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Date ....

Matrices and Linear operators

$$F : V \to U$$

$$T : V \to V$$

Let $\{e_1, e_2, \dots, e_n\}$ be a basis of a vector space V over a field K and for any vector $v \in V$, suppose $v = a_1e_1 + a_2e_2 + \dots + a_ne_n$.

Then the coordinate vector of v relative to $\{e_i\}$ which we ~~written~~ write as a column vector

$$[v]_e = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = (a_1, a_2, \dots, a_n)'$$

$\boxed{T : V \to V}$

Matrix representation of a linear operator:

Let T be a linear operator on a vector space V over a field K. ~~and~~ Suppose $\{e_1, e_2, \dots, e_n\}$ is a basis of V. Here $T(e_1), T(e_2), \dots, T(e_n)$ are vectors in V and so each is a linear combination of $\{e_1, e_2, \dots, e_n\}$.

That is,

$$T(e_1) = a_{11}e_1 + a_{12}e_2 + \dots + a_{1n}e_n$$

$$T(e_2) = a_{21}e_1 + a_{22}e_2 + \dots + a_{2n}e_n$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$T(e_n) = a_{n1}e_1 + a_{n2}e_2 + \dots + a_{nn}e_n$$

Coefficient matrix $= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$

The transpose of the above coefficient matrix is called the matrix representation of T relative to the basis $\{e_i\}$ or the matrix of T in the basis $\{e_i\}$. It is denoted by $[T]_e$ or $[T]$.

$$\begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}$$

Problem: Find the matrix representation of each of the following operators T on $\mathbb{R}^2$ relative to the usual basis $\{e_1 = (1, 0),\ e_2 = (0, 1)\}$:

[If $(a, b) \in \mathbb{R}^2$, then $(a, b) = ae_1 + be_2$]

Here $T: \mathbb{R}^2 \to \mathbb{R}^2$

① $T(x, y) = (2y,\ 3x - y)$

② $T(x, y) = (3x - 4y,\ x + 5y)$

Solⁿ: ① $T(e_1) = T(1, 0) = (0, 3) = 0e_1 + 3e_2$

and $T(e_2) = T(0, 1) = (2, -1) = 2e_1 + (-1)e_2$

$= 2e_1 - e_2$

Coefficient matrix $= \begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}$

$\therefore$ Matrix representation $= [T]_e = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$.

② $T(e_1) = T(1, 0) = (3, 1) = 3e_1 + e_2$

$T(e_2) = T(0, 1) = (-4, 5) = -4e_1 + 5e_2$

Here $[T]_e = \begin{pmatrix} 3 & -4 \\ 1 & 5 \end{pmatrix}$

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Problem: Find the matrix representation of each of the following operators T on $\mathbb{R}^2$ relative to the basis $\{f_1 = (1,3),\ f_2 = (2,5)\}$:

① $T(x,y) = (2y,\ 3x-y)$

② $T(x,y) = (3x-4y,\ x+5y)$

Solⁿ: First we find the coordinates of an arbitrary vector $(a,b) \in \mathbb{R}^2$ w.r. to the basis $\{f_1, f_2\}$.

$$(a,b) = xf_1 + yf_2 = x(1,3) + y(2,5)$$

$$\text{or } (a,b) = (x+2y,\ 3x+5y)$$

$$\Rightarrow x+2y = a \text{ and } 3x+5y = b$$

$$x = 2b-5a \text{ and } y = 3a-b$$

$$\therefore \boxed{(a,b) = (2b-5a)f_1 + (3a-b)f_2}$$

$a=6,\ b=0$

① $T(x,y) = (2y,\ 3x-y)$

$$T(f_1) = T(1,3) = (6,0) = -30f_1 + 18f_2$$