রাজশাহী বিশ্ববিদ্যালয়

প্রশ্নোত্তরের অতিরিক্ত উত্তরপত্র

রোল নম্বর

বিষয়

পত্র/কোর্স

No 074797

পরীক্ষা কেন্দ্রের সীলমোহর

তদারককারীর স্বাক্ষর

রাঃ বিঃ প্রেস—১,৩০,০০০/১৪-০১-০৮/৫৫ পঃ নিঃ ০৬-০১-২০০৮

Page 162

8.1 = The determinant of a matrix A and its transpose A^t are equal $|A| = |A^t|$.

For any operators $S, T \in A(V)$, $[T+S]_e = [T]_e + [S]_e$ and $[kT]_e = k[T]_e$

Theorem 7.2:— Let $\{e_1, e_2, \dots, e_n\}$ be a basis of V over K and let $\mathbb{A}$ be the algebra of n-square matrices over K. Then the mapping $T \mapsto [T]_e$ is a vector space isomorphism from $A(V)$ onto $\mathbb{A}$. That is, the mapping is one-one and onto and for any $S, T \in A(V)$ and any $k \in K$, $[T+S]_e = [T]_e + [S]_e$ and $[kT]_e = k[T]_e$

Proof:— The mapping is one-one since, by Theorem 8.1 a linear mapping is completely determined by its values on a basis. The mapping is onto since each matrix $M \in \mathbb{A}$ is the image of the linear operator

$$F(e_i) = \sum_{j=1}^{n} m_{ij} e_j \qquad i = 1, 2, \dots, n$$

where (m_{ij}) is the transpose of the matrix M.

Now, suppose, for $i = 1, 2, \dots, n$,

$$T(e_i) = \sum_{j=1}^{n} a_{ij} e_j \quad \text{and} \quad S(e_i) = \sum_{j=1}^{n} b_{ij} e_j .$$

Let A and B be the matrices $A = (a_{ij})$ and $B = (b_{ij})$: Then $[T]_e = A^t$ and $[S]_e = B^t$.

Now $(T+S)(e_i) = T(e_i) + S(e_i)$

$$= \sum_{j=1}^{n} (a_{ij} + b_{ij}) e_j$$

Clearly $A+B$ is the matrix $(a_{ij} + b_{ij})$.

Accordingly, $[T+S]_e = \boxed{(A+B)^t = A^t + B^t}$

$$= [T]_e + [S]_e .$$

Also, we have, for $i = 1, 2, \dots, n$

$$(kT)(e_i) = kT(e_i) = k \sum_{j=1}^{n} a_{ij} e_j$$

$$= \sum_{j=1}^{n} (k a_{ij}) e_j .$$

Clearly kA is the matrix $(k a_{ij})$.

Accordingly, $[KT]_e = (KA)^t = KA^t = K[T]_e$

কিমনার স্যার ৩য় Lec.. Change of Basis:—

Def^n:– Let $\{e_1, e_2, \cdots, e_n\}$ be a basis of V and let $\{f_1, f_2, \cdots, f_n\}$ be another basis.

Suppose

$$f_1 = a_{11}e_1 + a_{12}e_2 + \cdots + a_{1n}e_n$$

$$f_2 = a_{21}e_1 + a_{22}e_2 + \cdots + a_{2n}e_n$$

$$\cdots\cdots$$

$$f_n = a_{n1}e_1 + a_{n2}e_2 + \cdots + a_{nn}e_n.$$

Then the transpose P of the above matrix of coefficients is termed the transition matrix from the old basis $\{e_i\}$ to the new basis $\{f_i\}$:

$$P = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}$$

Theorem 7.3 (P→152) $[ST]_e = [S]_e[T]_e$

(P→162)

Since the vectors $f_1, f_2, \cdots, f_n$ are linearly independent, so the matrix P is invertible.

by P→113 Problem 5.47

P→153 7.5 Ex.:– Consider the following two bases of $\mathbb{R}^2$: $\{e_1 = (1,0),\ e_2 = (0,1)\}$ and $f_1 = \{1,1\},\ f_2 = (-1,0)\}$.

$P = (\quad)$ and $Q = (\quad)$. $PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

Theorem:- 7.4:- Let P be the transition matrix from a basis $\{e_i\}$ to a basis $\{f_i\}$ in a vector space V. Then for any vector $v \in V$,

$$P[v]_f = [v]_e \text{ and hence } [v]_f = P^{-1}[v]_e .$$

Proof:- For, $i = 1, 2, \dots, n$, suppose

$$f_i = a_{i1} e_1 + a_{i2} e_2 + \dots + a_{in} e_n$$

$$= \sum_{j=1}^{n} a_{ij} e_j .$$

Then P is the n-square matrix whose jth row is $(a_{1j}, a_{2j}, \dots, a_{nj})$. $\longrightarrow$ (1)

Again, suppose

$$v = k_1 f_1 + k_2 f_2 + \dots + k_n f_n = \sum_{i=1}^{n} k_i f_i$$

Then $[v]_f = (k_1, k_2, \dots, k_n)^t \longrightarrow$ (2) [transpose of a row vector]

Also ~~Now~~, $v = \sum_{i=1}^{n} k_i f_i = \sum_{i=1}^{n} k_i \left(\sum_{j=1}^{n} a_{ij} e_j \right)$.

$$= \sum_{j=1}^{n} \left(\sum_{i=1}^{n} a_{ij} k_i \right) e_j$$

$$= \sum_{j=1}^{n} (a_{1j} k_1 + a_{2j} k_2 + \dots + a_{nj} k_n) e_j$$

Hence, $[v]_e$ is the column vector whose jth entry is $a_{1j}k_1 + a_{2j}k_2 + \cdots + a_{nj}k_n \rightarrow$ (3)

On the other hand, the jth entry of $P[v]_f$ is obtained by multiplying ~~the~~ (1) and (2). But the product of (1) and (2) is (3) and hence $P[v]_f$ and $[v]_e$ have the same entries.

Thus $P[v]_f = [v]_e$.

Also, multiplying $P[v]_f = [v]_e$ by P^{-1} gives $P^{-1}P[v]_f = P^{-1}[v]_e$

i.e. $[v]_f = P^{-1}[v]_e$.

P-7.1 & 2

7.12, 7.13

Theorem 7.3:— For any operators $S, T \in A(V)$, $[ST]_e = [S]_e[T]_e$

Theorem 7.5:— Let P be the transition matrix from a basis $\{e_i\}$ to a basis $\{f_i\}$ in a vector space V. Then for any linear operator T on V,

$$[T]_f = P^{-1}[T]_e P.$$

2

Theorem 7.1: Suppose $\{e_1, e_2, \cdots, e_n\}$ is a basis of V and T is a linear operator on V. Then for any $v \in V$,

$$[T]_e [v]_e = [T(v)]_e .$$

Proof: For $i = 1, 2, \cdots, n$,

suppose $T(e_i) = a_{i1} e_1 + a_{i2} e_2 + \cdots + a_{in} e_n$

$$= \sum_{j=1}^{n} a_{ij} e_j$$

Then $[T]_e$ is the n-square matrix whose jth row is $(a_{1j}, a_{2j}, \cdots, a_{nj}) \longrightarrow (1)$

Again suppose $v = k_1 e_1 + k_2 e_2 + \cdots + k_n e_n$

$$= \sum_{i=1}^{n} k_i e_i .$$

Here $[v]_e = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = (k_1, k_2, \cdots, k_n)^t \rightarrow (2)$

Also, $T(v) = T\left(\sum_{i=1}^{n} k_i e_i\right) = \sum_{i=1}^{n} k_i T(e_i)$

$$= \sum_{i=1}^{n} k_i \left(\sum_{j=1}^{n} a_{ij} e_j\right) = \sum_{j=1}^{n} \left(\sum_{i=1}^{n} a_{ij} k_i\right) e_j$$

$$= \sum_{j=1}^{n} (a_{1j} k_1 + a_{2j} k_2 + \cdots + a_{nj} k_n) e_j$$

Hence $[T(v)]_e$ is the column vector whose jth entry is

3/

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$$a_{1j}k_1 + a_{2j}k_2 + \cdots + a_{nj}k_n \longrightarrow (3)$$

On the other hand, the jth entry of $[T]_e[v]_e$ is obtained by multiplying the jth row of $[T]_e$ by $[v]_e$; i.e. (1) by (2).
But the product of (1) and (2) is (3).
Hence $[T]_e[v]_e$ and $[T(v)]_e$ have the same entries.
Therefore $[T]_e[v]_e = [T(v)]_e$.