Quantum Computing Review II

Wednesday, September 9, 2020

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Outline

- ***A minimal example of Hadamard gate?***
- Measurement
- Recapping the Bell state creation circuit from yesterday
- Unitary matrices

A game

How far can we push this analogy??

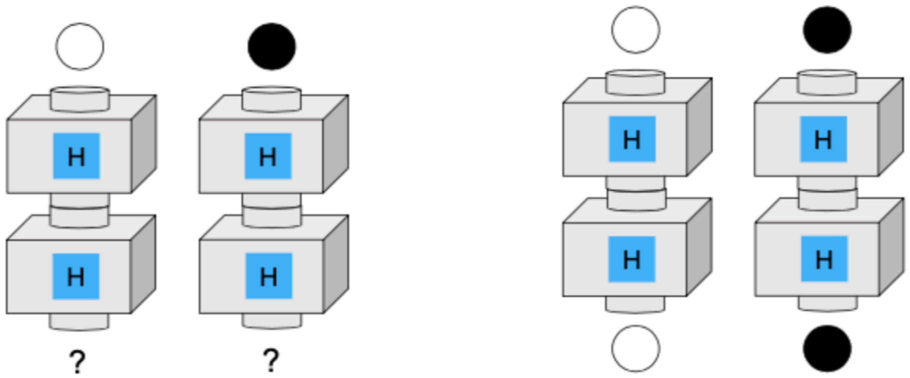
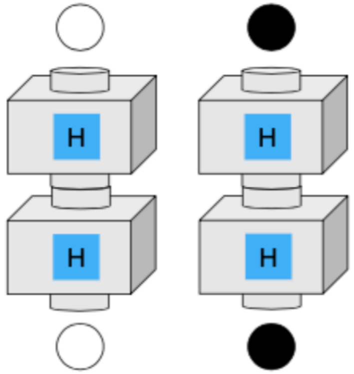


FIG. 6. Showing that the random output of a Hadamard gate goes away when two of them are stacked together.

Is this real?

A GREAT question.



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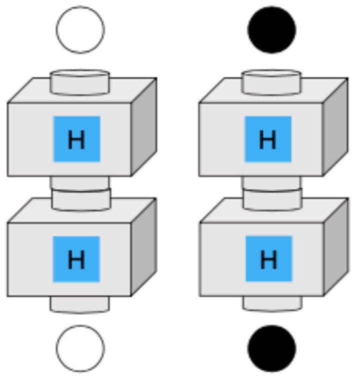

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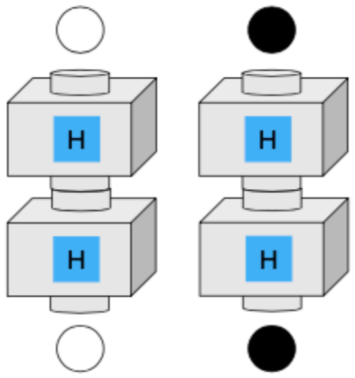
Is this real?

A GREAT question.

Researchers point to double-slit as a minimal real-life example… but the analogy is not quite exact.



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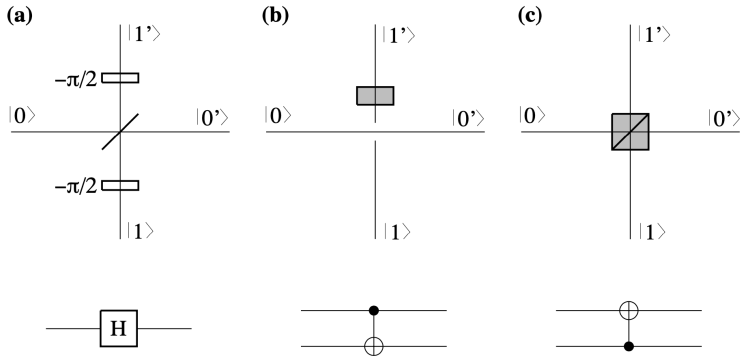
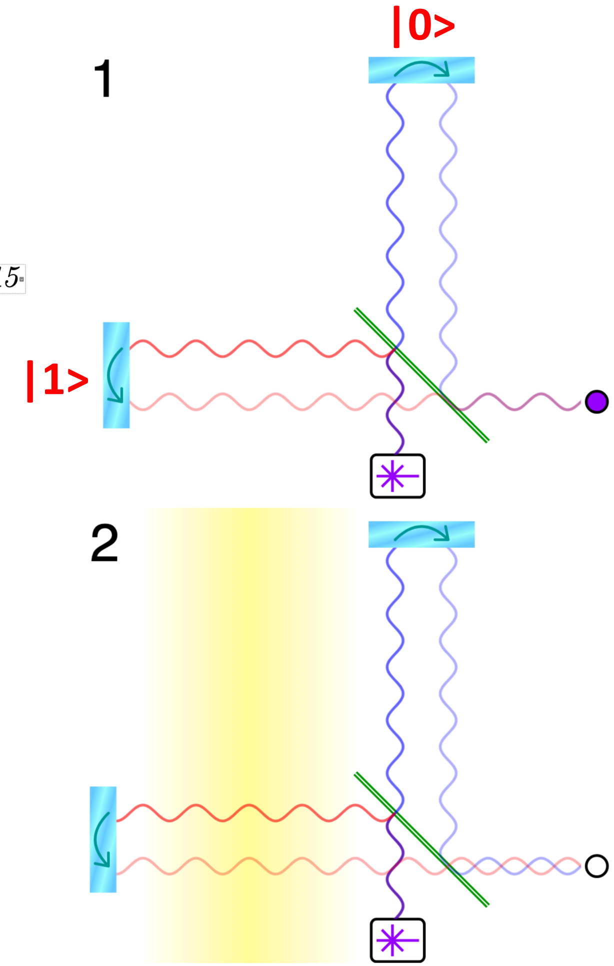
Optical Simulation of Quantum Logic

N. J. Cerf[1], C. Adami[1,2], and P. G. Kwiat[3]
[1] *W. K. Kellogg Radiation Laboratory and* [2] *Computation and Neural Systems*
California Institute of Technology, Pasadena, California 91125
[3] *Physics Division, P-23, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
(March 1997)

FIG. 1. Example of optical simulation of basic quantum logic gates. (a) Hadamard gate on a "location" qubit, using a lossless symmetric beam splitter. (b) Controlled-NOT gate using a polarization rotator. The location and polarization are the control and target qubit, respectively. (c) Same as (b) but the control and target qubits are interchanged by the use of a polarizing beam splitter.





https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Supplemental_Modules_(Physical_and_Theoretical_Chemistry)/Quantum_Tutorials_(Rioux)/Quantum_Optics/277%3A_A_Quantum_Circuit_for_a_Michelson_Interferometer

https://en.wikipedia.org/wiki/LIGO

Minimal physical examples of qubits and gates

- In previous slide, |0> and |1> are encoded as location
- Phase is due to propagation delay
- Enforces idea that qubit is a time and value discretized abstraction
- To be sure, getting a physical device to offer the illusion of “qubit” abstraction is very hard.
- For example, what would be an X gate in this model where we encode it as location?
- DiVincenzo's criteria about whether a quantum mechanical system could make a qubit.

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Measurement

- If we measure the state $|\psi\rangle = \sum_i c_i |v_i\rangle$
- Then the probability of observing an outcome i: $\mathrm{Pr[i]} = |c_i|^2$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

Probability of observing 0: $|a_0|^2$
Probability of observing 1: $|a_1|^2$

- You can also in non-standard basis; we will revisit when discussing VQE

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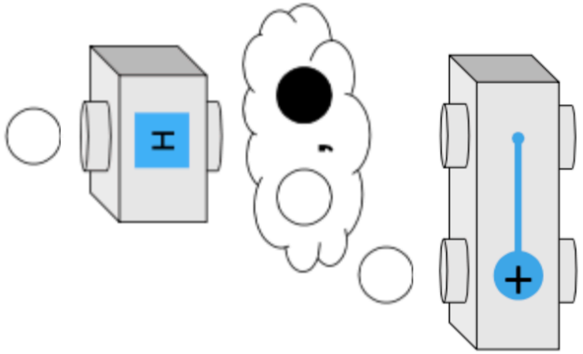
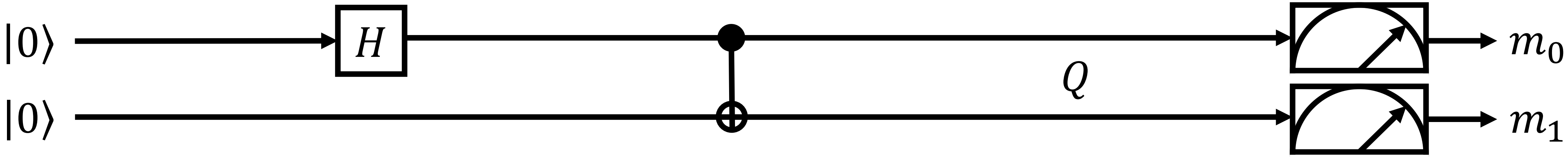
2 qubit gates: entanglement



$$\mathrm{CNOT}(\mathrm{H} \otimes \mathrm{I}|00\rangle) = \mathrm{CNOT}(\mathrm{H}|0\rangle \otimes \mathrm{I}|0\rangle) = \mathrm{CNOT}\begin{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} \\ \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



Two qubits
Tensor product

$$|0\rangle \otimes |0\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} = |00\rangle$$

Product state
Can be factored

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

Controlled-NOT
Two-qubit operator

$$\mathrm{CNOT} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$

Entangled state
Cannot be factored

$$Q = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Measurement
Results correlated

$$(m_0, m_1)$$

$$= \begin{cases}(0,0), P = {}^1\!/_2\\(1,1), P = {}^1\!/_2\end{cases}$$

- Can |psi>= $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ be factored such that |psi> = |a> $\otimes$|b>?
- …

- Can |psi>= $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ be factored such that |psi> = |a> $\otimes$|b>?
- Suppose |a> $\otimes$|b> = $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} a_0 \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \\ a_1 \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$=|psi>
- …

- Can |psi>=$\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ be factored such that |psi> = |a> $\otimes$|b>?
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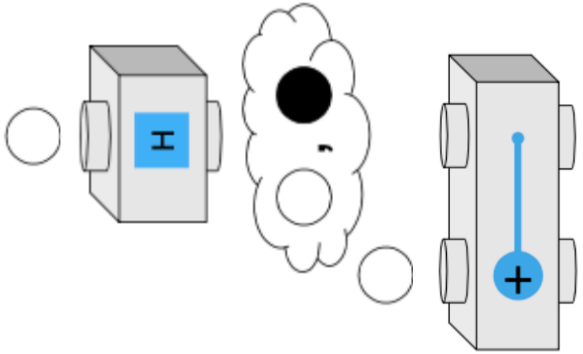
- Because $a_0 b_1 = 0$, therefore either $a_0 = 0$ or $b_1 = 0$
- But neither can be the case because $a_0 b_0$≠0 and $a_1 b_1$≠0
- Therefore supposition is wrong.

What about determining whether arbitrary state has entanglement or not?

- Determining whether states are product states
- Testing product states, quantum Merlin-Arthur games and tensor optimisation. Aram W. Harrow, Ashley Montanaro. https://arxiv.org/abs/1001.0017

2 qubit gates: entanglement



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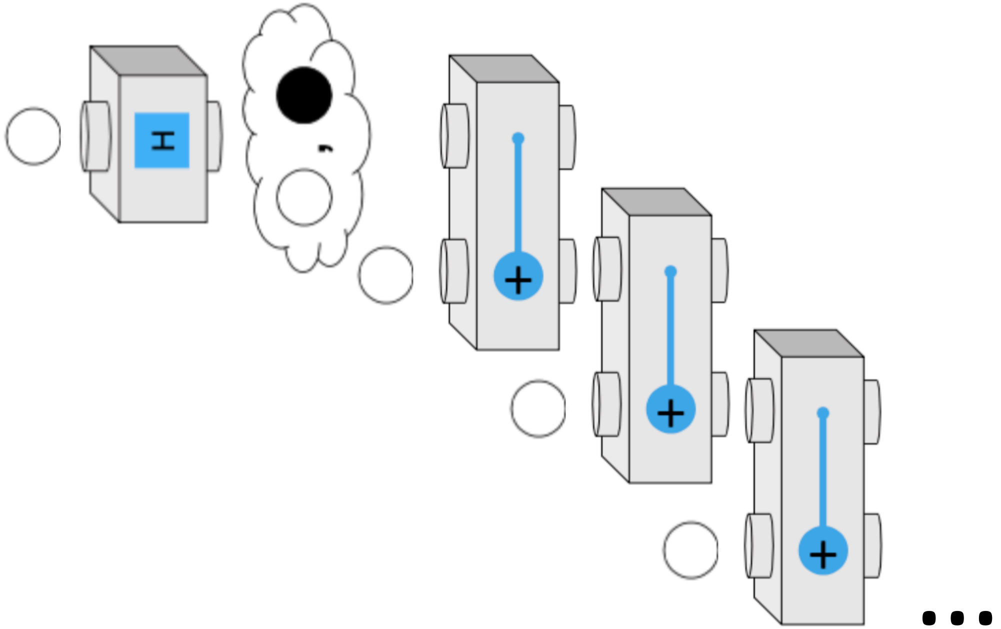
What is the significance??
Entanglement means that you need the full state vector to describe states

2 qubit gates: entanglement



$$\frac{1}{\sqrt{2}}|00\ldots0\rangle + \frac{1}{\sqrt{2}}|11\ldots1\rangle$$

What does Preskill say about this?
Is this plausible? Why not? What if we can pull it off?

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Unitary matrices

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$H = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$\mathrm{CNOT}|00\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Schrodinger's equation $\dfrac{d|\psi(t)\rangle}{dt} = -iH(t)|\psi(t)\rangle$

H is the Hamiltonian, a Hermitian matrix
i.e., H is its own conjugate transpose

Discrete time updating $|\psi(t)\rangle = U|\psi(0)\rangle$

Where U is the operator unitary matrix.
$U^{\dagger}U = UU^{\dagger} = I$
i.e., the adjoint (conjugate transpose) of U is U's inverse

Significance:
Must conserve energy
Must be reversible
Enforces the summation of probabilities = 1

Some topics that we may revisit as needed

- Universality of the {H, T, CNOT, S} quantum gate set
- Mixed states and quantum noise
- First-step quantum algorithms: teleportation and superdense coding. Deutsch Jozsa.

Recommended reading

- Quantum Computer Science: An Introduction. David Mermin.
- UCSD Summer school notes Quantum computing cheat sheet