Goat Discovery Bot

CS 520 Final Question 4

Spring 2019

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GOAT DISCOVERY BOT

a) Logically, how could you (LogicalGoatDiscoveryBot) model this information? Probabilistically, how could you (ProbabilisticGoatDiscoveryBot) model this information? Hint: Consider the statements In A, In B, In C.

LogicalGoatDiscoveryBot: $(A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C) = True$

ProbabilisticGoatDiscoveryBot: $P(In\ A \mid \neg B \wedge \neg C) + P(In\ B \mid \neg A \wedge \neg C) + P(In\ C \mid \neg A \wedge \neg B) = 1$

b) Under the logical formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

Under the logical formulation, there are no difference among 'Select A', 'Select B' and 'Select C'. Because our knowledge base is only that we know there is a goat in these three locations. Therefore, these three locations are same to us. So obviously, we don't have a best action among these three.

c) Under the probabilistic formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

Same as logical formulation. Since we don't have further information, there's no difference among these three choice. Hence, no best action.

d) Update your logical formulation to reflect this new information.

LogicalGoatDiscoveryBot: $(A \wedge \neg C) \vee (\neg A \wedge C) = True$

e) Update your probabilistic formulation to reflect this new information. Hint: The CBMHBot's decision to tell you the goat is not in B depended both on which location you selected, and where the goat actually is.

ProbabilisticGoatDiscoveryBot: $P(In\ A \mid Open\ B \wedge \neg C) + P(In\ C \mid Open\ B \wedge \neg A) = 1$

$$\frac{P(In\ A \wedge Open\ B \wedge \neg C)}{P(Open\ B \wedge \neg C)} + \frac{P(In\ C \wedge Open\ B \wedge \neg A)}{P(Open\ B \wedge \neg A)} = 1$$

f) Under the logical formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

Since we already know that goat is not in B, therefore 'Re-Select B' makes no sense. 'Re-Select A' and 'Re-Select C' here makes no difference. We know that there must be one, A or C, that has goat, so currently the possibility we can choose the correct one is 1/2.

g) Under the probabilistic formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

By doing 'Re-Select A' is actually making no difference. Since we choose A when we have no further information, in which case the possibility of choosing the right one is 1/3, the possibility applies now. However, if we change to 'Re-Select C', because we know that there must be a goat in A or C, therefore $P(A) + P(C) = 1$ stands. We can easily know that at this time, 'Re-Select C' will give us 2/3 of chances to get the right answer.

h) Under the logical formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

There's no difference to change or not. Because in this case, the goat being in A or C is equal to 1/2.

i) Under the probabilistic formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

Obviously we need to change. Because at this case, the possibility that goat in A is 1/3 and in C is 2/3. We need to change our initial choice.

j) Who is more successful in their mission, LogicalGoatDiscoveryBot or ProbabilisticGoatDiscoveryBot? Justify your answer.

ProbabilisticGoatDiscoveryBot is more successful.

As explained above, when we facing a second chance to change our choice, for LogicalGoatDiscoveryBot, we have equally 1/2 possibility to choose the right answer and it doesn't matter if we change our choice or not. However, for ProbabilisticGoatDiscoveryBot, we will have a possibility of 2/3 to win if we change our initial answer. Overall, when we get the chance to re-select, probabilistic method is a better way to think this obviously.

Bonus: You initially select location A. Suppose that you know CouldBeMoreHelpfulBot is biased, in the following way: if CBMHBot has a choice between telling you B and C, then CBMHBot tells you B with probability p, and C with probability 1–p. CBMHBot tells you that the goat is not in location B. What is the utility of sticking with your initial selection? What is the utility of switching to C? What is the rational choice, and does it depend on p?

In this case, probability of sticking with initial selection will be:

$$P(In\ A \mid Open\ B \wedge Select\ A) = \frac{P(Open\ B \wedge Select\ A \mid In\ A) \cdot P(In\ A)}{P(Open\ B \wedge Select\ A)} = \frac{p \cdot \frac{1}{3}}{\frac{1}{3} \cdot (1+p)} = \frac{p}{1+p}$$

Changing to C will be:

$$P(In\ C \mid Open\ B \wedge Select\ A) = \frac{P(Open\ B \wedge Select\ A \mid In\ C) \cdot P(In\ C)}{P(Open\ B \wedge Select\ A)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{3} \cdot (1+p)} = \frac{1}{1+p}$$

Since 0<p<1, therefore no matter how p changes, ‘Re-Select C’ will always be a better choice.