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Classification and regression with neural networks

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Contents

Abstract

Classification of credit card data was studied by developing a logistic regression code and a feed-forward neural network. Both models were compared to each other, as well as tested against similar codes using Scikit-Learn and keras. The neural network was then modified to solve the Franke function using linear regression. The network was compared to the linear regression model developed in project 1, as well as tested against a keras model. It was found that the logistic regression code underperformed compared to the Scikit-Learn model, while both neural networks performed equally well as their keras counterparts. The optimal input parameters for each model was studied in detail, where it was found that a learning rate in the range of $\eta = [0.001 - 0.01]$ and regularisation hyper-parameter in the range of $\lambda = [0.0001, 0.001]$ provided the best results. For the classification problem, it was found that the neural network achieved a higher accuracy score than the logistic regression code. For the linear regression problem, the neural network achieved a similar R^2 score as the one obtained with Lasso regression from project 1.

1 Introduction

Machine learning is a fairly new concept to most people, but its early history dates back to 1943 when Warren McCulloch and Walter Pitts introduced the first neural network (Mayo et al., 2018). In the following decade, the Turing test was made known, and in 1958, Frank Rosenblatt designed the first artificial neural network called the *Perceptron*. However, it was not until the early 1980s the interest in neural networks gained new momentum. In 1986, the *back propagation* algorithm was developed, an important step in any neural network. Since then, the world has seen a large increase in both data and computing power. These factors have contributed to the rapid progress in machine learning that we see today.

The aim of this project is to study classification and regression problems by developing our own neural network and comparing it to a logistic regression code and the linear regression code developed in Project 1. We will use a credit card data set from UCI for the classification problem. This data set is well studied, which is useful when validating our models. For the linear regression problem we will revisit the Franke function from Project 1.

The structure of the report is as follows. In section 2, we start by explaining the data preprocessing, before taking a deep dive into logistic regression methods and the machine learning algorithms that go into the feed-forward neural network. The results are presented in section 3. Section 4 contains a discussion of our main findings, and section 5 gives the concluding remarks.

2 Method

2.1 Data preprocessing

The studied data set contains information on default payments, credit data, payment history, bill statements and other demographic factors of credit card clients in Taiwan from April 2005 to September 2005 (Yeh and Lien, 2009). The data set contains 23 explanatory variables and the response variable. Each variable contains 30000 observations, but not all of them are relevant. If the amount of bill statements and/or the amount of previous payments are 0, we remove the observations from all features of that client. Additionally, we remove uncategorised observations as long as they are not a major class. A categorical feature is an explanatory variable with levels. An example is the feature MARRIAGE, where 1=married, 2=single and 3=others. In this data set, however, many of the categorical features of the data set include levels that are not specified. Figure 1 shows histograms of the history of past payments, and shows that level 0 is a major class. Normally, this would be fine, but in this data set level 0 is undocumented. Most undocumented values can safely be removed, but in this case that would mean loosing the majority of observations. Hence, we decide to keep the observations as an NA class.

After stripping the data set for observations, we are left with ~ 28000 observations. We define features and targets, where the features are normalised using the standard scaler from Scikit-Learn. For the classification neural network, we one-hot encode the categorical features and the targets. The advantage of one-hot encoded categorical features is to remove assumed dependencies between categories. The categorical feature MARRIAGE can

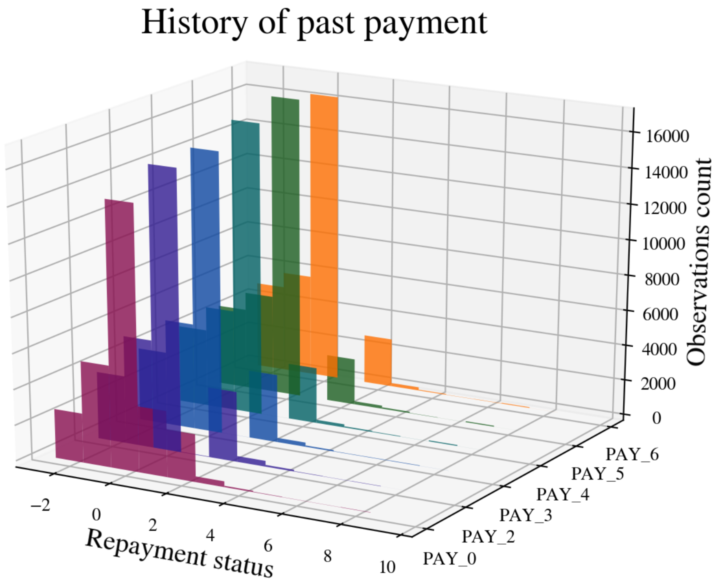

Figure 1: Histograms showing the history of past payment for the credit card holders.

be represented as

$$\begin{bmatrix}1\\2\\2\\\vdots\\1\\0\end{bmatrix} \rightarrow \begin{bmatrix}0&1&0\\0&0&1\\0&0&1\\\vdots&&\\0&1&0\\1&0&0\end{bmatrix}$$

while the binary targets can be represented as

$$\begin{bmatrix}1\\1\\0\\\vdots\\1\\0\end{bmatrix} \rightarrow \begin{bmatrix}0&1\\0&1\\1&0\\\vdots&\\0&1\\1&0\end{bmatrix}$$

In principle, all categorical features should be one-hot encoded. However, since the history of past payment features have 12 levels each, we find it too extensive to one-hot encode them (this would lengthen the x-dimension of the feature matrix by $12 \times 6!$). The matrix gets sparse, and the computation time inceases.

After preprocessing the data set, the ratio of default payment is 78.5%.

2.2 Logistic regression

Logistic regression is used when dealing with a categorical dependent variable, or *target*. In classification problems, the targets take the form of discrete variables such as categories. In many cases the target is binary, meaning that it can be either yes or no, true or false, positive or negative etc. The credit card data set that we are exploring has binary targets.

Logistic regression is an important first step in order to understand the neural network algorithms and how supervised deep learning works. In logistic regression, we want to find $\hat{\beta}$ such that the cost function $C(\hat{\beta})$ is minimised. This leads to gradient descent methods, which are important parts of nearly all machine learning algorithms.

Figure 2 shows a simple logistic regression model with three features. The $\hat{\beta}$-values of each feature are passed to an *activation function*, before the

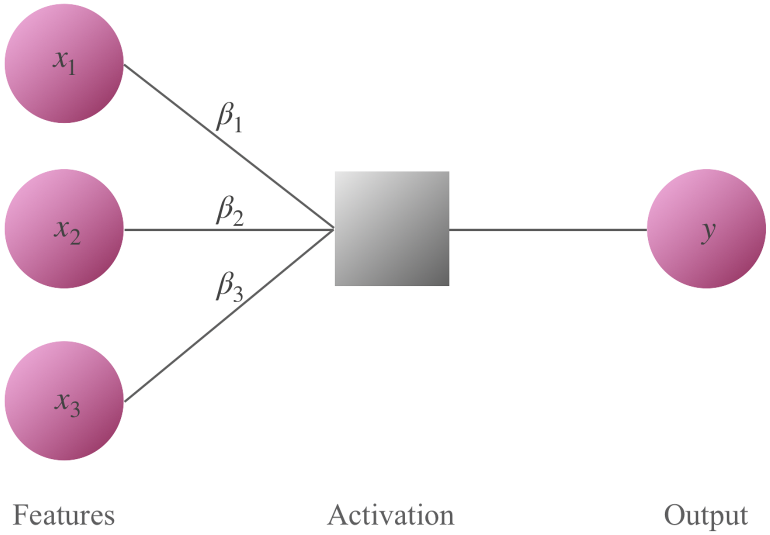

Figure 2: Simple illustration of a logistic regression model.

output is given. In the following, we go into the details of setting up a logistic regression model.

2.2.1 Cost function

In order to find the $\hat{\beta}$-values that minimise $C(\hat{\beta})$, we need to define the cost function. The target of our classification problem is binary, hence we start by defining the probability, or likelihood, for a given event. The probability of a data point belonging to a category $y = \{0, 1\}$ is given by the Sigmoid function

$$p(\theta) = \frac{1}{1 + e^{-\theta}} = \frac{e^{\theta}}{1 + e^{\theta}} \tag{1}$$

where θ is the prediction of y (in logistic regression, this reduces to the linear equation $\hat{\tilde{y}} = \hat{X}\hat{\beta}$). Further, the probabilities of y_i being either 0 or 1 can be expressed as

$$\begin{aligned} p(y_i = 1|\hat{X}^i, \hat{\beta}) &= \frac{e^{\beta_0 + x_1^i\beta_1 + x_2^i\beta_2 + \ldots + x_p^i\beta_p}}{1 + e^{\beta_0 + x_1^i\beta_1 + x_2^i\beta_2 + \ldots + x_p^i\beta_p}} \\ p(y_i = 0|\hat{X}^i, \hat{\beta}) &= 1 - p(y_i = 1|\hat{X}^i, \hat{\beta}) \end{aligned}$$

where p is the number of predictors. Next, we use the above probabilities to define the log-likelihood

$$C(\hat{\beta}) = -\sum_{i=1}^{n}(y_i \text{log} p(y_i = 1|\hat{X}^i, \hat{\beta}) + (1 - y_i)\text{log}[1 - p(y_i = 1|\hat{X}^i, \hat{\beta})]) \quad (2)$$

which is the cost function. In statistics, this equation is known as the *binary cross-entropy*, where y is the target (either 0 or 1) and $p(y)$ is the predicted probability of the data point being 1. ‘As an example, we see that when $y_i = 1$, the cost function reduces to

$$C(\hat{\beta}) = -\sum_{i=1}^{n} \text{log} p(y_i = 1|\hat{X}^i, \hat{\beta}),$$

and when $y_i = 0$ the cost function reduces to

$$C(\hat{\beta}) = -\sum_{i=1}^{n} \text{log}[1 - p(y_i = 1|\hat{X}^i, \hat{\beta})].$$

2.2.2 Gradient descent

Gradient descent is an optimisation algorithm used to find the local minima of a function. The idea is to minimise $C(\hat{\beta})$, where $\hat{\beta}$ is a vector with elements for each feature value (Marsland, 2014). We want to find a sequence of new points $\hat{\beta}(i)$ that move towards a solution. This is done by taking the derivative of the cost function. However, the direction of the derivative is not arbitrary. In steepest gradient descent, we always choose to go downhill as fast as possible for each point. This leaves us with the equation

$$\hat{\beta}_{k+1} = \hat{\beta}_k - \eta \nabla_\beta C(\hat{\beta}) \quad (3)$$

where η is the learning rate and the derivative of the cost function is

$$\nabla_\beta C(\hat{\beta}) = -\hat{X}^T(\hat{y} - \hat{p}) \quad (4)$$

Here, $\hat{X}$ is the feature matrix, $\hat{y}$ is a vector of the targets and $\hat{p}$ is a vector of the fitted probabilities (Sigmoid function). A drawback of the steepest gradient descent method is that many of the directions that $\hat{\beta}$ travel are directly towards the centre (local minima).

In general, gradient descent methods have limitations, and we can address some of the shortcomings by considering the stochastic gradient descent (SGD) method. We keep equation (3) as it is, but add stochasticity by randomly shuffle the data. Stochastic gradient descent converges faster for larger data sets, but since the method uses one example at a time, the computations can slow down (Patrikar, 2019). By adding mini-batches to the model, we can vectorise the SGD, making the computations faster. Additionally, the gradient is computed against more training samples, which in turn means that it is averaged over more training samples. This may then lead to a smoother convergence.

For each *epoch*[1], we loop over all the mini-batches and calculate $\hat{\beta}_{k+1}$ using equation (3). After we have calculated $\hat{\beta}_{k+1}$ for the training samples in all the mini-batches, we randomly shuffle the data before continuing to the next epoch. The process is repreated until we reach the end of the last epoch.

To summarise and connect the above theory to Figure 2, the logistic regression model for classification problems is as follows:

- Divide the data into features and targets.
- Split the data into training and test samples.
- Feed a randomly generated $\hat{\beta}$-value to the activation layer, where we for each epoch calculate $\hat{\beta}_{k+1}$ for the training samples in all the mini-batches. Randomly shuffle the training data.
- Calculate the prediction of $\hat{y}$.

2.3 Neural network

A neural network is a collection of neurons that can learn to recognise patterns in data. In biological terms, a neuron is a nerve cell. They are the processing units of the brain, and each neuron is a separate processor performing simple tasks (Marsland, 2014). An artificial neuron sums the incoming signals, and an activation function/threshold decides whether or not an output is given. If the threshold is not overcome, the neuron has zero output (Hjorth-Jensen, 2019).

[1] A single iteration over all the data.

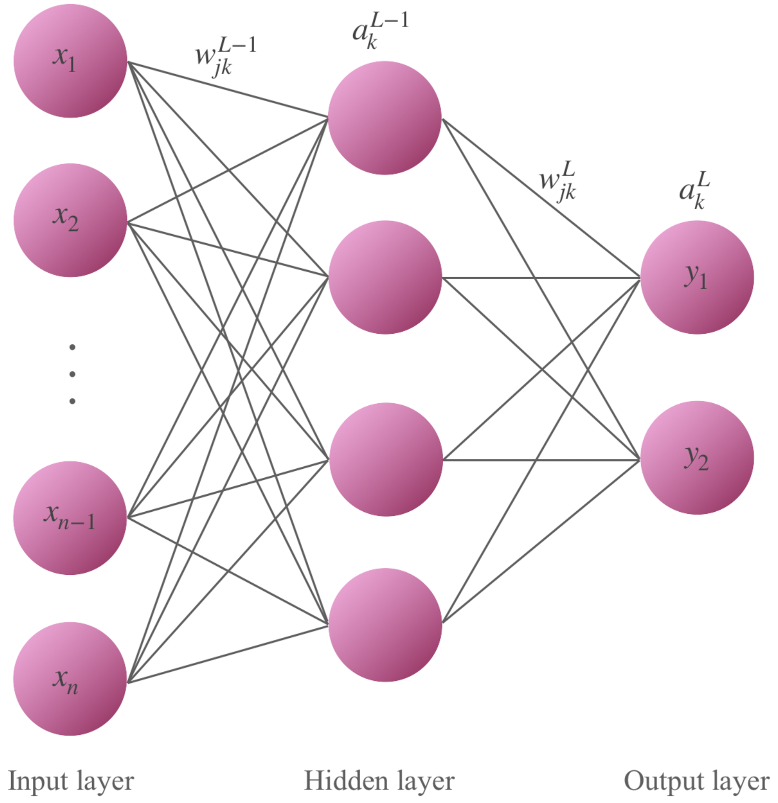

Figure 3: Simple illustration of a feed-forward neural network with one hidden layer.

In this project, we are creating a feed-forward neural network. That means that the information moves forward through the layers. Figure 3 shows a Multi-layer Perceptron (MLP) with an input layer consisting of n neurons, one hidden layer consisting of 4 neurons and an output layer consisting of 2 neurons. An MLP is a fully-connected feed-forward neural network with three or more layers, and consists of neurons with non-linear activation functions (such as the Sigmoid function). Training the MLP consists of finding the outputs given the inputs and current weights and biases, and then updating the weigths and biases according to the output error. In the following, we will take a deeper dive into the mathematical model comprising the Multi-layer Perceptron.

2.3.1 Feed-forward pass

The feed-forward pass refers to the process of passing values from the input neurons through the network, layer by layer, until the output layer is reached. The activation values of each neuron in each layer is defined as

$$z_j^l = \sum_{k=1}^{N_{l-1}} w_{jk}^l a_k^{l-1} + b_j^l \tag{5}$$

where $\hat{w}^l$ are the weights, $\hat{a}^{l-1}$ are the forward passes/outputs from the previous layer, $\hat{b}^l$ are the biases and N_{l-1} represents the total number of neurons in layer $l-1$. With the activation values from the current layer, we can define the output $\hat{a}^l$ of the current layer as

$$\hat{a}^l = f(\hat{z}^l) \tag{6}$$

where the function f is the *activation function*. In the logistic regression model, we used the Sigmoid function in equation (1) as our activation function. This is a non-linear activation function, and we can use it in order to obtain the output of each neuron in each layer such as

$$a_j^l = f(z_j^l) = \frac{e^{z_j^l}}{1 + e^{z_j^l}} \tag{7}$$

In the case of linear regression, we change the activation function from a Sigmoid function to a Rectified Linear Unit (ReLU). The ReLU activation function is defined as

$$f(z_j^l) = \max(0, z_j^l), \tag{8}$$

and is linear for all positive values of z_j^l and zero for all negative values of z_j^l.

2.3.2 Back propagation

Back propagation refers to the algorithm used to train a feed-forward neural network. Back propagation computes the gradient of the cost function with respect to the weights of the network. In the following section, we derive the back propagation algorithm by first defining the cost function. Then, we compute the gradients and error terms, before updating the weights and biases.

For the classification problem, we use binary cross-entropy defined as

$$C(\hat{W}^L) = -\sum_{k=1}^{N_L} (y_k \text{log} a_k^L + (1 - y_k)\text{log}[1 - a_k^L]) \tag{9}$$

where y_k is the target and a_k^L is the output from neuron k in layer L (the output layer). The derivative of the cost function with respect to the weights can be written as

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^L} = \frac{\partial C(\hat{W}^L)}{\partial a_k^L} \frac{\partial a_k^L}{\partial w_{jk}^L} \tag{10}$$

where we have applied the chain rule. We can apply the chain rule to last derivative on the right-hand side as well, obtaining

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^L} = \frac{\partial C(\hat{W}^L)}{\partial a_k^L} \frac{\partial a_k^L}{\partial z_k^L} \frac{\partial z_k^L}{\partial w_{jk}^L} \tag{11}$$

Now, we have three derivatives that we can solve. The first derivative on the right-hand side of equation (11) is solved as

$$\begin{aligned} \frac{\partial C(\hat{W}^L)}{\partial a_k^L} &= -\frac{y_k}{a_k^L} + \left(\frac{1 - y_k}{1 - a_k^L}\right) \\ &= \frac{a_k^L(1 - y_k)}{a_k^L(1 - a_k^L)} - \frac{y_k(1 - a_k^L)}{a_k^L(1 - a_k^L)} \\ &= \frac{a_k^L(1 - y_k) - y_k(1 - a_k^L)}{a_k^L(1 - a_k^L)} \\ &= \frac{a_k^L - y_k}{a_k^L(1 - a_k^L)} \end{aligned}$$

The second derivative on the right-hand side of equation (11) can be solved as

$$\begin{aligned} \frac{\partial a_k^L}{\partial z_k^L} &= f'(z_k^L) \\ &= a_k^L(1 - a_k^L) \end{aligned}$$

The last term on the right-hand side of equation (11) is just

$$\frac{\partial z_k^L}{\partial w_{jk}^L} = a_k^{L-1}$$

Next, we insert the separate solutions of the derivatives into equation (11) and obtain

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^L} = a_k^{L-1}(a_k^L - y_k) \tag{12}$$

We can then define the output error

$$\delta_k^L = f'(z_k^L)\frac{\partial C(\hat{W}^L)}{\partial a_k^L} = a_k^L - y_k \tag{13}$$

This will be the starting equation of the back propagation algorithm. The error terms of the hidden layers now be defined as

$$\delta_j^l = f'(z_j^l)\frac{\partial C(\hat{W}^l)}{\partial a_j^l} = \frac{\partial a_j^L}{\partial z_j^l}\frac{\partial C(\hat{W}^l)}{\partial a_j^l} = \frac{\partial C(\hat{W}^l)}{\partial z_j^l}$$

In addition, the error can also be interpreted in in terms of biases b_k^L, such as

$$\delta_j^l = \frac{\partial C(\hat{W}^l)}{\partial b_j^l}\frac{\partial b_j^l)}{\partial z_j^l} = \frac{\partial C(\hat{W}^l)}{\partial b_j^l}$$

However, we want to express the error in terms of the equation for layer $l+1$. This can be done by employing the chain rule so that

$$\delta_j^l = \sum_k^{N_{l+1}} \frac{C(\hat{W}^l)}{\partial z_k^{l+1}}\frac{\partial z_k^{l+1}}{\partial z_j^l} = \sum_k^{N_{l+1}} \delta_k^{l+1}\frac{\partial z_k^{l+1}}{\partial z_j^l}$$

From equation (5), we see that the derivative $\frac{\partial z_k^{l+1}}{\partial z_j^l} = w_{kj}^{l+1}f'(z_j^l)$. The hidden layer error can finally be written as

$$\delta_j^l = \sum_k^{N_{l+1}} \delta_k^{l+1}w_{kj}^{l+1}f'(z_j^l) \tag{14}$$

After computing the errors of the hidden layers, the weights and biases are updated using stochastic gradient descent as

$$w_{jk}^l \leftarrow w_{jk}^l - \eta\delta_j^l a_k^{l-1} \tag{15}$$

and

$$b_j^l \leftarrow b_j^l - \eta \delta_j^l \tag{16}$$

For the linear regression neural network model, we define the cost function as the Mean Squared Error

$$C(\hat{W}^L) = \frac{1}{2} \sum_{k=1}^{N_L} (y_k - a_k^L)^2 \tag{17}$$

where the derivative is given as

$$\frac{\partial C(\hat{W}^L)}{\partial a_k^L} = a_k^L - y_k \tag{18}$$

The derivative of the ReLU activation function is defined as

$$f'(z_k^L) = \begin{cases} 1 & \text{if } z_k^L > 0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

2.4 Regularisation

The model is overfitted when the network performs well on training data, but poorly on the test data. The network has a high variance, and can not generalise well to the data it has been trained on (Peixeiro, 2019). One way of addressing overfitting is to add regularisation to the cost function. In addition to minimising the cost, we also want to penalise the large weights by adding an L_2 regularisation term. We can redefine any cost function as

$$C(\hat{W}^L) = C(\hat{W}^L) + \frac{\lambda}{2N_L} ||\hat{W}^L||^2 \tag{20}$$

where λ is a regularisation hyper-parameter that can be tuned. If the weights are large, they will be penalised more if λ is large as well. On the other hand, if λ is small, the effect of the regularisation decreases. Because the size of the weights are reduced, the effect of the activation function decreases and a less complex function is fitted to the data.

Since we have added regularisation to the cost function, we need to reformulate the update of the weights. The derivative of the cost function then becomes

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^l} = \delta_j^l a_j^{l-1} + \lambda w_{jk}^l \tag{21}$$

where the weights are updated as

$$w_{jk}^{l} \leftarrow w_{jk}^{l} - \eta(\delta_j^l a_k^{l-1} + \lambda w_{jk}^{l}) \tag{22}$$

2.5 Model evaluation

The performance of the classification models are measured using an *accuracy score*. The accurcy score is defined as

$$\text{Accuracy} = \frac{\sum_{i=1}^{n} I(t_i = y_i)}{n} \tag{23}$$

where t_i represents the target and y_i represents the output of either the logistic regression code or the classification neural network. The function I is an indicator function, and is defined as

$$I = \begin{cases} 1 & \text{if } t_i = y_i \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

This means that a classifier that is able to predict all the targets has an accuracy of 100%.

The model performance of the linear regression neural network is measured using the R^2 score given by

$$R^2(t_i, y_i) = 1 - \frac{\sum_{i=1}^{n}(t_i - y_i)^2}{\sum_{i=1}^{n}(t_i - \bar{y}_i)^2} \tag{25}$$

where $\bar{y}_i$ is the mean value of the predicted variable.

Further, we can use the accuracy score to verify that the neural network is learning. By reducing the data set to a few samples and train the neural network on this, the training set should immediately reach an accuracy of 100% (the training set is overfitted). At the same time, the accuracy of the test set should go to 0%. As a second test, we keep the full data set, but shuffle the labels. By doing this, the neural network can only learn by memorising the training set, and the training cost decrease slowly. The test cost will increase quite rapidly, until it reaches the random chance test cost[2].

The above tests are relatively quick and easy to implement while developing the code, and work as a way of benchmarking the neural network. However, the best method of validation is to compare the obtained results against a similar code. In this project, we have tried to replicate our neural network with keras.

[2] The *Golden Tests* have been found at stackexchange.

2.6 Code

In the previous project, we discussed bootstrapping as a resampling method. In this project, we use k-fold cross-validation as the main assessment method. The training and test data are randomly partitioned into k subsamples. The model is then trained on each of the training subsamples, and tested on the test subsamples. The variance is reduced since we are using different cross-validation training and test sets.

All the developed codes in this project utilise k-fold cross-validation. The input features $\hat{X}$ and targets $\hat{y}$ are randomly shuffled and split into training and test subsamples. For each k-fold, we loop over epochs, and for each epoch we calculate the new weights ($\hat{\beta}$ for logistic and linear regression, $\hat{w}^l$ for neural networks) using stochastic gradient descent with mini-batches (see Section 2.2.2). Below is a pseudocode of the k-fold cross-validation of the neural network.

```python
import numpy as np
from sklearn.model_selection import KFold

X, y = data_preprocessing()

# shuffle X and y
random_index = np.arange(X.shape[0])
np.random.shuffle(random_index)
X = X[random_index,:]
y = y[random_index,:]

kfolds = KFold(n_splits=n_folds)

train_index = []
test_index  = []
for i_train, i_test in kfolds.split(X):
  train_index.append(np.array(i_train))
  test_index.append(np.array(i_test))

for k in range(n_folds):
  # split into training and test data
  X_train = X[train_index[k]]
  y_train = y[train_index[k]]

  X_test = X[test_index[k]]
  y_test = y[test_index[k]]
```

```
# define weights and biases
weights_biases()

for j in range(len(epochs)):
  for i in range(0,X_train.shape[0],minibatch_size):
    feed_forward(X_train[i:i+minibatch_size,:])
    backpropagation(y_train[i:i+minibatch_size,:])

# prediction from training and test data
feed_forward(X_train)
feed_forward(X_test)
```

3 Results

3.1 Logistic regression

The performance of our logistic regression model is measured using the accuracy score in equation (23). In order to find the input parameters that give the highest accuracy score, we experiment with different values of the learning rate η. Figure 4 shows the accuracy score for values of $\eta \in [10^{-4}, 10^{0}]$. We have used 10-fold cross-validation with 100 epochs and 100 minibatches. When the learning rate is small, the model converges slowly. In the figure, we see that in order to obtain a higher accuray score, we need to run the code for more epochs. Oppositely, when the learning rate is large, the model converges faster, and we risk overfitting the model if the number of epochs is large. However, there exists a middle ground where the model learns at a relatively good pace. This happens when $\eta = 0.001$. We observe that the number of epochs can be reduced by approximately half, as the accuracy flattens around 80%.

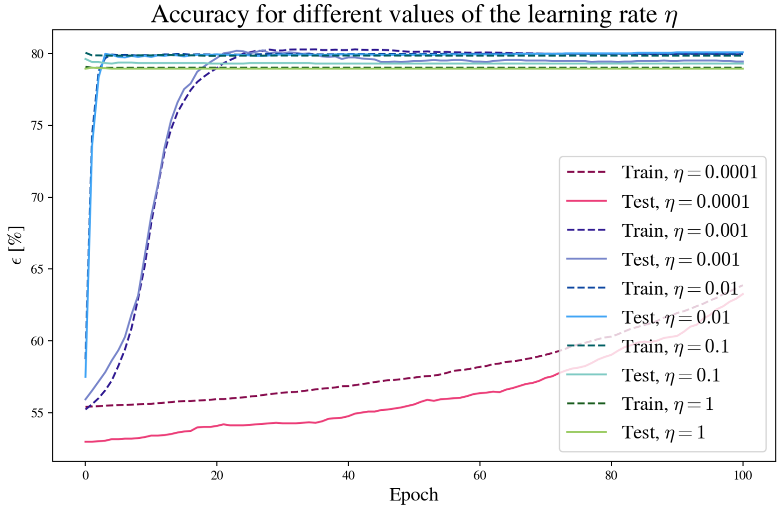

Figure 4: Accuracy of the logistic regression model for different values of the learning rate η.

Results of experiments with different mini-batch sizes are shown in figure 5. When we decrease the size of the mini-batches, we decrease the amount of data that goes into each batch. As a result, we loop over a larger amount of mini-batches every epoch, slowing down the computation time. The figure clearly illustrates that larger mini-batch sizes do not affect the accuracy of the model negatively. For the sake of computation time, we can therefore safely use a mini-batch size of 150.

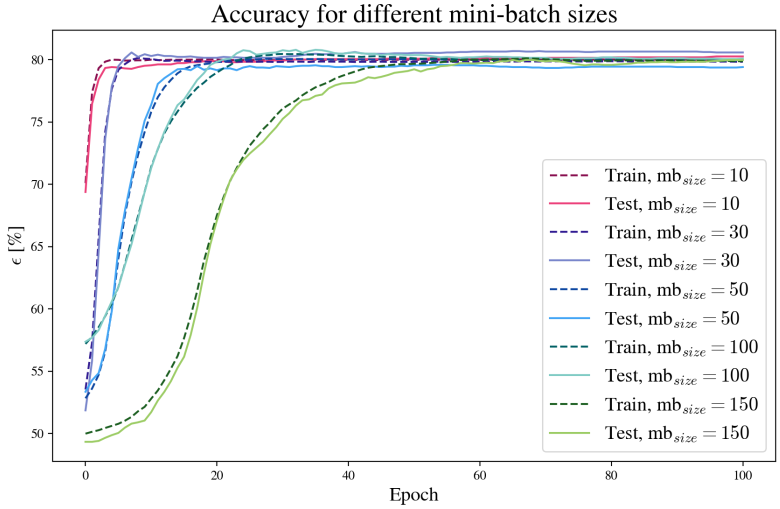

Figure 5: Accuracy of the logistic regression model for different mini-batch sizes.

Based on the results of figures 4 and 5, we run the logistic regression model with $\eta = 0.001$, 60 epochs, 150 mini-batches and 10 folds. For each k-fold, we check the last accuracy score against the previous ones to find the maximum and minimum accuracy from the cross-validation. The results are shown in figures 6 and 7. Figure 6 is a direct illustration of the maximum and minimum accuracy for the training and test data, while figure 7 shows the cost as a function of epochs for the maximum and minimum accuracy. Both figures show that as the number of epochs increase, the accuracy and cost converges to their maxima and minima, respectively.

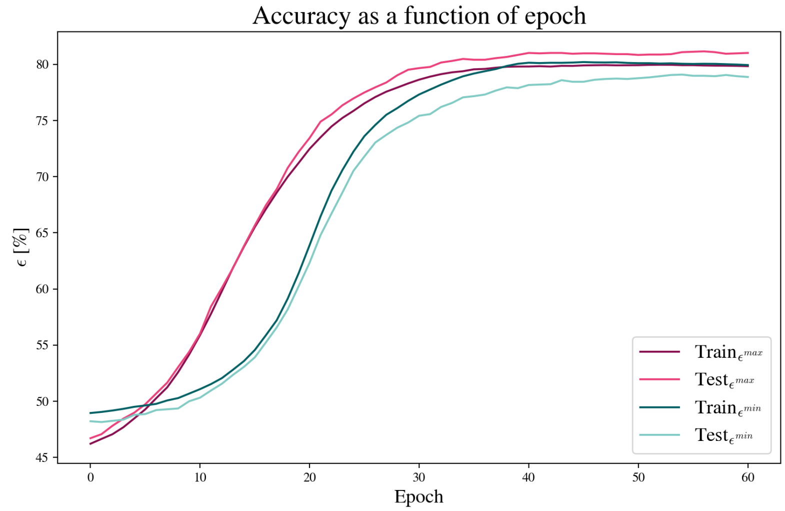

Figure 6: Maximum and minimum accuracy from the logistic regression model.

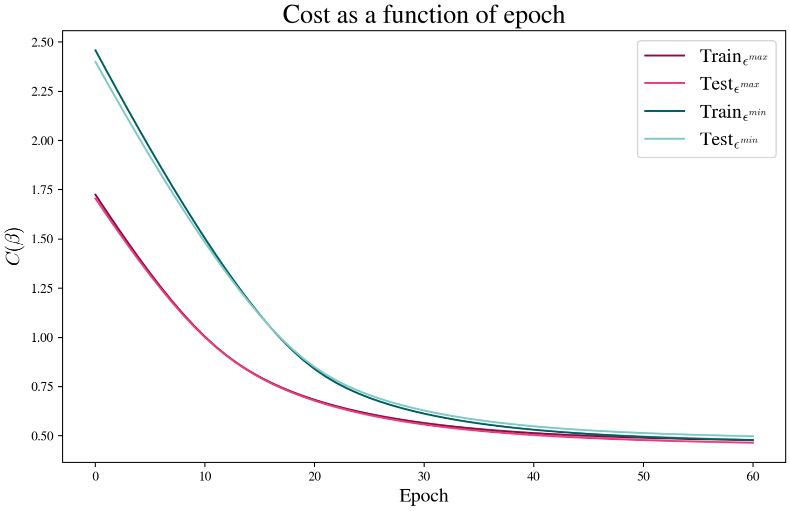

Figure 7: Cost as a function of epochs for the logistic regression model. The cost originates from the same iterations as the maximum and minimum accuracy scores.

We verify our model by comparing it to a Stochastic Gradient Descent Classifier (SGDClassifier) from Scikit-Learn. Figure 8 shows the training and test accuracy from both models, where the last training and test accuracies from each k-fold have been stored. Although the accuracy difference between the two models is relatively small, the figure shows that the SGDClassifier from Scikit-Learn is able to produce better accuracy scores.

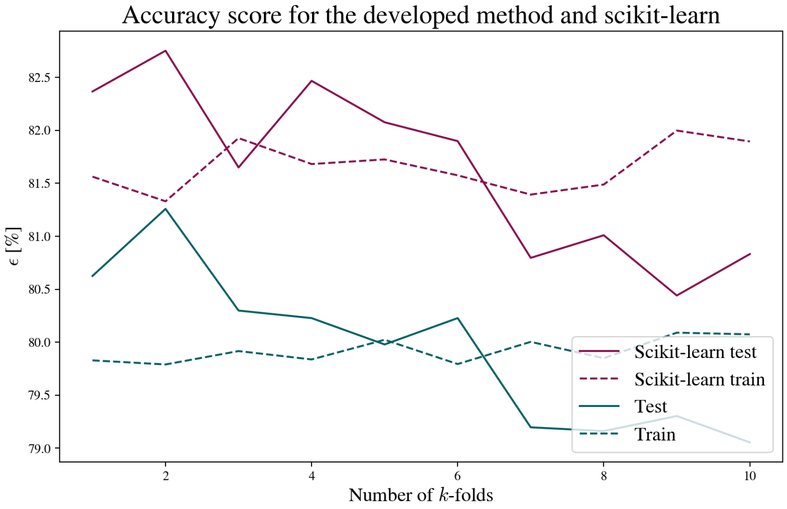

Figure 8: Comparison of the developed logistic regression model and Scikit-Learn.

3.2 Neural networks

3.2.1 Verification of the neural network

We verify that the neural network is able to learn by training the network with few samples. The result is seen in figure 10, where the neural network immediately overfit the training set. The training accuracy goes to 100% immediately, while the test accuracy stays at 0%. Figure 10 shows the accuracy of the neural network for a reduced portion of the total data set. The labels are shuffled, and the only way for the neural network to learn is by memorising the training set. The test set should reach the ratio of default payments, which is $\sim 68\%$ for the reduced data set. From the figure, we see that this indeed happens, and we are confident that our neural network is able to learn.

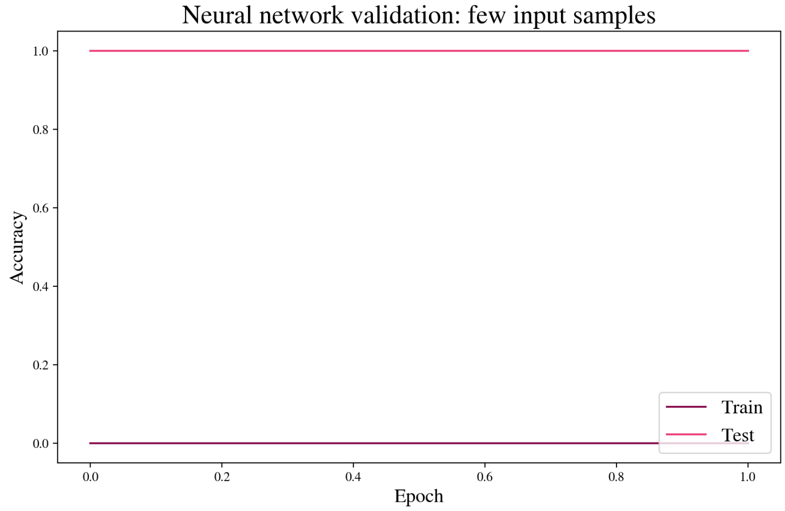

Figure 9: Accuracy of neural network with few samples.

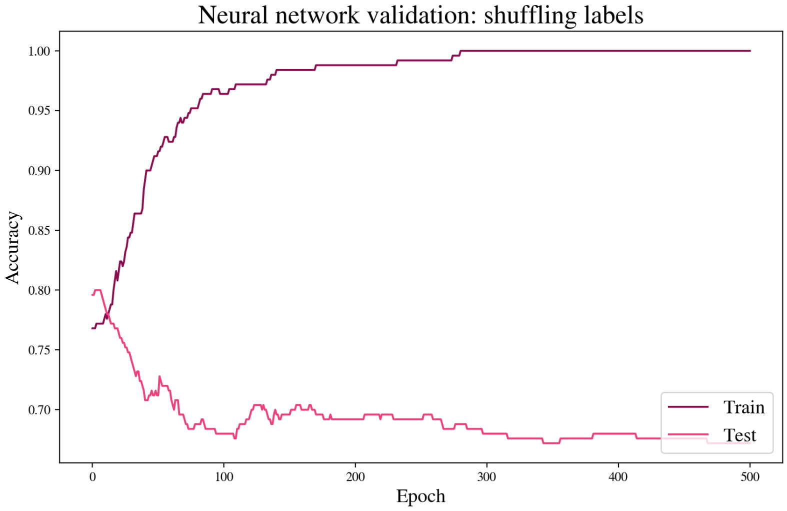

Figure 10: Accuracy of neural network when the labels are shuffled.

3.2.2 Classification

As with the logistic regression model, we want to find the input parameters that give the highest accuracy score. Figure 11 shows the accuracy scores for different values of λ. The model has been run with a constant learning rate $\eta = 0.01$, 100 mini-batches, 50 epochs and 10 folds for the cross-validation. The figure shows that increasing values of λ decreases the accuracy, and vice versa. When $\lambda = 0.0001$, we obtain the largest accuracy score. As a matter of fact, the number of epochs does not appear to be sufficient enough in order to obtain the maximum accuracy score, hence we should increase it.

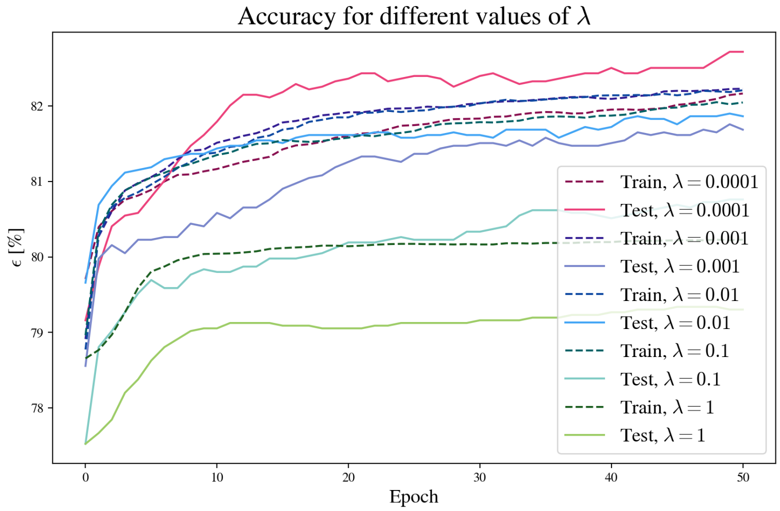

Figure 11: Accuracy score of the neural network for different regularisation hyper-parameters λ.

Further, we explore the effects of changing the learning rate η. We exclude the regularisation hyper-parameter ($\lambda = 0$), and set the mini-batch size to 100, number of epochs to 50 and use 10-fold cross-validation. The result is seen in figure 12. By employing the smallest learning rate, the accuracy is kept more or less constant for the first $8-9$ epochs. Additionally, the lowest learning rate increases the computation time, hence we dismiss this value. Larger learning rates produce higher accuracies, but we also note that the accuracy is nearly constant over all epochs. Based on the figure, we settle at a middle ground where $\eta = 0.01$.

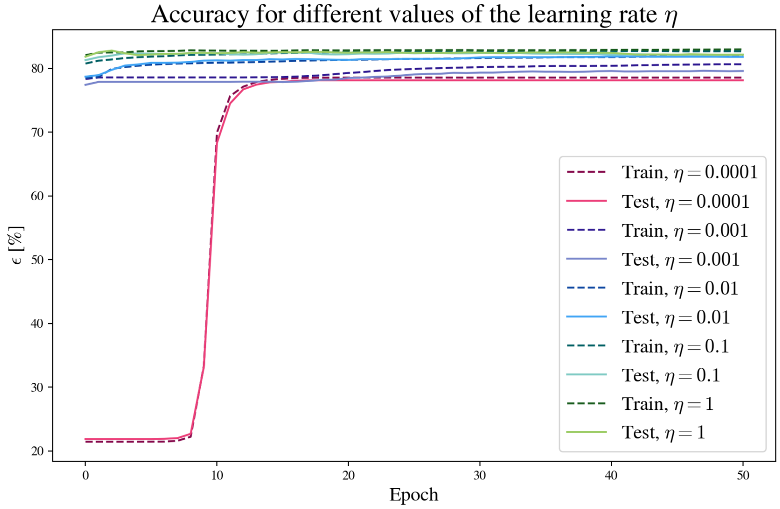

Figure 12: Accuracy of the neural network for different values of the learning rate η.

We observe what happens when we vary the size of the mini-batches using a constant learning rate $\eta = 0.01$, zero regularisation, 50 epochs and 10-fold cross-validation. Figure 13 shows that there are larger differences in the accuracy scores compared to figure 5. We should aim at using smaller mini-batch sizes, but still keep in mind that this increases the computation time. According to the figure, a mini-batch size of ~ 50 should yield a relatively good accuracy score.

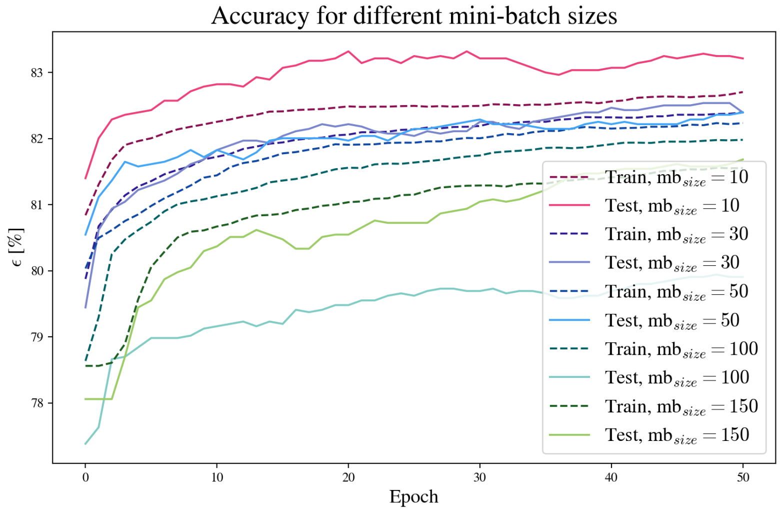

Figure 13: Accuracy of the neural network for different mini-batch sizes.

Based on the above assessment of the input parameters, we have run our network with $\eta = 0.01$, $\lambda = 0.0001$, a mini-batch size of 50, 100 epochs and 10-fold cross-validation. Figure 14 illustrates the maximum and minimum accuracy from the cross-validation, where we see that the highest accuracy score obtained from the network is $\sim 83.5\%$. Figure 15 shows the cost from the same iterations as the maximum and minimum accuracy.

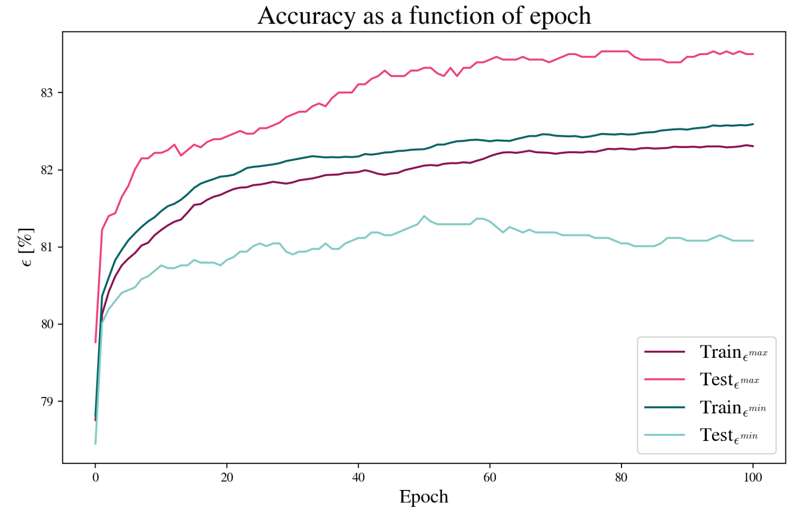

Figure 14: Maximum and minimum accuracy as a function of epochs.

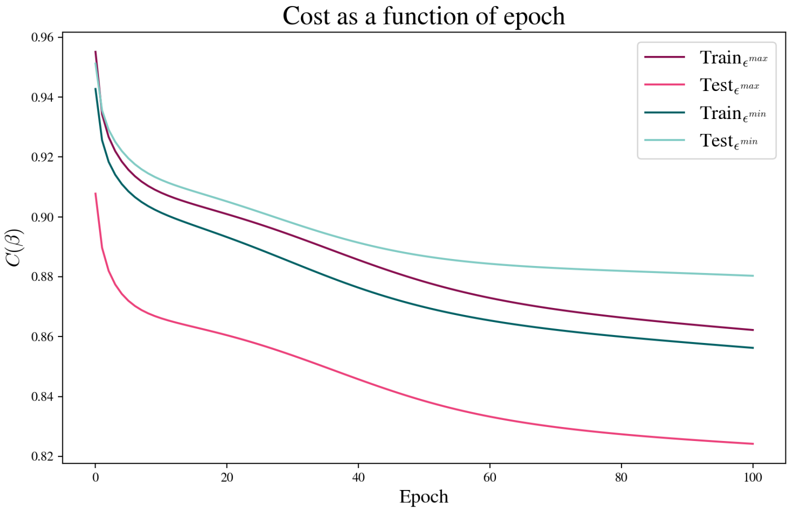

Figure 15: Cost for the maximum and minimum accuracy iterations as a function of epochs.

We compare our developed neural network with a neural network from keras. The result is seen in figure 16, where the performance of both networks are equally good.

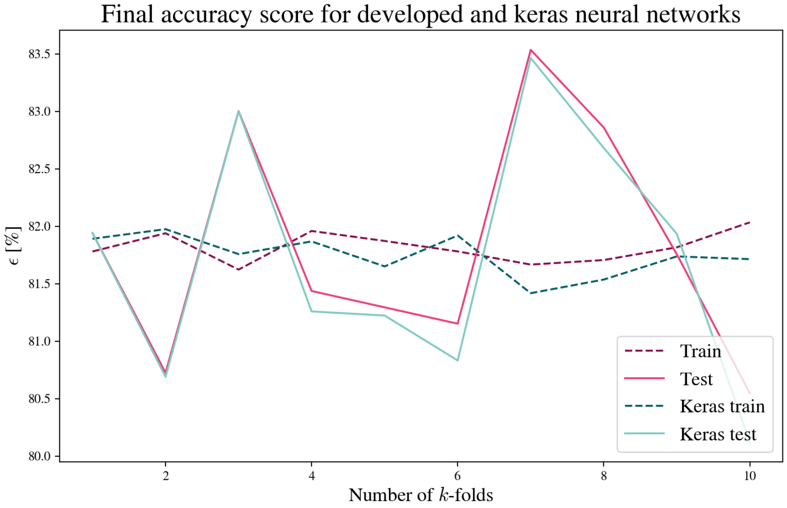

Figure 16: Final accuracy output of the developed and keras neural networks.

3.2.3 Linear regression

For the linear regression neural network, we use the Franke function as input data. Compared to the credit card data, preprocessing of the Franke function is relatively easy. We define a meshgrid that is (100×100), and calculate the Franke function. It is important to note that the entire Franke function acts as our target, while the meshgrid is the features. The network performance is measured using the R^2 score in equation (25).

In order to find the best input parameters of the network, we test different values of the regularisation hyper-parameter λ, learning rate η and size of the mini-batches. Figure 17 shows the R^2 score as a function of epochs for different values of λ. The network was run with $\eta = 0.01$, a mini-batch size of 100, 300 epochs and 10 folds for cross-validation. For the largest value of λ, the R^2 score converges at 60%. Further, smaller λ-values lead to higher R^2 scores.

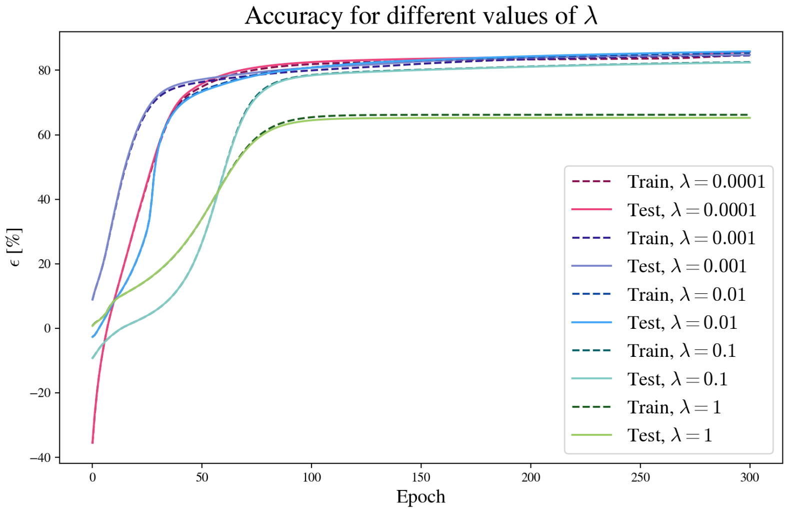

Figure 17: R^2 score of the linear regression neural network for different regularisation hyper-parameters λ.

Figure 18 shows the R^2 score when varying the learning rate, η. The network was run with zero regularisation, a mini-batch size of 100, 300 epochs and 10 folds. We see that the R^2 score for the smallest learning rate is almost constant. As the learning rate increases, the R^2 score converges to a maximum. Based on the figure, a learning rate of $\eta = 0.01$ should provide a sufficiently good R^2 score.

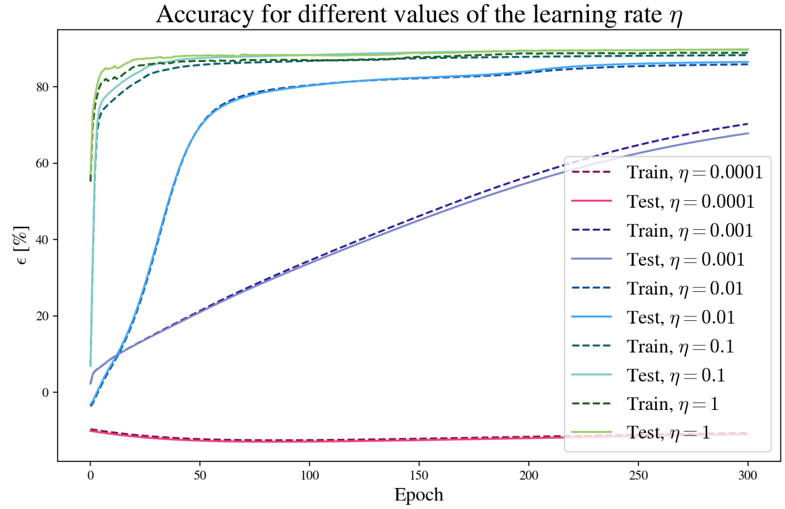

Figure 18: R^2 score of the linear regression neural network for different values of the learning rate η.

Figure 19 shows the R^2 score when varying the mini-batch size. The network was run with a learning rate of $\eta = 0.01$, zero regularisation, 300 epochs and 10 folds. The figure shows that smaller mini-batch sizes yield higher R^2 scores, but the difference in R^2 score between the sizes 10 and 150 is not large. Increasing the mini-batch size speeds up the computation time, hence in this case it is advantageous to choose a larger size.

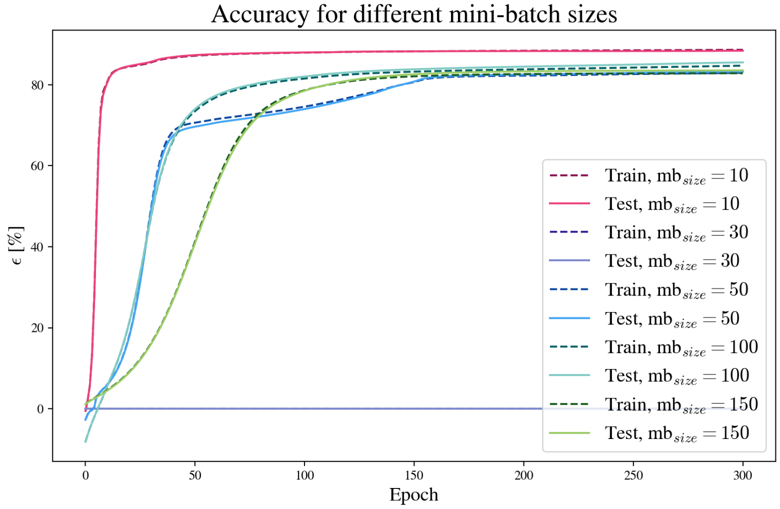

Figure 19: R^2 score of the linear regression neural network for different mini-batch sizes.

Based on the above assessment of the input parameters, we run our network with $\eta = 0.01$, $\lambda = 0.001$, mini-batch size of 150, 200 epochs and 10 folds. The results are displayed in figures 20, 21 and 22, showing the R^2 score, cost and prediction, respectively. Compared to the previous models of this project, the cost and maximum and minimum R^2 scores from the k-fold cross-validation takes on the same behavior. While the R^2 score increases and converges to a maximum, the cost function decreases and converges to a minimum. The most interesting result of the linear regression is seen in figure 22. The figure shows the Franke function together with the prediction from the neural network, where the target Franke function includes noise. The network is able to predict the most prominent features, such as large peaks. However, it is not able to predict the smaller features, such as the small dip at the bottom of the Franke function.

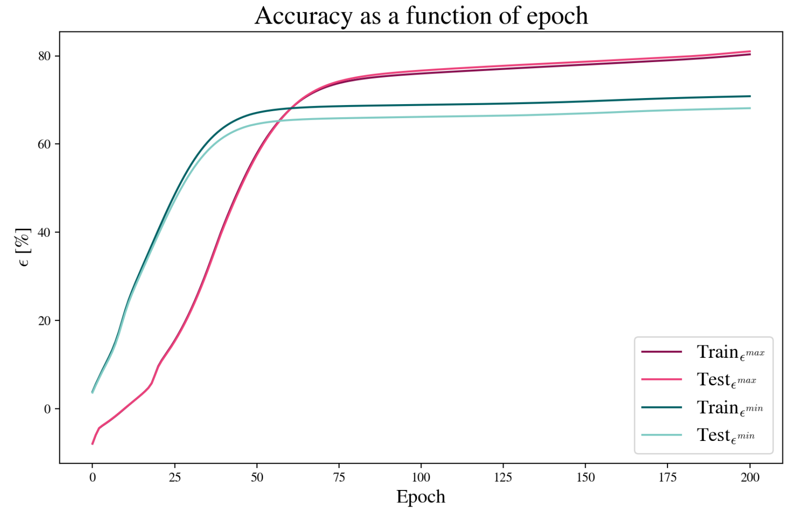

Figure 20: Maximum and minimum R^2 scores as a function of epochs.

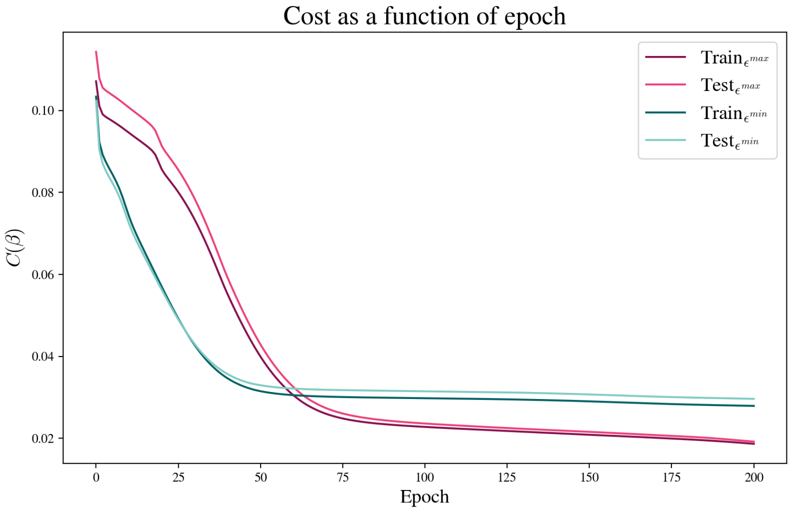

Figure 21: Cost for the maximum and minimum R^2 scores as a function of epochs.

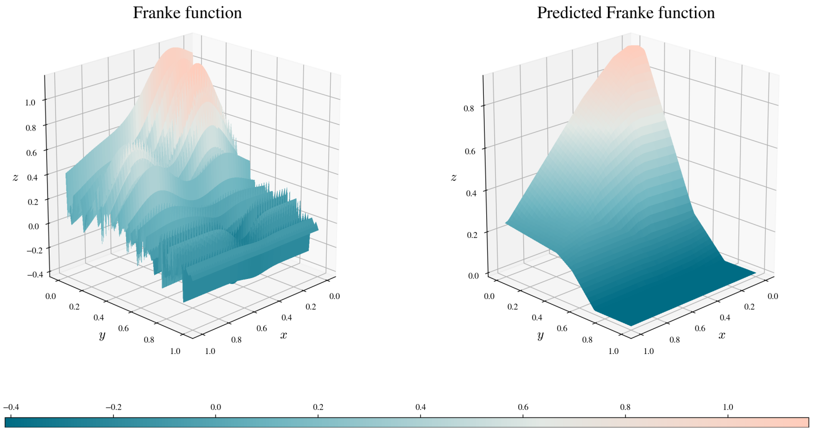

Figure 22: Target and predicted surface from the linear regression neural network.

We compare the developed neural network for linear regression with a neural network from keras. The result is seen in figure 23, where we have stored the final output R^2 score for each k-fold. The figure shows that for most k-folds, both networks perform equally well. There is an exception at the 5th k-fold, where the developed neural network has a dip in the R^2 score that deviates from the keras model.

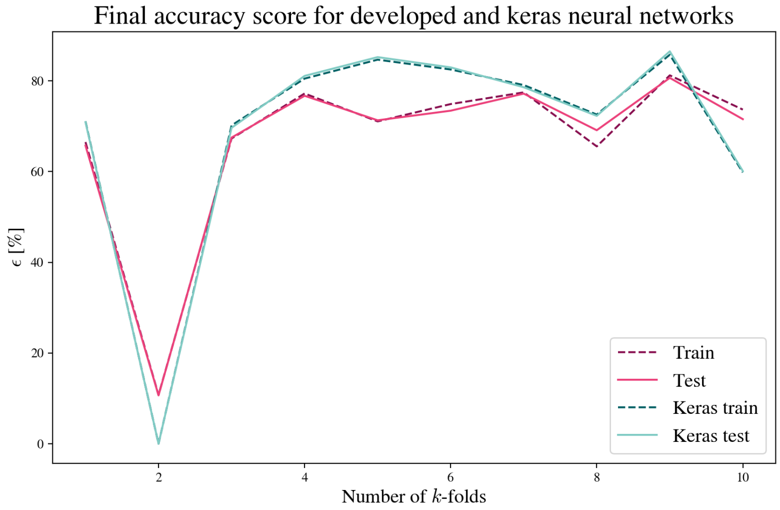

Figure 23: Final R^2 score of the developed linear regression neural network and keras.

3.2.4 Hidden layers and neurons

The number of hidden layers and neurons in each layer was tested with a trial and error method. For the classification problem, we experienced a decrease in the accuracy when increasing the number of hidden layers. As a result, we ran the neural network with a single hidden layer with 50 neurons. The number of neurons was a bit arbitrary, but we observed a decrease in the accuracy when this number was decreased. For the linear regression problem, we experienced a small increase in the accuracy when increasing the number of hidden layers. The neural network was run with 3 hidden layers with 8, 4 and 3 neurons, respectively. The number of neurons in each layer was first tested based on the classification network, but we obtained the best accuracy score when reducing this number by a large factor.

4 Discussion

After performing logistic regression on the credit card data, we have obtained an accuracy score of $\sim 80\%$. The logistic regression model in Yeh and Lien (2009) achieves an R^2 score of 79.4% (in the following, we refer to the models of this paper as `yeh_log` and `yeh_nn`). Our developed neural network for classification problems achieves an accuracy of $\sim 83.5\%$, and `yeh_nn` obtains an R^2 score of 96.5%. Even though these measurements are not comparable, we look at the numbers and make an observation. The accuracy scores of our logistic regression model and neural network are almost equal, while the R^2 scores of `yeh_log` and `yeh_nn` have larger differences. `yeh_nn` has an almost perfect R^2 score, leading us to believe that it should also achieve a high accuracy score. Based on this, we suppose that our neural network underperforms compared to `yeh_nn`.

Generally, a comparison of our models to the models in Yeh and Lien (2009) is difficult. There is a drawback of the credit card data set, as it consists of uncategorised data comprising major classes in 6 of the features. This is an issue that is not addressed by the authors of the paper. We have also noted another difference with the data set in the paper and our data set. The authors describe the data as having 25000 observations, while our data set contains 30000 observations before preprocessing. While the authors might have reduced the data set without explicitly mentioning it, we have not found any combination of data removal that leads to a reduction of 5000 observations. Even though the paper is not about the actual data, but rather the data mining techniques, we have found it difficult to make detailed comparisons to their models and results.

Additionally, we get a lower ratio of default payment than Yeh and Lien (2009). After reducing the data set by approximately 2000 observations, we get a ratio of 78.5% compared to their ratio of 87.88%. This implies that there are observable differences in our data sets. When it comes to the results of this project, it also implies that our logistic regression model has approximately the same accuracy as labelling all inputs as 1. We have tried to reduce the data set to 4000 observations by removing the uncategorised major classes in the PAY features. This gives a ratio of default payment of 66%, and an accuracy score between 78-80% for both the logistic regression model and neural network. Even though this gives a better prediction of the data, it comes at the cost of major data removal. Hence, we have decided to keep the ”full” data set where only minor uncategorised observations are removed.

By comparing the developed logistic regression model and neural network for classification, we see that the neural network has a higher accuracy score. However, based on the above discussion, this does not mean that the network is performing well. The difference between the accuracy score of the neural network and ratio of default payments is 5%. Further, this implies that the prediction of the neural network is slightly better than labelling all inputs to 1.

By comparing the linear regression neural network to the results of project 1, we see that the predicted Franke function is similar to the prediction made with Lasso regression. As we increase the value of λ, the R^2 score decreases. This is seen in both the current and the previous project. Additionally, the R^2 scores obtained in this project are comparable to those of project 1. In the previous project, we obtained an R^2 score from Lasso regression of 80% for a 5th degree polynomial with $\lambda = 0.001$. In this project, we obtain an R^2 score that is approximately the same, but with a value of λ that is one order of magnitude larger. It is important to keep in mind that the neural network is predicting the function from scratch, while the linear regression model from project 1 has the advantage of being fitted to a polynomial of high degree. In project 1, we observed that fitting of polynomial of low degree produced predictions with low R^2 scores. Hence, the neural network is doing a great job in predicting the Franke function. Better R^2 scores might have been obtained by exploring other activation functions, such as the non-linear tahn function.

All the developed models of this project are performing relatively well compared to their Scikit-Learn/keras counterparts. The biggest difference in accuracy is seen in the logistic regression model, where the SGDClassifier from Scikit-Learn achieves a larger accuracy on both the training and test sets. For the classification neural network, the developed code is performing equally well as keras for all k-folds. The developed linear regression neural network is also performing equally well to keras, with the exception of a dip at the 5th k-fold.

Deciding on the number of hidden layers is not an intuitive task, especially since increasing this number may or may not improve the accuracy. Generally, it depends on the complexity of the problem. More complex functions might need additional hidden layers for the neural network to be able to predict the problem to a higher accuracy. However, less complex functions might be overfitted if the number of hidden layers is chosen too high. In the linear regression problem, we found that by increasing the number of hidden layers, a higher R^2 score was achieved. This is intuitive, as the Franke function is relatively complex. In the classification problem, we found that a single hidden layer was sufficient in order to obtain the highest accuracy. Compared to the linear regression problem, this is somewhat counter-intuitive. A reason for this result can be that the majority of the prediction is based on a few features, while the rest of the features only contribute a small part. Another reason can be that the classification problem is not nearly as complex as we think. After all, we do not know the underlying function explaining the data set.

5 Conclusion

We have developed a logistic regression model, as well as a neural network for studying classification problems. The neural network has also been reformulated to perform regression analysis on the Franke function, where the results have been compared to those of project 1. Due to time restrictions, we have not been able to directly compare our logistic regression model and neural network to the models in Yeh and Lien (2009). However, we have concluded that the logistic regression model and neural network in the paper most likely perform better than the developed models of this project. The linear regression neural network is comparable to the Lasso regression model in project 1. Looking back at project 1, we saw that the Ordinary Least Squares method performed best on the Franke function, and it was the method with least computation time. From the current project, we have learned that using a neural network for linear regression is not the right solver for the problem. For classification problems, a feed-forward neural network performs better than a logistic regression model. The main reasoning behind this conclusion lies in Yeh and Lien (2009), but we have also seen that the developed neural network performs slightly better than the logistic regression model. We have learned that developing a neural network is not a straight-forward process, and that the best prediction is obtained by finding the optimal parameters and methods such as the number of hidden layers, activation functions etc.

Additional work on this project would be to investigate other activation functions in order to observe the effect on the accuracy score, especially with regards to the classification problem. It would also be interesting to fine-tune the input parameters even more, as well as implement a dynamic learning rate. We would also like to implement a validation-based early stopping routine in order to stop the training when no improvements are made.

As a final remark, we believe that we would have had better success with another data set. The current data set was skewed and had issues regarding uncategorised data. It would have been interesting to explore additional data sets, but due to time restrictions this what not possible. Future work would include testing both small and large data sets on the developed models, and hopefully see clear differences in terms of which model to use for classification problems.

6 Appendix

Relevant programs developed to solve this project can be found at the GitHub address

https://github.com/hellmb/FYS-STK4155/tree/master/Project_2

References

Hjorth-Jensen, M.
2019. Data analysis and machine learning: Neural networks, from the simple perceptron to deep learning. https://compphysics.github.io/MachineLearning/doc/pub/NeuralNet/html/NeuralNet.html. Accessed: 2019-07-11.

Marsland, S.
2014. *Machine Learning: An Algorithmic Perspective*. Chapman and Hall/CRC.

Mayo, H., H. Punchihewa, J. Emile, and J. Morrison
2018. History of machine learning. https://www.doc.ic.ac.uk/~jce317/history-machine-learning.html. Accessed: 2019-07-11.

Patrikar, S.
2019. Batch, mini batch & stochastic gradient descent. https://towardsdatascience.com/batch-mini-batch-stochastic-gradient-descent-7a62ecba642a. Accessed: 2019-07-11.

Peixeiro, M.
2019. How to improve a neural network with regularization. https://towardsdatascience.com/

how-to-improve-a-neural-network-with-regularization-8a18ecda9fe3.
Accessed: 2019-07-11.

Yeh, I.-C. and C.-h. Lien
2009. The comparisons of data mining techniques for the predictive accuracy of probability of default of credit card clients. *Expert Systems with Applications*, 36(2, Part 1):2473 – 2480.