Linear Algebra — Matrix

1. Define Matrix. Example.
2. Equality of two Matrix.
3. Square Matrix, Rectangular Matrix, Row Matrix, Column Matrix, Horizontal Matrix, Vertical Matrix, Zero Matrix, Identity Matrix.
4. Sums of matrices. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ -1 & 2 & 5 \end{bmatrix}$; $A+B=?$ / $A-B=?$
5. Multiplication of matrices.

   * Cases when 2 matrices has not satisfied the value $AB \neq BA$.

   $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ find AB and show that $AB \neq BA$.
6. The rank of a matrix, minor, co-factor.
7. Non-Singular matrix, Singular matrix.
8. Equivalent matrices.
9. Echelon form of a Matrix.
10. Different kind of matrix:

    a. Idempotent matrix,

    b. Nilopotent matrix,

    c. Involuntory matrix,

    d. Periodic matrix,

    e. Symmetric matrix (skew Symmetric matrix)

    f. Hermistian matrix (Skew Hermistian matrix)
11. Transpose of a matrix.
12. Conjugate of a matrix.

13. Normal form of a Matrix.

* Find the rank of the given matrix:

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$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 2 \end{bmatrix}$ or $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ or $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$

14. Adjoint of a matrix.

15. Inverse of a matrix.

* $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}$ find the Adj A and A^{-1}.

16. Find the rank of the given matrices:

a. $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 2 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

d. $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$

Suggestions

1. Nilopotent Matrix. Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is a nilopotent matrix of order 2.

2. Show that $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is a nilopotent matrix of order 3.

3. Show that, ① $(A^t)^t = A$

② $(AB)^t = B^t A^t$

3. Prove that, every square matrix can be uniquely expressed as the sum of a symetric and skew-symetric matrix.

4. If $A = [a_{ij}]$ be an $n \times n$ matrix, then prove that $A \cdot (\mathrm{Adj}\, A) = (\mathrm{Adj}\, A) A = |A| I$. where I is an $n \times n$ identity matrix.

5. If A and B are two $n \times n$ matrices, then show that, $\mathrm{Adj}\,(AB) = (\mathrm{Adj}\, B) \cdot (\mathrm{Adj}\, A)$

6. If A and B be two non-singular matrices of the same order than AB is also non-singular and $(AB)^{-1} = B^{-1} A^{-1}$.

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17. Eigen values and Eigen vectors.

18. Properties of Eigen values and Eigen vectors.

* find eigen values and eigen vectors of a matrix,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

Conjugate of a ma

19. Solve the following system of equations by the Crammers rule,

$$x+y+z=6 \qquad \text{and} \qquad x+y+z=1$$
$$x-y+z=2 \qquad\qquad x+2y+3z=2$$
$$2x+y-z=1 \qquad\qquad x+4y+9z=4$$

20. Using matrix method, solve the following systems of equations:

$$2x-y+3z=9 \qquad \text{and} \qquad x+y+z=6$$
$$x+y+z=6 \qquad\qquad x+2y+3z=14$$
$$x-y+z=2 \qquad\qquad x+4y+9z=36$$

"Additionals"

1. Find A^{-1} for the matrix, $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

2. Harmitian matrix: $\bar{A}' = A$

3. Orthogonal matrix: $AA^T = I_n$ (square).

4. Define similarity of two matrices. Prove that two $n \times n$ similar matrices A and B have the same characteristic polynomial and hence the same eigenvalues.

Linear Algebra. Linear Algebra.

[1]

a. Define a vector space and subspace with example.

b. Let $V = \mathbb{R}^n$. Prove that $W = \{(a,b,c) : a^2+b^2+c^2 \le 1\}$ not a subspace of V.

[4.9 and 4.10] [7.23]

c. Define direct sum. (4.37-81) / Theorem of sum and direct sum.

[2]

a. Define Basis and Dimension of a vector space. Determine whether the basis and dimension form a basis of $\mathbb{R}^3$

$\rightarrow \{(1,1,1), (1,2,3), (2,-1,1)\}$

b. Theorem 5.3. Prove that every basis of vector space have same number of element.

c. 5.72 → 117

[3]

a. Define a linear mapping. Show that the following mapping F is linear or non linear.

$F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ define by $f(x,y) = (x+y, x)$

b. Define Image and kernal of a linear mapping. Let $F: V \rightarrow U$ a linear mapping prove that $\operatorname{Img} F$ is a subspace of U.

c. 6.15 — 136

[4]

a. Define co-ordinate vector and matrix representation of a linear operator. Find the matrix representation of a linear operator: $T(x,y) = (3x-4y, x+5y)$ relative to the usual basis $\mathbb{R}^2$ $\{e_1=(1,0), e_2=(0,1)\}$.

b. Define Transition matrix. Find the transition matrix P and Q form one to another of the basis $\{e_1=(1,0,0), e_2=(0,1,0), e_3=(0,0,1)\}$ and $\{f_1=(1,1,1), f_2=(1,1,0), f_3=(1,0,0)\}$ and show that $P = Q^{-1}$ or $PQ = I_n$