Extended Essay

Investigating a Tuned Mass Damper System

Name:

Session number:

School:

Subject: Physics

Supervisor's name:

Word Count:

Abstract

A tuned mass damper system is a pendulum-like structure found in tall buildings that is designed to reduce vibrations. It works by transferring the resonating vibrations of the building to that of the suspended mass found at the top of the building.

The vibrations of the resonating building were investigated experimentally with the use of an accelerometer, an arduino board and an excel data acquisition system. The purpose of this experiment is to compare the effectiveness of a tuned mass damper system to that of a building with no damping system.

In order to calculate the damping efficiency of the system, we must examine the time and acceleration of the system from maximum acceleration to no acceleration. Using this data we are able to calculate the necessary length to calibrate the pendulum, the period of oscillation and the damping ratio. The damping ratio of the building measures how fast the building's amplitude will decay. The optimal damping ratio of a building is 1, which means the system is critically damped, it returns to equilibrium with no oscillations. However, this is not attainable in real life due to material constraints. A more realistic system is one with a damping ratio between 0 and 1 (underdamped). What this means is that the building will continue to oscillate after it has been disturbed, eventually it will return to its equilibrium position.

Contents

1. Introducing

I was walking around Toronto one day and I noticed that one of the buildings had a giant sphere hanging from the ceiling. I was instantly curious whether this was purely for aesthetic or if it served a real purpose. Days later, I was perusing the internet and found that the Taipei 101, which is one of the world's tallest buildings, also has a giant mass hanging from the ceiling. I quickly learned that this ball served a critical purpose; it was meant to dampen the effects of wind and natural disasters such as earthquakes. My interest was piqued and I wanted to know how effective a tuned mass damper system is at reducing vibrations in a building.

Research Question:

At what height does a tuned mass damper system become better at dissipating vibrations when compared to a building without one?

My hypothesis is that the short buildings with no pendulum will dampen faster than the short buildings with a pendulum. I also hypothesize that the taller buildings with a pendulum will dampen faster than the tall buildings with no pendulum. I made these educated assumptions based upon the fact that short buildings are good dampeners at relatively slow vibrations [1]. The tuned mass damper will become more impractical as the building height is decreased because the pendulum will not have enough time to react to the sudden vibrations. Tall buildings are more affected by slow vibrations [1]. The taller buildings will oscillate slower and thus the tuned mass damper will be able to better dissipate the kinetic energy of the building compared to a shorter building. I am aiming to determine whether this hypothesis is true by means of an experiment.

2. Design and Calibration of the Experimental Setup

I designed and built a testing rig where the base of a building was shaken, in order to simulate the effects of an earthquake.

The experimental setup can be broken up into 5 parts:

- Motor
- Slider Crank Mechanism
- Pendulum
- Accelerometer
- Building height

a) Slider Crank Mechanism

I designed an earthquake simulator by connecting a motor to a crank wheel with a radius of 2cm. A bearing was placed on the radius of the crank wheel, which is then connected to a connecting rod. This connecting rod is then attached to the base of the building using another bearing. A drawer rail was taken from an old cabinet and used in order for the base of the building to move in a straight line. All of the parts were then screwed to a particle board to prevent any unwanted movement from occurring. Essentially I have created a modified version of a slider crank mechanism which could simulate the linear harmonic motion of an earthquake and apply it to the base of a building. A stepper motor with an rpm of 100 is connected to an arduino in order to accurately maintain a consistent linear velocity for all trials.

Note: All parts in black were 3D printed and designed by me using Solidworks 2018.

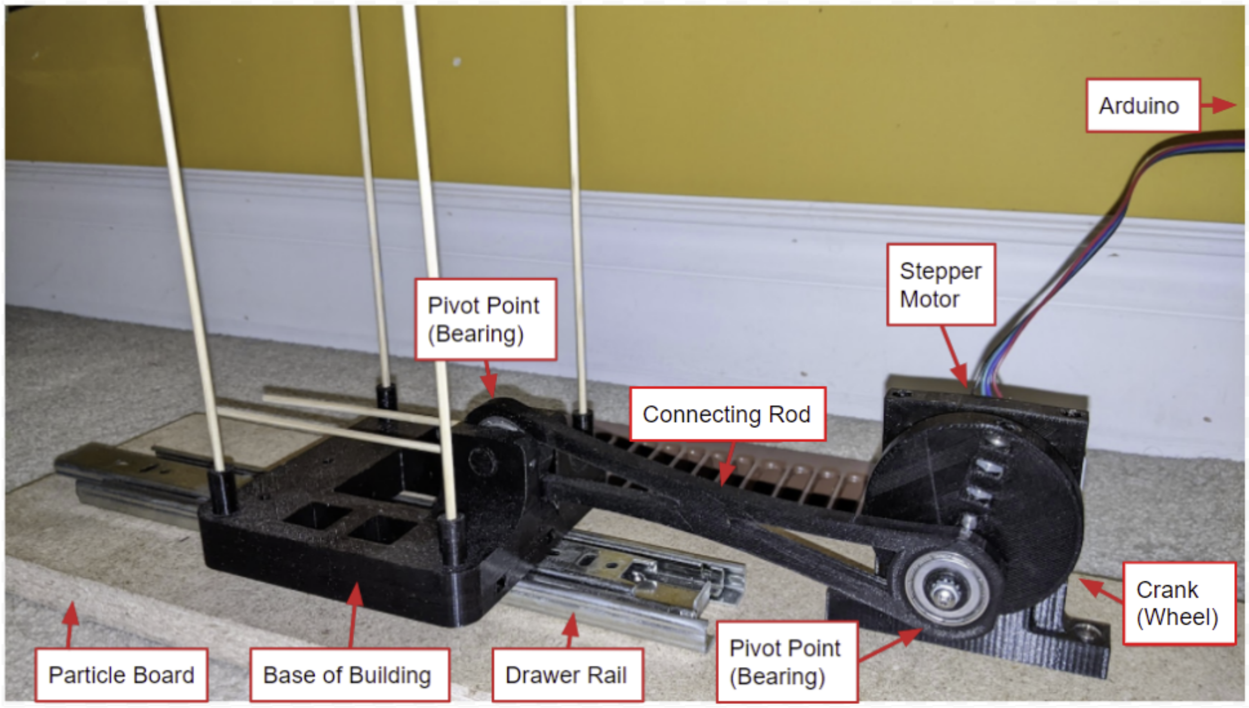


Figure 1: *The construction of the slider crank mechanism and base*

The mechanism of a piston inspired me to design my slider crank mechanism in its current configuration.

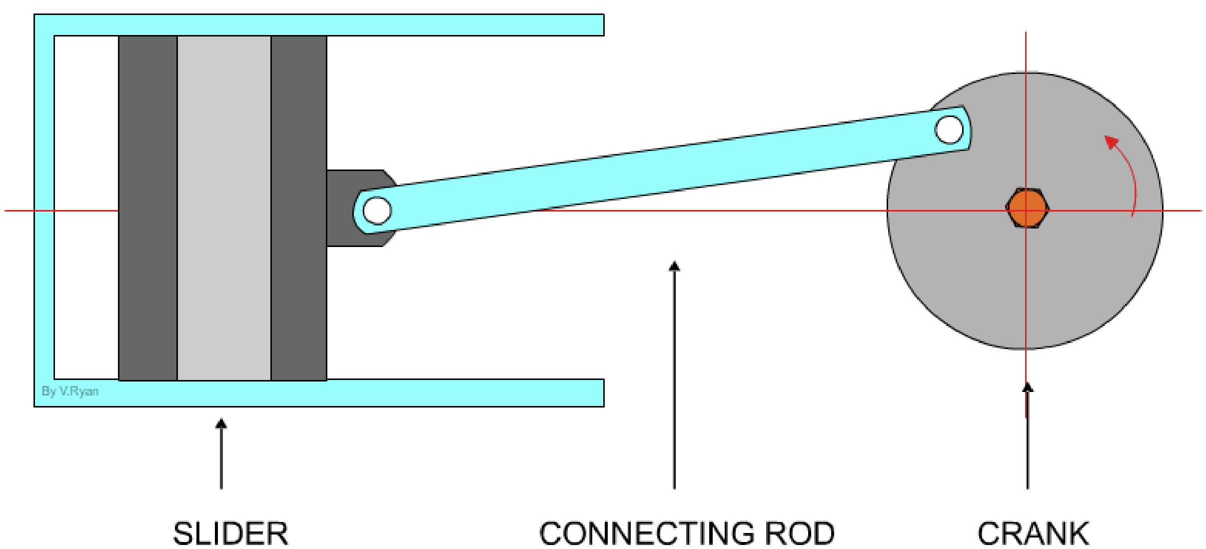


Figure 2: [2] *Inspiration behind design of the slider crank mechanism*

b) Motor

I am using a stepper motor in order to control the speed of the moving base. Through the Arduino IDE, I programmed the stepper motor to rotate at 100 rpm.

The materials used are:

- A4988 motor driver board
- 12V power supply
- Arduino board
- Capacitor (100 µF)
- Small breadboard
- NEMA 17 stepper motor

The schematic for the motor is illustrated in figure 3:

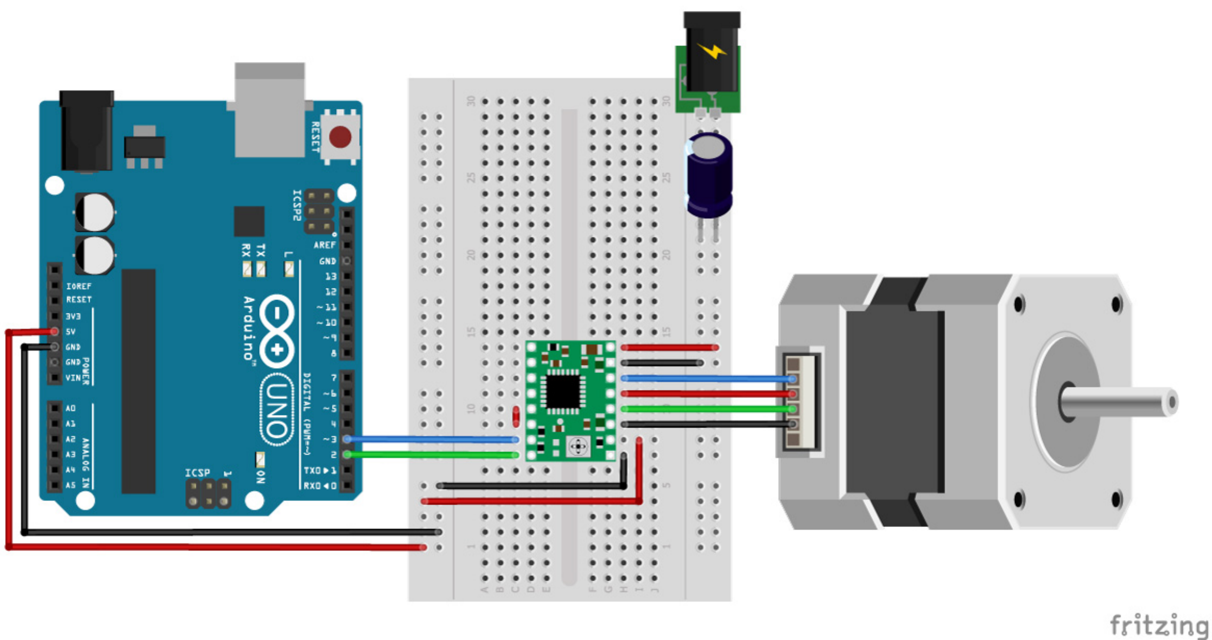


Figure 3: [3] *Schematic of arduino board and A4988 motor driver board*

c) Accelerometer

An ADXL335 accelerometer was used in order to measure the acceleration of the vibrations of the building. This was securely placed at the top of the building using screws. The accelerometer is then connected to a computer and I use the Excel macro PLX-DAQ to collect my data.

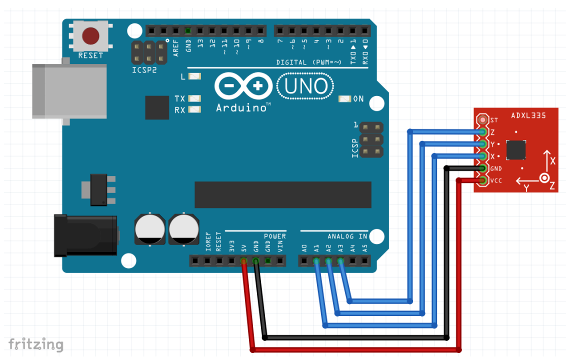


Figure 4: *Schematic of arduino board and ADXL335 accelerometer (Made using Fritzing)*

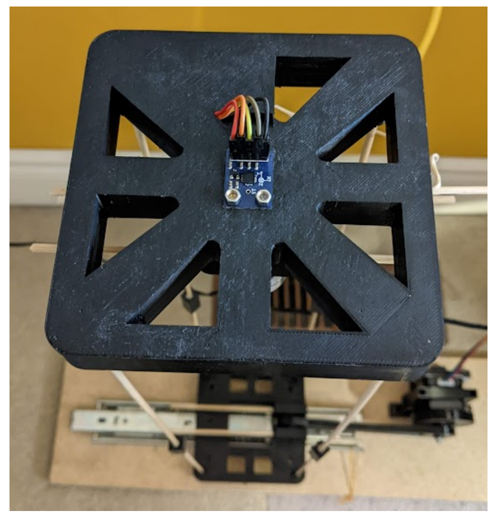


Figure 5: *Accelerometer placed at the top of the building.*

Calibration of Accelerometer

The output of the accelerometer is in millivolts, however our desired unit is ms^{-2}. Using the data sheet of the accelerometer, we are able to convert millivolts to units of gravity, and then units of gravity to units of acceleration.

First, we must lay the accelerometer flat on the ground and measure the voltage using the arduino board. Essentially, this measures the signal produced by the accelerometer when it experiences 1.00g due to gravity (Note: g here represents the gravitational acceleration constant). The voltage I received when the accelerometer is under 1.00g is 384mV. Secondly, we must flip the accelerometer on its other side, and measure the voltage using the arduino board again. This measures the signal produced when the accelerometer is under -1.00g. The voltage I received when the accelerometer is under -1.00g is 260mV. Using this information, we can now convert the range of voltage to a range of acceleration in terms of gravitational acceleration constant. Next we convert from units of gravitational acceleration to units of acceleration by simply multiplying by 9.81. The code found in figure 5 demonstrates the calibration steps above.

```
xvalue = analogRead(xpin);                  //reads values from x-pin & measures acceleration in X direction
float x = map(xvalue, 260, 384, -100, 100); //maps the extreme ends analog values from -100 to 100 for our understanding
float xgInitial = (float)x / (-100.00);     //converts the mapped value into acceleration in terms of "g"
xgFinal = xgInitial * 9.81;
```

<u>Figure 6:</u> *Code used to calibrate the accelerometer. (annotations of each step are provided on the right)*

<u>Accuracy of Accelerometer</u>

It is important to realize that although calibrating the accelerometer may reduce systematic error, random error may still occur due to the accuracy of the accelerometer. The accelerometer's readings are accurate to around ±10%. This can be calculated from the data sheet of the accelerometer:

SPECIFICATIONS

T_A = 25°C, V_S = 3 V, C_X = C_Y = C_Z = 0.1 μF, acceleration = 0 *g*, unless otherwise noted. All minimum and maximum specifications are guaranteed. Typical specifications are not guaranteed.

Table 1.

Parameter	Conditions	Min	Typ	Max	Unit
SENSOR INPUT	Each axis				
Measurement Range		±3	±3.6		*g*
Nonlinearity	% of full scale		±0.3		%
Package Alignment Error			±1		Degrees
Interaxis Alignment Error			±0.1		Degrees
Cross-Axis Sensitivity [1]			±1		%
SENSITIVITY (RATIOMETRIC) [2]	Each axis				
Sensitivity at X_{OUT}, Y_{OUT}, Z_{OUT}	V_S = 3 V	270	300	330	mV/*g*
Sensitivity Change Due to Temperature [3]	V_S = 3 V		±0.01		%/°C

<u>Figure 7:</u> [4] *Data sheet for ADXL 335 accelerometer*

Accuracy for max: $\frac{330-300}{300} = 0.1mV/g \times 100\% = 10\%$

Accuracy for min: $\frac{270-300}{300} =- 0.1mV/g \times 100\% = - 10\%$

Therefore the accelerometer has an uncertainty of ±10%.

d) Pendulum

For the pendulum, I am using 2 bearings for the mass, and an extra bearing for the pivot point. Since I will be calibrating the length of the pendulum, I decided to use a long screw as the rod to easily adjust the position of the mass. The following materials and tools were used to construct the pendulum:

- 3mm nut, quantity:4
- 3mm screw, 30cm length
- 3 Metric Ball Bearings Model: 6000ZZ
- 3D printer

A tuned mass damper system works by dissipating the kinetic energy of the building into heat, this is done by introducing friction at the pivot point of the pendulum. This was achieved by using two 3D printed washers to create friction between the bearing and the pendulum pivot point.

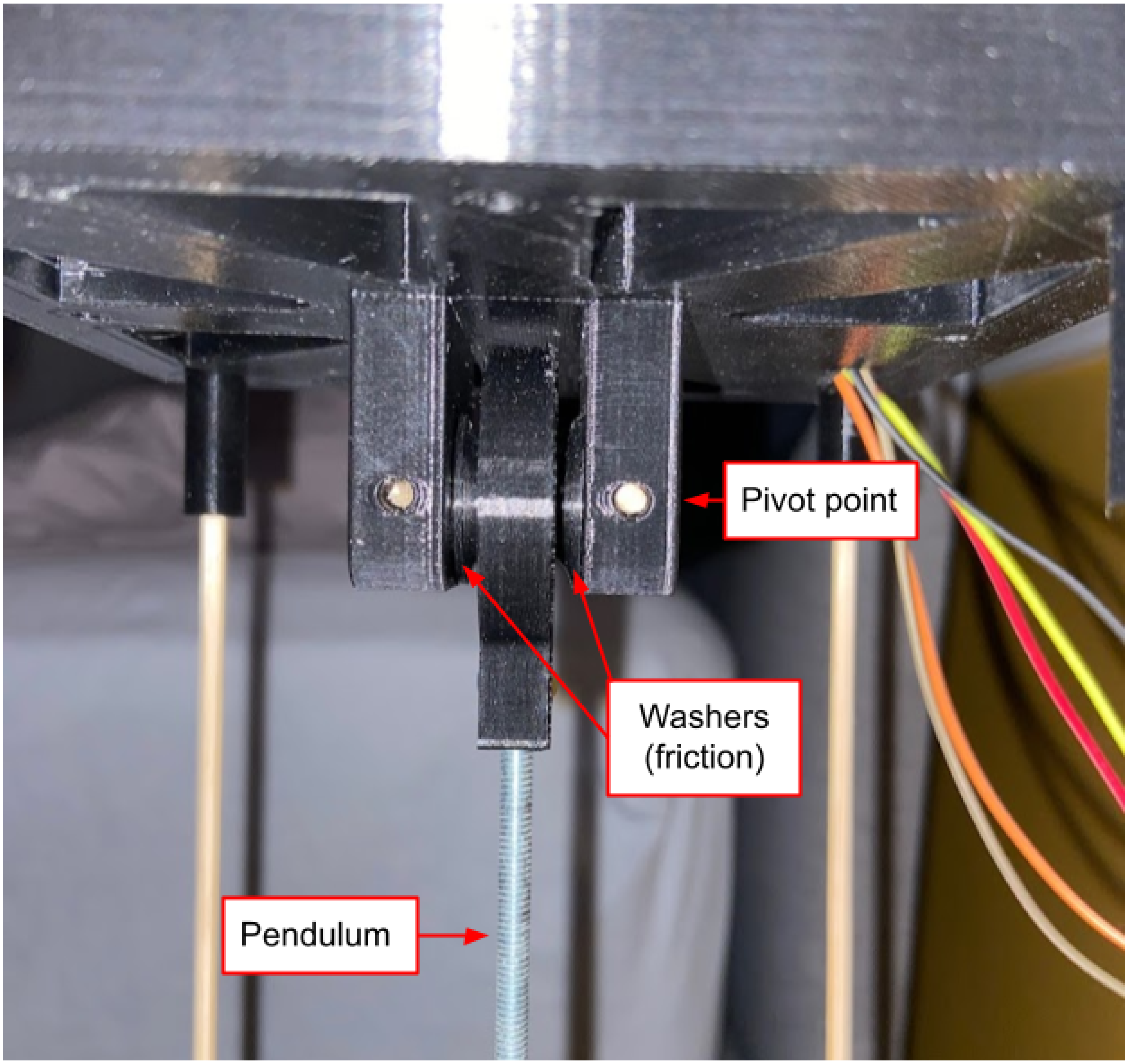


Figure 8: *Washers and mechanism holding the pendulum to the top of the building.*

Calibration of Pendulum

The length of the pendulum must be found in order to calibrate the pendulum. This needs to be repeated for each change in height. The mass that I am using is equal to 0.046kg. The mass of the pendulum does not have to be calibrated, since mass has no impact over period T (since mass is not in the formula $T = 2\pi\sqrt{\frac{L}{g}}$).

Figure 9: *Picture of adjustable pendulum.*

e) Building height

The building height is my independent variable, thus it will be changed during the experiment multiple times. To easily switch between the various heights, I 3D printed connectors that can attach to bamboo skewer sticks to vary the height. To keep the sticks from falling out, I used paper to tighten them into the connector.

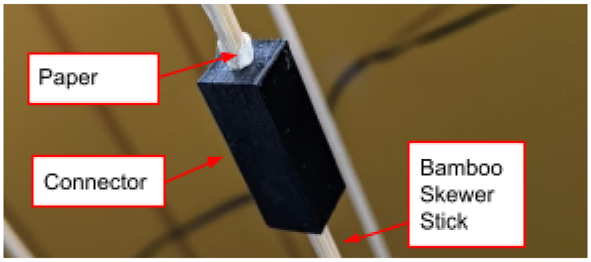


Figure 10: *Picture of building height adjustment system.*

The height of the building is measured from the top of the building (where the accelerometer is placed) and the base of the building.

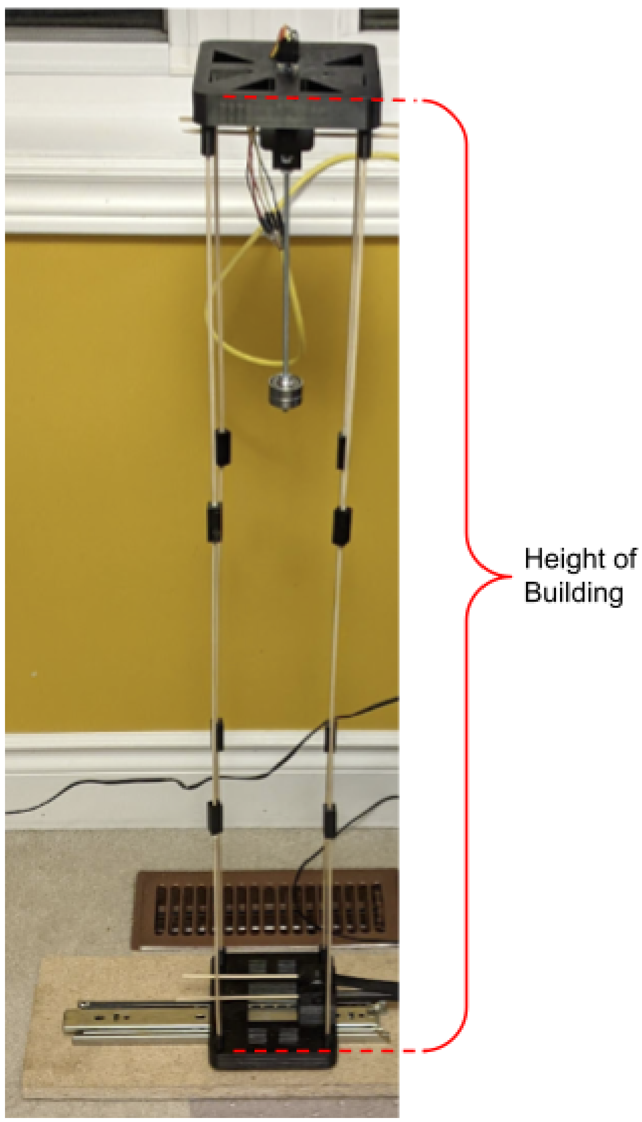
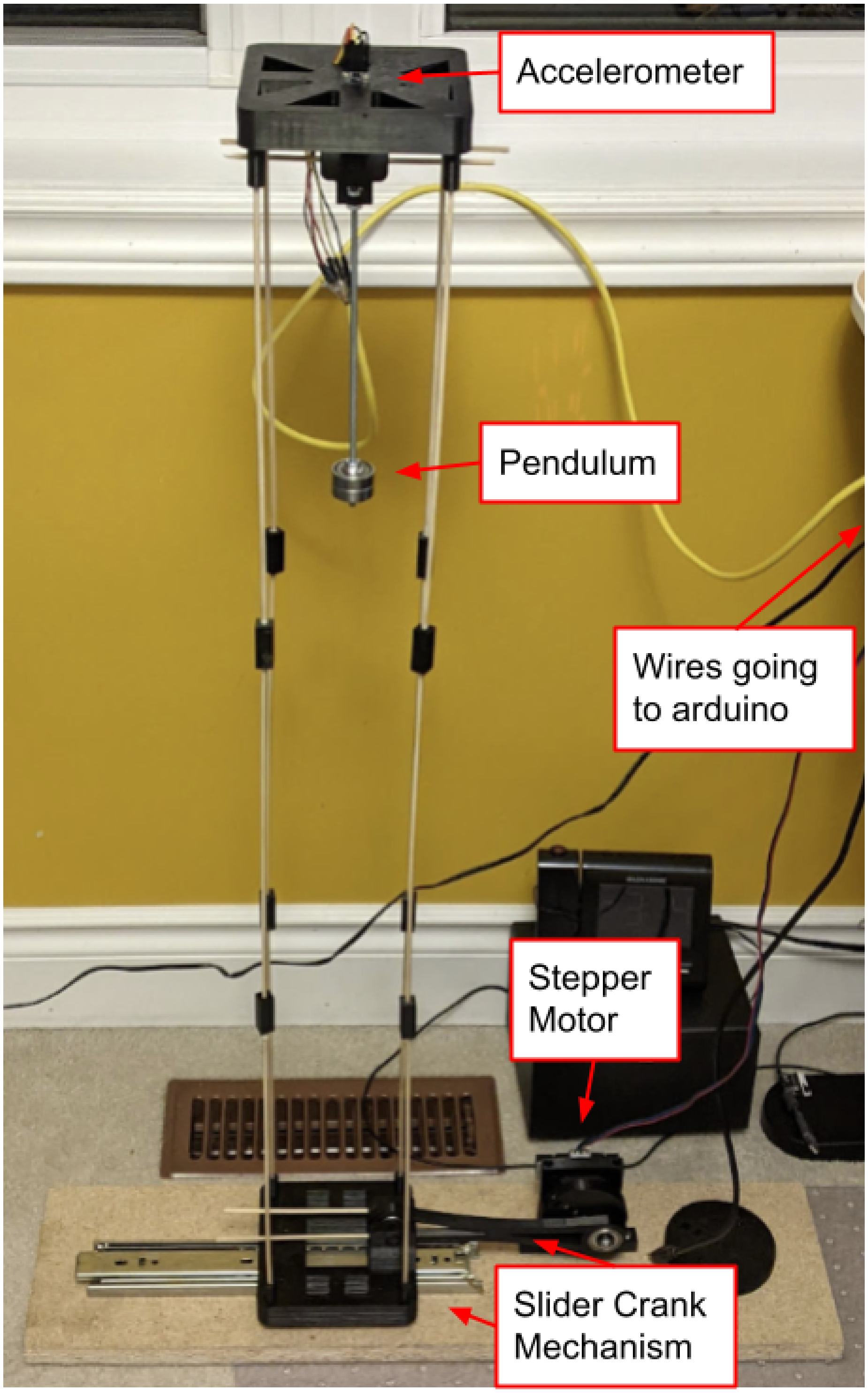


Figure 11: *Picture of how the height of the building is going to be measured.*

Final Testing setup:



<u>Figure 12:</u> *Picture of final testing setup*

3. Control of Variables

a) Independent Variables

1. The height of the building will be altered for each experiment. The experiments will be conducted at the following heights: 90 cm, 85 cm, 80 cm, 75 cm, 70 cm, with an uncertainty of ± 0.05 cm for each.

2. Because the building height is changing, we must also calibrate the pendulum for each height individually. The pendulum heights will be approximately: 25.2 cm, , , , respectively. The uncertainty will be negligible. Check page 14 to understand why uncertainty is negligible.

b) Dependent Variables

1. Period T must be collected in order to determine the pendulum height of each building. This is collected by measuring the distance between the first 3 peaks and then taking their average. Check page 13 to better understand how period T is collected.

2. The first and third amplitudes of the buildings vibrations are collected in order to calculate the damping ratio of each building.

c) **Control Variables**

1. **Temperature** - Temperature can affect the accuracy of the accelerometer, thus the experiment is conducted on the same day in order to avoid temperature variations.

2. **Stepper Motor** - The motor rotates at 100 rpm for all trials in the experiment. This ensures that all buildings will shake at the same magnitude.

3. **Mass & shape of pendulum bob** - Although in theory the mass of a pendulum does not change its amplitude or period, a mass with a larger surface area will experience more air drag. To prevent air drag from interfering with the experiment, the mass and shape of the pendulum bob will be kept constant.

4. **The friction in pivot point** - In order to ensure that the same amount of kinetic energy is dissipated into heat, the same washers will be used for all trials.

5. **Bamboo skewers** - Each structure will be made of the same bamboo material. Different materials have varying degrees of flexibility, which influences the damping ratio.

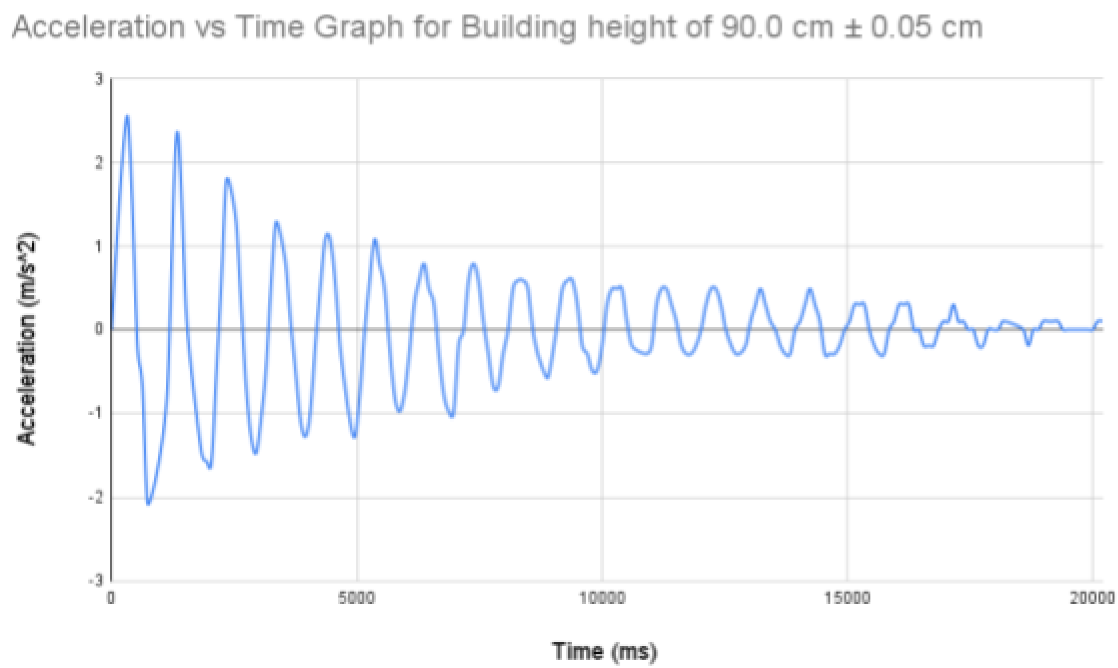
6. **Baud rate of Arduino will be kept at 9600 bps** - The Baud rate is the rate at which data is collected per second. This is kept constant to ensure we collect the same amount of data for each trial.

4. Experimental Procedure:

The steps for gathering data are outlined below. This is an example for the 90 cm-tall building. The experiment is divided into three sections. The first section involves finding the length of the pendulum. The second section involves finding the damping coefficient of the building with a pendulum. The third section involves finding the damping coefficient of the building with no pendulum.

<u>Section 1 of Experimental Procedure</u>

1.	Set building height to 90 cm. Remove the pendulum.
2.	Run the code that spins the motor at 100 rpm for 5 seconds and then stops. This will cause the building to oscillate until it eventually returns to its equilibrium.
3.	The accelerometer will automatically start recording as soon as the motor stops spinning. The accelerometer will track the oscillations of the building until it stops oscillating. The data from the accelerometer will be collected and recorded via PLX-DAQ (this is an excel macro).
4.	In order to solve this, I must find the period of natural frequency of the building. I will achieve this by looking at the oscillations of the building. For this example, I am going to use a building height of 90.0 cm ± 0.05 cm (a tape measure was used to measure, therefore uncertainty is 0.05 because $\frac{smallest\ increment}{2} = \frac{0.1}{2} = 0.05cm$).



<u>Figure 13:</u> *Acceleration (units: ms^{-2}) vs time (units: ms) graph*

In order to find period T, I found the average time for the first 3 periods. The uncertainty will be 0.5ms or 0.0005s because we are using a digital timer with a least significant digit of 1ms (0.001s). Therefore uncertainty is 0.0005s because

$$\frac{least\ significant\ digit}{2} = \frac{0.001s}{2} = 0.0005s$$

Distance between peak 1 & 2: T_1 = 1334 - 327 = 1007 ms = 1.007 s ± 0.0005 s

Distance between peak 2 & 3: T_2 = 2340 - 1334 = 1006 ms = 1.006 s ± 0.0005 s

Distance between peak 3 & 4: T_3 = 3349 - 2340 = 1009 ms = 1.009 s ± 0.0005 s

$T_{Avg} = \frac{T1+T2+T3}{3}$

	$T_{Avg} = \frac{(1.007 \pm 0.0005) + (1.006 \pm 0.0005) + (1.009 \pm 0.0005)}{3}$ $T_{Avg} = \frac{3.022 \pm 0.0015}{3}$ $T_{Avg} \approx 1.007\text{ s} \pm 0.0005\text{ s}$
5.	Now I use the period of a pendulum formula to calculate the pendulum length. $T = 2\pi\sqrt{\frac{L}{g}}$ T = period of natural frequency of building π = 3.14 L = pendulum length g = acceleration due to gravity (using $9.81 m/s^2$ from data booklet)
6.	I am solving for L, therefore I must rearrange the given equation. $L = \frac{T^2 \cdot g}{4\pi^2}$
7.	Find L using the formula found in step 6. $L = \frac{T^2 \cdot g}{4\pi^2}$ $L = \frac{(1.007 \pm 0.0005)^2 \cdot 9.81}{4\pi^2}$ $L = \frac{(1.007 \pm 0.05\%)^2 \cdot 9.81}{4\pi^2}$ Convert absolute uncertainty to % uncertainty $L = \frac{(1.014 \pm 0.10\%) \cdot 9.81}{4\pi^2}$ Square Uncertainty

	$L \approx 0.2521\ m \pm 0.10\%$ $L \approx 0.2521\ m \pm 0.00025\ m$ $L \approx 25.21\ cm \pm 0.025\ cm$ $L \approx 25.21\ cm \pm 0.03\ cm$ Length of the pendulum is thus 25.21 cm.

Section 2 of Experimental Procedure (With Pendulum)

<table>
<tr><td>1.</td><td>Attach the pendulum to the building and adjust the pendulum length to 25.2cm.


<u>Figure 14:</u> Picture of calibrated pendulum. Length is measured from the middle of the pivot point (bearing) to the middle of the mass.</td></tr>
<tr><td>2.</td><td>Steps 2 and 3 from section 1 must be repeated 5 times. Record the first and third amplitudes, there is no need to collect the periods since they are no longer necessary for following calculations.</td></tr>
</table>

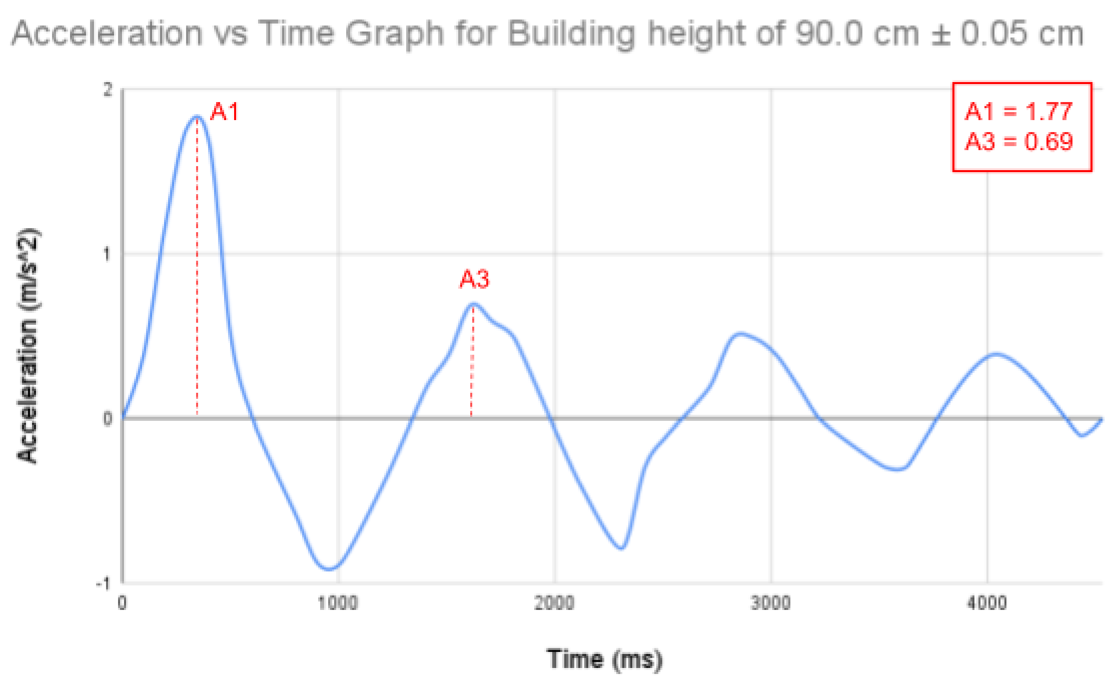


Figure 15: *Picture of trial 1 demonstrating amplitudes being collected for building with pendulum.*

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Amplitude 1 (m s^{-2}) (±10%)	1.77	1.77	1.86	1.86	1.86
Amplitude 3 (m s^{-2}) (±10%)	0.69	0.88	0.88	0.88	0.88

Figure 16: *Trials 1-5 for 90 cm-tall building with pendulum.*

3. Take the mean value of the first and third trials separately.

Average of amplitude 1 (m s^{-2}) = 1.824 ± 10%

Average of amplitude 3 (m s^{-2}) = 0.842 ± 10%

4.	Using the averages from the previous step, we can calculate the logarithmic decrement. $\delta = ln(\frac{Amplitude\ 1}{Amplitude\ 3})$ $\delta = ln(\frac{1.824 \pm 10\%}{0.842 \pm 10\%})$ $\delta \approx ln(2.17 \pm 20\%)$ $\delta \approx ln(2.17 \pm 0.43)$ Use the half range method of finding uncertainty. Uncertainty = $\frac{ln(max) - ln(min)}{2}$ Uncertainty = $\frac{ln(2.17 + 0.43) - ln(2.17 - 0.43)}{2}$ Uncertainty = 0.20 $\delta \approx 0.77 \pm 0.20$
5.	Find the damping ratio. $\zeta = \frac{1}{\sqrt{1+(\frac{2\pi}{\delta})^2}}$ $\zeta = \frac{1}{\sqrt{1+(\frac{2\pi}{0.77 \pm 0.20})^2}}$ $\zeta = \frac{1}{\sqrt{1+(\frac{2\pi}{0.77 \pm 26.2\%})^2}}$

$$\zeta = \frac{1}{\sqrt{1 + \frac{4\pi^2}{0.60 \pm 52.5\%}}}$$

$$\zeta = \frac{1}{\sqrt{1 + 66.07 \pm 52.5\%}}$$

$$\zeta = \frac{1}{\sqrt{1 + 66.07 \pm 34.65}}$$

$$\zeta = \frac{1}{\sqrt{67.07 \pm 34.65}}$$

$$\zeta = \frac{1}{\sqrt{67.07 \pm 51.67\%}}$$

$$\zeta = \frac{1}{8.19 \pm 25.84\%}$$

$$\zeta \approx 0.12 \pm 25.84\%$$

$$\zeta \approx 0.12 \pm 0.03$$

Section 3 of Experimental Procedure (No Pendulum)

1. Remove the pendulum from the building.

2. Steps 2 and 3 from section 1 must be repeated 5 times. Record the first and third amplitudes, there is no need to collect the periods since they are no longer necessary for following calculations.

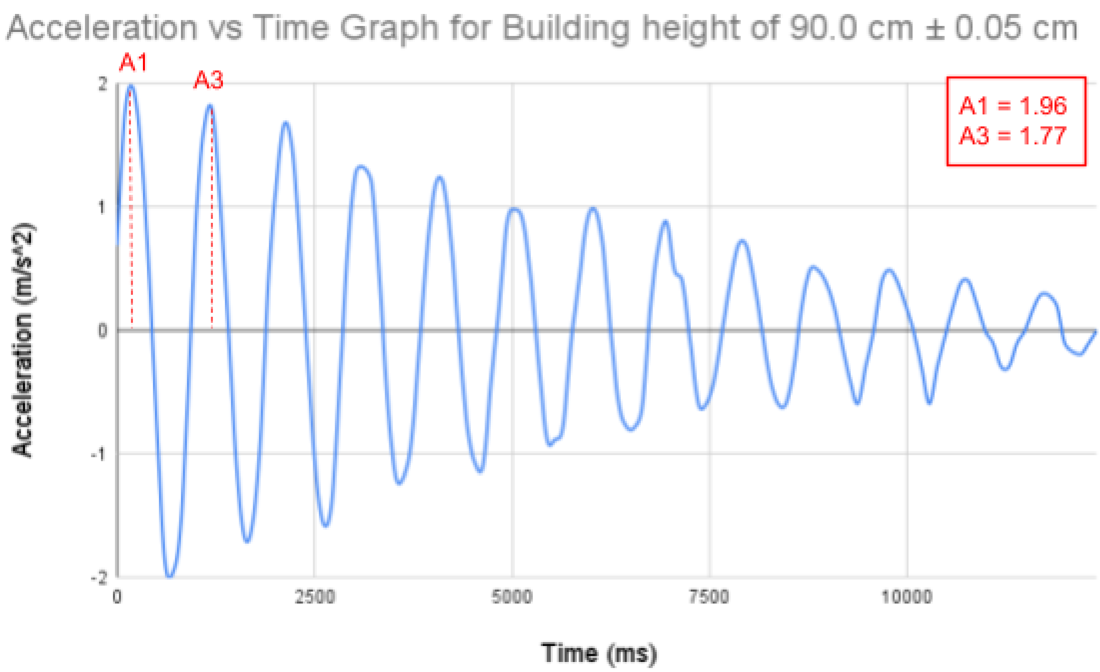


Figure 17: *Picture of trial 1 demonstrating amplitudes being collected for building without pendulum.*

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Amplitude 1 (m s^{-2}) (±10%)	1.96	1.96	1.96	2.06	1.96
Amplitude 3 (m s^{-2}) (±10%)	1.77	1.67	1.77	1.77	1.67

Figure 18: *Trials 1-5 for 90 cm-tall building with no pendulum.*

3.	Take the mean value of the first and third trials separately. Average of amplitude 1 (m s^{-2}) = 1.98 ± 10% Average of amplitude 3 (m s^{-2}) = 1.73 ± 10%
4.	Using the averages from the previous step, we can calculate the logarithmic decrement. $\delta = ln(\frac{Amplitude\ 1}{Amplitude\ 3})$ $\delta = ln(\frac{1.98 \pm 10\%}{1.73 \pm 10\%})$ $\delta \approx ln(1.14 \pm 20\%)$ $\delta \approx ln(1.14 \pm 0.23)$ Use the half range method of finding uncertainty. Uncertainty = $\frac{ln(max) - ln(min)}{2}$ Uncertainty = $\frac{ln(1.14+ 0.23) - ln(1.14-0.23)}{2}$ Uncertainty = 0.20 $\delta \approx 0.135 \pm 0.203$
5.	Find the damping ratio $\zeta = \frac{1}{\sqrt{1 + (\frac{2\pi}{\delta})^2}}$ $\zeta = \frac{1}{\sqrt{1 + (\frac{2\pi}{0.13 \pm 0.20})^2}}$

$$\zeta = \frac{1}{\sqrt{1 + (\frac{2\pi}{0.13 \pm 150.2\%})^2}}$$

$$\zeta = \frac{1}{\sqrt{1 + \frac{4\pi^2}{0.018 \pm 300.4\%}}}$$

$$\zeta = \frac{1}{\sqrt{1 + 2166.95 \pm 300.4\%}}$$

$$\zeta = \frac{1}{\sqrt{1 + 2166.95 \pm 6509.52}}$$

$$\zeta = \frac{1}{\sqrt{2167.95 \pm 6509.52}}$$

$$\zeta = \frac{1}{\sqrt{2167.95 \pm 300.4\%}}$$

$$\zeta = \frac{1}{46.56 \pm 150.2\%}$$

$$\zeta \approx 0.0215 \pm 150.2\%$$

$$\zeta \approx 0.0215 \pm 0.0323$$

Uncertainty of the damping ratio will be high due to the nature of the calculations. Excel was used to facilitate the calculation process.

5. The results of the experiment:

Period T Raw Data

Building Height (cm) (± 0.05 cm)	**Period T1 (s) (± 0.0005 s)**	**Period T2 (s) (± 0.0005 s)**	**Period T3 (s) (± 0.0005 s)**
90.0	1.007	1.006	1.009
85.0	0.908	0.907	0.907
80.0	0.807	0.805	0.806
75.0	0.704	0.706	0.706
70.0	0.604	0.605	0.605

Figure 19: *Table with raw data collected in order to find pendulum length.*

Length of Pendulum Results

Building Height (cm) (± 0.05 cm)	**Length of Pendulum (cm)**
90.0	25.21 ± 0.0252
85.0	20.5 ± 0.0205 (negligible uncertainty)
80.0	16.1 ± 0.0161 (negligible uncertainty)
75.0	12.4 ± 0.0124 (negligible uncertainty)
70.0	9.09 ± 0.00909 (negligible uncertainty)

Figure 20: *Table demonstrating calculated results from figure 19. The same significant figures from the raw data are used for the calculated results.*

Amplitudes 1 and 3 Raw Data

90 cm Tall Building no Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	1.96	1.77
Trial 2	1.96	1.67
Trial 3	1.96	1.77
Trial 4	2.06	1.77
Trial 5	1.96	1.67

90 cm Tall Building With Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	1.77	0.69
Trial 2	1.77	0.88
Trial 3	1.86	0.88
Trial 4	1.86	0.88
Trial 5	1.86	0.88

85 cm Tall Building no Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	3.24	2.75
Trial 2	3.14	2.65
Trial 3	3.14	2.75
Trial 4	3.24	2.75
Trial 5	3.24	2.85

85 cm Tall Building With Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	1.67	0.88
Trial 2	1.67	0.88
Trial 3	1.77	0.98
Trial 4	1.77	0.88
Trial 5	1.77	0.88

80 cm Tall Building no Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	3.53	3.04
Trial 2	3.73	3.24
Trial 3	3.63	3.14
Trial 4	3.73	3.04
Trial 5	3.53	2.94

80 cm Tall Building With Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	1.77	0.98
Trial 2	1.67	0.98
Trial 3	1.77	0.98
Trial 4	1.77	1.18
Trial 5	1.77	1.08

75 cm Tall Building no Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	3.83	3.14
Trial 2	3.83	3.24
Trial 3	3.73	3.14
Trial 4	3.73	3.14
Trial 5	3.83	3.24

75 cm Tall Building With Pendulum		
	Amplitude 1 (m s^{-2}) (±10%)	Amplitude 3 (m s^{-2}) (±10%)
Trial 1	1.96	1.37
Trial 2	1.96	1.28
Trial 3	1.96	1.28
Trial 4	2.06	1.37
Trial 5	1.96	1.37

70 cm Tall Building no Pendulum		
	Amplitude 1 ($m\ s^{-2}$) (±10%)	Amplitude 3 ($m\ s^{-2}$) (±10%)
Trial 1	4.91	4.02
Trial 2	5.01	4.02
Trial 3	4.91	4.12
Trial 4	4.81	4.02
Trial 5	4.81	4.02

70 cm Tall Building With Pendulum		
	Amplitude 1 ($m\ s^{-2}$) (±10%)	Amplitude 3 ($m\ s^{-2}$) (±10%)
Trial 1	2.75	2.16
Trial 2	2.75	2.06
Trial 3	2.75	2.16
Trial 4	2.55	2.06
Trial 5	2.75	1.96

Figure 21-30: *Table with raw data collected with information needed to find the damping ratio.*

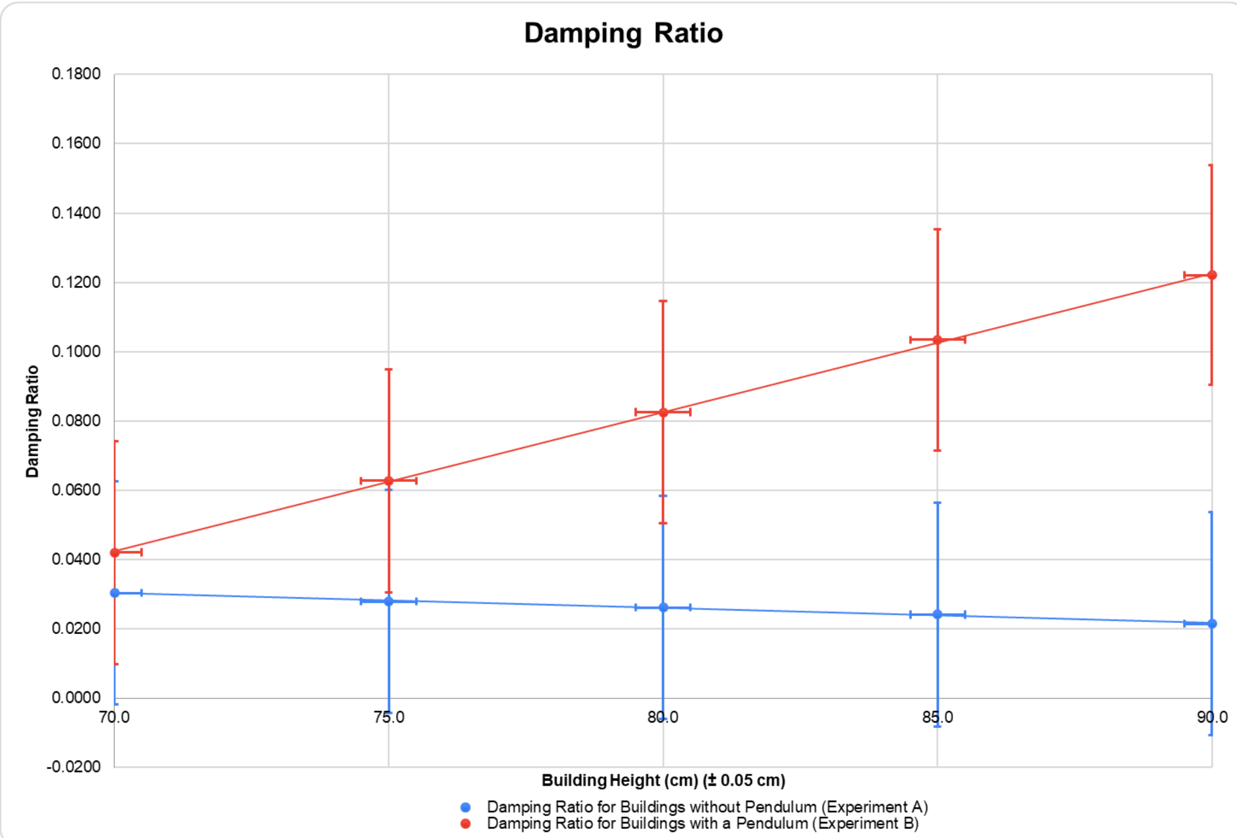
Damping Ratio Results

Buildings Without Pendulum	
Building Height (cm) (± 0.05 cm)	**Damping Ratio**
70.0	0.0304 ± 0.0322
75.0	0.0279 ± 0.0322
80.0	0.0261 ± 0.0322
85.0	0.0241 ± 0.0322
90.0	0.0215 ± 0.0323

Buildings With Pendulum	
Building Height (cm) (± 0.05 cm)	**Damping Ratio**
70.0	0.0421 ± 0.0322
75.0	0.0627 ± 0.0321
80.0	0.0825 ± 0.0320
85.0	0.1034 ± 0.0319
90.0	0.1221 ± 0.0318

Figure 31-32: *Table demonstrating calculated results from figures 21-30. The same significant figures from the raw data are used for the calculated results.*

6. Analysis



<u>Figure 33:</u> *Graph compiling data from figures 31, 32.*

<u>Analysis of Results</u>

From this graph it can be observed that the damping ratio in relation to height for experiment A is linear. It is also observed that the damping ratio decreases as height increases. When observing the damping ratio for experiment B, we see the opposite correlation, damping ratio increases when there is an increase in height. Although the line is linear, the magnitude of the slope in experiment B is greater than that in experiment A. Although the system in experiment B is not critically damped (damping ratio of 1), it is evident that the pendulum greatly increased the damping ratio of the experiment.

Determining Which System is Better

In order to determine which system is more efficient, we must extend the trendlines further.

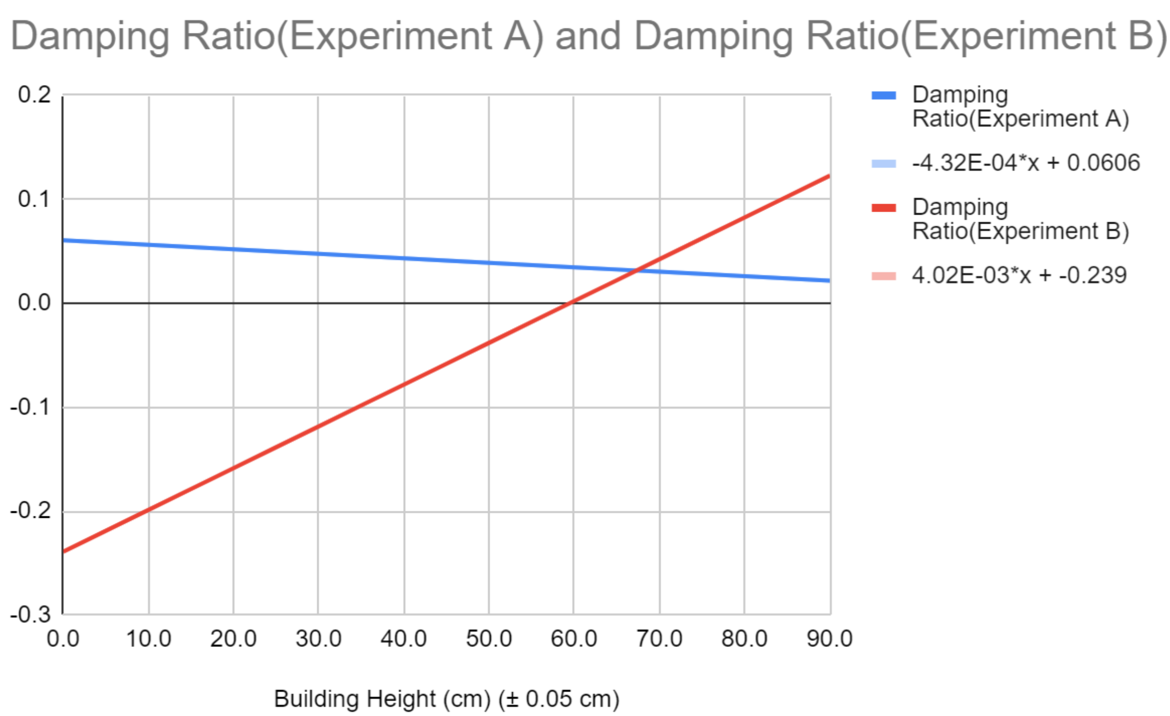


Figure 34: *Graph with extended trendlines.*

By finding the point of intersection, we can find the building height where it becomes obsolete to have a pendulum system.

$$equation\ 1:\ y = -0.000432x + 0.0606$$

$$equation\ 2:\ y = 0.00402x - 0.239$$

$$0.00402x - 0.239 = -0.000432x + 0.0606$$

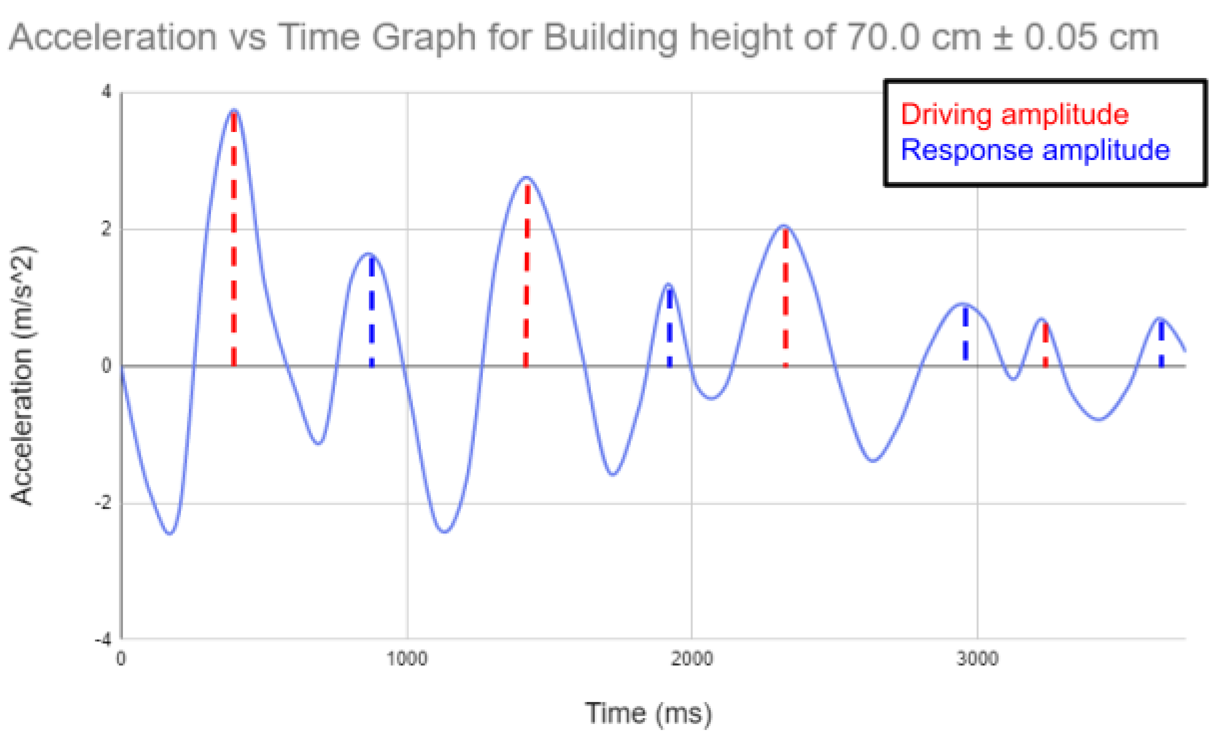
$$0.004452x = 0.2996$$

$$x \approx 67.30\ cm$$

At a building height of around 67.30 cm, experiment A and B will have the same damping ratio. For heights ≳ 67.30 cm, experiment B will have a higher damping ratio, and for heights ≲ 67.30 cm, experiment A will have a higher damping ratio. This explains why shorter buildings do not use a tuned mass damper system. Smaller buildings are good natural dampeners because of their smaller dimensions, a TMD is only effective for buildings of a certain height. For this particular experiment, a TMD system is better suited for heights greater than 67.30 cm. For heights lower than 67.30 cm, having no pendulum is more beneficial as the damping ratio will be higher.

<u>Flaws of the TMD System</u>

An interesting observation that I noticed when testing out the smallest building height (70 cm-tall) with the TMD system, were some unusual amplitude changes.



<u>Figure 34:</u> *Accelerometer readings for 70.0 cm tall building with TMD.*

These readings were very interesting, because it demonstrates two fatal flaws of the TMD system at lower heights. The first flaw is that period T decreases as height decreases, thus the oscillations are occurring much faster. This becomes a problem because the tuned mass damper can no longer react to the vibrations and a phase shift between the oscillations of the building and the pendulum start to form. This leads to the second flaw of this system, when the pendulum is not in phase with the building, the pendulum acts more like a heavy mass on top of the building. This would pose a serious danger hazard in real life, as the TMD system would only lead to more oscillations. I conducted one more experiment at a height of 50 cm to examine what would occur, however I was unable to record the values because the bamboo sticks on my test building collapsed due to the immense magnitude of the amplitudes. This perfectly demonstrates why the damping ratio becomes negative for experiment B, the response amplitudes were amplified by the tuned mass damper acting as a mass, resulting in the building's collapse.

7. Conclusion

Although the uncertainty of the damping ratio is quite high, there is an evident trend in the data. Even with a more accurate accelerometer, the uncertainty will still be large because of the nature of the calculations. This experiment is applicable to real life because it is able to determine when a TMD is needed depending on the building height. However this experiment can be improved by examining the vibrations occurring in the y direction.

My hypothesis that the TMD will become impractical as building height is decreased is correct, for this particular experiment this height is approximately 67.30 cm ± 0.05 cm. The results of this experiment conclude that having no TMD is more suitable for smaller buildings, while taller buildings should utilize a TMD system.

8. Bibliography

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