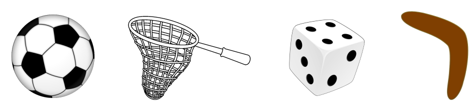
We will strictly enforce the following rules, ask questions now if something is unclear.

Guidelines. Students may discuss problems in the assignment. In fact, discussions are encouraged across HW/MP groups. However, each group must write down the answers independently. If discussions are held, one should also note down the people involved in the discussion (this will help especially if multiple groups make identical mistakes). For the written part of the solution, submit as a **SINGLE TYPESET PDF** (either from latex or converted from a word editing software). You may use scanned hand drawn figures. See course website for the rules on late submissions.

In your submission, name your PDF file as NETID.pdf where NETID is your actual netid. For a group with two people, use NETID1_NETID2.pdf as the name. We do not allow more than two people to form a submission group for this course. To make it flexible, we will not use the "group" function on Sakai. Any member of a group can submit. If we receive more than one submission from a group, we will only grade the one with a later timestamp.

Problem 1 [6 points]. Provide examples of three robots and/or automation devices in existence before the year 1900AD, with pictures and brief description of their purposes (you may use your favorite search engine).

Problem 2 [4 points]. Which of the objects on the right are convex? They are: a soccer, a net, a dice, a boomerang. Ignore small features such as shallow grooves on a soccer ball.



(a) (b) (c) (d)

Problem 3 [10 points]. In class, we use $\mathcal{P}(S)$ to denote the *powerset* of a set S. Let $\mathcal{P}^n(S)$ be defined as $\mathcal{P}^n(S) = \mathcal{P}(\mathcal{P}^{n-1}(S))$ for $n > 1$ and $\mathcal{P}^1(S) = \mathcal{P}(S)$. What is $\mathcal{P}(\varnothing)$ and $\mathcal{P}^2(\varnothing)$ in which $\varnothing$ is the empty set? What is $|\mathcal{P}^n(\varnothing)|$? Given set S with $|S| = k$. What is $|\mathcal{P}^n(S)|$?.

Problem 4 [10 points]. Recall that a **group** is a set G with a binary operation $\cdot$ satisfying the following **group axioms**:

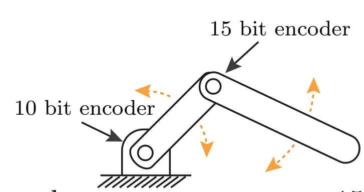
1. Closedness: $\forall a, b \in G$, $a \cdot b \in G$.
2. Associativity: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
3. Existence of an identity element: $\exists e \in G$ s.t. $\forall a \in G$, $a \cdot e = e \cdot a = a$.
4. Existence of inverses: $\forall a \in G, \exists b \in G$ s.t. $a \cdot b = b \cdot a = e$.

Using the group axioms, prove if G is a group then:

1. G has a **unique** identity element.
2. For each $a \in G$, a has a **unique** inverse.

For each step in your proof, you should explain which group axiom is applied. Proofs without explanation will not receive full credits.

Problem 5 [15 points]. See the figure on the right for a two link robot arm. The two arm segments only rotate in the xy-plane. The first (shorter) and second (longer) arm segments have lengths $0.5m$ and $0.8m$, respectively. The motor that moves the first segment has a 10 bit absolute wheel encoder (recall that the absolute encoder discussed in class uses 3 bits). The motor between the first and the second arm segments uses a 15 bits absolute wheel encoder. Theoretically, what is the maximum position uncertainty at the tip of the second segment?



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Problem 6 [10 points]. Suppose that you have a 10-sided fair dice (e.g., a dodecahedron). What is the expected number of tosses needed to get all 10 sides? What if it is an n-sided dice? Show your work.

Problem 7 [10 points]. Determine whether the following functions are injective, surjective, and bijective. Provide brief justifications for your answer. If a function is not bijective, modify either the domain or the co-domain (but not both) to make the function bijective.

1. $f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [0, 1], x \mapsto \cos x$.
2. $f : \mathbb{R} \to \mathbb{R}, x \mapsto e^x$.

Problem 8 [15 points]. Each pair of the spaces given below are homeomorphic. Provide a continuous bijective map to establish this. Justify your answer (showing how you derived your function is sufficient).

1. The interval $(-1, 1)$ and the real line $\mathbb{R}$. As a hint, consider first mapping $(-1, 1)$ to the circle $x^2 + (y - 1)^2 = 1$ with its north pole removed. Then map this circle to the real line by projecting from the north pole onto the x axis.
2. The circle $\{(x, y) \mid x^2 + y^2 = 1\}$ and the square $\{(x, y) \mid \|(x, y)\|_\infty = 1\}$. $\|\cdot\|_\infty$ refers to the infinity norm.

Problem 9 [10 points]. Consider the space X formed by chaining two circles together. More formally, we may describe the two circles in three dimensions as

1. C1: $x^2 + y^2 = 1, z = 0$.
2. C2: $(x - 1)^2 + z^2 = 1, y = 0$.

Following the definition of topological manifolds covered in class, determine whether X is a manifold and if so, its dimension.

Problem 10 [10 points]. Using the sampling procedure covered in class and python (or any other way you see fit), generate N samples for the cumulative distribution function

$$\Phi(x) = \frac{1}{2} + \frac{sign(x)}{2}\sqrt{1 - e^{-\frac{2x^2}{\pi}}}.$$

$sign(x)$ is the sign of x, e.g., $sign(-1.6) = -1$, $sign(5.3) = 1$, and $sign(0) = 0$. Discard any sample if $|x| > 5$. Plot a histogram of your data from -5 to 5 with 0.2 increments (i.e., you should have 50 bins). Do this for $N = 50, 100, 200$, and 500. You should submit four figures. Note that you can easily do histograms in python using matplotlib (Hint: it can be slightly tricky to compute x from the CDF; but you don't really need to).

Problem * [5 bonus points]. Prove that Gray code used in an absolute encoder is always possible to construct for n bits with $n \geq 1$. That is, it is always possible to order binary numbers from 0 to $2^n - 1$ on a circle such that two neighbors differ at at most one bit.

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