_2 - x_1\|_2^2$$

$$\Rightarrow f(x_2) \leq f(x_1) + \nabla f^T(x_1)(x_2 - x_1) + \frac{L}{2} \|x_2 - x_1\|_2^2$$