$T(n)$

Correct

$\star$ $O(n)$
$O(\log n)$
$O(n \log n)$
$O(n^2)$

$a * m$

$T(m,n) = O(m \log n)$

$\Rightarrow O(m \log n) \le C * m \log n$

$\Rightarrow C * k \log \frac{n}{2} + C(m-k) \log \frac{n}{2} \le C * m \log n$

$\Rightarrow C * m(\log n - \underline{\underline{\log 2}}) + a * m \le C * m \log n$

$\hookrightarrow 1$

$\Rightarrow C * m \log n - C * m + a * m \le C * m \log n$

$\Rightarrow -C * m + a * m \le 0$

$\Rightarrow \boxed{a \le C}$

$O(\sqrt{n})$

C exits ?

$$a_n = 6a_{n-2} - a_{n-1}$$

$$\Rightarrow a_n x^n = 6a_{n-2} x^n - a_{n-1} x^n$$

$$\begin{cases} a_0 = 1 \\ a_1 = 2 \end{cases}$$

$$\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} 6a_{n-2} x^n - \sum_{n=2}^{\infty} a_{n-1} x^n \quad ①$$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n \qquad \sum_{n=0}^{\infty} \quad \sum_{n=2}^{\infty}$$

$$G_2(x) = \sum_{n=2}^{\infty} a_n x^n + a_1 x + a_0 \quad ②$$

$$\Rightarrow$$

$$①② : G(x) - 2x - a_0 = 6x^2 \cdot G(x) - x \cdot G(x) + a_0 x$$

$$G(x) = \frac{3x+1}{-6x^2 + x + 1}$$

$$G(x) = \frac{1}{-2x+1} = \sum_{n=0}^{\infty} a_n x^n$$

$$= \frac{1}{1-2x} = \sum_{n=0}^{\infty} a_n x^n$$

$$a_n = 2^n$$

$$
\begin{cases}
T(n) = 3T\left(\frac{n}{3}\right) + a \cdot (\log n)^3 & n > 2 \\
T(n) = b & n \leq 2
\end{cases}
$$

$$\underbrace{(\log n)^3}_{\downarrow}$$

$$a * n^{\frac{1}{2}}$$

Master Theorem

first rule of Mater theorem.

Divide & Conqure $\underline{\underline{M(n,m)}}$

First, find the middle row j of matrix

Linearly Scan row j to find leftmost minimum element, x and corresponding index, k, the leftmost min element for all rows having index less than j can only be exist at position $< k$.

Hence, we can recursiely call the top-left part of $M[1 \cdots (j-1), 1 \cdots k]$

$M[(j+1 \cdots n, k \cdots m)$

Pseudo code:

findLeftmostMinval(M):

if $n == 1$,

linear search for min value x

return x

else:

$j = \frac{n}{2}$

linear search for row j of M, which is $x(j, k)$

$M_1 = M[1 \cdots (j-1), 1 \cdots k]$

$M_2 = M[(j+1) \cdots n, k \cdots m]$

I_1 = findLeftmostMinval (M_1)

I_2, findLeftmostMinval (M_2)

return X, I_1, I_2

$O(m \log n)$