

Midterm: likely on 10/11 or 11/06

Fall 560 \rightarrow Spring 561/562

What is a robot?

A robot is an autonomous machine handling repetitive/complex tasks.
sensing - computing - control

Mathematic foundation

A set is a collection of elements.

n -dimensional Euclidean spaces, \mathbb{R}^n

symmetric difference: $A \oplus B = A \cup B - A \cap B$

powerset: $\mathcal{P}(S) = \{A \mid A \subset S\}$

e.g. $S = \{1, 2\}$ $\mathcal{P}(S) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

A function is **surjective** if $f(x) = y$. E.g. $\mathbb{R} \rightarrow \mathbb{R}^+ \cup \{0\}$, $x \mapsto x^2$

injective

$\forall x_1 \neq x_2, f(x_1) \neq f(x_2)$

bijective

it's both surjective and injective.

In general, the size of $\mathcal{P}(S)$ is $|\mathcal{P}(S)| = 2^{|S|}$

if $|A| \leq |B|$ and $|A| \geq |B|$, then $|A| = |B|$ useful when trying to prove equality.

A set X and a collection \mathcal{P} of subsets (called open sets) of X form a topological space if:

- $\emptyset \in \mathcal{P}$ and $X \in \mathcal{P}$

- Arbitrary union (\cup) of elements of \mathcal{P} is again in \mathcal{P} .

- Finite intersection (\cap) of elements of \mathcal{P} is again in \mathcal{P} .

E.g. $\mathcal{P} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$



$\{\emptyset, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$



$\{1, 2\} \cap \{2, 3\} = \{2\} \notin \mathcal{P}$

A set is closed if $(X-A)$ is open.

One can build a series of homeomorphisms to deform between two objects, that is.

$F_t, t \in [0, 1]$, is a bijective continuous function with domain X .

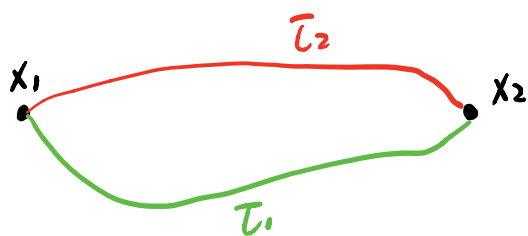
$F_0(x) = X, F_1(x) = Y \quad F_x, x \in [0, 1]$ here is known as deformation.

classic example: coffee mug \Leftrightarrow donut

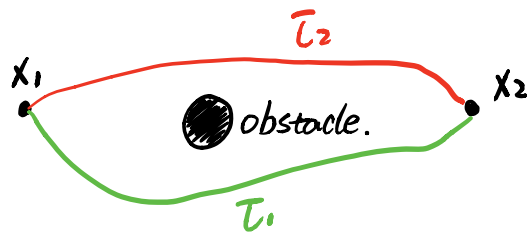
Simply connected space

A topological space X is simply connected if $\forall x_1, x_2 \in X$ and any τ_1, τ_2 with $\tau_1(0) = \tau_2(0) = x_1$ and $\tau_1(1) = \tau_2(1) = x_2$, τ_1 and τ_2 are homotopic. Otherwise, X is multiply connected.

Do not understand!



↑ simply connected



↑ multiply connected.