3/7 Notes

Correlation-based learning in a firing rate formalism:
Let w represent the weight of the efficacy with which neuron I talks to neuron J. Here, we're dealing in **firing rates**

Question To Address: how does this weight fluctuate in response to the activity of the pre- and post-synaptic neuron?
Activity = firing rate

How can we mathematically describe "Neurons that fire together wire together"? [Hebbian learning]

Record the spikes of a pre- and post- synaptic neuron, with the goal of observing how the firing rate changes between the two.

Hebbian learning has two requirements: local rule and coactivity
What does it mean for a rule to be local?

It takes into account things that are happening nearby in order to change the weight. These include the activity of the **presynaptic neuron**, the activity of the **postsynaptic neuron**, and the **current weight** of the observed synapse.

The function for weight change postulates that when activity for pre- and post- synaptic neuron is there, something will change the weight; **this function must be sufficiently well behaved for us to use the following method to attempt to describe it**. Well-behaved refers to the ability to probe the value of the function over time.

Mathematical method:
We don't know the function, but we do know that $f(0) = C$. Let's hypothesize that the polynomial P approximating this unknown function is simply C: $P(x) = C$. Now, let's say that you move a bit in time, and can observe how that value changes to C': we now know that the value of $f'(0) = C'$. With this information, we can attempt a better polynomial: $P(x) = f(0) + f'(0)x$. Remember, we know that $P(0) = f(0)$. What if we also know

f”(0)? Then p”(0) = f”(0). Supplement the previous polynomial: P(x) = P(0) + p’(0) + 1/2 f”(0)x^2. When this process is infinitely repeated, it follows the rule that p(n) = f^n(0) * (x^n)/(n!)
This is the Taylor approximation in 2 dimensions

(Raise your hand if you forgot how math works!
Taylor expansions for Dummies: https://medium.com/@andrew.chamberlain/an-easy-way-to-remember-the-taylor-series-expansion-a7c3f9101063

The Idiot’s Guide to Taylor Expansions: https://www.mathsisfun.com/algebra/taylor-series.html

Taylor Expansions with Friends: http://tutorial.math.lamar.edu/Classes/CalcII/TaylorSeries.aspx

The bilinear term in the Taylor series expansion of a Hebbian rule implements the **AND** condition for joint activity. Were the expansion to be stopped before this term, the learning rule would be non-Hebbian. **This is because pre- or post- synaptic activity alone would induce the change in synaptic efficacy and joint activity would be irrelevant**. A Hebbian learning rule must therefore have either the bilinear term or a higher order term that **involves the activity of *both* pre- and post- synaptic neurons.**

Taking a closer look at the pre- synaptic variable:
The PSV depends on the weight variable. This dependance can be used to limit the growth fo weights at a maximum value w_max. There are two standard choices of weight dependence.

Hard Bound:
weight growth stops abruptly if the PSV reaches the upper bound w_max

Soft Bound:
A change tends to zero as its weight approaches the maximum value w_max

***note:** for Hebbian learning, weights can only increase, so eventually they'll always reach a maximum. This isn't used in practice, but is rather used to manipulate the polynomial.