Finite Space Construction

Ugur Buyukdurak

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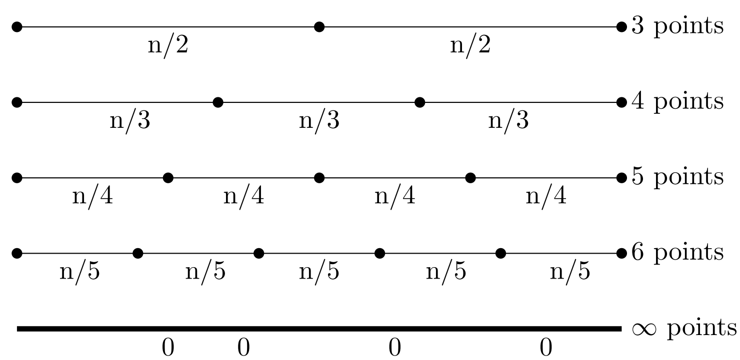
1 Introduction

The former teachings of mathematical spaces taught us that spaces can be both finite and infinite. For example, we are told that the real number line ($\mathbb{R}$) extends infinitely while constituting an infinite space. This paper says that when a space is constructed by the smallest entity, a point, then finite spaces of all dimensions are well defined, whereas infinite spaces are undefined. Hence, this paper is going to show you how an infinite number of points constitutes a finite space and that you cannot formally construct an infinite space from an infinite number of points.

2 Proof



As seen in the illustration above, as you keep adding points, the gaps will become equally smaller but will never become zero in length. However, you need to make the gaps zero in order to construct a continuous space, and the only way to do this is to use a limit that reaches infinity. But you first need a function for that.

When there is x number of points that are equally distributed on the line, there is $x - 1$ number of divisions and the length of each division is $n/(x - 1)$.

We are trying to fully close the gaps between all consecutive points to construct a finite continuous space; therefore, we use the limit:

$$\lim_{x\to\infty} \frac{n}{x-1} = 0$$

The above statement is a function that models the behavior in the illustration and tells us that as the number of points equally distributed on the line, x, reaches infinity, $(x-1)$ approaches infinity, thus making the gaps between all consecutive points infinitely approach zero in length —which is effectively zero, which means that as the number of points reaches infinity between any two points, they constitute a continuous finite line. You will see from the formula that if there is a finite number of points, the gaps never become zero in length, which means that a finite number of points cannot constitute a continuous finite line.

3 Results

We can show that, unlike finite spaces, infinite spaces cannot be formally constructed using the proof, that is, if the length of the line reaches infinity as well:

$$\lim_{\substack{n\to\infty \\ x\to\infty}} \frac{n}{x-1} = \frac{\infty}{\infty}$$

which is an indeterminate form, which, in its current form, is not solvable because n and x are not defined in terms of each other —they are unrelated. The two variables n and x, representing the length of the line and the number of points equally distributed on the line, respectively, do not have a direct physical relationship. Hence, the limit does not exist. When n is a constant, which means that the length is finite, the equation is well defined and equal to zero, which means that all gaps between points are fully closed.