

Topics:

1. Matrix multiplication
 - (a) Lower bound
 - (b) Analysis of the usual matrix multiplication algorithm
 - (c) Strassen's Matrix Multiplication algorithm
 - (d) Recurrence relation for Strassen's Algorithm
2. Graph representation
 - (a) Definition of a graph
 - (b) Properties: degree, size of a graph
 - (c) Undirected and directed graphs
 - (d) Representation using Adjacency list and Adjacency matrix
 - (e) Types of graphs: sparse, complete, K_i , etc.
3. Undirected graphs
 - (a) DFS using explore
 - (b) postvisit and previsit numbers
 - (c) Types of edges: Tree and back
 - (d) Connected components
 - (e) Using DFS to find connected components
 - (f) Using DFS to determine if a graph is acyclic
4. Directed graphs
 - (a) DFS pre and post visit number computation
 - (b) Types of edges: back, tree, cross, and forward
 - (c) Properties of pre and post visit numbers
5. Directed Acyclic Graphs (DAG)
 - (a) Definition
 - (b) Topological sorting (linearization)
 - (c) Proof of correctness of linearization using type of edges and the properties of post visit numbers
 - (d) Finding Source and sink vertices in a DAG
6. Strongly Connected components (SCC)
 - (a) Definition of the "connected" relation
 - (b) Definition of SCC as equivalence classes
 - (c) Decomposing a digraph into a DAG of SCC
 - (d) Proof using properties of post visit numbers, that explore can find a vertex in a sink SCC
 - (e) Algorithm to find SCC on a digraph

7. Shortest paths on undirected graphs
 - (a) The breadth first search algorithm (BFS)
 - (b) Using BFS to find all shortest paths from one node
 - (c) Proof that BFS correctly finds all shortest paths from a given node
 - (d) Analysis of the BFS and DFS algorithms
8. Priority Queues
 - (a) Min/Max binary Heap definition and properties
 - (b) Operations on Heaps and analysis
 - (c) Heapsort and analysis
9. Shortest paths
 - (a) Weighted graph representation
 - (b) Dijkstra's shortest paths algorithm
 - (c) Correctness of Dijkstra's algorithm
 - (d) Binary heap (min-heap) implementation
 - (e) Worst case analysis of Dijkstra's algorithm
 - (f) Dijkstra's in the case of negative weight edges
 - (g) All pairs shortest paths (Bellman-Ford)
 - (h) Negative cycle detection
10. Disjoint sets
 - (a) Union-find problem
 - (b) Union by rank
 - (c) Find using path compression
 - (d) Analysis of union-find (without path compression)
 - (e) Complexity of union-find with path compression
11. Greedy algorithms
 - (a) Definition of the greedy approach
 - (b) Definition of a MST
 - (c) Definition and properties of a cut
 - (d) Kruskal's MST algorithm
 - (e) Prim's MST algorithm and its difference with respect to Kruskal's algorithm
 - (f) Correctness of Prim's and Dijkstra's algorithms
 - (g) Analysis of both MST algorithms