Neural Computation

Theories of Learning

Brain-Inspired Computing

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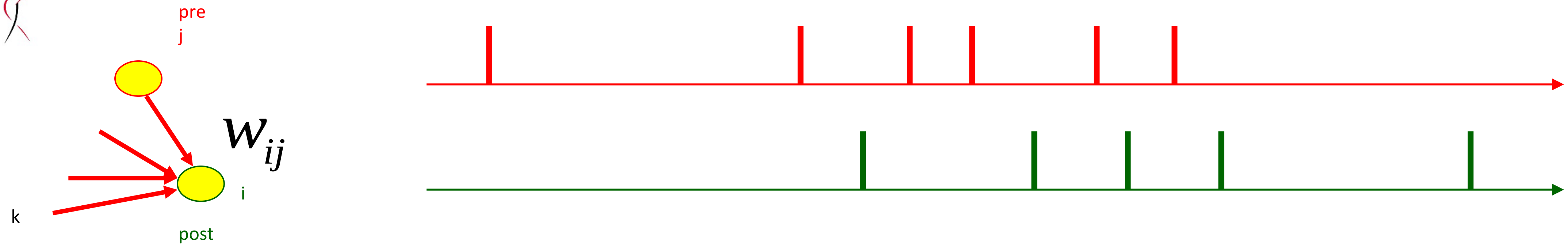
Correlation-based learning in a firing rate formalism

- For the time being, we content

ourselves with a description in terms of

mean firing rates

Hebbian Learning: Rate Models



When an axon of cell j repeatedly or persistently takes part in firing cell i, then j's efficiency as one of the cells firing i is increased

Hebb, 1949

- local rule
- simultaneously active (correlations)

Rate model: active = high rate = many spikes per second

General formula for the change of the synaptic efficacy

2 important aspects of Hebb's plasticity

- *Locality*: the change of the synaptic efficacy can only depend on local variables, i.e., on information that is available at the site of the synapse, such as pre- and postsynaptic firing rate, and the actual value of the synaptic efficacy, but not on the activity of other neurons.

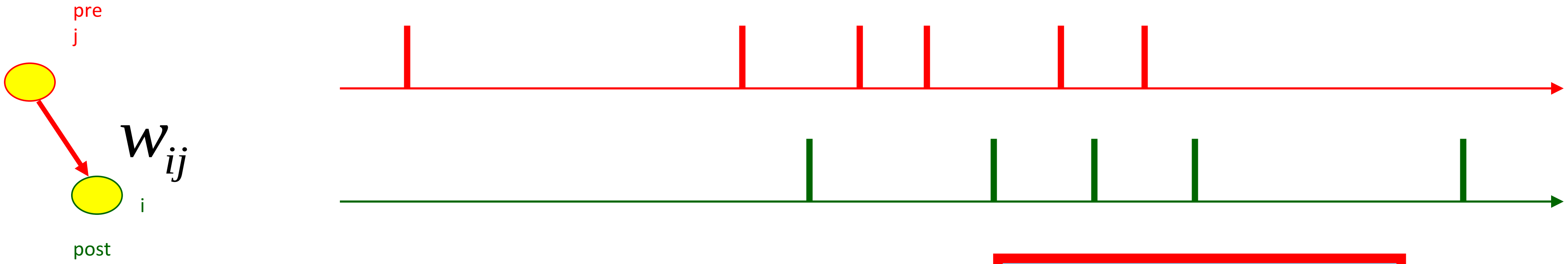
A sufficiently "well-behaved" function, not yet determined

$$\frac{d}{dt} w_{ij} = F(w_{ij}; \nu_j^{pre}, \nu_i^{post})$$

- *Joint activity: implies that pre- and postsynaptic neurons have to be active simultaneously for a synaptic weight change to occur.*

  We can use this property to learn something about the function F

Rate based Hebbian Learning

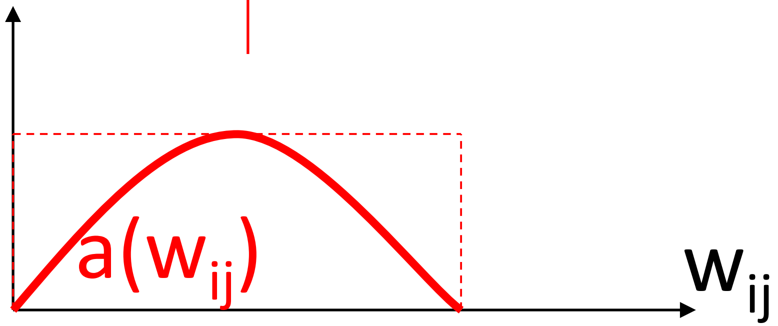


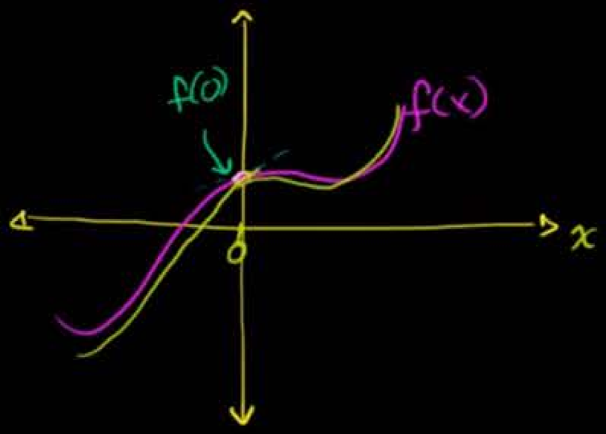
Blackboard

$$\frac{d}{dt} w_{ij} = F(w_{ij}; \nu_j^{pre}, \nu_i^{post})$$

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} \nu_j^{pre} + a_1^{post} \nu_i^{post} + a_2^{corr} \nu_j^{pre} \nu_i^{post} + \dots$$

$a = a(w_{ij})$





$f(0),\ f'(0),\ f''(0),\ f'''(0)\ \ldots$

$p(0) = f(0)$ ✓ $\qquad p(x) = \underline{\underline{f(0)}}$ (1)

$p'(0) = f'(0) \qquad * \ \underline{p(x) = f(0) + f'(0)x}$ (2)

$p(0) = f(0)$ ✓

$p'(x) = \underline{\underline{f'(0)}}$

$\underline{p'(0) = f'(0)}$

(3) $P(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2$

$P(0) = f(0) \qquad p'(x) = f'(0) + f''(0)x$

$p'(0) = f'(0) \qquad p''(x) = f''(0)$

$p''(0) = f''(0)$

$$p(x) = f(0) + f'(0)x + f''(0)\cdot\frac{1}{2\cdot 1}\cdot x^2 + f'''(0)\cdot\frac{1}{2\cdot 3\cdot 1}\cdot x^3$$

$3\cdot 2\cdot 1$

$$+ f''''(0)\cdot\frac{1}{4\cdot 3\cdot 2\cdot 1}\cdot x^4 + \cdots\cdots f^n(0)\cdot\frac{x^n}{n!}$$

$4! = 4\cdot 3\cdot 2\cdot 1$

* Maclaurin Series

Hebbian Rule and Taylor series Expansion

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} \nu_j^{pre} + a_1^{post} \nu_i^{post} + a_2^{corr} \nu_j^{pre} \nu_i^{post} + \dots$$

This term implements the AND condition for joint activity.

- If the Taylor expansion had been stopped *before the bilinear term*, the learning rule would be called 'non-Hebbian', because pre- or postsynaptic activity alone induces a change of the synaptic efficacy and joint activity is irrelevant.

Therefore, a Hebbian learning rule needs either the bilinear term

$$a_2^{corr} \nu_j^{pre} \nu_i^{post}$$

or a higher-order term such as

$$a_3^{corr} \nu_j^2 \nu_i$$

that *involves the activity of both pre- and postsynaptic neurons*.

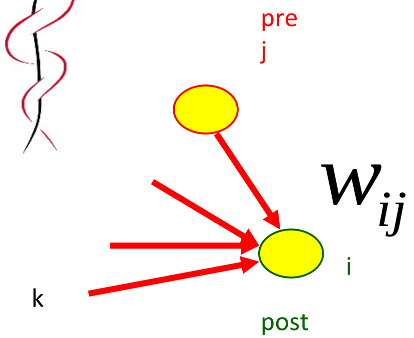
The simplest choice for a Hebbian learning rule within the Taylor expansion

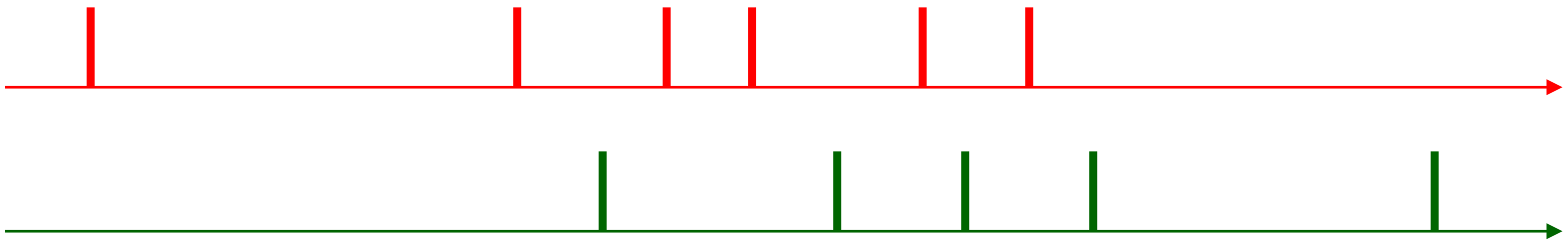
$$\frac{d}{dt}w_{ij} = a_0 + a_1^{pre}\nu_j^{pre} + a_1^{post}\nu_i^{post} + a_2^{corr}\nu_j^{pre}\nu_i^{post} + \dots$$

Fix $a_2^{corr} = c > 0$

And then set all other terms to zero

$$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}\nu_i^{post} = c\,\nu_j^{pre}\nu_i^{post}$$





pre	on	off	on	off
post	on	on	off	off
	+	0	0	0

$$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}\nu_i^{post}$$

The coefficient a_2^{corr} *depends on* w_{ij}

- This dependence can be used to limit the growth of weights at a maximum value w_{max}
- Two standard choices of weight-dependence

’hard bound’

$$a_2^{corr} = \gamma$$

$$0 < \gamma < w_{\max}$$

weight growth stops abruptly if γ *reaches the upper bound* $wmax$

‘soft bound’

$$a_2^{corr}(w_{ij}) = \gamma(w_{\max} - w_{ij})^{\beta}$$

a change tends to zero as its wij approaches its maximum value

where γ and β are positive constants (typically β=1)

Problem

No possibility for a decrease of synaptic weights

In a system where synapses can only be strengthened, all efficacies will eventually saturate at their upper maximum value - how do we solve this?

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} \nu_j^{pre} + a_1^{post} \nu_i^{post} + a_2^{corr} \nu_j^{pre} \nu_i^{post} + \ldots$$

soft bound

IF
$$\begin{cases} a_2^{corr}(w_{ij}) = \gamma (w_{\max} - w_{ij})^{\beta} \\ w_{\max} = \beta = 1 \\ a_0(w_{ij}) = -\gamma_0 w_{ij} \end{cases}$$

In the absence of stimulation, synapses spontaneously decay back to zero

$$\frac{d}{dt} w_{ij} = \gamma (1 - w_{ij}) \nu_i \nu_j - \gamma_0 w_{ij}$$