problem 1.5

In the convergence proof of GD with constant step size and strongly convex objective function proof the coercivity of the gradient:

$$\left(\nabla f(x) - \nabla f(y)\right)^T (x-y) \geq \frac{\mu L}{\mu + L} \|x-y\|_2^2 + \frac{1}{\mu + L} \|\nabla f(x) - \nabla f(y)\|_2^2$$

Solution:

Let define $g(x) = f(x) - \frac{\mu}{2}\|x\|_2^2$

from strong convexity of $f(x)$, we get $g(x)$ is convex

based on convergence of GD with constant step size and μ-strongly convex and L-smooth, we can say that the function $f(x)$, is differentiable and its gradient is L-Lipschitz continuous,

So we get $g(x)$ is also L-Lipschitz continuous and smooth with parameter $(L-\mu)$

Now we can apply inequality in problem 1.2c to $g(x)$: co-coercivity

$$\left(\nabla g(x) - \nabla g(y)\right)^T (x-y) \geq \frac{1}{L-\mu} \|\nabla g(x) - \nabla g(y)\|_2^2$$

$$\left(\nabla f(x) - \mu x - \nabla f(y) + \mu y\right)^T (x-y) \geq \frac{1}{L-\mu} \|\nabla f(x) - \nabla f(y) - \mu(x-y)\|_2^2$$

$$\left(\nabla f(x) - \nabla f(y) - \mu(x-y)\right)^T (x-y) \geq \frac{1}{L-\mu} \|\nabla f(x) - \nabla f(y) - \mu(x-y)\|_2^2$$