

Problem 2.1 c) 1/1

Q: Is f strongly convex? If so, find a high μ .

A: f is twice differentiable, so it is strongly convex if $\nabla^2 f(w) \succeq \mu I, \forall w$

$$\text{We have } \nabla^2 f(w) = \frac{1}{N} \sum_i \nabla^2 f_i(w) + 2\lambda,$$

so we need μ :

$$x^T (\nabla^2 f(w) - \mu I) x \geq 0, \forall x, w$$

$$\Leftrightarrow x^T \nabla^2 f(w) x \geq \mu x^T x, \forall x, w$$

$$\rightarrow \mu \leq \frac{x^T \nabla^2 f(w) x}{x^T x} = \frac{1}{N} \sum_{i \in [N]} \underbrace{\frac{x^T \nabla^2 f_i(w) x}{x^T x}}_{(A)} + 2\lambda, \forall x \neq 0, w$$

where (A) is lower bounded by zero.

Thus, we need $\mu \leq 2\lambda$, \rightarrow choose $\mu = 2\lambda$.

Answer: $f(w)$ is μ -strongly convex, with μ

$$\mu = 2\lambda.$$