CS 460/560
Introduction to Computational Robotics
Fall 2019, Rutgers University

# Lecture 14
# Intro To Sampling-Based Planning Methods (2)

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# Outline (for Next 3-4 Lectures)

Drawbacks of combinatorial motion planning methods

Probabilistic roadmap (PRM) introduction

Components of sampling-based motion planning methods

- ⇨Sampling
- ⇨$k$-d tree and nearest neighbor search
- ⇨Distance metric
- ⇨Collision detection

PRM in more detail

A new notion of completeness

Rapidly-exploring random trees (RRT)

When would sampling-based method work well?

Optimality issues

# Drawbacks of Combinatorial Methods

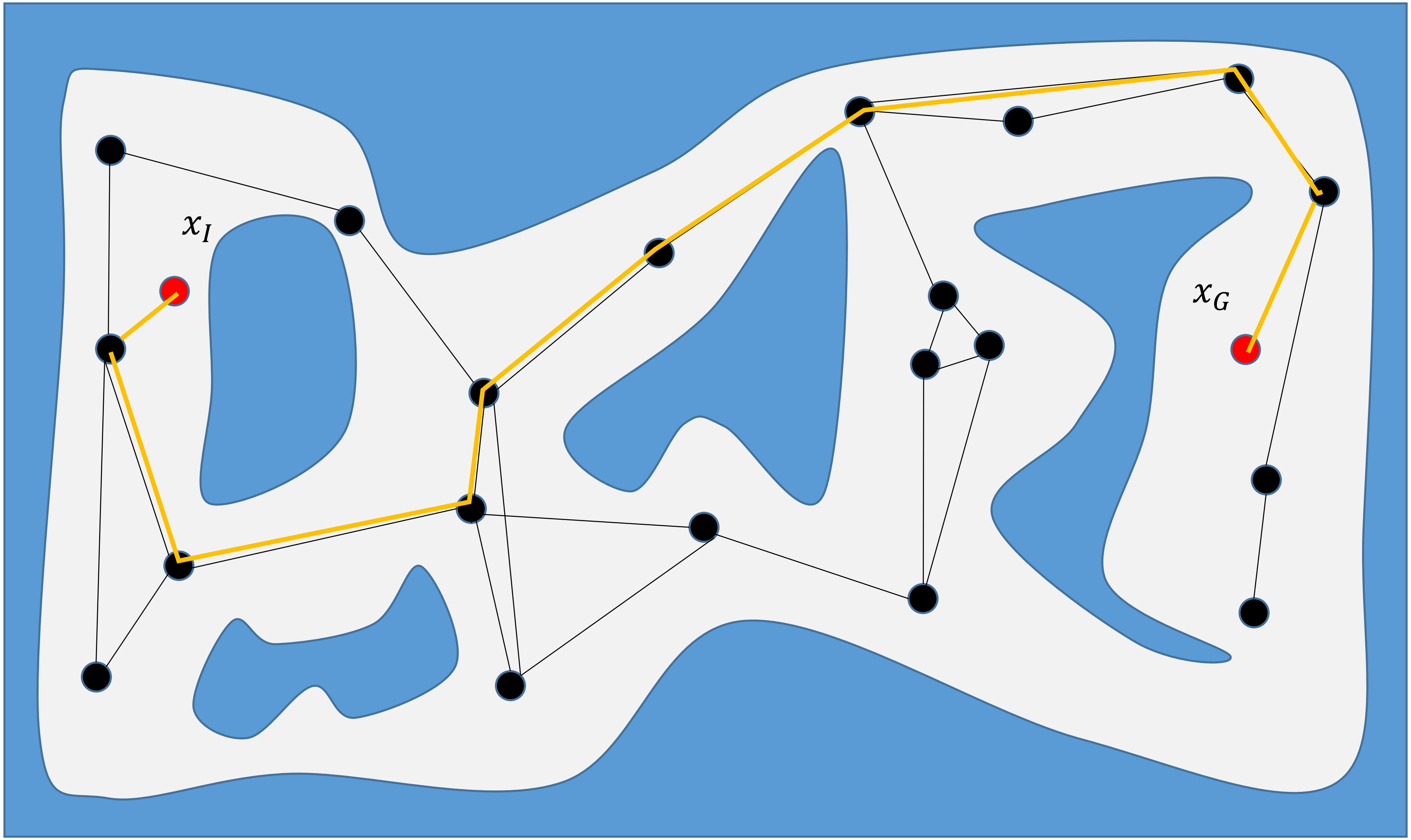
Recall the illustration of the Piano Mover's Problem

- ⇨ Modeling of the free configuration space $C_{free}$ can be a daunting task – they have to be represented as semi-algebraic sets
- ⇨ The associated computation is also prohibitive for even just a few degrees of freedom

To the rescue: **sampling based methods** – instead of representing $C_{free}$ explicitly and globally, we instead "probe" the space locally, as necessary

Image source: Planning Algorithms by LaValle

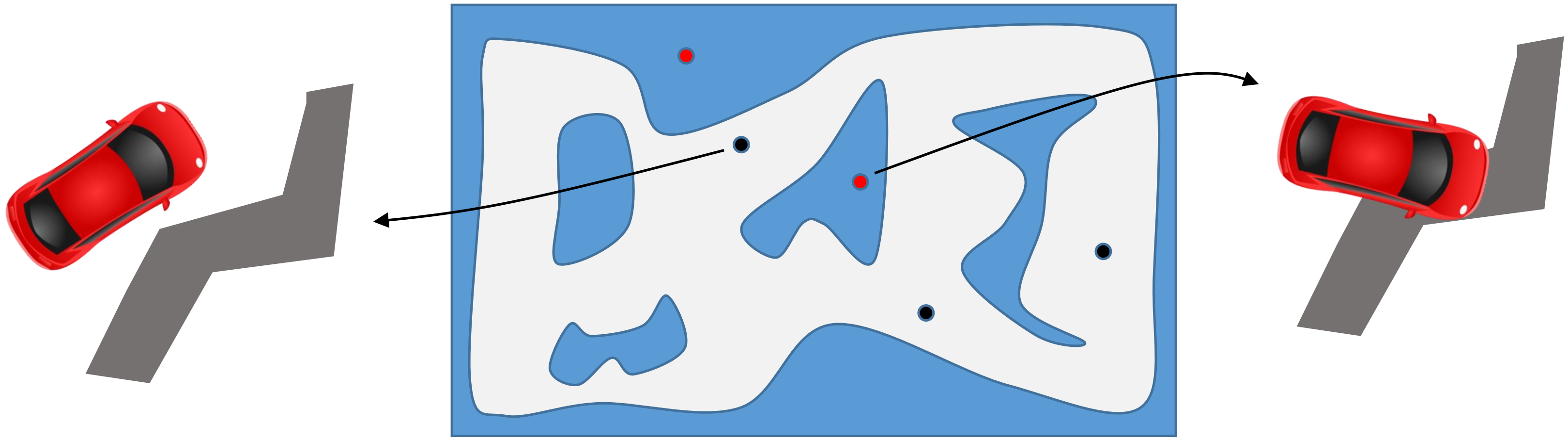
# Sampling-Based Planning



# Key Components of Sampling-Based Planning

Sampling-based planning requires several important subroutines

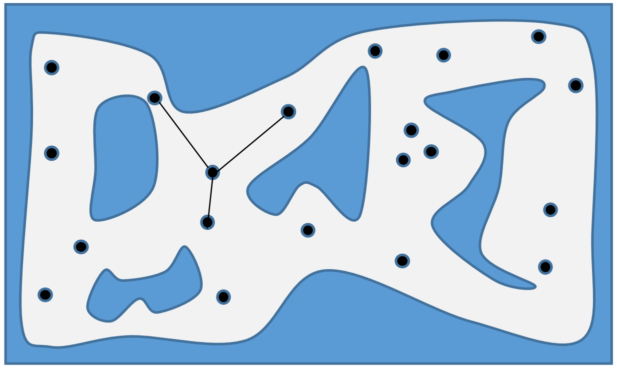
- ⇨An **efficient sampling routine** is needed to generate the samples. These samples should **cover** $C_{free}$ well in order to be effective
- ⇨**Efficient nearest neighbor search** is necessary for quickly building the roadmap: for each sample in $C_{free}$ we must find its $k$-nearest neighbors
- ⇨The neighbor search also requires a **distance metric** to be properly defined so we know the distance between two samples
  - ⇨ This can be tricky for certain spaces, e.g., $SE(3)$
- ⇨**Collision checking** - Note that $C_{free}$ is not computed explicitly so we actually are checking collisions between a complex robot and a complex environment



# Nearest Neighbor Search

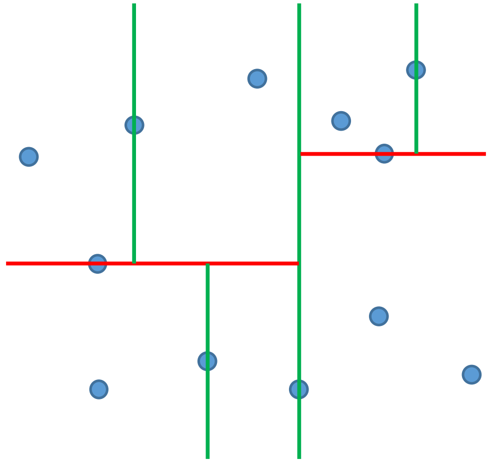
## Connecting the samples

- ⇨ Building the graph requires connecting the samples
- ⇨ This cannot be done for all pairs of points!
  - ⇨ For $N$ sample points, this requires $N^2$ operations
  - ⇨ But $N$ can be very large, e.g., $> 10^5$
  - ⇨ For $N = 10^6$, $N^2 = 10^{12}$
- ⇨ We have to do it more efficiently!
- ⇨ This is known as nearest neighbor search



## Variants of useful nearest neighbor search

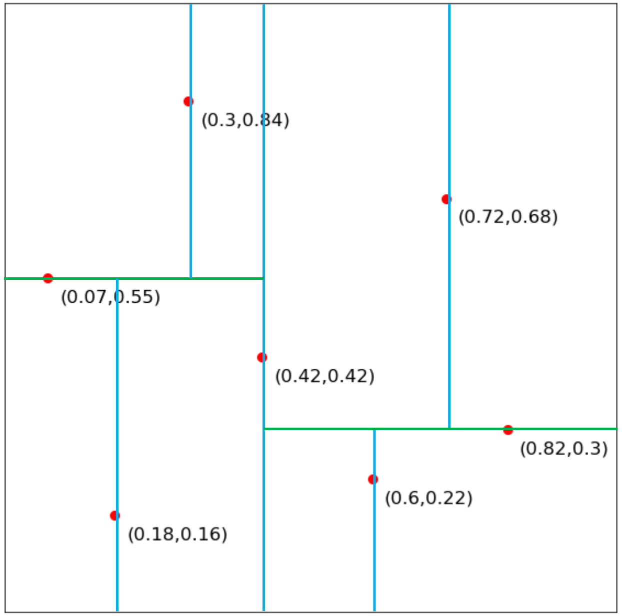
- ⇨ 1-NN: finding a single nearest neighbor
  - ⇨ Can be done with $k$-d trees
  - ⇨ We will look at this in more detail
- ⇨ $k$-NN: finding $k$ nearest neighbors
  - ⇨ Note the $k$ here is not the same as the $k$ in $k$-d trees
  - ⇨ Can run 1-NN algorithms $k$ times



# $k$-d Tree

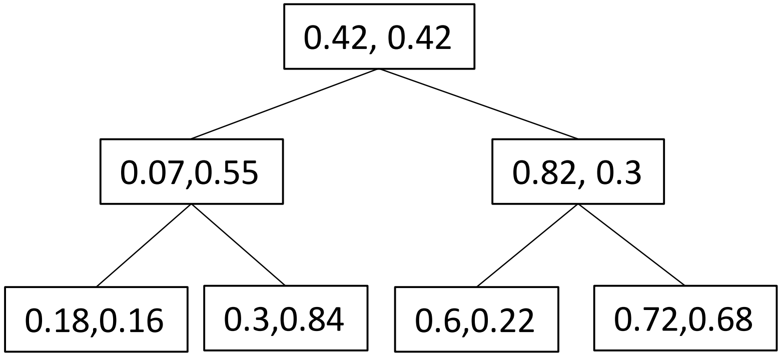
## $k$-d tree stands for $\boldsymbol{k}$**-dimensional trees**

- ⇨A data structure for storing points in $k$ dimensions
- ⇨Assumes a tree like structure
- ⇨Useful for finding points w/ certain properties
- ⇨Can be used for solving 1-NN




## Construction of a $k$-d tree for $n$ points

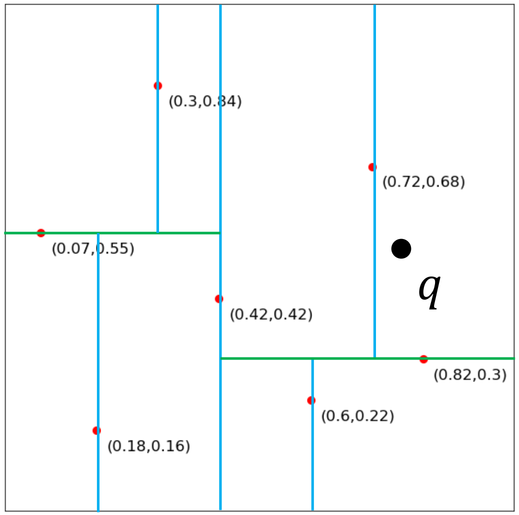
- ⇨Pick dimension $i$, pick a point with coordinates $x = (x_1, \ldots, x_i, \ldots, x_k)$
- ⇨Split the points based on $x_i$ (greater or less than)
- ⇨Repeat the above two steps recursively
  - ⇨ Increase $i$ (modulo $k$) each time
  - ⇨ I.e., pick a new dimension each time
- ⇨Depth: $\log n$ if balanced
- ⇨Construction takes $O(kn \log n)$ time
  - ⇨ Each dimension needs sorting $\sim O(n \log n)$
- ⇨Can speed up to $O(n \log n)$
- ⇨Balancing is important



# Nearest Neighbor Search w/ $k$-d Tree

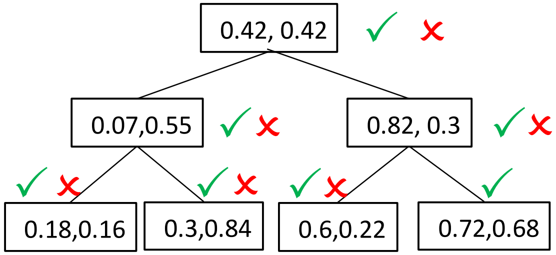
## Finding nearest neighbor of a query point $q$

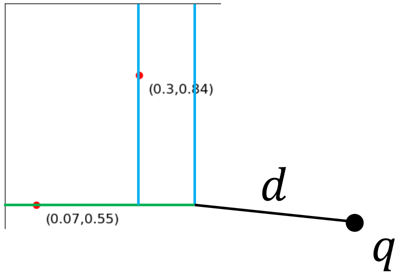
- ➩ Basically, traverse the tree, e.g., using BFS
- ➩ Maintain a current best **candidate** $x$
- ➩ Also maintain a queue of subtree distances to $q$
- ➩ Uses the subtree distances to prioritize search




## Example

- ➩ Start with root (0.42,0.42)
  - ➩ $x = (0.42,0.42)$, both left and right subtrees are active
- ➩ Examine (0.07, 0.55)
  - ➩ Three trees on the queue afterward
- ➩ Examine (0.82, 0.3), update $x = (0.82,0.3)$
  - ➩ Truncate the left subtrees
  - ➩ Two subtrees left
- ➩ Examine (0.72, 0.68), update $x = (0.72,0.68)$
  - ➩ We are done since the last subtree is further from $q$ than $x$



# Performance of $k$-d Tree and Generalization

General performance of balanced $k$-d tree

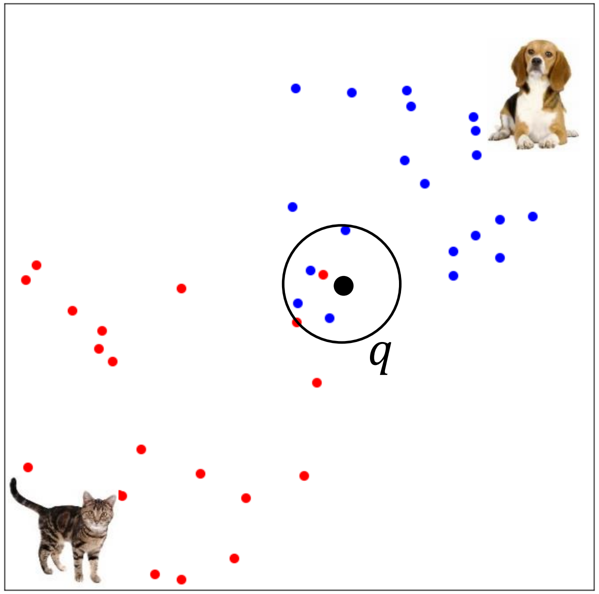
- ⇨ Construction: $O(n \log n)$ w/ $O(n)$ median computation
- ⇨ Construction through presorting the points: $O(kn \log n)$
- ⇨ Inserting/deletion of a new point: $O(\log n)$
- ⇨ Nearest neighbor search: $O(\log n)$ for randomly distributed points

$k$-d trees can be used for $k$-NN ($k$ nearest neighbor search) as well

- ⇨ Naïve implementation: simply run 1-NN $k$ times
- ⇨ This yields $O(k \log n)$ running time
- ⇨ Improvement
  - ⇨ Keep up to $k$ candidates
  - ⇨ Only discard a subtree if worse than all $k$ candidates

Applications of $k$-NN

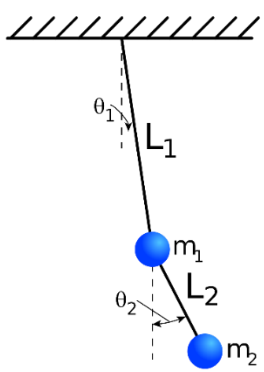
- ⇨ Widely used in classification tasks, e.g.,
  - ⇨ Optical character recognition (OCR)
  - ⇨ Pattern recognition

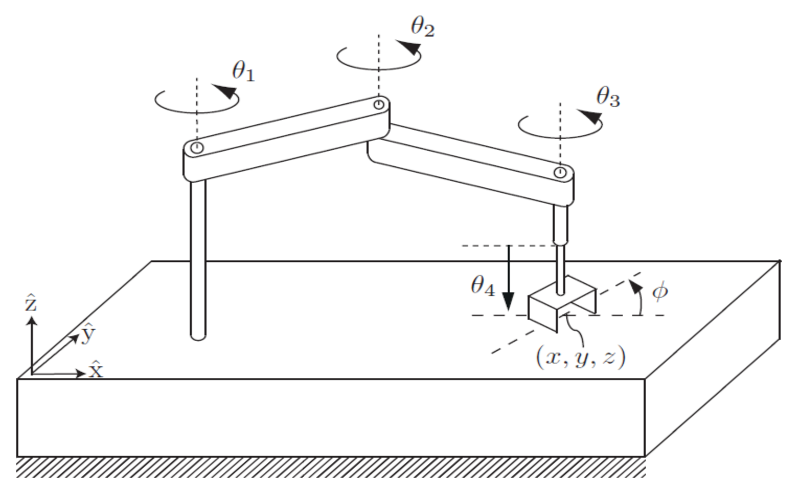


# A Brief Look at the Issue of Distance Metric

## Nearest neighbor queries requires a distance metric

- ⇨ Given two points $x$ and $q$, need to know their distance $d(x, q)$
- ⇨ Otherwise, cannot compare!
- ⇨ This is easy in Euclidean space: $d(x, q) = ||x - q||_2 = \sqrt{\sum (x_i - q_i)^2}$
- ⇨ But what about $T^2$ or $\mathbb{R}^2 \times S^1$, or more complex settings?
  - ⇨ For $T^2$, $\theta_1$ seems to be more important
  - ⇨ For $\mathbb{R}^2 \times S^1$, a small change in $\theta$ can be hard to make
  - ⇨ Sometimes, we can work with the workspace or task space
    - ⇨ This however will make the sampling more difficult
- ⇨ There is no universal solution – requires some creativity



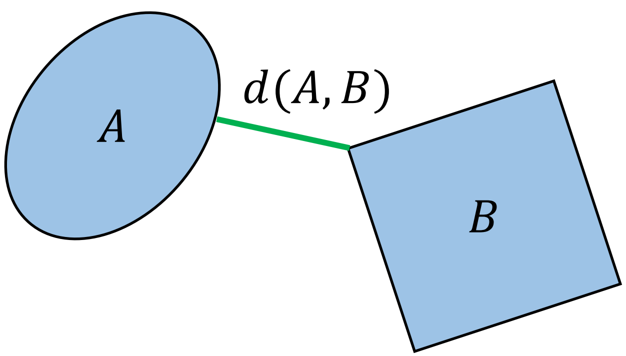
# Collision Detection

Sampling based methods need to check whether robot is in collision

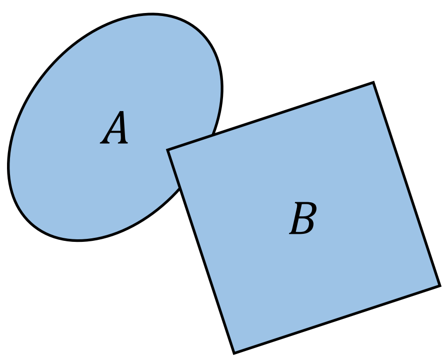
⇨Generally, given two sets of points $A$ and $B$, we want to check the distance between them

$$d(A, B) = \min_{a \in A, b \in B} |a - b|$$

⇨Clearly, $A$ and $B$ intersect (collide) if and only if $d(A, B) = 0$



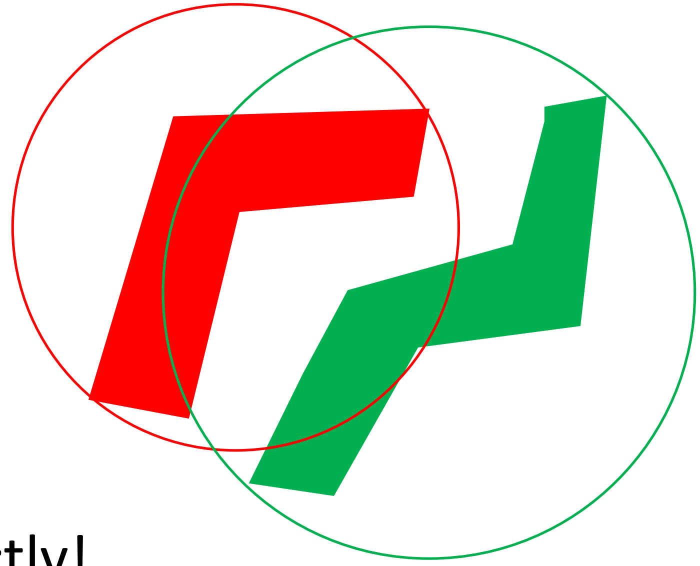

$d(A, B) > 0$: no collision




$d(A, B) = 0$: collision

# Bounded Volume Hierarchy (BVH)

Collision checking can be difficult for general objects, e.g.,



$d(A, B)$ are hard to compute directly!

Often, simpler **bounding volumes** are used to approximate the shapes

⇨However, bounding volumes **over approximate** the shapes

⇨No collision between bounding volumes → no collision between the shapes

⇨Collision between bounding volumes → **possible** collision

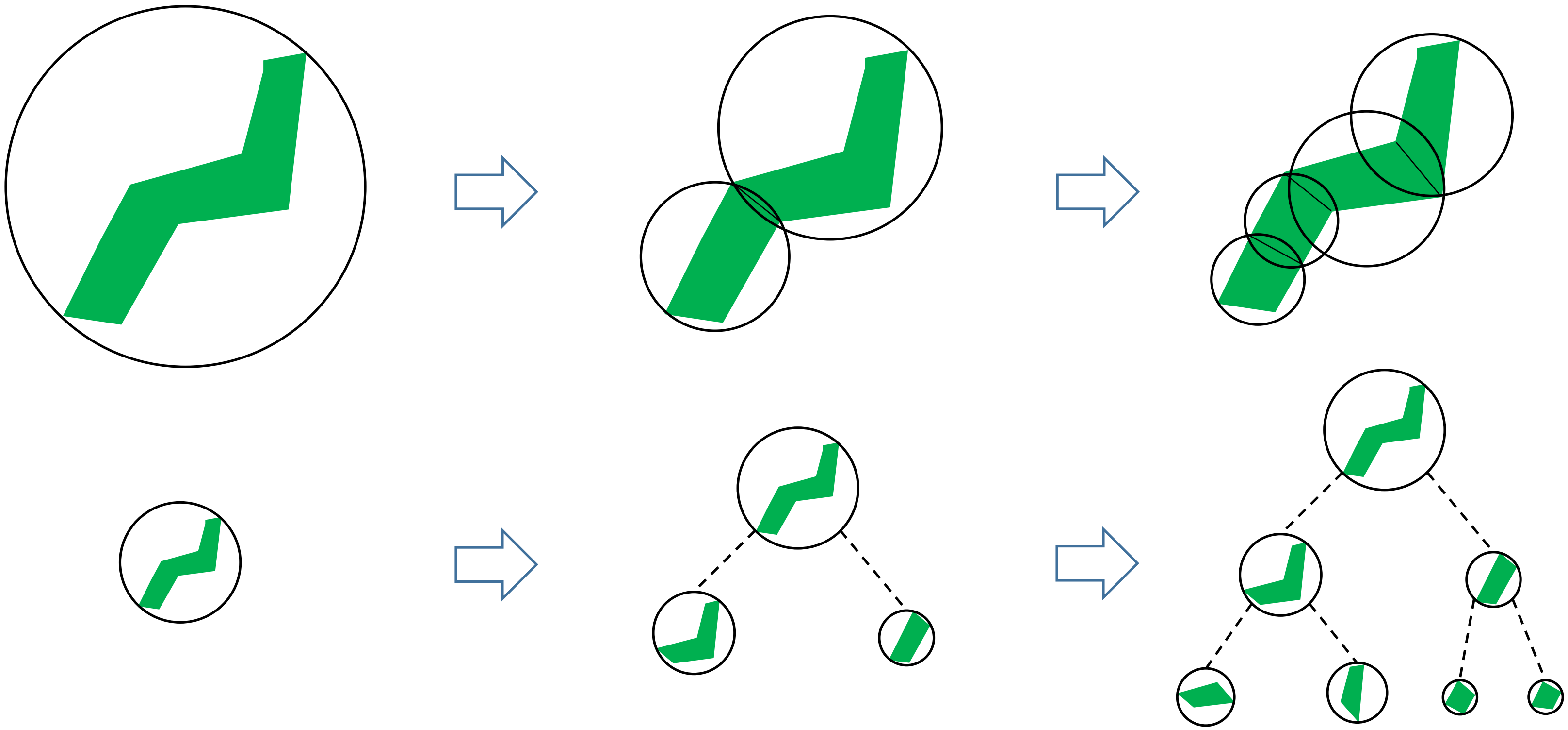
⇨Need to refine hierarchically if a possible collision is detected

⇨Such a method is called **bounded volume hierarchy** (BVH)

# Bounded Volume Hierarchy (BVH)

BVH breaks objects into smaller pieces

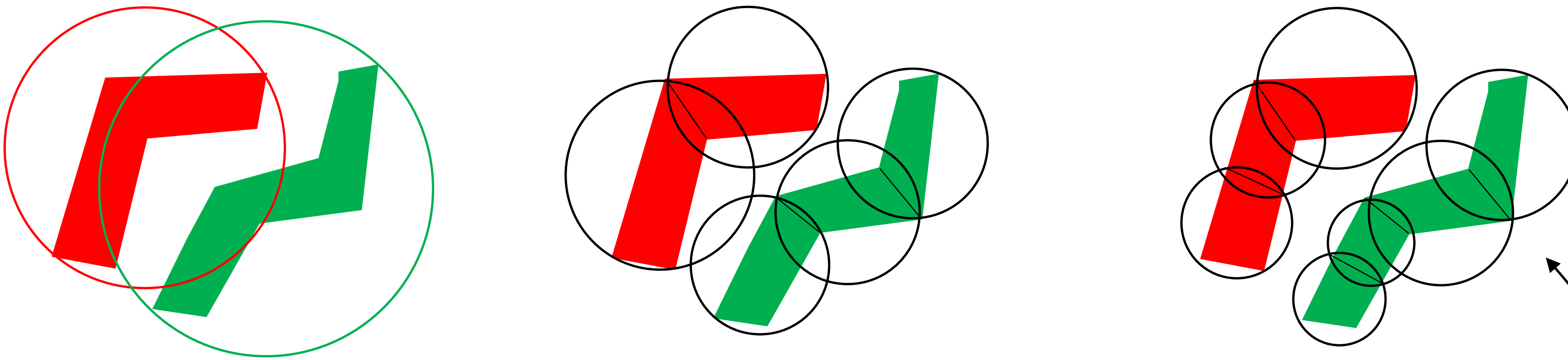
Which yields a hierarchy, represented as a **tree**

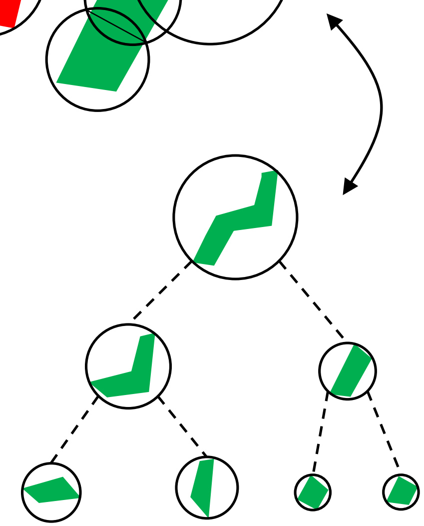


This is carried out incrementally

⇨Finer hierarchies are created as needed and then saved for later

# Bounded Volume Hierarchy (BVH), Continued



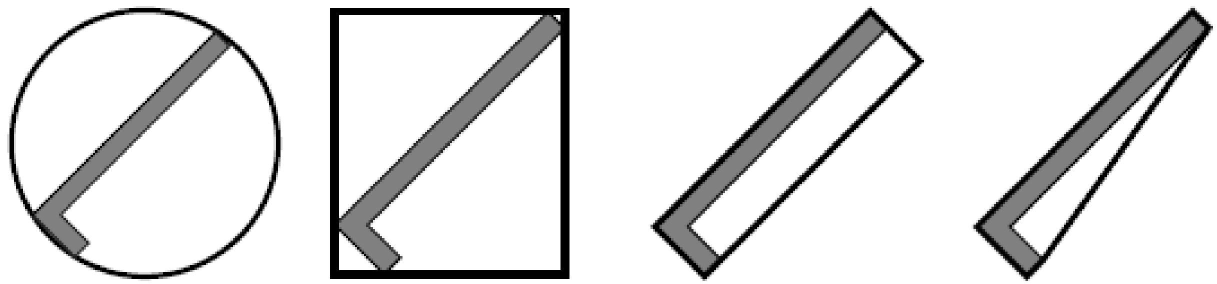


For collision checking, it works with two BVH trees

- ⇨Starting from the roots and check for collision (how?)
  - ⇨No collision → done with the branch
  - ⇨Otherwise, check pairs of children on the trees
- ⇨Recursively call the procedure
- ⇨Traverse down the tree
- ⇨How many possible checks in total (say each object has $n$ pieces)?
  - ⇨At most $n^2$ checks
  - ⇨Using BVH can save some checks

# Types of Bounding Volumes

## Many types of bounding volumes are possible



(a) (b) (c) (d)

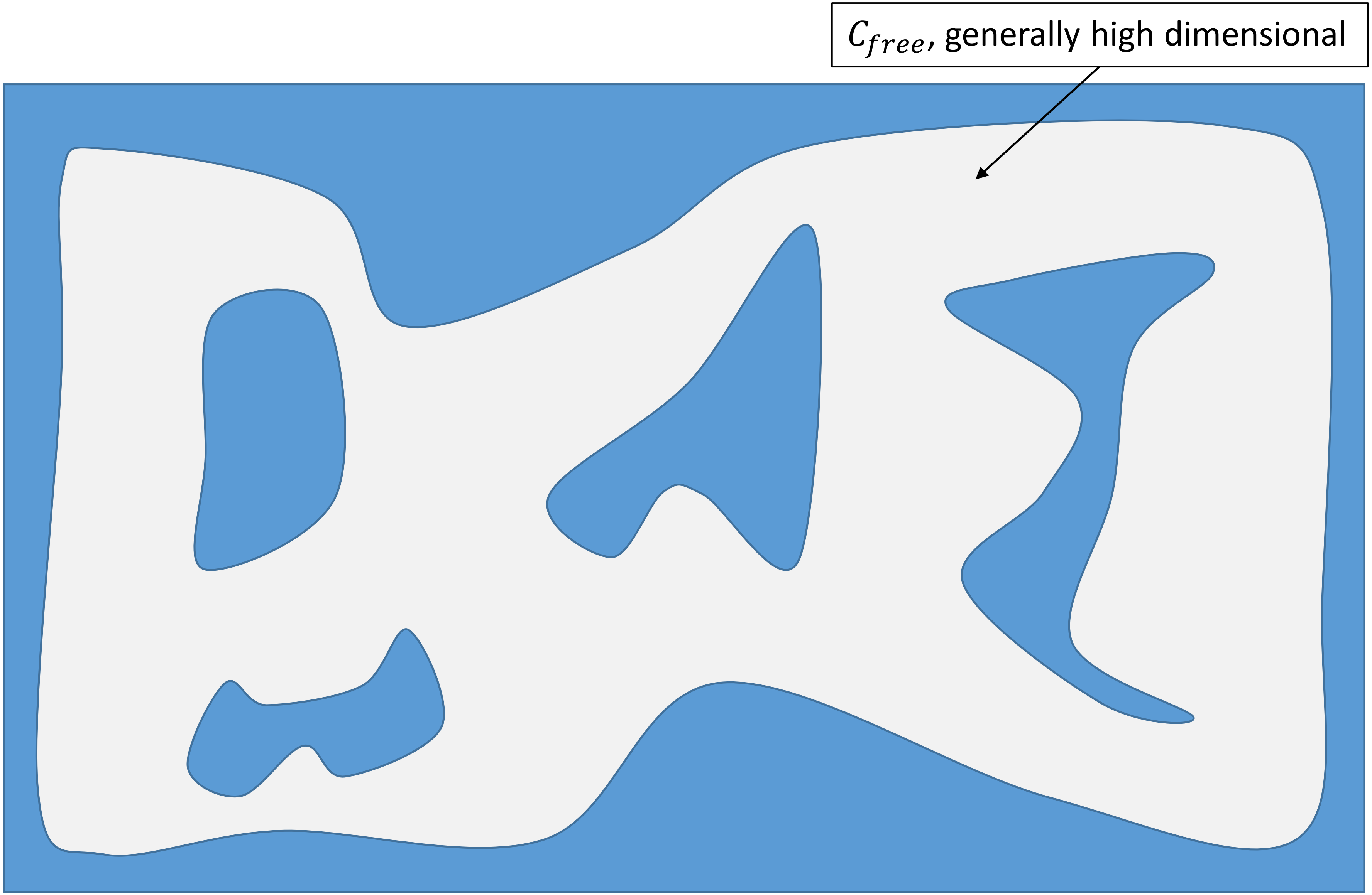
Figure 5.9: Four different kinds of bounding regions: (a) sphere, (b) axis-aligned bounding box (AABB), (c) oriented bounding box (OBB), and (d) convex hull. Each usually provides a tighter approximation than the previous one but is more expensive to test for overlapping pairs.

## Each has benefits and issues

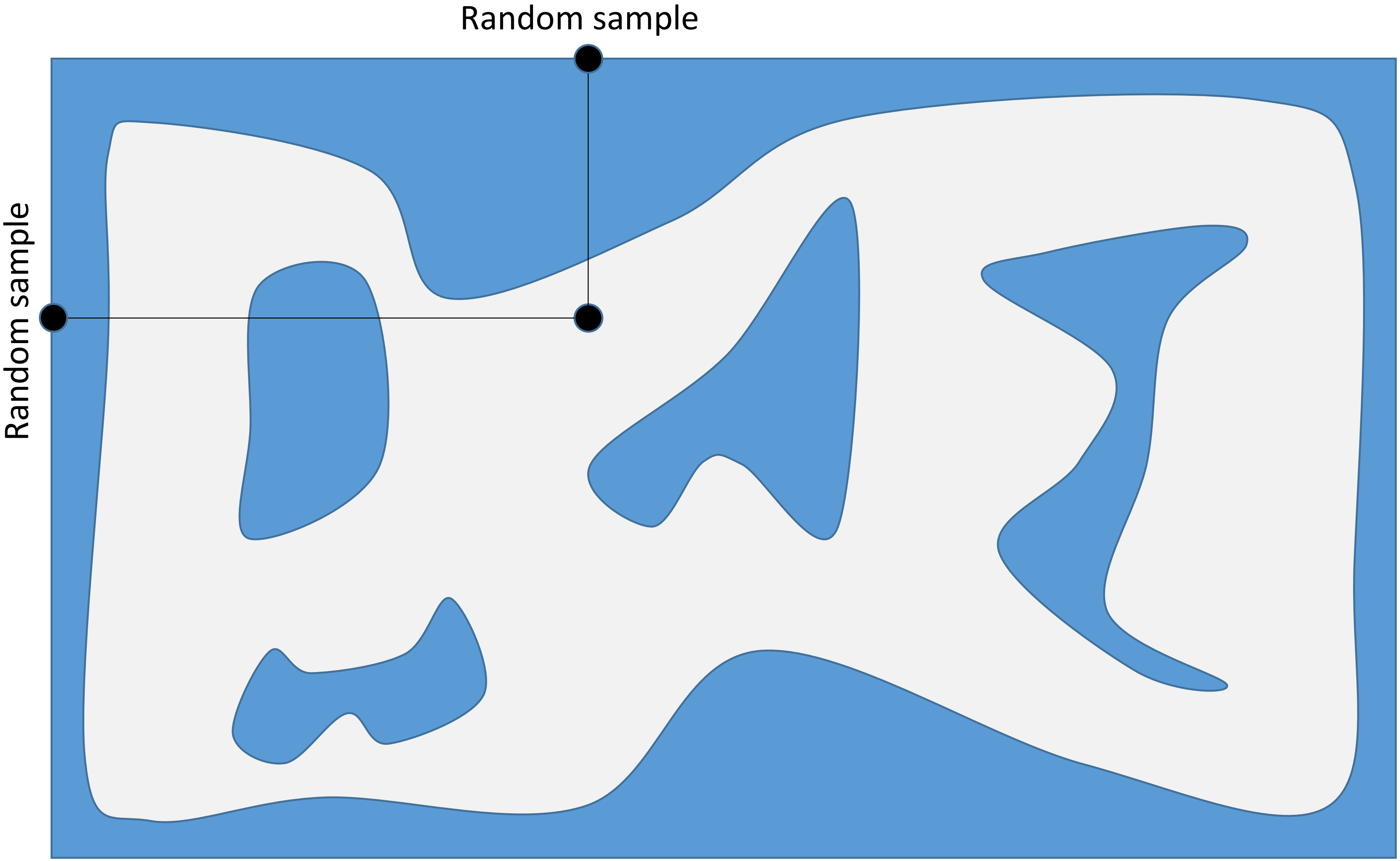
- ⇨Spheres are simple and **orientation invariant** but do not fit tightly
- ⇨AABBs are even simpler, but not orientation invariant, not tight
- ⇨OBBs are orientation invariant, reasonably tight
- ⇨Convex hulls are tight and orientation invariant, but require more computation

Image sources: Planning Algorithms by LaValle

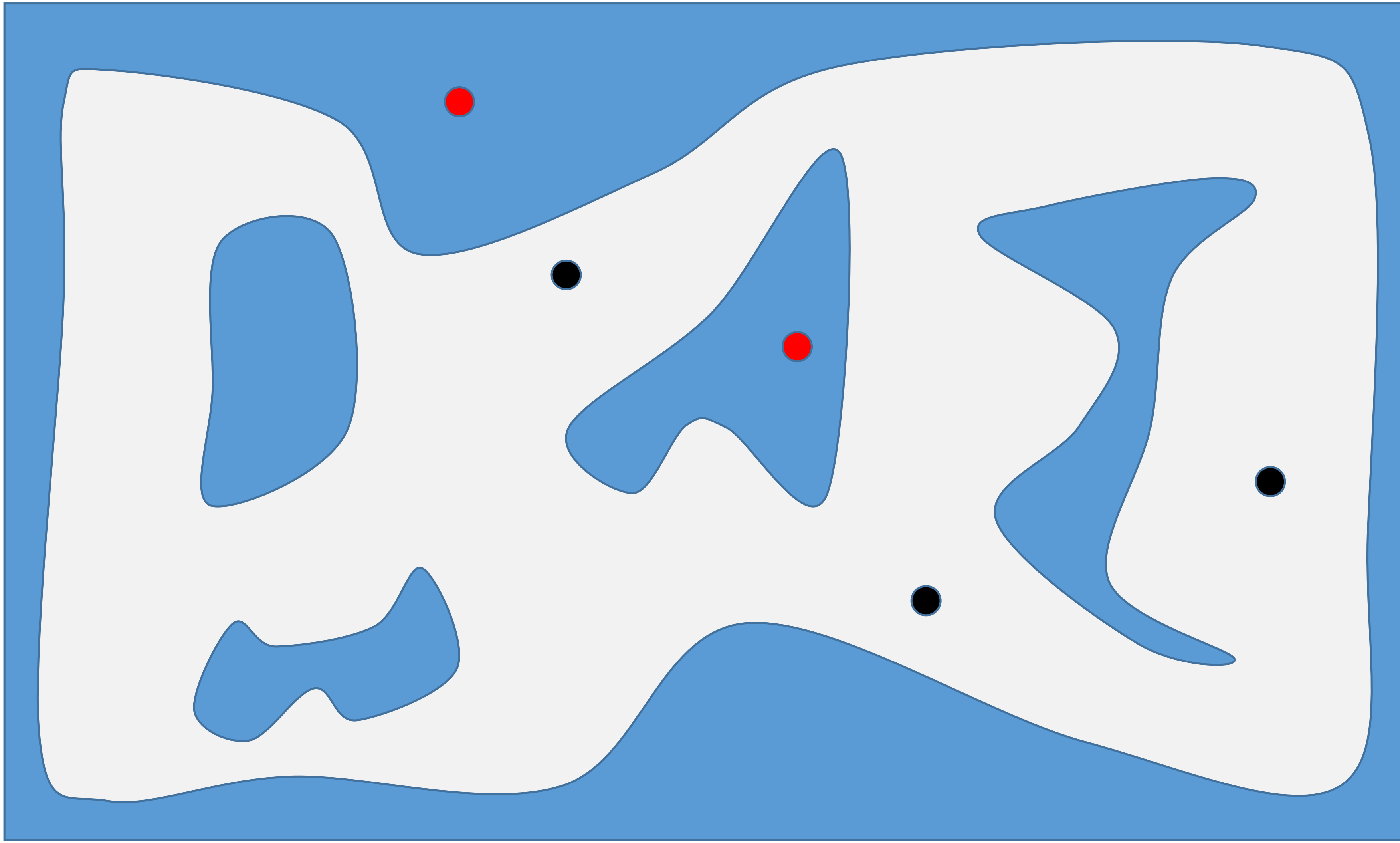
# Probabilistic Roadmap in More Detail



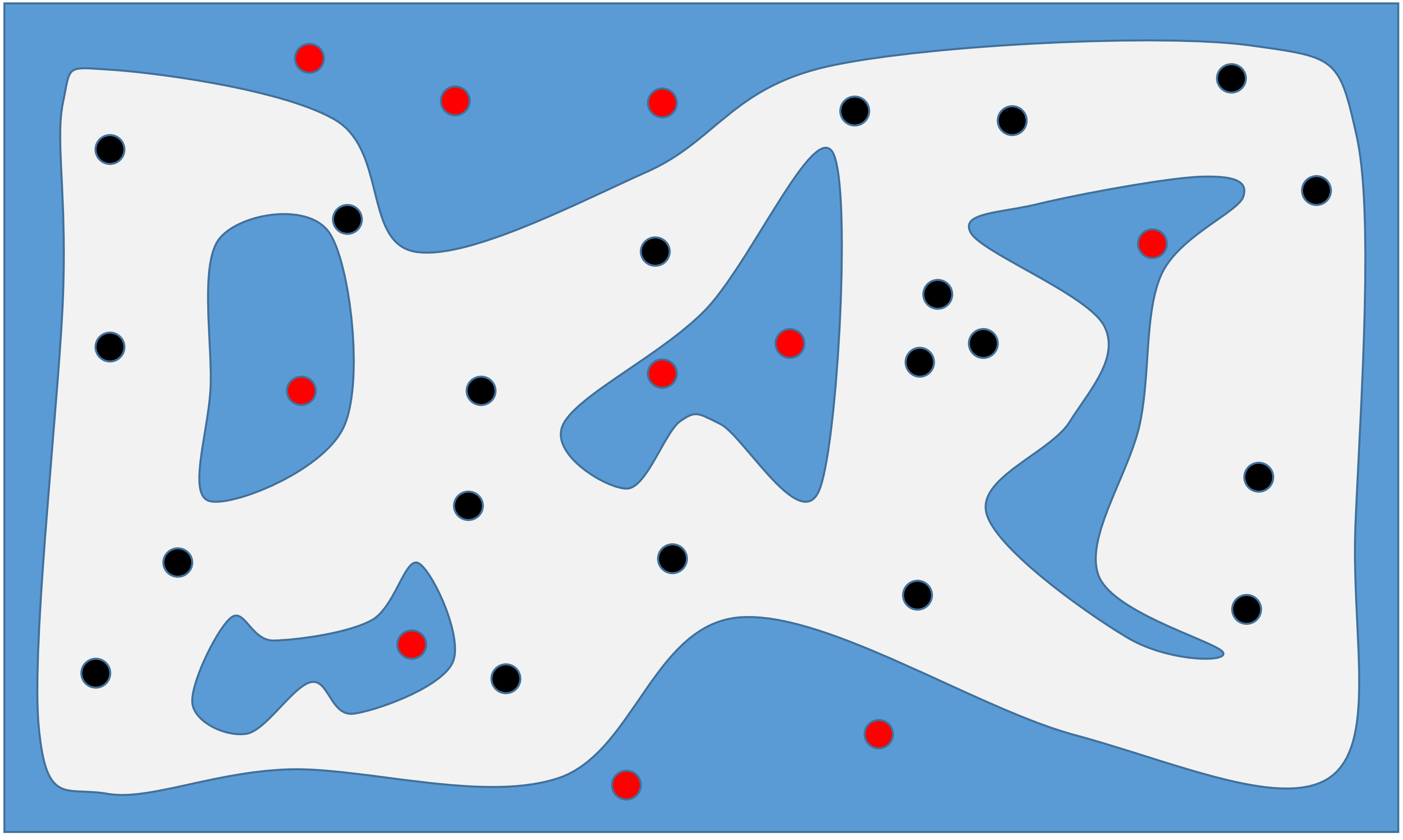
# Generating Random Samples



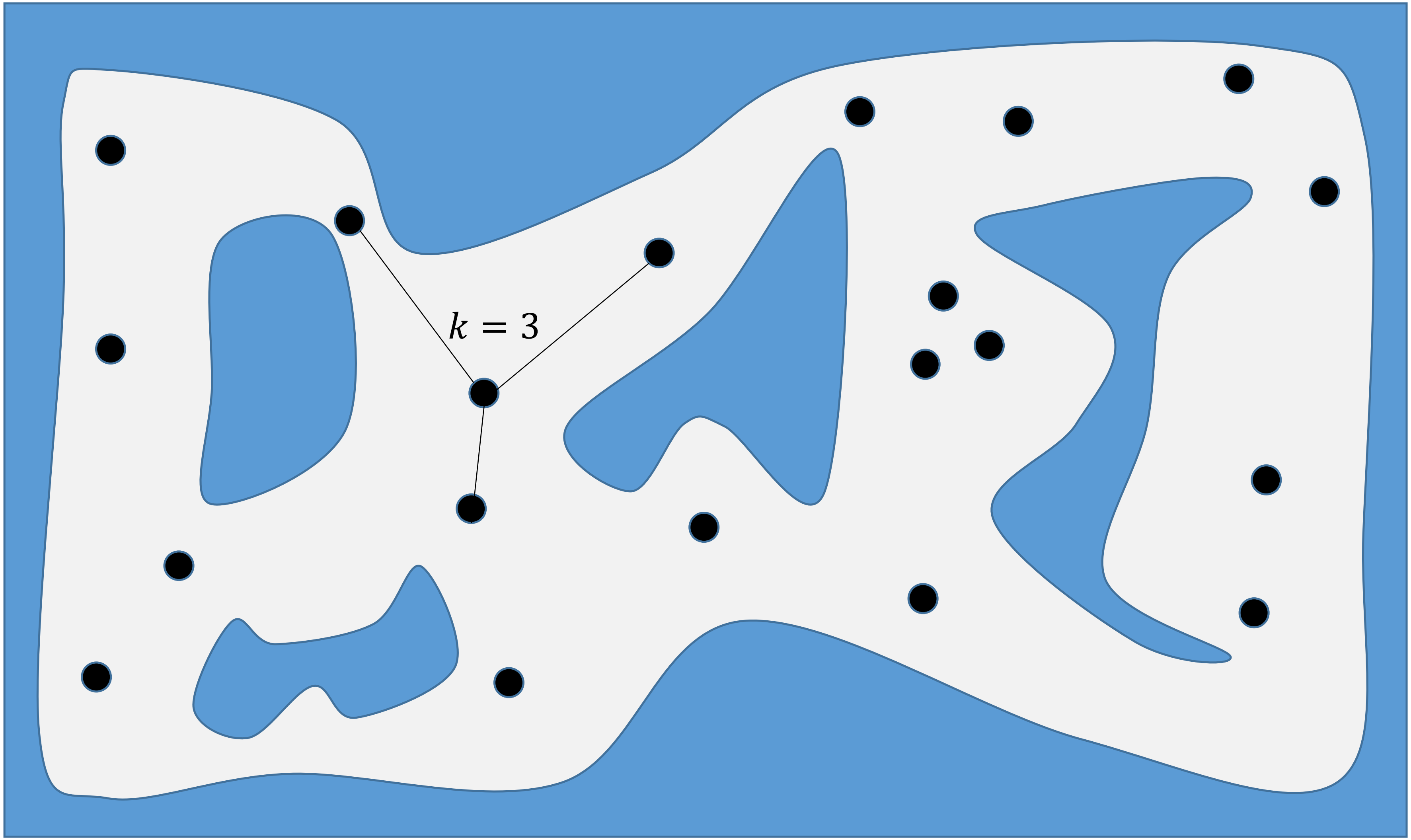
# Rejecting Samples Outside $C_{free}$



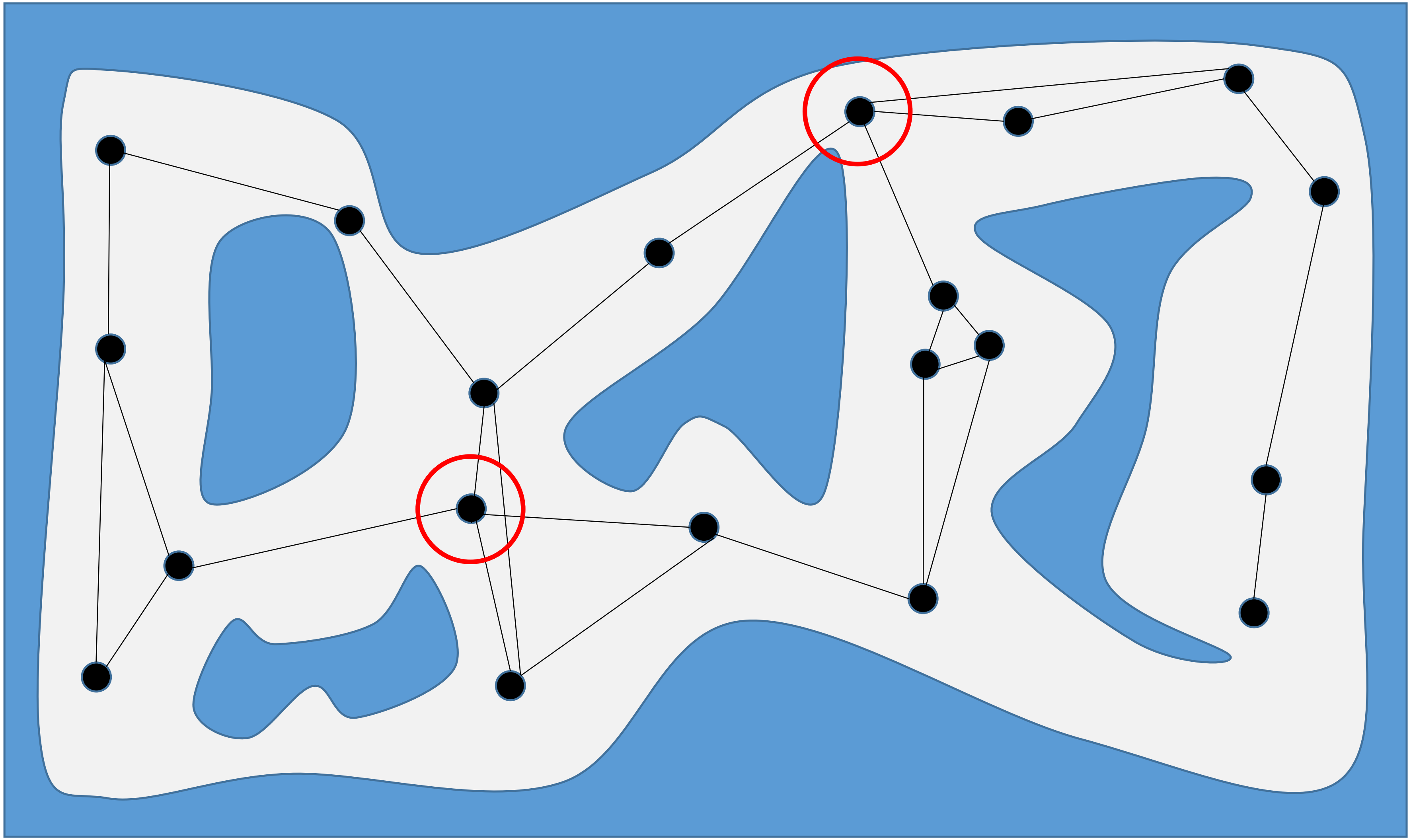
# Collecting Enough Samples in $C_{free}$



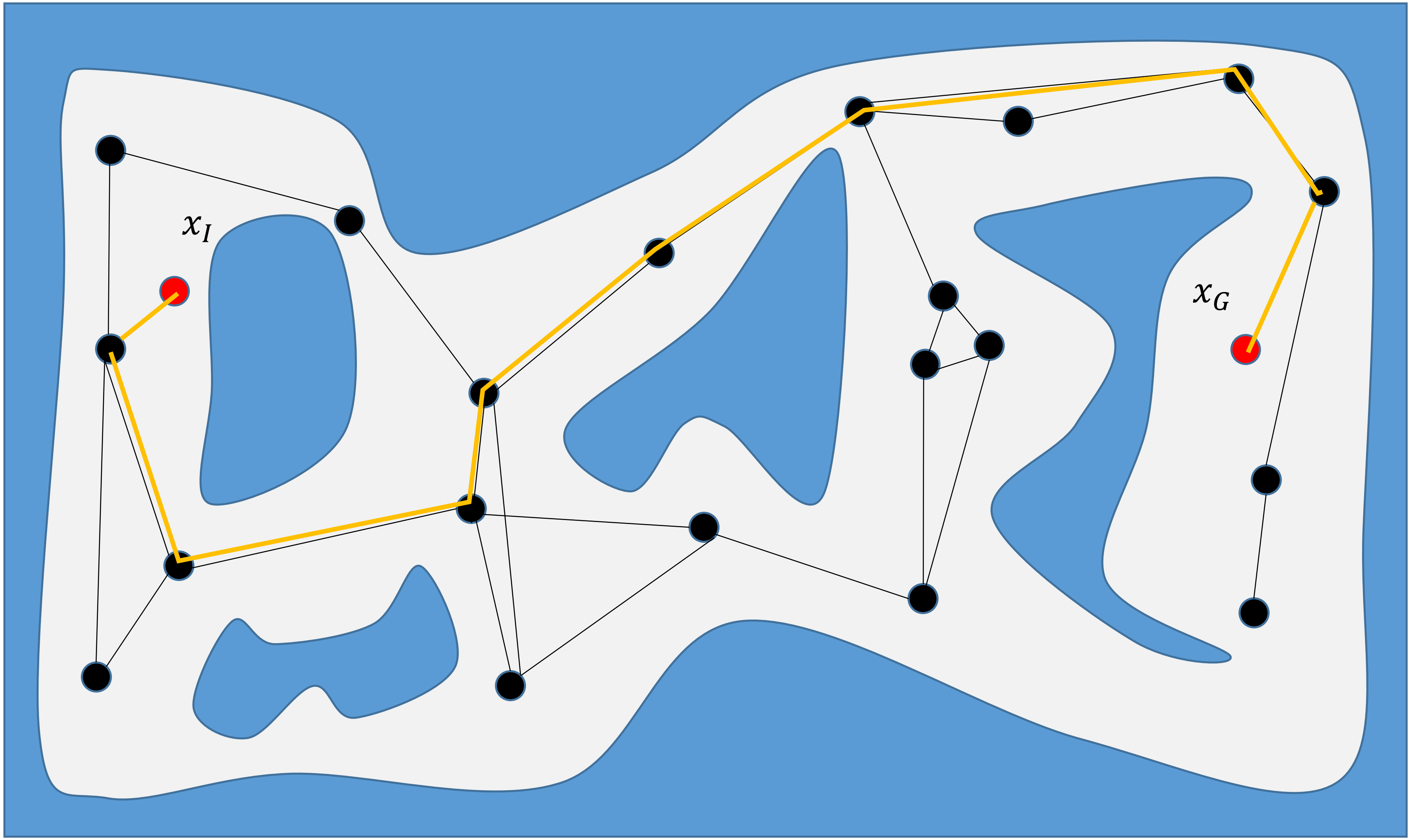
# Connect to $k$ Nearest Neighbors (If Possible)



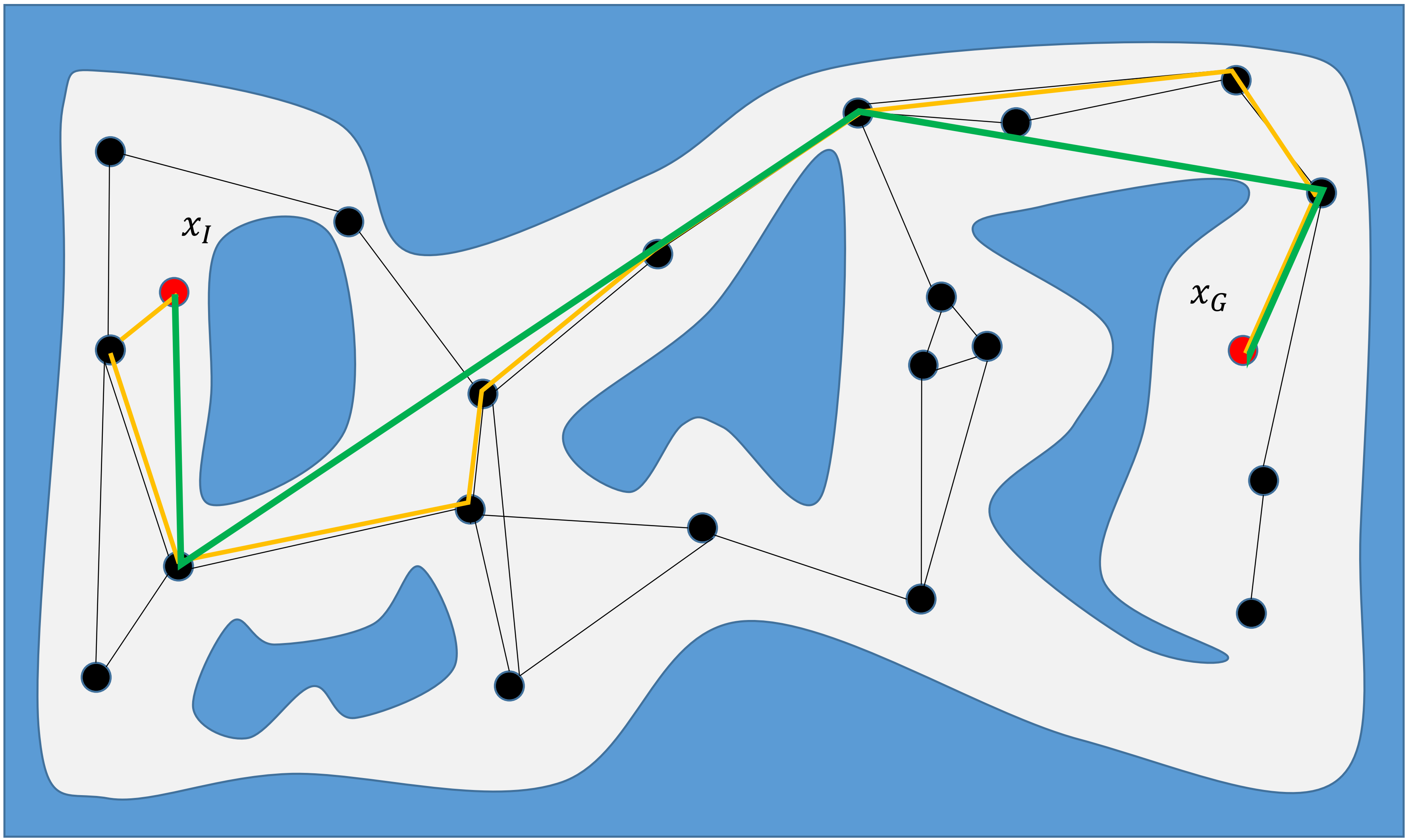
# Connect to $k$ Nearest Neighbors (If Possible)



# Query Phase



# Path Smoothing



# PRM Algorithm – Roadmap Construction

First proposed by Kavraki et al.

**Algorithm 6** Roadmap Construction Algorithm

**Input:**

$n$ : number of nodes to put in the roadmap

$k$ : number of closest neighbors to examine for each configuration

**Output:**

A roadmap $G = (V, E)$

$V \leftarrow \emptyset$<br>
$E \leftarrow \emptyset$<br>
**while** $|V| < n$ **do**<br>
    **repeat**<br>
        $q \leftarrow$ a random configuration in $\mathcal{Q}$<br>
    **until** $q$ is collision-free<br>
    $V \leftarrow V \cup \{q\}$<br>
**end while**<br>
**for all** $q \in V$ **do**<br>
    $N_q \leftarrow$ the $k$ closest neighbors of $q$ chosen from $V$ according to *dist*<br>
    **for all** $q' \in N_q$ **do**<br>
        **if** $(q, q') \notin E$ **and** $\Delta(q, q') \neq \mathtt{NIL}$ **then**<br>
            $E \leftarrow E \cup \{(q, q')\}$<br>
        **end if**<br>
    **end for**<br>
**end for**

Image sources: Principles of Robot Motion

# PRM Algorithm – Query Solving

**Algorithm 7** Solve Query Algorithm

**Input:**

$q_{\text{init}}$: the initial configuration
$q_{\text{goal}}$: the goal configuration
$k$: the number of closest neighbors to examine for each configuration
$G = (V, E)$: the roadmap computed by algorithm 6

**Output:**

A path from $q_{\text{init}}$ to $q_{\text{goal}}$ or failure

$N_{q_{\text{init}}} \leftarrow$ the $k$ closest neighbors of $q_{\text{init}}$ from $V$ according to *dist*
$N_{q_{\text{goal}}} \leftarrow$ the $k$ closest neighbors of $q_{\text{goal}}$ from $V$ according to *dist*
$V \leftarrow \{q_{\text{init}}\} \cup \{q_{\text{goal}}\} \cup V$
set $q'$ to be the closest neighbor of $q_{\text{init}}$ in $N_{q_{\text{init}}}$
**repeat**
    **if** $\Delta(q_{\text{init}}, q') \neq \texttt{NIL}$ **then**
        $E \leftarrow (q_{\text{init}}, q') \cup E$
    **else**
        set $q'$ to be the next closest neighbor of $q_{\text{init}}$ in $N_{q_{\text{init}}}$
    **end if**
**until** a connection was succesful or the set $N_{q_{\text{init}}}$ is empty
set $q'$ to be the closest neighbor of $q_{\text{goal}}$ in $N_{q_{\text{goal}}}$
**repeat**
    **if** $\Delta(q_{\text{goal}}, q') \neq \texttt{NIL}$ **then**
        $E \leftarrow (q_{\text{goal}}, q') \cup E$
    **else**
        set $q'$ to be the next closest neighbor of $q_{\text{goal}}$ in $N_{q_{\text{goal}}}$
    **end if**
**until** a connection was succesful or the set $N_{q_{\text{goal}}}$ is empty
$P \leftarrow$ shortest path$(q_{\text{init}}, q_{\text{goal}}, G)$
**if** $P$ is not empty **then**
    **return** $P$
**else**
    **return** `failure`
**end if**

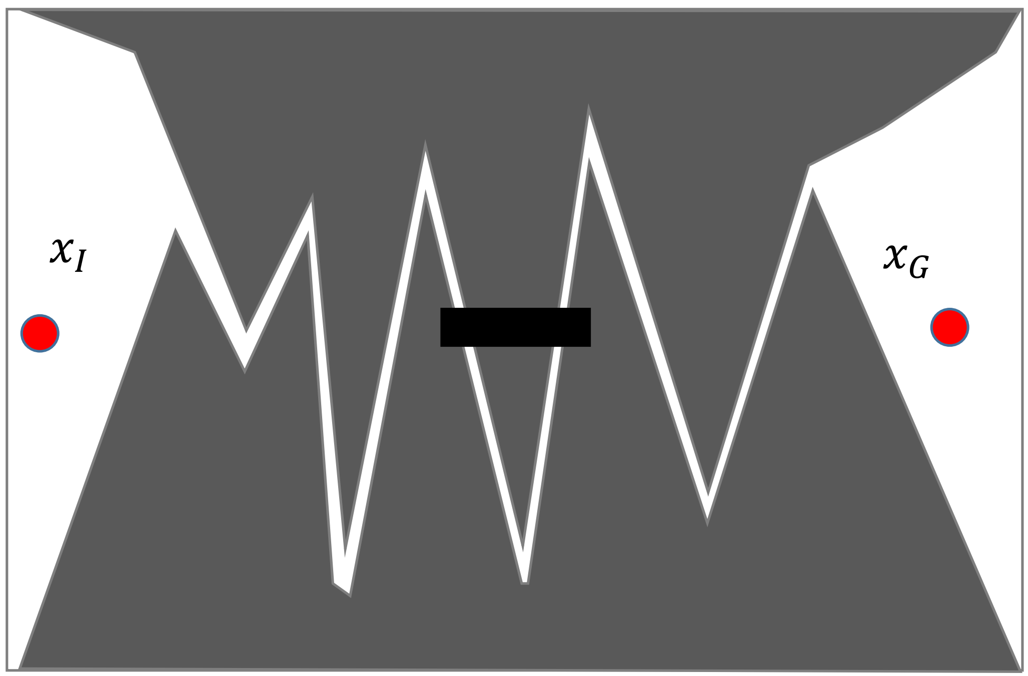
Image sources: Principles of Robot Motion

# A Look at Completeness

Sampling-based algorithms are no longer **complete**!

⇨ If a solution exists, it will eventually find one and stop

⇨ When there is no solution, the algorithm may keep running forever (so we need to have a timeout for these methods)




We need a new notion of completeness

# A New Notion of Completeness

Define a new notion of completeness based on **denseness** of sampling

⇨ A set of samples is **dense** if dispersion $\delta(P) \rightarrow 0$ as $|P| \rightarrow \infty$

⇨ This means that the roadmap will get into any opening

⇨ But it is hard to predict when if we do not know how big is the opening

Resolution completeness

⇨ For deterministic sampling (e.g., using a Halton sequence)

⇨ An algorithm is **resolution complete** if it samples deterministically and densely

Probabilistic completeness

⇨ For probabilistic methods

⇨ An algorithm is **probabilistic complete** if it samples probabilistically, e.g., uniformly random, and densely