রাজশাহী বিশ্ববিদ্যালয়

প্রশ্নোত্তরের অতিরিক্ত উত্তরপত্র

No 0548025

পরীক্ষার রোল নম্বর/শিক্ষার্থী পরিচিতি নম্বরঃ

বিষয়ঃ

পত্র/কোর্সঃ

পরীক্ষা তদারককারীর স্বাক্ষর
ও
পরীক্ষা কেন্দ্রের সীলমোহর

রাবি প্রেস - ১০,০০,০০০/ প:নি: ১৩০৪/ তাং- ০১/০৯/২০২২

Problem: Solve the following system of equations by the Cramer's rule:

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

Sol^n: By Cramer's rule,

$$\frac{x}{D_1} = \frac{y}{D_2} = \frac{z}{D_3} = \frac{1}{D}$$

Here $D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 6(1-1) - 1(-2-1) + 1(2+1) = 3+3 = 6$

$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(-2-1) - 6(-1-2) + 1(1-4)$
$= -3 + 18 - 3 = 12$

$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 1(-1-2) - 1(1-4) + 6(1+2) = -3 + 3 + 18 = 18$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(1-1) - 1(-1-2) + 1(1+2) = 3 + 3 = 6$

Now $\frac{x}{6} = \frac{1}{6}$ $\therefore x = 1$

$\frac{y}{12} = \frac{1}{6}$ $\therefore y = 2$

$\frac{z}{18} = \frac{1}{6}$ $\therefore z = 3$

* By Cramer's rule, solve the following system of equations

$$x+y+z=1$$
$$x+2y+3z=2$$
$$x+4y+9z=4$$

* Using matrix method, solve the following system of equations:

$$2x-y+3z=9$$
$$x+y+z=6$$
$$x-y+z=2$$

Soln : Let $A=\begin{bmatrix}2 & -1 & 3\\ 1 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}$ and $K=\begin{bmatrix}9\\6\\2\end{bmatrix}$.

Assume that there exists a matrix $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ such that

$$AX=K.$$

Then $\begin{bmatrix}2 & -1 & 3\\ 1 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}9\\6\\2\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3 & 0 & 4\\ 1 & 1 & 1\\ 2 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}15\\6\\8\end{bmatrix}$ by $R_1\to R_1+R_2$, $R_3\to R_3+R_2$

$\Rightarrow 3x+4z=15$, $\quad x+y+z=6$, $\quad 2x+2z=8$

$x+z=4$

$3x+4(4-x)=15$ $\quad y+4=6$

or $3x+16-4x=15$ $\quad y=2$

or $-x=-1$ $\quad \therefore z=3$

$\therefore x=1$

Hence $x=1, y=2, z=3$

* Solve by matrix method

$x+y+z=6$
$x+2y+3z=14$
$x+4y+9z=36$

Problem: Find eigenvalues and associated nonzero eigenvectors of the matrix $A=\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}$.

Sol^n: We seek a scalar t and a nonzero vector $X=\begin{pmatrix}x\\ y\end{pmatrix}$ such that $AX=tX$; i.e. $AX-tX=0$

$$\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=t\begin{pmatrix}x\\ y\end{pmatrix}$$

The above matrix equation is equivalent to the homogeneous system

$x+2y=tx$
$3x+2y=ty$

or $\left.\begin{aligned}(t-1)x-2y&=0\\ -3x+(t-2)y&=0\end{aligned}\right\} \longrightarrow (1)$

The homogeneous system has a non-zero solution iff the determinant of the matrix of coefficient is 0.

$$\begin{vmatrix}t-1 & -2\\ -3 & t-2\end{vmatrix}=0$$

$\Rightarrow t^2-3t-4=0 \quad \Rightarrow (t-4)(t+1)=0$

$\therefore t=4$ or $t=-1$

Putting $t=4$ in (1)

$$3x - 2y = 0$$

$$-3x + 2y = 0 \quad \text{or} \quad 3x - 2y = 0$$

$\therefore$ $v = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is a nonzero eigenvector

belonging to $t = 4$

$t = -1$ in 1

$$x + y = 0$$

$$w = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$