Chapter 6 : Frequent Itemsets

the discovery of frequent itemsets =“association rules”를 발견하는 것!

▼ ***Course Outline***

> https://www.youtube.com/watch?v=2NyZmnuIicw&list=PLoCMsyE1cvdVnCgHk43vRy7PVTVWJ6WVR&index=2

1) “market-basket” model

2) First: Define

   Frequent itemsets
   Association rules:
   Confidence, Support, Interestingness

3) Then: Algorithms for finding frequent itemsets

   Finding frequent pairs
   A-Priori algorithm
   PCY algorithm

6.1. THE MARKET-BASKET MODEL

- *Baskets* = “**transactions**”
- items ⊂ Basket

6.1.1 Definition of Frequent Itemsets

> *"frequent"*
>
> ⇒ s, called the *support threshold*
>
> I : set of items → support for I : I가 포함된 Basket의 수
>
> if support for I > s : I는 빈번하다(frequent)!

Example 6.1 :

1. {Cat, and, dog, bites}
2. {Yahoo, news, claims, a, cat, mated, with, a, dog, and, produced, viable, offspring}
3. {Cat, killer, likely, is, a, big, dog}
4. {Professional, free, advice, on, dog, training, puppy, training}
5. {Cat, and, kitten, training, and, behavior}
6. {Dog, &, Cat, provides, dog, training, in, Eugene, Oregon}
7. {"Dog, and, cat", is, a, slang, term, used, by, police, officers, for, a, male–female, relationship}
8. {Shop, for, your, show, dog, grooming, and, pet, supplies}

- Since the empty set is a subset of any set, the support for ∅ is 8.⇒ 하지만 일반적으로 공집합은 아무것도 말해주지 않기 때문에 신경 쓰지 X
- Example) "Dog":(5)번을 제외 모든 basket에서 등장 ⇒ "Dog"의 support는 7
- Suppose that we set our threshold at s = 3 : Then there are five frequent singleton itemsets: {dog}, {cat}, {and}, {a}, and {training}

- doubletons

	training	a	and	cat
dog	4, 6	2, 3, 7	1, 2, 7, 8	1, 2, 3, 6, 7
cat	5, 6	2, 3, 7	1, 2, 5, 7	
and	5	2, 7		
a	none			

Figure 6.2: Occurrences of doubletons

- There are five frequent doubletons if s = 3; they are {dog, a} {dog, and} {dog, cat} {cat, a} {cat, and}
    - Each appears at least three times

- frequent triple은 frequent doubletone의 조합으로만 가능
- 해당 예제에서는 하나의 frequent triple만 존재하므로,no frequent quadruples or larger sets.

6.1.2 Applications of Frequent Itemsets

: If someone buys diaper and milk, then he/she is likely to buy beer.

1. *Related concepts:* Let 1)*items* → *words*, and 2)*baskets* → *documents* (e.g., Web pages, blogs, tweets).

- stopword를 제외하면, 공통 개념을 나타내는 두개의 단어 쌍이 자주 발견될 수 있음.

2. *Plagiarism:* Let 1)the *items* → *documents* and 2)the *baskets* → *sentences. :* An item/document is “in” a basket/sentence if the sentence is in the document.

- we should remember that the relationship between items and baskets is an arbitrary many-many relationship : "in"은 "~의 일부"라는 전통적인 의미를 가질 필요가 없음.
- 여러 문장을 공유하는 두 개의 문서 → 표절의 좋은 지표!

3. _Biomarkers:_ Let 1) the *items* be of two types – *biomarkers* such as genes or blood proteins, and *diseases.* 2) Each *basket* is the set of data about a *patient*: their genome and blood-chemistry analysis, as well as their medical history of disease.

- 질병 검사로 활용 가능

6.1.3 Association Rules

: extracting frequent sets of items from data = *association rules*라고 부르는 **if-then 규칙**의 모음으로 표시

*** *association rules***

- form : $I \to j$, where I : set of items, j : an item
- $\{i_1, i_2, \ldots, i_k\} \to j$ means: "**if** a basket contains all of $i_1, \ldots, i_k$ **then** it is likely to contain j"
- In practice there are many rules, want to find **significant/interesting** ones!

- 1) Defining the ***confidence*** of the rule $I \to j$

> 📌 conf($I \to j$) $\overset{\text{def}}{=} \frac{\text{support } (I \cup \{j\})}{\text{support } (I)}$

: j가 포함되는 모든 basket I 의 비율

= support(I) is given → how often does j appear next to it?

$= P(j|I) = \frac{P(I,j)}{P(I)}$

Example) Fig 6.1에서
The confidence of the rule {cat, dog} → and = 3/5.

- {cat, dog}이 언급된 basket의 support = 5
- {cat, dog}이 and 와 함께 언급된 basket = 즉, {cat, dog, and}의 support = 3

- I의 support가 매우 크다면, confidence 하나만으로도 매우 useful 할 수 있음.
- However, I가 j에 (어떻게든) 영향을 미치는 실제 관계에서는 Association Rule이 더 중요함.
- 2) Thus, define the ***interest*** of an association rule $I \rightarrow j$: (rule $I \rightarrow j$의 confidence)와 (j를 포함하는 basket의 비율)의 차이

> 📌 $Interest(I \rightarrow j) = |conf(I \rightarrow j) - Pr[j]|$

  - if I가 j에 영향을 미치지 않는다면,

$$\frac{I\text{와 } j\text{를 포함하는 basket}}{I\text{를 포함하는 basket}} = \frac{j\text{를 포함하는 basket}}{\text{모든 basket}}$$

    → rule의 interest = 0

    - Example) I가 있으면 항상 J가 따라오는 경우 → not very interesting!
  - rule이 high(+) interest = basket의 I가 → j가 존재하는 것을 유발함
    - the rule $\{diapers\} \rightarrow beer$ has high interest.
  - rule이 highly negative(-) interest = basket의 I가 → j가 존재하는 것을 억제함을 알 수 있음.
    - the rule $\{pepsi\} \rightarrow coke$ can be expected to have negative interest.
  - interest가 매우 낮거나 매우 높은 경우 → 둘다 interesting!

6.1.4 Finding Association Rules with High Confidence

basket의 합리적인 정도(reasonable fraction)에 적용되는 Association Rules $I \rightarrow j$를 찾기 위해서는,

1) I의 support가 상당히(reasonably) 높아야 한다.

- 오프라인 매장에서의 마케팅과 같이 실제로 “reasonably high”한 비율은 종종 바구니의 약 1%

2) rule의 confidence 상당히 높아야 한다.

- usually above 0.5 = 50%, 그렇지 않으면 규칙이 실제 효과가 거의 X
- 결과적으로 집합 $I \cup \{j\}$도 상당히 높은 support를 가져야 함.

💡 Problem: Find all association rules with support ≥s and confidence ≥c

→ This means: support($I \cup \{j\}$) $\geq s$ ⇒ Conf $\geq c$

why?

- Note: Support of an association rule is the support of the set of items in the rule (left and right side)
- Hard part: Finding the frequent itemsets!

Suppose) 1. s와 c : given to us by user or by the data analyst.

2. threshold of support(s)를 달성하는 모든 itemsets를 찾았고, 각 itemset에 대한 support를 계산했다고 가정 → 빠르게 conf 계산 가능

→ 높은 support && 높은 confidence를 가진 Association Rules를 찾을 수 있음

That is) if J 가 frequent($\geq s$)한 n개의 items을 가지고 있다면,
→ only n possible association rules involving this set of items
= J에 존재하는 모든 j에 대하여 $J-\{j\} \rightarrow j$ (총 n개 존재)

→ J가 frequent($\geq s$) 이면, $J-\{j\}$도 frequent($\geq s$)

Why? If $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ will be "frequent"

⇒ J 와 $J-\{j\}$의 support 비율
= 규칙 $J-\{j\} \rightarrow j$의 신뢰도 = $\frac{\text{the support for } J-\{j\} \cup \{j\}}{\text{the support for } J-\{j\}}$ = $\frac{\text{the support for } J}{\text{the support for } J-\{j\}}$

Assumed that) there are not too many frequent itemsets and thus
not too many candidates for high-support, high-confidence

association rules. → 너무 많은 frequent itemsets을 얻지 않도록 support threshold(s)를 조정하는 것이 일반적!

*** How to make Algorithm ?**

Step 1) Find all frequent itemsets I
(we will explain this next)

Step 2) Rule generation

- For every subset A of I, generate a rule $A \to I \setminus A$
  Since I is frequent, A is also frequent
  - **Variant 1**: Single pass to compute the rule confidence
    confidence(A,B→C,D) = support(A,B,C,D) / support(A,B)
  - **Variant 2**:
    - Observation: If A,B,C→D is below confidence, so is A,B→C,D

      why? support({A,B}) > support({A,B,C})
      conf(A,B,C→D) = support(A,B,C,D) / support(A,B,C)
      conf(A,B→C,D) = support(A,B,C,D) / support(A,B)

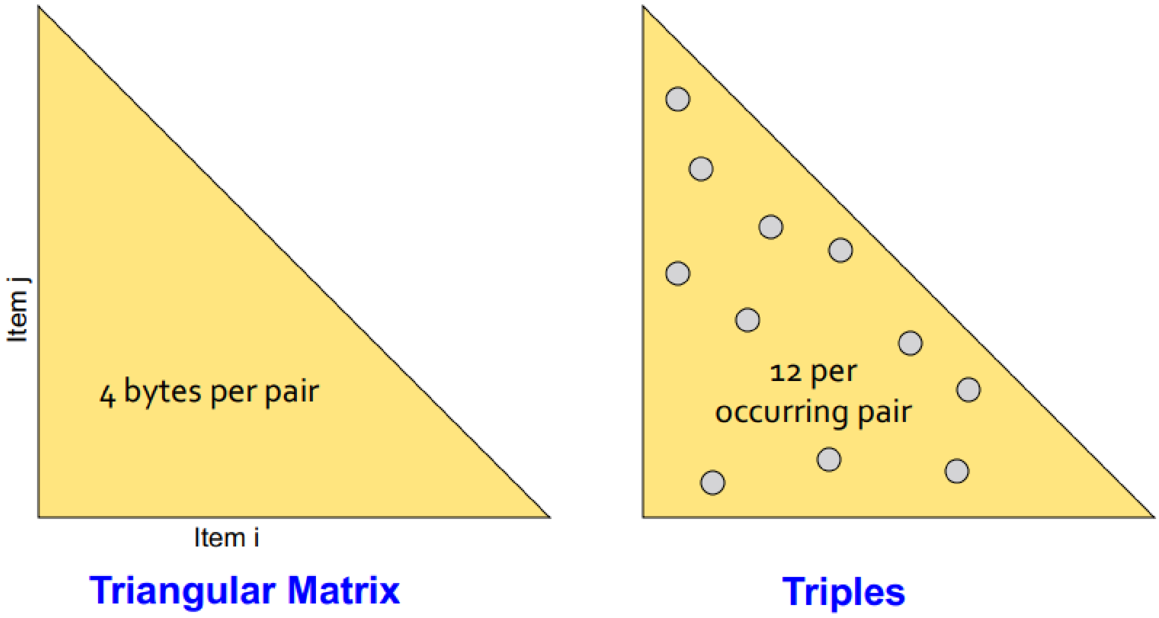
      ∴ in every case, conf(A,B,C→D) < conf(A,B→C,D)
    - Can generate "bigger" rules from smaller ones!
      Output the rules above the confidence threshold

→ Finding Frequent Itemsets

- Itemsets Computation Model
  - The true cost of mining diskresident data is usually **the number**
    of disk I/Os : disk에 저장된 파일을 처음~끝까지 읽어내리는 시간!
  - Bottleneck : main-memory
  - Question : How do we know if something cannot be frequent if we haven't counted it yet?

1) Naive Algorithm : data file을 한번 스캔하여 각 pair가 몇번 등장하는지 count한다. → impossible

2) Counting Pairs in Memory



Approach 1) Triangular Matrix : Dense

- 장점; 4 bytes per every pair
- 단점; preallocating every element

Approach 2) Triples : Sparse

- 장점; preallocating nothing
- 단점; 12 bytes per occuring pair

→ “how many different pairs actually occured in the data”에 따라 사용할 방법 결정.

- all possible pairs almost all occur : App1
- lots of items, but only few pairs tent to occur : App2

& Approach 2 beats Approach 1 if less than 1/3 of possible pairs actually occur. (가능한 쌍의 1/3 미만이 발생할 경우 App2가 더 효율적)

💥 **Problem** : if we have too many items so the pairs do not fit into memory. Can we do better? → A-Priori Algorithm !

6.2 Market Baskets and the A-Priori Algorithm

- `Key idea` : Monotonicity of Frequent

⇒ if set of items I가 최소 s번 나타난다면, I의 subset J도 최소 s번 나타남. J가 less frequent 할 수 없음.

대우 ⇒ if item i가 s개의 basket에서 나타나지 않는다면(support threshold를 넘지 못한다면), i를 포함하는 모든 집합은 s를 넘을 수 없음.

- `method` : frequent singletones to frequent pairs!

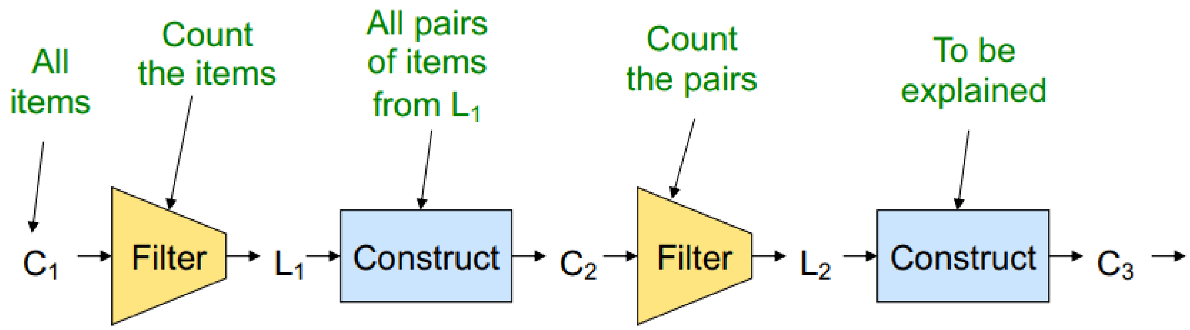
Pass 1)Read baskets and count in main memory the # of occurrences of each individual item

→ Items that appear $\geq s$ times are the **frequent items**

Pass 2) Read baskets again and keep track of the count of only those pairs where both elements are frequent (from Pass 1)

→ Requires memory proportional to square of **frequent items** only (not all items)

→ K-tuple로 일반화 가능



: single → pairs → triples → ... k-tuples

Example)

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent. We get: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent. L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} {b,c,j} {b,m,j} {c,m,j} } **
  - Count the support of itemsets in C_3
  - Prune non-frequent. L_3 = { {b,c,m} }

C3의 {b,c,j} {b,m,j} {c,m,j} 는 non-frequent 할 수밖에 없음

why? {b,c,j}에서 {b,c}와 {c,j}는 frequent하지만, {b,j}는 frequent 하지 않음. (by L2)

⇒ {b,c,j}는 frequent할 수 없으므로, generate하지 않아도 됨.

6.3 Handling Larger Datasets in Main Memory

→ to cut down on the size of candidate set C_2

6.3.1 PCY(Park, Chen, and Yu) Algorithm

= Also known as “DHP(Direct Hashing and Pruning)”

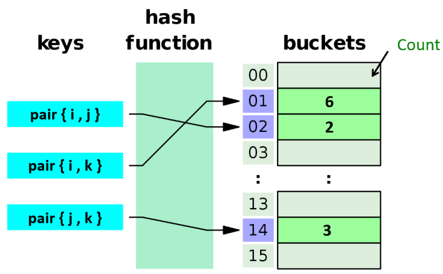
- **Pass 1**

```
FOR (each basket) :
    FOR (each item in the basket) :
```

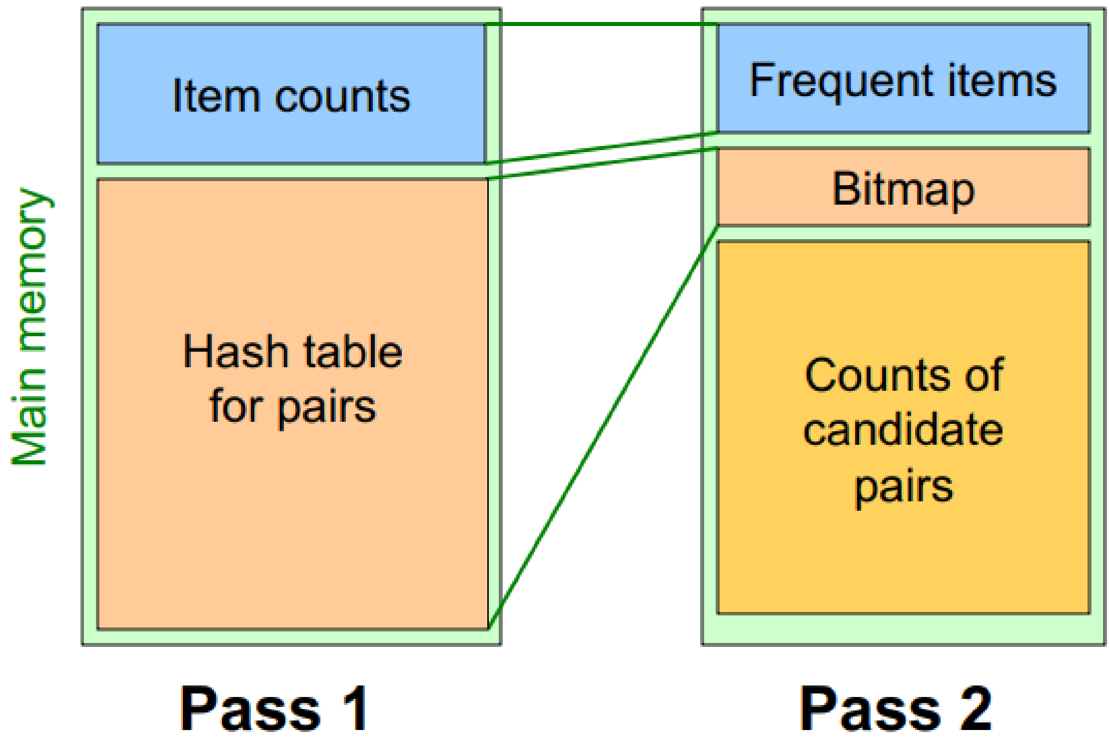
```
        add 1 to item's count;
    <!-- new in PCY , Hashing Process -->
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

PCY - Hash Table

- keys : Pairs
- buckets : integers(Count)



- **Between Passes**



Observation:

1) If a bucket contains a frequent pair, then the bucket is surely frequent

- However, even without any frequent pair, a bucket can still be frequent ☹️
  § So, we cannot use the hash to eliminate any member (pair) of a “frequent” bucket

2) But, for a bucket with total count less than s, none of its pairs can be frequent 🙂

- Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- Pass 2: **Only count** pairs that hash to frequent buckets

cf> hash table을 만들 때, (메모리가 충분하다면) 가능한 fewer collision을 만드는 것이 좋음!

implementation : Replace the buckets by a bit-vector!

→ **1** means the bucket count exceeded the support s(call it a frequent bucket); **0** means it did not

→ count를 위한 4byte Integer 가 bit-vector로 대체되면서 1/32 메모리만 사용 가능!

- pass1에서 pass2로 넘어가면서 hash table이 frequent bucket인지 여부($\geq s$) 를 Bitmap으로 기록.

- **Pass 2**

Count all pairs {i, j} that meet the conditions for being a candidate pair:

1. Both i and j are frequent items
2. The pair {i, j} hashes to a bucket whose bit in the bit vector is 1 (i.e., a frequent bucket)

⇒ 두가지 조건이 모두 만족되면 Tracking :
Both conditions are *necessary* for the pair to have a chance of being frequent.

**** Example***

PCY Example

- Support $s = 3$
- Items: milk (1), Coke (2), bread (3), Pepsi (4), juice (5).
- Transactions are
- t1 = {1, 2, 3} → milk, Coke, bread
- t2 = {1, 4, 5}
- t3 = {1, 3}
- t4 = {2, 5}
- t5 = {1, 3, 4}
- t6 = {1, 2, 3, 5}
- t7 = {2, 3, 5}
- t8 = {2, 3}

Pass 1 :

1. Item’s Count : 각 item이 몇번 등장하는지 count한다.

    * Hash Table 아님

    *** Item Count 결과**

Item	Count
1	5
2	5
3	6
4	2
5	4

    - item4는 s 를 넘지 못함.

2. Make Hash Table for bucket counts

    a. step 1) 모든 basket의 가능한 Pair를 생성

b. step 2) 각 Pair를 Hash Table에 해싱

→ 여기서는 Hashing Rule을 다음과 같이 정의

: Hashing a pair {i, j} to a bucket k, where k = hash(i, j) = (i + j) / 5

예를 들어, {1,2}를 hashing한다면 (1+2)/5 = 3 이니까 3번 bucket에 hashing됨.

```
(1, 4) and (2, 3) ->  k = 0
 (1, 5) and (2, 4) ->  k = 1
 (2, 5) and (3, 4) ->  k = 2
 (1, 2) and (3, 5) ->  k = 3
 (1, 3) and (4, 5) ->  k = 4
```

- For each pair in each transaction:
- t1 = $(1,2)^3$ $(2,3)^0$ $(1,3)^4$
- t2 = $(1,4)^0$ $(1,5)^1$ $(4,5)^4$
- t3 = $(1,3)^4$
- t4 = $(2,5)^2$
- t5 = $(1,3)^4$ $(3,4)^2$ $(1,4)^0$
- t6 = $(1,2)^3$ $(1,3)^4$ $(1,5)^1$ $(2,3)^0$ $(2,5)^2$ $(3,5)^3$
- t7 = $(2,3)^0$ $(2,5)^2$ $(3,5)^3$
- t8 = $(2,3)^0$

Total: 21 pairs

*** Hash Table 결과**

Bucket	Count
0	6
1	2
2	4
3	4
4	5

- 1번 버킷으로 hashing되는 pair는 t2의 (1,5)와 t6의 (1,5)만 존재
  → Count : 2
- 1번 버킷에 속하는 pair는 s를 넘지 못함.
  → 1번 버킷에 속하는 (1,5)와 (2.4)는 not frequent!

Pass 2 :

Frequent items : {1,2,3,5} (By Pass 1 - item's count)

→ 이에 따라서, 가능한 candidate pair는 (1,2) (1,3) (1,5) (2,3) (2,5) (3,5)

⭐ (1,5)는 폐기 : because bucket 1 is not frequent! (By Pass 1 - hash Table)

→ Surviving Pairs = (1,2) (1,3) (2,3) (2,5) (3,5)

→ Counts of the Surviving Pairs

Pair	Count
(1,2)	2
(1,3)	4
(2,3)	4
(2,5)	3
(3,5)	2

- (1,2) (3,5)는 s를 넘지 못함

⇒ Result : Frequent itemsets are {1} {2} {3} {5} {1,3} {2,3} {2,5}

6.3 Handling Larger Datasets in Main Memory

The MMDS book covers several other extensions beyond the PCY idea: "Multistage" and "Multihash"

- Recommended video (starting about 10:10): https://www.youtube.com/watch?v=AGAkNiQnbjY

6.4 Limited-Pass Algorithms

: Can we use fewer passes? (in $\leq k$ passes)

Frequent Itemsets in ≤ 2 Passes

Use 2 or fewer passes for all sizes,but may miss some frequent itemsets

1. Random sampling
2. SON (Savasere, Omiecinski, and Navathe) Algorithm
3. Toivonen Algorithm

6.4.1 The Simple, Randomized Algorithm

: 1) Take a random sample of the market baskets
→ 2) Run a-priori or one of its improvements in main memory

- 장점 : Disk I/O 시간 필요 X
- Sample size에 맞게 support threshold(s)를 감소시켜야 함.
    - Example) if your sample is 1/100 of the baskets, use $s/100$ as your support threshold instead of s.
    - Smaller threshold, e.g., $s/125$, helps catch more truly frequent itemsets (But requires more space)
- To avoid *false positives*: 추가로, candidate pairs가 전체 데이터에 대해서 frequent한지 확인하기 위해서 **second pass**에서 추가로 sampling한 데이터를 가지고 검증!

6.4.4 The SON Algorithm and MapReduce

: Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets

- Note: Sampling 방법과 비슷하지만, sampling하는 것 아님(6.1.1과 다르게), but 모든 file data를 in memory-sized **chunks**로 쪼갬.

Path 1) An itemset becomes a candidate if it is found to be frequent in any one or more subsets of the baskets. (: make Candidate itemsets)

pass 2) count all the candidate itemsets and determine which are frequent in the entire set.(: Verify)

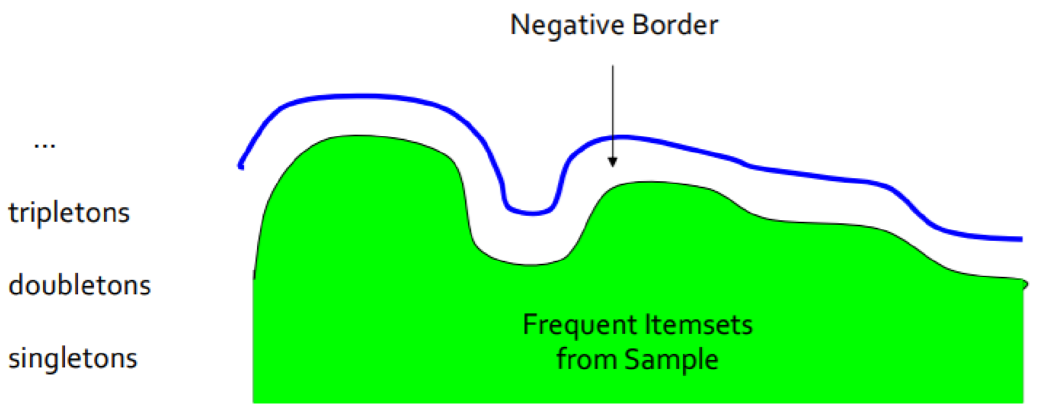
→ SON 알고리즘은 병렬 컴퓨팅 환경에 적합. 각 pass를 MapReduce 작업으로 표현하여 두 단계의 MapReduce-MapReduce 시퀀스로 실행시킬 수 있음.

6.4.5 Toivonen's Algorithm

Pass 1)

1. Start with a random sample
    - but lower the threshold(s) slightly for the sample
2. Find frequent itemsets in the sample
3. construct the ***negative border***
    - Negative border : An itemset is in the negative border *if it is not frequent in the sample, but all its immediate subsets are frequent.*
    - Immediate subset = "delete exactly one element"

- {*A*,*B*,*C*,*D*} is in the negative border if and only if:
    1. It is not frequent in the sample, but
    2. All of {*A*,*B*,*C*}, {*B*,*C*,*D*}, {*A*,*C*,*D*}, and {*A*,*B*,*D*} are.



Pass 2) new sampling → Count all candidate frequent itemsets from the first pass, and also count sets in their **negative border.**

- If no itemset from the negative border turns out to be frequent, then we found all the frequent itemsets.
- What if we find that something in the negative border is frequent?
  → We must start over again with another sample!

※ 대체로, Pass 1에서는 frequent할 수 있는 candidate pairs를 만들고 → Pass 2는 candidate들을 전체 dataset에 대해서 verify 하는 알고리즘!