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FYS4150

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# Introduction to numerical projects

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## Summary

- Tease the reader
- Write last

## Introduction

- Motivate the reader
- What have we done
- Structure of report

The aim of this project is to solve the one-dimensional Poisson equation with Dirichlet boundary conditions by rewriting it as a set of linear equations. We will be solving the equation

$$-u''(x) = f(x), \quad x \in (0, 1), \quad u(0) = u(1) = 0$$

where we define the discretized approximation to  $u$  as  $v_i$  with grid point  $x_i = ih$  in the interval from  $x_0$  to  $x_{n+1} = 1$ , and the step length as  $h = 1/(n+1)$ .

## Methods

- Describe methods and algorithms
- Explain
- Calculations to demonstrate the code, verify results(benchmarks)

With the boundary condition  $v_0 = v_{n+1} = 0$ , the approximation of the second derivative of  $u$  was written as

$$-\frac{v_{i+1} + v_{i-1} - 2v_i}{h^2} = f_i, \quad i = 1, \dots, n$$

where  $f_i = f(x)$ . We then rewrote the equation as a linear set of equations:

$$-(v_{i+1} + v_{i-1} - 2v_i) = h^2 f_i$$

We solved this equation for a few values of  $i$ . We also set  $h^2 f_i = d_i$ .

$i = 1$ :

$$\begin{aligned} -(v_{1+1} + v_{1-1} - 2v_1) &= d_1 \\ -(v_2 + v_0 - 2v_1) &= d_1 \\ -v_2 - 0 + 2v_1 &= d_1 \end{aligned}$$

$i = 2$ :

$$\begin{aligned} -(v_{2+1} + v_{2-1} - 2v_2) &= d_2 \\ -v_3 - v_1 + 2v_2 &= d_2 \end{aligned}$$

$i = 3$ :

$$\begin{aligned} -(v_{3+1} + v_{3-1} - 2v_3) &= d_3 \\ -v_4 - v_2 + 2v_3 &= d_3 \end{aligned}$$

We saw that this could be written as a linear set of equations  $\mathbf{A}\mathbf{v} = \mathbf{d}$ ,

$$\begin{pmatrix} 2 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & & -1 & 2 & -1 \\ 0 & \dots & & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \dots \\ v_{n-1} \\ v_n \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ \dots \\ d_{n-1} \\ d_n \end{pmatrix}$$

## Results

- Present results
- Critical discussion
- Put code etc. on GitHub and explain to reader where they can find it
- Explanatory figures with captions, labels etc.

## **Conclusion**

- Main findings
- Perspectives on improvement and future work

## **Appendix**

- Additional calculations
- Selected calculations with comments
- Code, if necessary
- Appendix can be pushed to GitHub!

## **References**

- Reference to material we based our work on(lecture notes etc.)
- Find scientific articles, books etc.
- BibTex - extract references online