What is the Distribution of the Inaccuracy in a 3D Printer's Ability to Replicate a Ellipsoid of Different Sizes but with the Same Volume

1. Introduction

The goal of this investigation is to determine the probability of a 3D printer successfully replicating a desired volume. This topic has great significance to me because I constantly have calibration and tolerance problems with my 3D printer, and this problem is very persistent with others involved in the 3D printing community. Most people in this community have FDM printers because they are the cheapest and most accessible to get, however they are also the most inaccurate types of 3D printers. Although other types of 3D printers are highly accurate in replicating complex shapes and volumes, this issue has not been solved yet at a hobby level. The high costs of precision 3D printing technology are due to high raw material costs as well as a high assembly cost. It will take many more years for hobby printers to reach a higher level of precision, however there are ways to optimize FDM printers through fine tuning and calibration, which will lead to a more accurate volume and shape. My goal with this investigation is to simply analyze one factor that has an impact on the accuracy of the volume of a 3D printed part, which is the angle at which the object is being printed at. I intend to find the probability of creating an accurate part, and find possible solutions that can be taken in the design process of future 3D prints in order to reduce inaccuracies.

The 3D printer that will be used is a Flyingbear P902. 10 ellipsoids will be 3D printed, each with a different angle. This will cause possible random and systematic errors, which is what will be further examined in this investigation.

Figure 1: The ellipsoids after being successfully 3D printed.

2. <u>Design</u>

I. Variables

Independent Variable:	-Printing orientation of the ellipsoid An ellipsoid with a theoretical volume of 10 cm^3 will be printed 10 times, however for each different print the angle between the printing plate and the ellipsoid will increase by an increment of 10°. The ellipsoid will be printed with the following orientations: 0°, 10°, 20°, 30°, 40°, 40°, 60°, 70°, 80°, 90°. This will be changed through the Cura Slicer program, which allows the angle of the print to be changed.
Dependent Variable:	-Volume of the ellipsoid The volume of the 3D printed ellipsoid will be measured using the volume of an ellipsoid formula. To find the volume, the length, width and height of the ellipsoid must be measured for each of the 10 3D printed ellipsoids.

Control Variables:	• 3D printer The same 3D printer will be used to print all 10 of the ellipsoids. Since each 3D printer will be differently calibrated, it is important to use the same 3D printer in order to reduce the discrepancy of the accuracy of each ellipsoid. In this experiment, a Flyingbear P902 3D printer was used. • Filament Filament refers to the plastic material being melted by the 3D printer to create a new object. It is important to use the same filament material when doing this experiment because different filaments may expand and contract after undergoing the high temperatures of the 3D printer. Thus, using different filaments would result in random errors that would change the volume of the ellipsoids unpredictably. • Time All 10 ellipsoids were printed at the same time. This is done in order to reduce any potential random errors produced by the surrounding environment, such as changes in the room temperature and discrepancies in the voltage supplied to the 3D printer. • Measuring Instrument The same digital caliper with uncertainty of ±0.01 cm was used. The same digital caliper was used because each measuring instrument possesses its own unique random and systematic errors. To remove these possible discrepancies, the same digital caliper was used.

II. Apparatus

Figure 2: Picture of the digital caliper used, with uncertainty of ± 0.01 cm.

Figure 3: Picture of all 10 3D printed ellipsoids, with their angles labeled on them.

III. Procedure

1. 3D print all 10 ellipsoids at angles 0°-90°.
2. Remove any support materials that cover the ellipsoids.
3. Measure the length, width and height of the ellipsoid printed at an angle of 0°.
4. Record all of these measurements into excel (for further data processing).
5. Repeat steps 1-4 for all of the other angles.

3. Data Collection and Processing:

I. Collected data

Angle (°)	Length (cm)	Width (cm)	Height (cm)
0	3.982	2.161	2.192
10	3.987	2.165	2.196
20	3.995	2.167	2.201
30	3.991	2.164	2.195
40	3.987	2.164	2.187
50	3.992	2.169	2.189
60	3.995	2.175	2.195
70	4.000	2.182	2.191
80	3.997	2.181	2.182
90	3.984	2.169	2.171

Table 1: Raw Data collected

II. Data processing

Calculations to find the theoretical ellipsoid volume using integrals:

The theoretical volume of the ellipsoid refers to the ellipsoid volume set through the 3D modeling software, SolidWorks. The dimensions are as followed:

Length: 4 cm

Width: 2.185096861 cm

Height: 2.185096861 cm

Center point: (0,0)

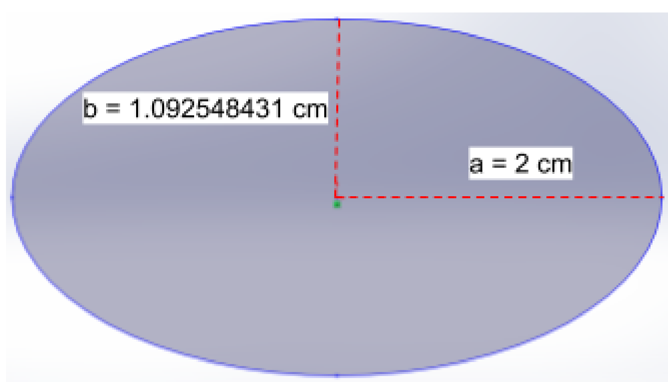


Figure 4: Picture demonstrating values a and b.

To find the theoretical volume of the ellipsoid, the equation of the ellipse must first be found.

Equation of any ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

A represents the distance from the center to the end of the major axis (x-axis), and b is the distance from the center to the end of the minor axis (y-axis).[1] For our values, a = 2 cm and b = 1.092548341 cm.

$$\frac{x^2}{2^2} + \frac{y^2}{1.09^2} = 1$$

$$\frac{y^2}{1.09^2} = 1 - \frac{x^2}{2^2}$$

$$y^2 = 1.09^2 - \frac{1.09^2 \cdot x^2}{2^2}$$

$$y^2 = 1.19 - 0.298x^2$$

$$V = \int_a^b \pi y^2 dx$$

Since we are revolving the ellipse equation around the x-axis, the values of b and a are going to be 2 and -2 respectively.

$$V = \pi \int_{-2}^{2} (1.19 - 0.298x^2)dx$$

$$V = \pi\left[1.19x - 0.298(\tfrac{1}{3}x^3)\right]_{-2}^{2}$$

$$V = \pi[(1.19 \cdot 2) - (0.298 \cdot \frac{2^3}{3})] - \pi\{[1.19 \cdot (-2)] - [0.298 \cdot \frac{(-2)^3}{3}]\}$$

$$V \approx 10.00\ cm^3$$

Calculations to find the experimental ellipsoid volume:

The following will be a demonstration of finding the volume of the ellipsoid printed at 0°. The dimensions of this ellipsoid are:

Length (cm)	A Value (cm)	Width (cm)	B Value (cm)	Height (cm)	C Value (cm)
3.982	1.991	2.161	1.0805	2.192	1.096

$V = \frac{4}{3}\pi abc$

$V = \frac{4}{3}\pi(1.991) \cdot (1.0805) \cdot 1.096$

$V \approx 9.876\ cm^3$

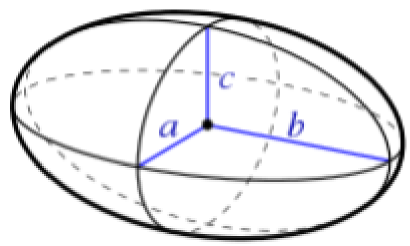
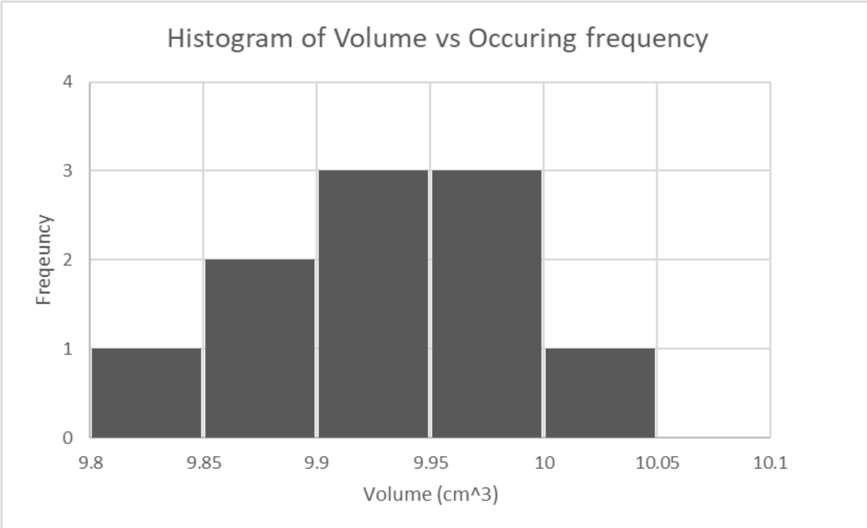


Figure 5: Image demonstrating variables a,b,c

This was then repeated for each of the other angles.

Angle (°)	Length (cm)	Width (cm)	Height (cm)	A Axis (cm)	B Axis (cm)	C Axis (cm)	Volume (cm^3)
0	3.982	2.161	2.192	1.991	1.081	1.096	9.876
10	3.987	2.165	2.196	1.994	1.083	1.098	9.925
20	3.995	2.167	2.201	1.998	1.084	1.101	9.977
30	3.991	2.164	2.195	1.996	1.082	1.098	9.926
40	3.987	2.164	2.187	1.994	1.082	1.094	9.880
50	3.992	2.169	2.189	1.996	1.085	1.095	9.924
60	3.995	2.175	2.195	1.998	1.088	1.098	9.986
70	4.000	2.182	2.191	2.000	1.091	1.096	10.013
80	3.997	2.181	2.182	1.999	1.091	1.091	9.960
90	3.984	2.169	2.171	1.992	1.085	1.086	9.823

Table 2: Calculations for values a,b,c and the volume of each ellipsoid.



Graph 1: Histogram used to better visualize the data from table 1.

Calculations to find the mean of the volumes:

This provides us with the average of all of the volumes collected.

$$\mu = \frac{\Sigma X}{N}$$

$$\mu = \frac{9.876+9.925+9.977+9.926+9.880+9.924+9.986+10.013+9.960+9.823}{10}$$

$$\mu = \frac{99.290}{10}$$

$$\mu = 9.929\ cm$$

Finding the Standard deviation:

The standard deviation provides the amount of variation in a dataset.

Variable	SD	x	μ	N
Unit	Standard deviation	Values of the population	Mean	Size of the population

$$SD = \sqrt{\frac{\Sigma \cdot |x-\mu|^2}{N}}$$

$$SD = \sqrt{\frac{|9.876-9.929|^2+|9.925-9.929|^2+|9.977-9.929|^2+|9.926-9.929|^2+|9.880-9.929|^2+|9.924-9.929|^2+|9.986-9.929|^2+|10.013-9.929|^2+|9.960-9.929|^2+|9.823-9.929|^2}{10}}$$

$$SD = 0.0548$$

Finding the Variation:

The variation measures the amount of spread between numbers in a dataset.

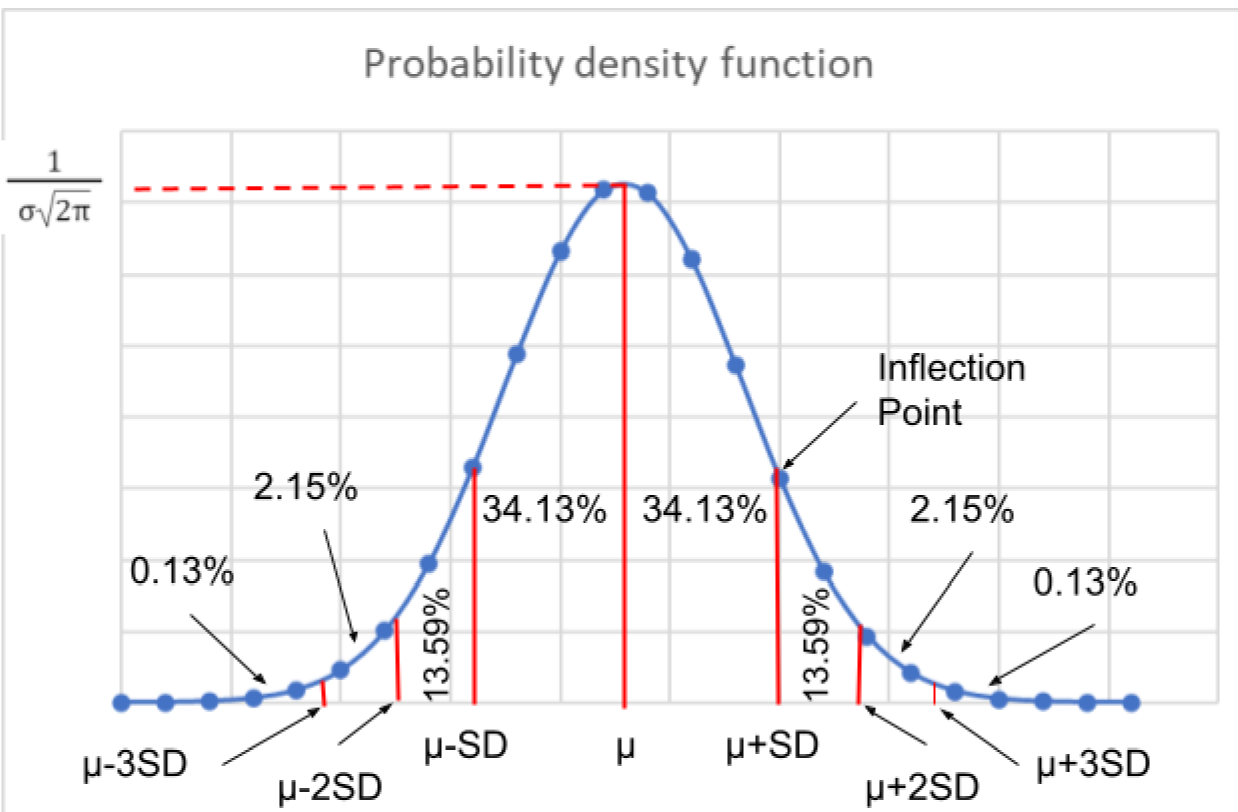
Variable	σ^2	x	μ	N
Unit	Variance	Values of the population	Mean	Size of the population

$$\sigma^2 = \frac{\Sigma\cdot|x-\mu|^2}{N}$$

$$\sigma^2 = \frac{|9.876-9.929|^2+|9.925-9.929|^2+|9.977-9.929|^2+|9.926-9.929|^2+|9.880-9.929|^2+|9.924-9.929|^2+|9.986-9.929|^2+|10.013-9.929|^2+|9.960-9.929|^2+|9.823-9.929|^2}{10}$$

$$\sigma^2 = 0.00301$$

Using the standard deviation and the mean, a normal distribution graph can be created.



Graph 2: Image showing how the probability density function is formed.

Find Correlation Coefficient and Coefficient of Determination

The correlation coefficient is a value from -1 to 1 which gives the strength and direction of the relationship between two variables.[2] This is important to analyze because it determines whether the angle at which the ellipsoids are printed has an effect on their volume.

The coefficient of determination determines how much variability of one factor can be caused by its relationship to another related factor, and is a value between 0.0 and 1.0.[3]

Variable	r	r^2	x	y	$\bar{x}$	$\bar{y}$	N
Unit	correlation coefficient	coefficient of determination	Values of the population	Values of the angles	Mean of the population	Mean of the angles	Size of the population

$$r = \frac{S_{xy}}{S_x \cdot S_y}$$

$$r = \frac{S_{xy}}{\sqrt{(S^2_x)} \cdot \sqrt{(S^2_y)}}$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$r = \frac{(9.876-9.929)(0-45)+(9.925-9.929)(10-45)+(9.977-9.929)(20-45)+(9.926-9.929)(30-45)+(9.880-9.929)(40-45)+(9.924-9.929)(50-45)+(9.986-9.929)(60-45)+(10.013-9.929)(70-45)+(9.960-9.929)(80-45)+(9.823-9.929)(90-45)}{\sqrt{(9.876-9.929)^2+(9.925-9.929)^2+(9.977-9.929)^2+(9.926-9.929)^2+(9.880-9.929)^2+(9.924-9.929)^2+(9.986-9.929)^2+(10.013-9.929)^2+(9.960-9.929)^2+(9.823-9.929)^2}\sqrt{(0-45)^2+(10-45)^2+(20-45)^2+(30-45)^2+(40-45)^2+(50-45)^2+(60-45)^2+(70-45)^2+(80-45)^2+(90-45}}$$

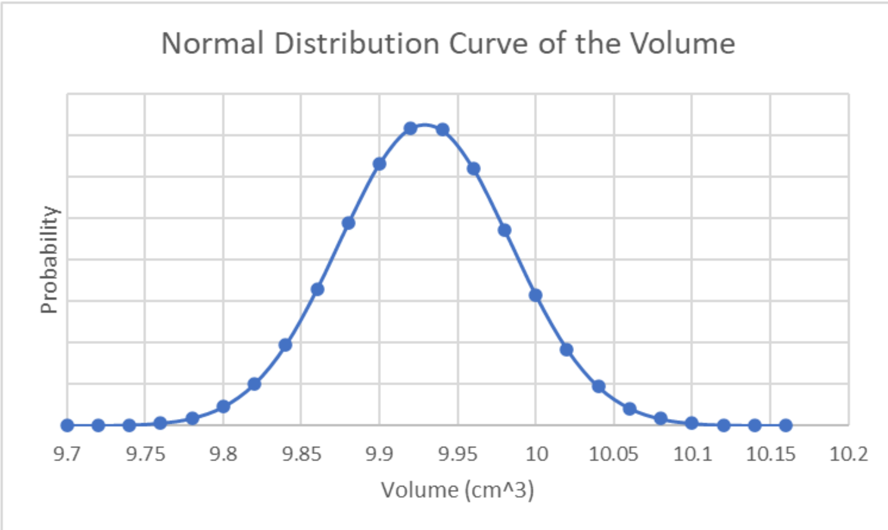
$$r = 0.0526$$

$$r^2 = 0.00277$$

A GDC was used to confirm that the calculated r and r^2 values were correct.

4. Evaluation of the Data:

Graph 3: Graph depicting the probability of 3D printing each volume.



Given the fact that the accuracy of the digital caliper used in this experiment was ± 0.01 cm, a volume from 9.990 cm^3 to 10.010 cm^3 would be within the range of the caliper's accuracy. To find the probability that the 3D printer can successfully replicate a volume of 9.990 cm^3 to 10.010 cm^3, we must find its quantile. This was done using a graphing calculator.

$$= P(9.990 \leq z \leq 10.010)$$

$$P(9.990 \leq z \leq 10.010) = normalcdf(9.99, 10.01, 9.928997, 0.054822)$$

$$P(9.990 \leq z \leq 10.010) = 0.0631$$

5. Conclusion and Analysis

From the histogram in graph 1, it is noticed that most of the volumes collected are below 10.00 cm^3. Only 1 of the trials collected in this experiment has a volume of over 10.00 cm^3. This trend is indicative of a possible systematic error that is affecting the data. Possible causes behind this systematic error include the improper calibration of the 3D printer, improper measuring techniques used when measuring with the caliper or the filament quality. A set of data without this systematic error would have a mean closer to 10.00 cm^3, however in this data set, the mean is 9.929 cm^3. It is important to note that random error does not play a large factor on the results of this experiment. This is known because the standard deviation of the collected volumes is low (0.054822), which indicates that the data collected is close to the mean. The data collected is close to the mean and is thus easier to predict.

From graph 3 and the calculations on page 11, the probability of accurately 3D printing a volume of 10.00 $cm^3 \pm 0.01\ cm^3$ was found to be around 6.31%, which means the printer used has a low accuracy at producing precise volumes. However, it is important to note that testing an FDM printer using lower volumes will produce larger discrepancies because the current FDM printer used lacks precision when 3D printing smaller volumes and shapes.

Further examining the processed data on page 10, it was found that the correlation coefficient is 0.0526 and the coefficient of determination is 0.00277. A correlation coefficient of 0.0526 is low, it indicates a very weak positive linear regression. This value indicates that there is a probability of around 5.26% that the results of the collected volume data were due to changes in the printing angle. The coefficient of determination of value 0.00277 indicates that the independent variable (printing angle) has a small impact on the average of the volume of the ellipsoid. However it is important to note that there were only 10 data points for volume, the accuracy of both the coefficient of determination and the correlation coefficient could have been improved if more data was collected.

Although no significant trends were observed in the raw data, one interesting observation was how close the volumes for the 60° and 70° ellipsoids were to the theoretical values. The explanation for why this may have happened is because the base of the ellipsoids was sitting on a layer of support material, which reduces its contact with the build plate (this is the area where the ellipsoids are printed on). Because it was not directly touching the build plate, the force of the ellipsoid was subject to less force. This leads to the length of the ellipsoid being less compressed while printing which results in a height that is closer to the theoretical length of 4.00 cm. This can even be seen in the raw data since the ellipsoid printed at a 70° angle has a perfect length of 4.00 cm. However, it is important to note that this is purely speculation, since not enough data was collected to accurately notice a significant trend.

6. Limitations and Extensions

One limitation that could have played a crucial role in the random error in this experiment is the material that was chosen when printing. The choice of material for this project was ABS. ABS is notorious for warping, due to the physical properties of the material. At first, when the ABS material is extruded out of the nozzle, the plastic expands slightly, but after it slightly cools down it contracts.[4] This poses a huge problem, because it directly affects the height, width and length dimensions of the ellipsoid, and thus will lead to random errors in the volume. If I were to repeat this experiment again, I would choose a material such as PLA which is less prone to warping.[5]

Another limitation that might have played a role in this experiment is using a heated printing bed. Using a heated printed bed helps with the adhesion of the ellipsoids to the printer. It is not always necessary, however it greatly reduces the printing failure of the machine. Using a printing bed also has its disadvantages such as heating the base of the material for long periods of time. This can cause warping to occur at the base, and possibly even expand the parts of the ellipsoids in unexpected places, due to heat having an expanding effect on plastic materials. To reduce this error in the future, the printer will be calibrated so that heat is applied to the base of the printer during the first 5 minutes of the printing process, and then turned off to avoid possible expansion in the plastic.

A systematic error that played a role in the inaccuracies of the volumes of the ellipsoids is the calibration of the 3D printer. Uncalibrated 3D printers occur because there is not an even amount of tension found in the belts that move the nozzle of the printer. In the raw data, it was noticed that almost all length's collected were under the desired 4.00 cm length, except during one case. This shows that there is systematic error present in the belt controlling the z-axis of the printer (the z-axis controls the height of the object being printed). To remove this systematic error, the printer must be recalibrated either through software or by tightening the belts controlling the nozzle.

An extension I would have liked to further explore is the effect of changing the material being used by the printer, and seeing how it impacts the volume of the print. This would be an interesting extension because we can observe the expanding and contracting properties of each type of plastic. Unfortunately I could not conduct this experiment because I am only limited to 2 types of printing materials, however as my desire to learn more about 3D printing grows I plan on testing other types of materials such as silicone or wood infused filaments.

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