c) $\left(\nabla f(x_2) - \nabla f(x_1)\right)^T (x_2 - x_1) \geq \frac{1}{L} \| \nabla f(x_2) - \nabla f(x_1) \|_2^2$

Let define two convex functions f_{x_1}, f_{x_2} with $\mathbb{R}^n$ domain

$$\begin{cases} f_{x_1}(z) = f(z) - \nabla f^T(x_1) \cdot z \\ f_{x_2}(z) = f(z) - \nabla f^T(x_2) \cdot z \end{cases}$$

This two functions have L- Lipschitz continuous gradient

we know that if $f: \mathbb{R}^n \to \mathbb{R}$ and f has a minimizer x^* then from the inequality in problem 1.2 b we have:

$$f(z) \geqslant f(x^*) + \cancelto{0}{\nabla f^T(x^*)} (z - x^*) + \frac{1}{2L} \| \nabla f(z) - \cancelto{0}{\nabla f(x^*)} \|_2^2$$

$$\Rightarrow f(z) - f(x^*) \geqslant \frac{1}{2L} \| \nabla f(z) \|^2$$

$\Rightarrow z = x_1$ minimize $f_{x_1}(z)$

$$(1) \quad f(x_2) - f(x_1) - \nabla f^T(x_1)(x_2 - x_1) = f_{x_1}(x_2) - f_{x_1}(x_1)$$

$$\geqslant \frac{1}{2L} \| \nabla f_{x_1}(x_2) \|_2^2$$

$$= \frac{1}{2L} \| \nabla f(x_2) - \nabla f(x_1) \|_2^2$$

similarly $z = x_2$ minimize $f_{x_2}(z)$

$$(2) \quad f(x_1) - f(x_2) - \nabla f^T(x_2)(x_1 - x_2) \geqslant \frac{1}{2L} \| \nabla f(x_2) - \nabla f(x_1) \|_2^2$$

now we can combine 1, 2 inequality:

$$\left(\nabla f(x_2) - \nabla f(x_1)\right)^T (x_2 - x_1) \geqslant \frac{1}{L} \| \nabla f(x_2) - \nabla f(x_1) \|_2^2$$