## **Topics:**

- 1. Complexity
  - (a) Definition of time and space complexity
  - (b) Counting operations of algorithms
  - (c) Role of the length of the input
  - (d) Worst case, Best case, Average case time complexity
- 2. Solving summations
  - (a) Using induction to prove results about summations
  - (b) Using integrals to find bounds of summations
  - (c) Common summations
- 3. Classification of functions O(g(x)),  $\Omega(g(x))$ ,  $\Theta(g(x))$ , O(g(x)), and O(g(x))
  - (a) Definition
  - (b) Using the definition to classify functions
  - (c) Proof that  $\Theta$  is an equivalence relation
  - (d) Properties: how the sets are related
  - (e) Using limits to compare function growth
  - (f) Ordering of common functions, including proofs for each one of the cases
- 4. Improving complexity
  - (a) Lower bound for a problem
  - (b) Upper bounds (algorithms)
  - (c) Finding better lower and upper bounds (gap)
- 5. Divide and Conquer Algorithms
  - (a) The steps of the divide and conquer strategy: divide, recursively solve, combine
  - (b) Finding a recurrence relation describing the time complexity of divide and conquer algorithms
  - (c) Example: Multiplication of two binary numbers
- 6. Recurrence Relations
  - (a) Solving recurrence relations by iteration
  - (b) Example: Mutliplication of two binary numbers
  - (c) Proof of the Master Theorem
  - (d) Solving recurrence relations using the Master Theorem

## 7. The sorting problem

- (a) Divide and conquer approaches
- (b) Quicksort using partition
- (c) Analysis of Quicksort (Best case, worst case)
- (d) Randomized Quicksort
- (e) The Mergesort algorithm
- (f) Analysis of the time complexity of Mergesort
- (g) Using Comparison Decision Trees to compute a lower bound for comparison based sorting
- (h) Optimality of algorithms

## 8. The Selection problem

- (a) Finding min, max, median as special cases of the selection problem
- (b) Lower bound for the Selection problem
- (c) Divide and Conquer algorithm for selection (randomized)
- (d) Divide and Conquer algorithm for selection (not randomized)
- (e) Recurrence relation for the (not randomized) Select algorithm