Power Method & Triangle Alg.

Yanbo Fang & Xuenan Wang

Target: Solve $Mx = x$

- Power Method
- Triangle Algorithm
- Jacobi & Gauss_Seidel & SOR

Power Method: Solve $Mx = x$

original Graph

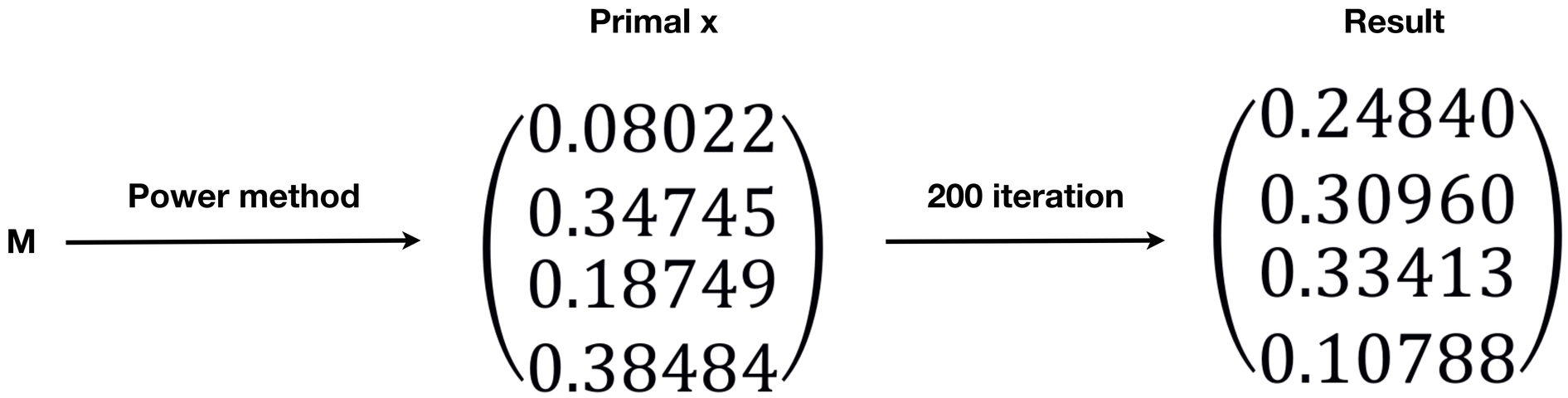
$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

process →

M

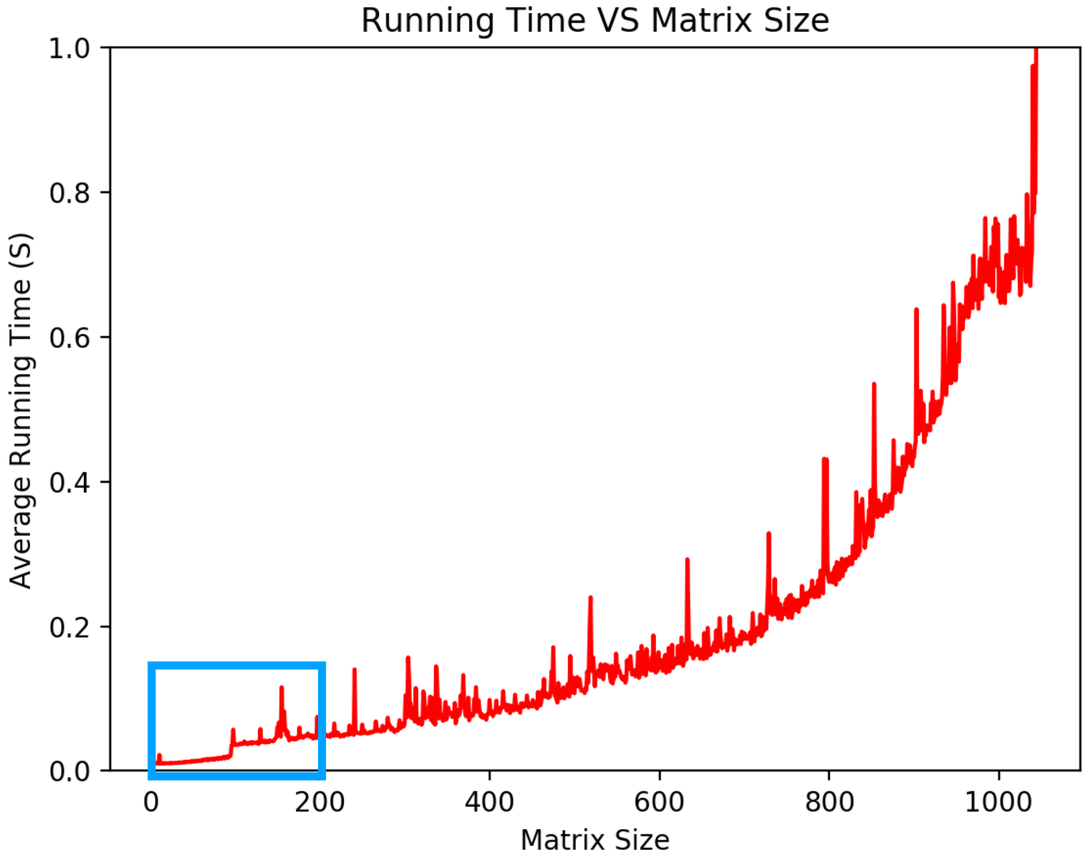
$$\begin{pmatrix} 0.32083 & 0.03750 & 0.32083 & 0.46250 \\ 0.03750 & 0.46250 & 0.32083 & 0.46250 \\ 0.32083 & 0.46250 & 0.32083 & 0.03750 \\ 0.32083 & 0.03750 & 0.03750 & 0.03750 \end{pmatrix}$$

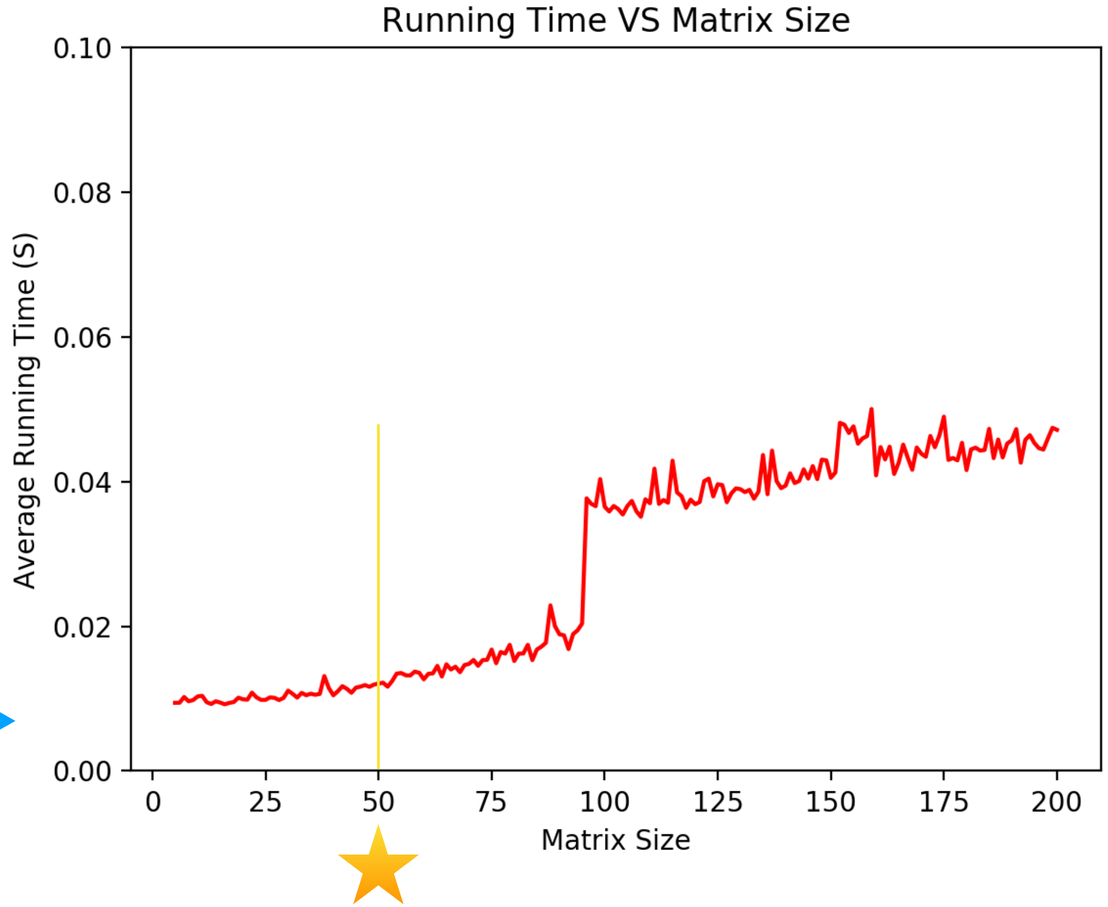
Power Method: Solve $Mx = x$



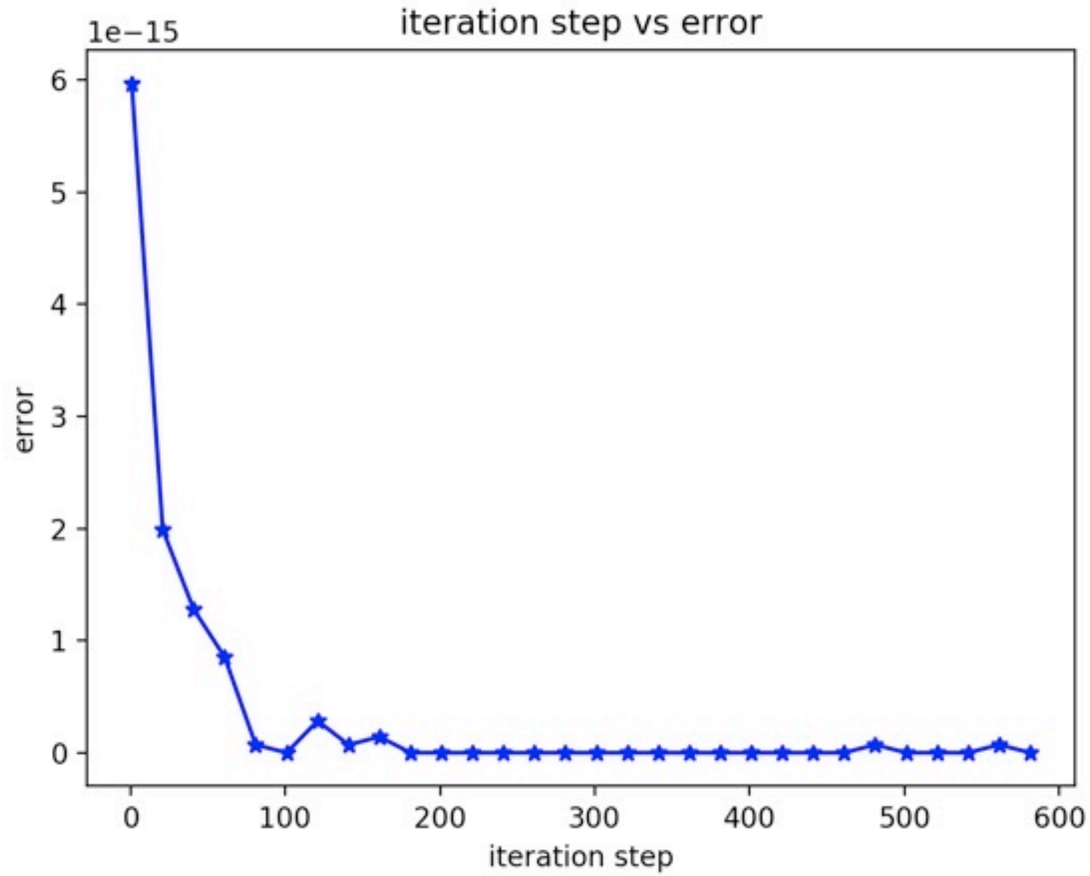
The error is: 0

Power Method: Solve $Mx = x$



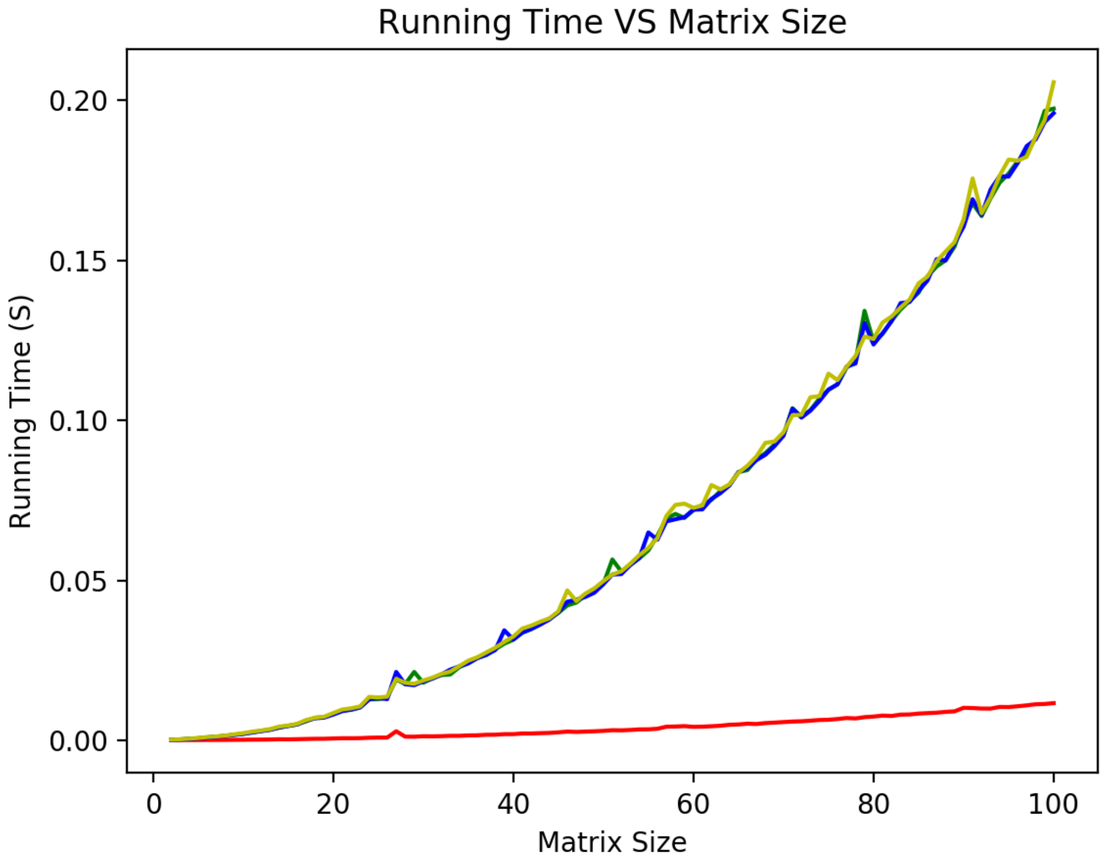


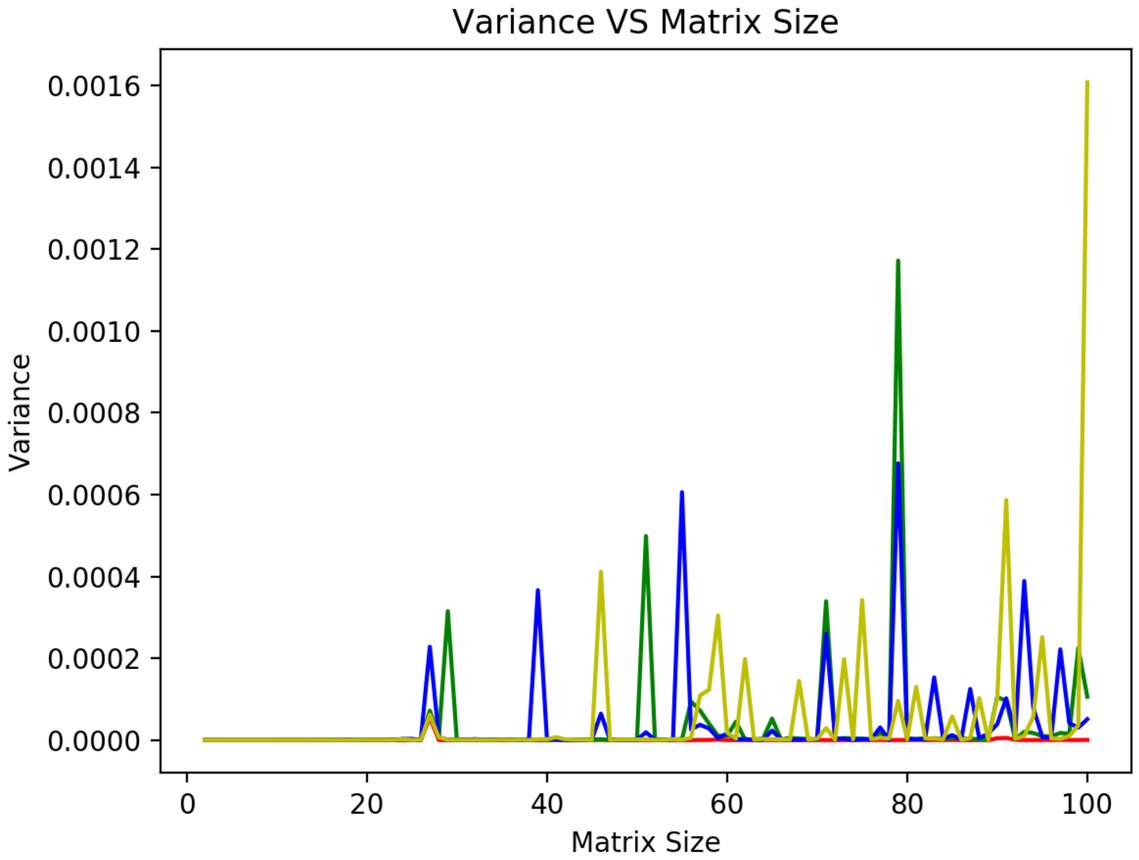
Power Method: Solve $Mx = x$



Matrix size: 50

Power Method: Solve $Mx = x$



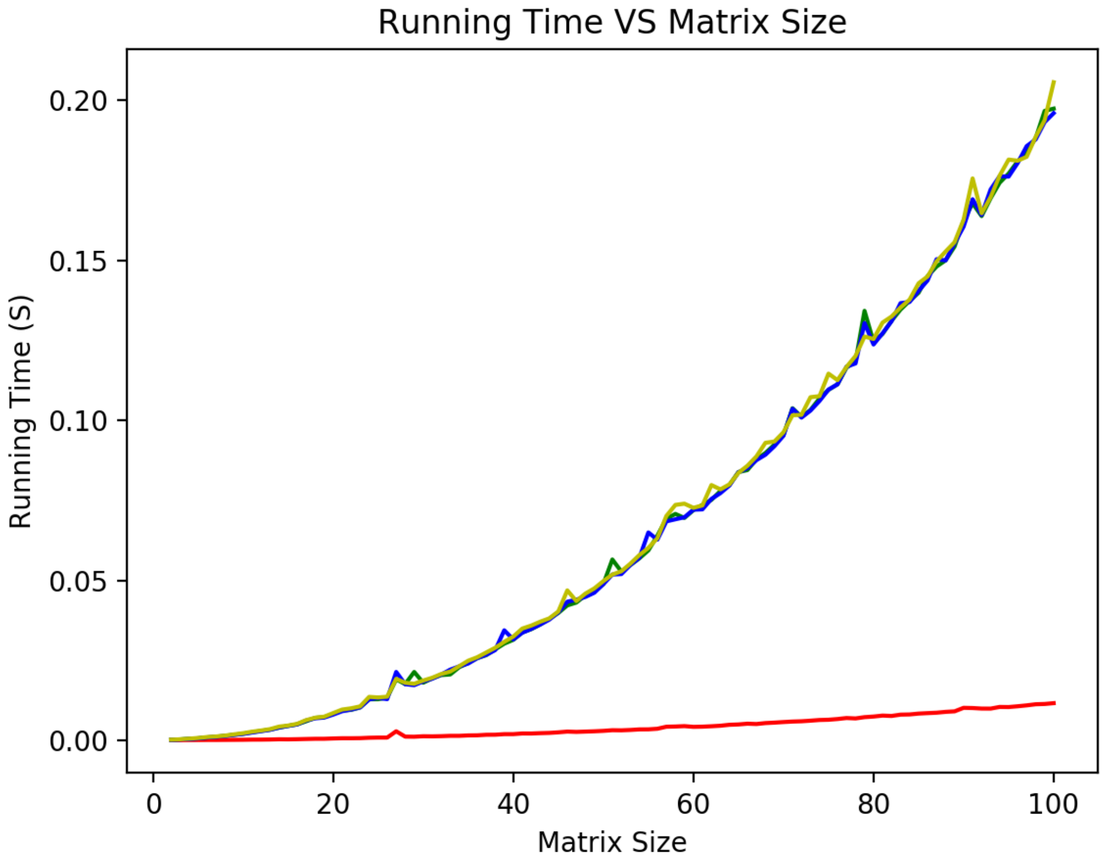


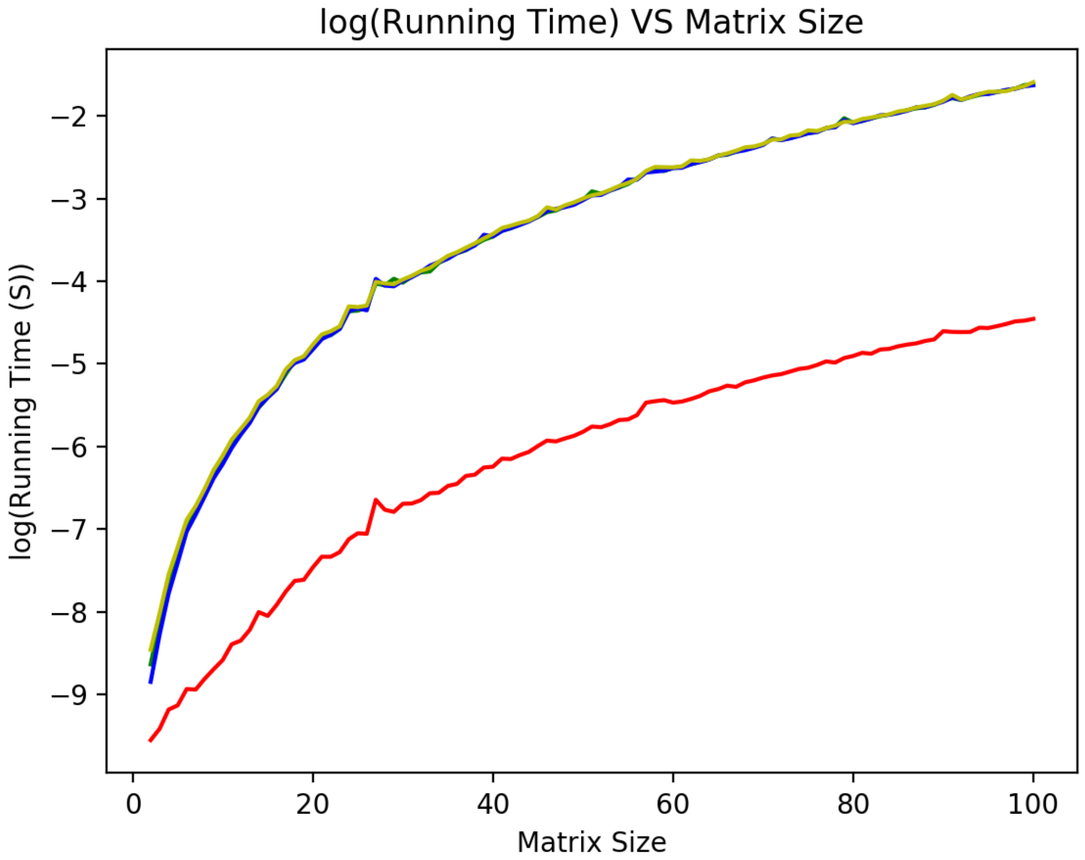
Power Method **Jacobi**

Iteration: 20

Gauss Seidel **SOR**

Power Method: Solve $Mx = x$



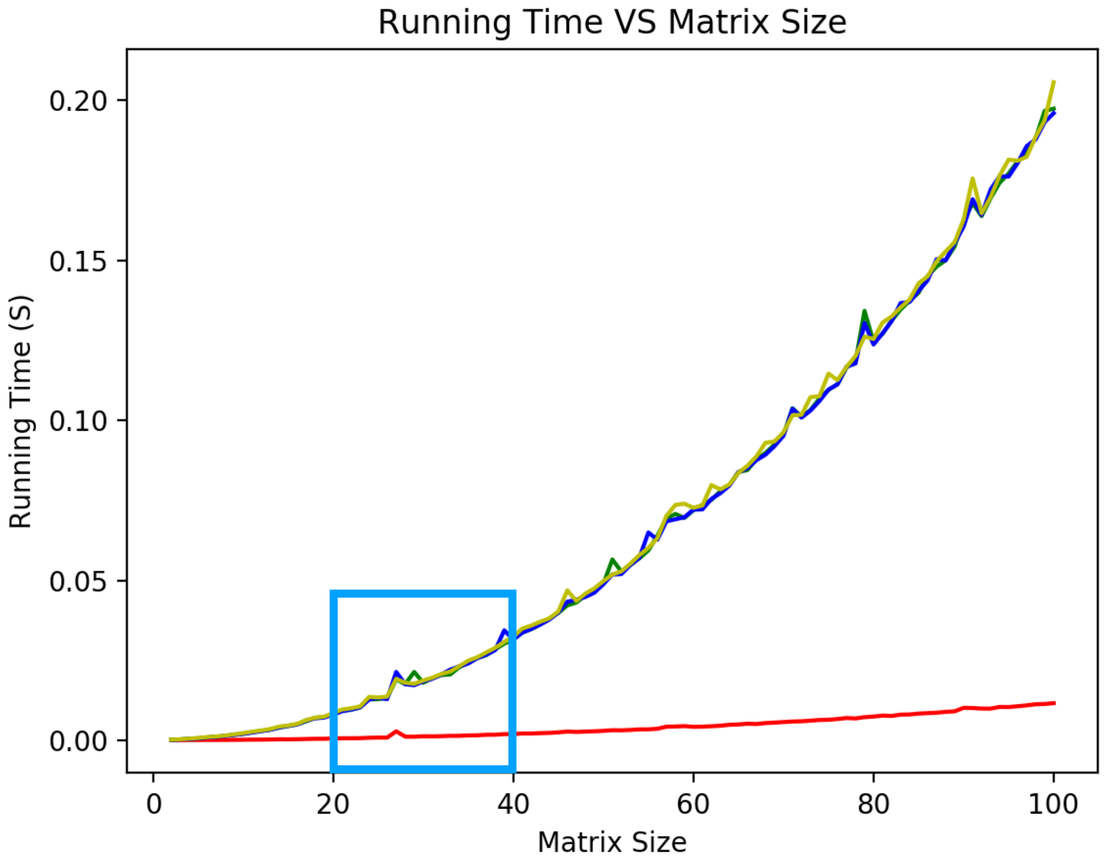


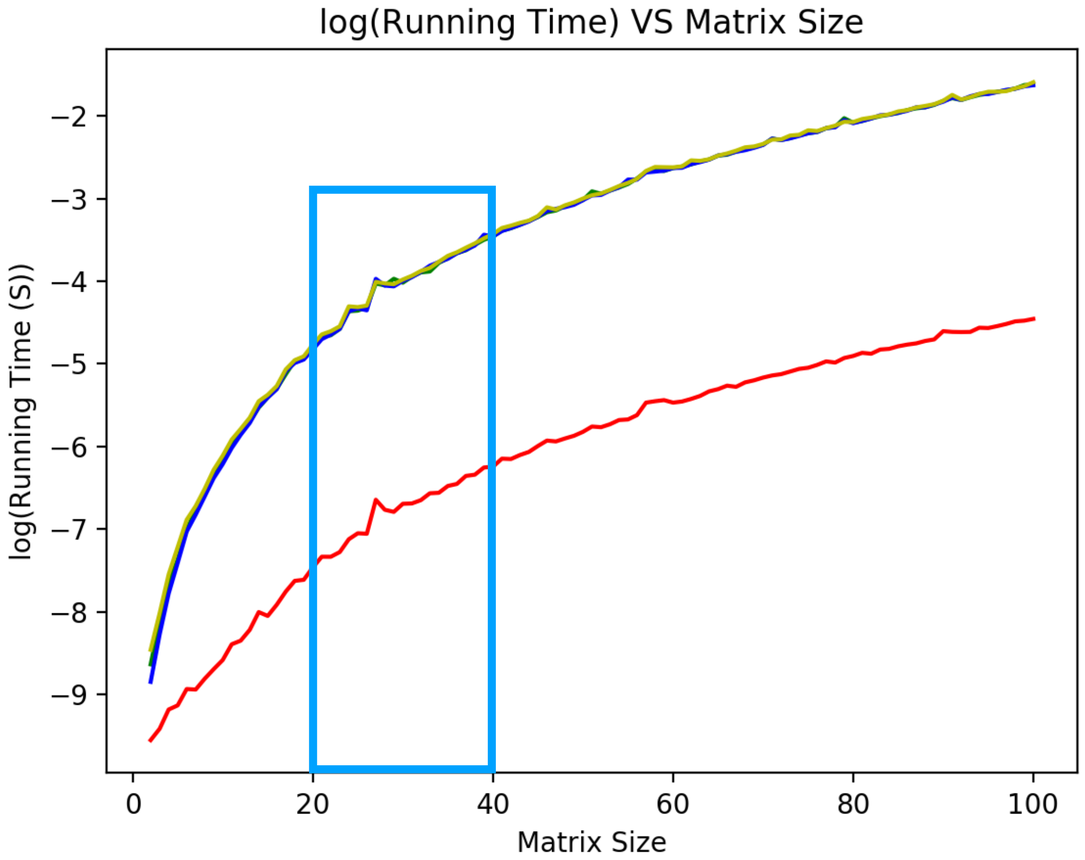
Power Method **Jacobi**

Iteration: 20

Gauss Seidel **SOR**

Power Method: Solve $Mx = x$

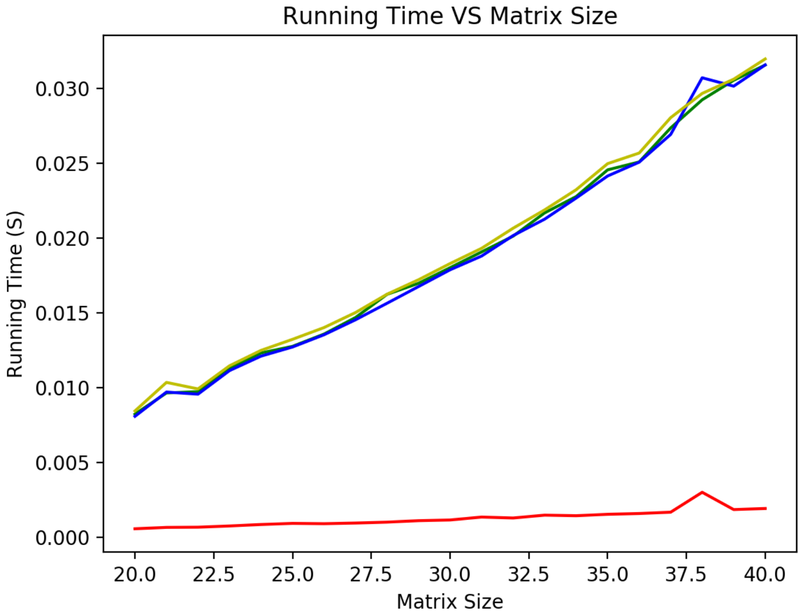


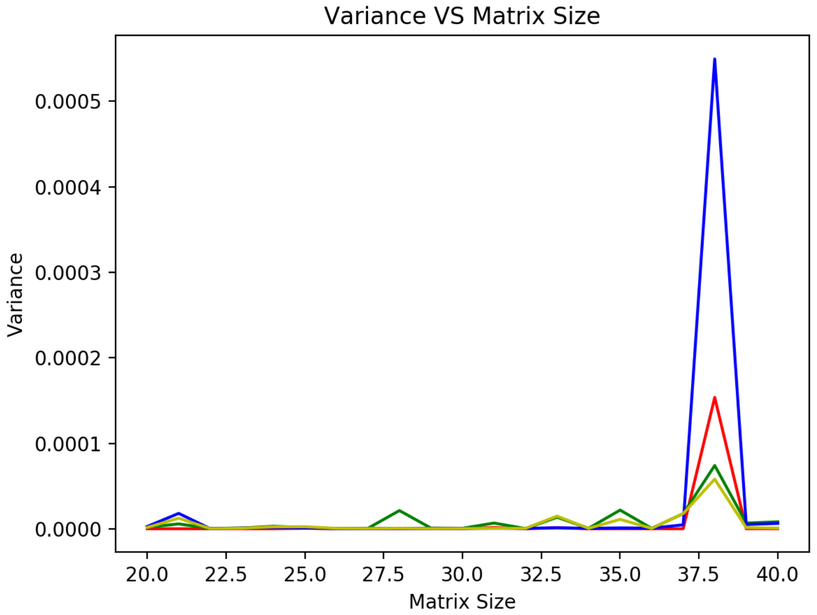


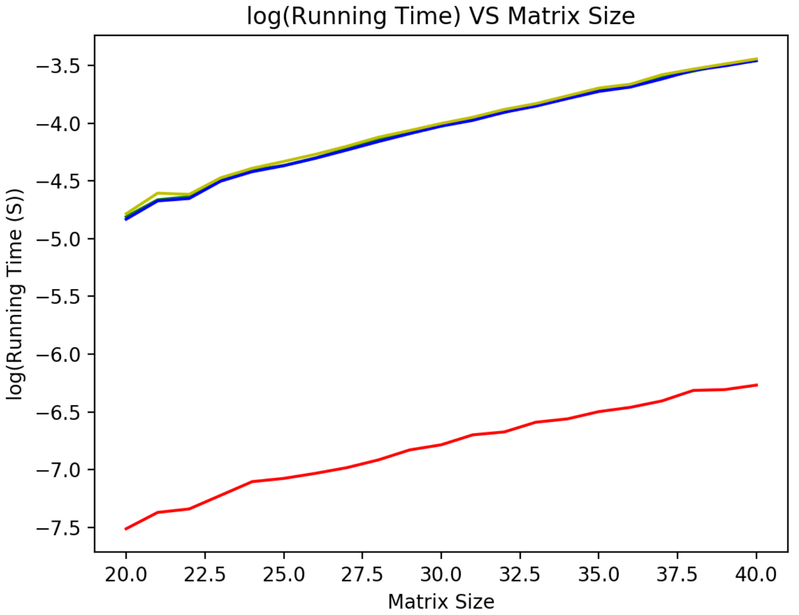
Power Method **Jacobi**

Gauss Seidel **SOR**

Power Method: Solve $Mx = x$







Iteration: 100

Power Method **Jacobi**

Gauss Seidel **SOR**

Power Method: Generate Symmetric Matrix

Connected graph

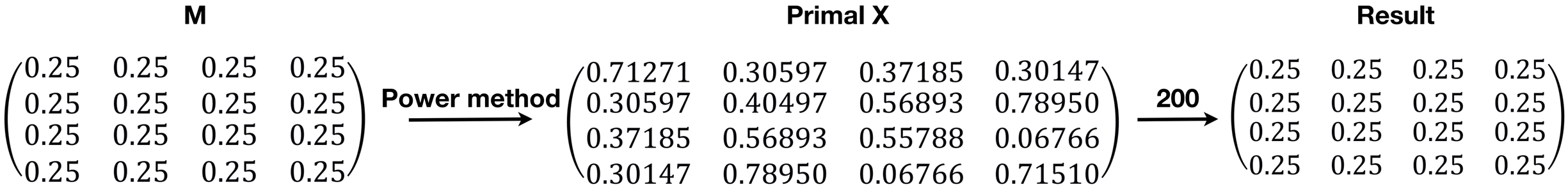
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Process

M

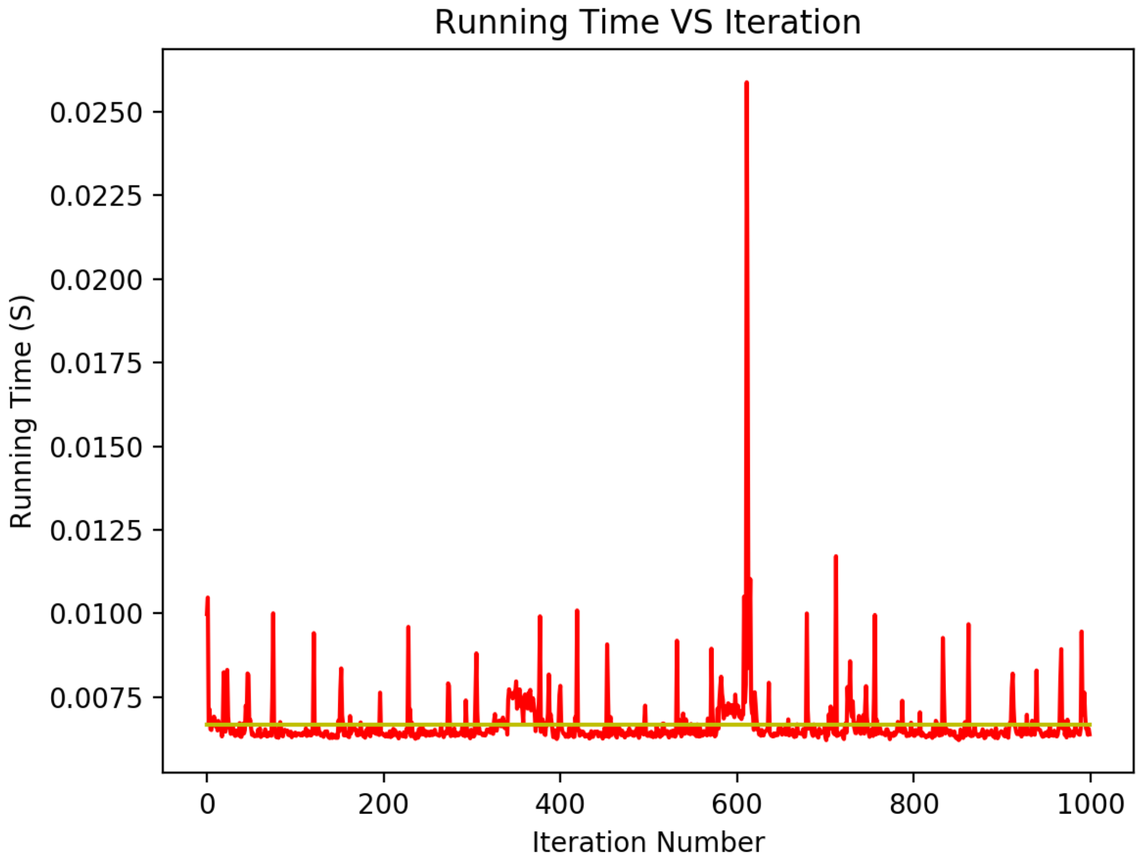
$$\begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}$$

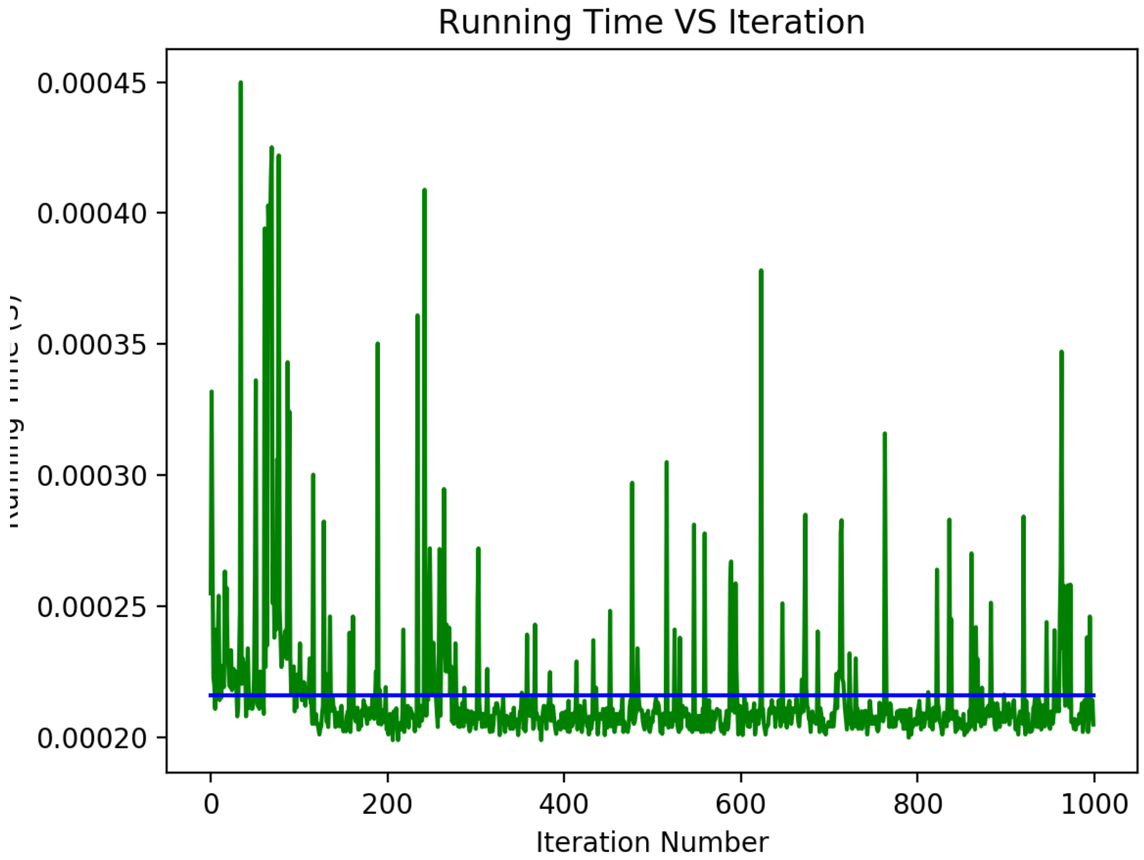
Power Method: Generate Symmetric Matrix



$$\begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix} \xrightarrow{\text{Power method}} \begin{pmatrix} 0.71271 & 0.30597 & 0.37185 & 0.30147 \\ 0.30597 & 0.40497 & 0.56893 & 0.78950 \\ 0.37185 & 0.56893 & 0.55788 & 0.06766 \\ 0.30147 & 0.78950 & 0.06766 & 0.71510 \end{pmatrix} \xrightarrow{200} \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}$$

Power Method: Apply on Symmetric Matrix





Random Matrix

mean:	**0.006680183172225950**
var:	8.55905404366638E-07

Random got 100% higher variance in a 100 times test.

Symmetric Matrix

mean:	**0.0002160489559173580**
var:	7.31300723089134E-10

Triangle Algorithm: Solve $Mx = x$

M

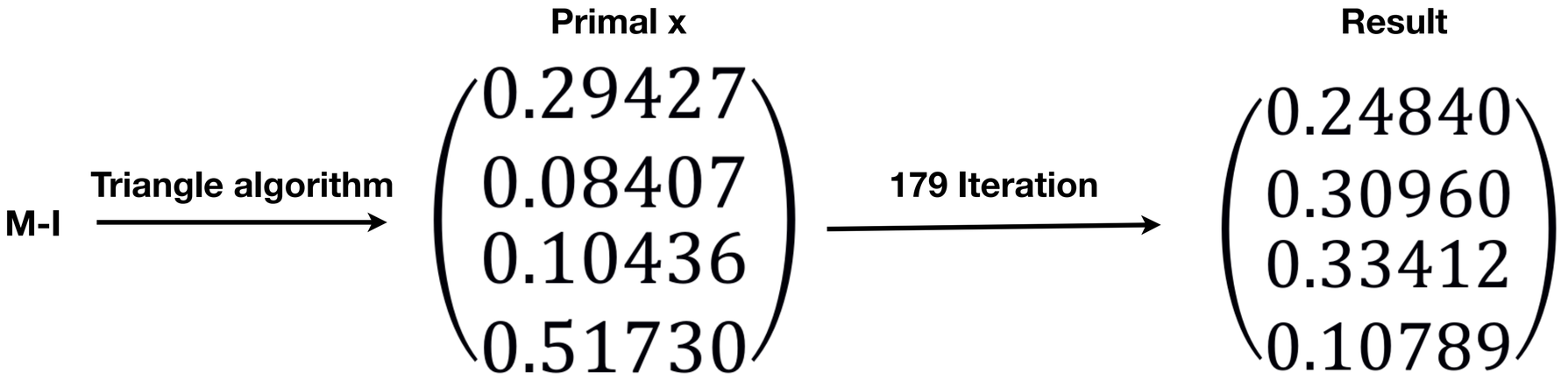
$$\begin{pmatrix} 0.32083 & 0.03750 & 0.32083 & 0.46250 \\ 0.03750 & 0.46250 & 0.32083 & 0.46250 \\ 0.32083 & 0.46250 & 0.32083 & 0.03750 \\ 0.32083 & 0.03750 & 0.03750 & 0.03750 \end{pmatrix}$$

(M-I)x = 0

M-I

$$\begin{pmatrix} -0.67916 & 0.03750 & 0.32083 & 0.46250 \\ 0.03750 & -0.53750 & 0.32083 & 0.46250 \\ 0.32083 & 0.46250 & -0.67916 & 0.03750 \\ 0.32083 & 0.03750 & 0.03750 & -0.96250 \end{pmatrix}$$

Triangle Algorithm: Solve $Mx = x$



The error is: 0.00001210521

Triangle Algorithm: Solve $Mx = x$