> $d := 9$

$$d := 9 \tag{1}$$

A) Solve the ode system (ode1, ode2) and matching the boundary condition.

A.1 Set up the ode system (ode1, ode2) = 0;

> $ode1 := simplify(t^2 \cdot diff(F(t), t, t) + (d-1) \cdot t \cdot diff(F(t), t) - (d-1) \cdot F(t) + 2 \cdot \text{lambda} \cdot t^2 \cdot diff(G(t), t) + \text{lambda}^2 \cdot t^2 \cdot F(t)) \text{ assuming } t > 0$

$$ode1 := t^2 \left(\frac{d^2}{dt^2} F(t) \right) + 8\, t \left(\frac{d}{dt} F(t) \right) - 8\, F(t) + 2\, \lambda\, t^2 \left(\frac{d}{dt} G(t) \right) + \lambda^2\, t^2\, F(t) \tag{2}$$

> $ode2 := -d \cdot \text{lambda}^2 \cdot t \cdot G(t) - 2 \cdot \text{lambda} \cdot t \cdot diff(F(t), t) - 2 \cdot \text{lambda} \cdot (d-1) \cdot F(t) - (d-1) \cdot diff(G(t), t) - t \cdot diff(G(t), t, t)$

$$ode2 := -9\, \lambda^2\, t\, G(t) - 2\, \lambda\, t \left(\frac{d}{dt} F(t) \right) - 16\, \lambda\, F(t) - 8 \frac{d}{dt} G(t) - t \left(\frac{d^2}{dt^2} G(t) \right) \tag{3}$$

Check the fundemental solution (sol:=(F(t), G(t)))to ode1 = 0 and ode2=0.

> $sol := dsolve(\{ode1, ode2\}, \{F(t), G(t)\})$

$$\begin{aligned} sol := \Big\{ & F(t) = \frac{1}{t^8} \Big(\sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^5\, t^5 - 3 \cos(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^5\, t^5 \\ & + \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C1\, \lambda^3\, t^5 + 15 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^4\, t^4 - 3 \cos(\lambda \sqrt{3}\, t)\, \_C2\, \lambda^3\, t^5 \\ & + 15 \cos(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^4\, t^4 + 6 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C2\, \lambda^2\, t^4 \\ & - 35 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^3\, t^3 + 6 \cos(\lambda \sqrt{3}\, t)\, \_C1\, \lambda^2\, t^4 + 105 \cos(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^3\, t^3 \\ & - 5 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C1\, \lambda\, t^3 - 140 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^2\, t^2 \\ & + 15 \cos(\lambda \sqrt{3}\, t)\, \_C2\, \lambda\, t^3 - 140 \cos(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^2\, t^2 - 5 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C2\, t^2 \\ & + 105 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C3\, \lambda\, t - 5 \cos(\lambda \sqrt{3}\, t)\, \_C1\, t^2 - 315 \cos(\lambda \sqrt{3}\, t)\, \_C4\, \lambda\, t \\ & + 105 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C4 + 105 \cos(\lambda \sqrt{3}\, t)\, \_C3 \Big), G(t) = \frac{1}{\lambda\, t^7} \Big(\\ & - \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^6\, t^4 - \cos(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^6\, t^4 - \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C2\, \lambda^4\, t^4 \\ & + 3 \sqrt{3}\, \sin(\lambda \sqrt{3}\, t)\, \_C3\, \lambda^5\, t^3 - \cos(\lambda \sqrt{3}\, t)\, \_C1\, \lambda^4\, t^4 - 9 \cos(\lambda \sqrt{3}\, t)\, \_C4\, \lambda^5\, t^3 \end{aligned} \tag{4}$$

$$+13\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C4\,\lambda^4\,t^2+13\cos(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^4\,t^2$$
$$-5\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C2\,\lambda^2\,t^2-10\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^3\,t$$
$$-5\cos(\lambda\sqrt{3}\,t)\,\_C1\,\lambda^2\,t^2+30\cos(\lambda\sqrt{3}\,t)\,\_C4\,\lambda^3\,t+5\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C1\,\lambda\,t$$
$$-10\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C4\,\lambda^2-15\cos(\lambda\sqrt{3}\,t)\,\_C2\,\lambda\,t-10\cos(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^2$$
$$+5\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C2+5\cos(\lambda\sqrt{3}\,t)\,\_C1)\}$$

> $F1 := eval(F(t), sol);$
> $G1 := eval(G(t), sol)$

$$F1 := \frac{1}{t^8}\Bigg(\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^5\,t^5-(3\cos(\lambda\sqrt{3}\,t)\,(6\sqrt{3}\,\sin(\lambda\sqrt{3})\,\lambda^2$$
$$-3\cos(\lambda\sqrt{3})\,\lambda^3-5\sqrt{3}\,\sin(\lambda\sqrt{3})+15\,\lambda\cos(\lambda\sqrt{3}))\,\lambda^6\,t^5)\Big/(27\sin(\lambda\sqrt{3})^2\,\lambda^8$$
$$+54\sin(\lambda\sqrt{3})^2\,\lambda^6-18\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^7+27\cos(\lambda\sqrt{3})^2\,\lambda^8$$
$$-2385\sin(\lambda\sqrt{3})^2\,\lambda^4+900\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^5-270\cos(\lambda\sqrt{3})^2\,\lambda^6$$
$$+3825\sin(\lambda\sqrt{3})^2\,\lambda^2-4500\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^3+2025\cos(\lambda\sqrt{3})^2\,\lambda^4$$
$$-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda-4725\cos(\lambda\sqrt{3})^2\,\lambda^2)$$
$$+\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,\_C1\,\lambda^3\,t^5+(15\sqrt{3}\,\sin(\lambda\sqrt{3}\,t)\,(6\sqrt{3}\,\sin(\lambda\sqrt{3})\,\lambda^2$$
$$-3\cos(\lambda\sqrt{3})\,\lambda^3-5\sqrt{3}\,\sin(\lambda\sqrt{3})+15\,\lambda\cos(\lambda\sqrt{3}))\,\lambda^5\,t^4)\Big/(27\sin(\lambda\sqrt{3})^2\,\lambda^8$$
$$+54\sin(\lambda\sqrt{3})^2\,\lambda^6-18\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^7+27\cos(\lambda\sqrt{3})^2\,\lambda^8$$
$$-2385\sin(\lambda\sqrt{3})^2\,\lambda^4+900\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^5-270\cos(\lambda\sqrt{3})^2\,\lambda^6$$
$$+3825\sin(\lambda\sqrt{3})^2\,\lambda^2-4500\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^3+2025\cos(\lambda\sqrt{3})^2\,\lambda^4$$

$$-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2)$$

$$+(3\cos(\lambda\sqrt{3}\,t)(15\sqrt{3}\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2$$

$$+105\cos(\lambda\sqrt{3})\lambda^3+105\sqrt{3}\sin(\lambda\sqrt{3})-315\lambda\cos(\lambda\sqrt{3}))\lambda^4t^5)/$$

$$(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6-18\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7$$

$$+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5$$

$$-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$

$$+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$

$$-4725\cos(\lambda\sqrt{3})^2\lambda^2)+15\cos(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^4t^4$$

$$-(6\sqrt{3}\sin(\lambda\sqrt{3}\,t)(15\sqrt{3}\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2+105\cos(\lambda\sqrt{}$$

$$-18\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4$$

$$+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2$$

$$-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2$$

$$+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2)$$

$$-35\sqrt{3}\sin(\lambda\sqrt{3}\,t)\,\_C3\,\lambda^3t^3+6\cos(\lambda\sqrt{3}\,t)\,\_C1\,\lambda^2t^4$$

$$+\left(105\cos\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda\right.$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$\left.-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)$$

$$-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\_C1\,\lambda\,t^3-\left(140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2\right.\right.$$

$$\left.\left.-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^3\,t^2\right)\Big/\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8\right.$$

$$+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8$$

$$-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$\left.-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)$$

$$-\left(15\cos\left(\lambda\sqrt{3}\,t\right)\left(15\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2\right.\right.$$

$$\left.\left.+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)-315\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^2\,t^3\right)\Big/$$

$$\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7\right.$$

$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$\left.-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3\right.$$

$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$

$$-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)-140\cos\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^2t^2$$

$$+\Big(5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\Big(15\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2+105\cos(\lambda\sqrt{}$$

$$-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$

$$-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$

$$+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)$$

$$+105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda t-5\cos\left(\lambda\sqrt{3}\,t\right)\_C1\,t^2$$

$$-\Big(315\cos\left(\lambda\sqrt{3}\,t\right)\Big(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\lambda^{}$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)$$

$$+\Big(105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\Big(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)$$
$$+15\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\lambda\Big)\Big/\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6$$
$$-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$
$$+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$
$$-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$
$$+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)+105\cos\left(\lambda\sqrt{3}\,t\right)\_C3\Big)$$

$$
\begin{aligned}
G1 := \frac{1}{\lambda t^7}\Bigg(& -\left(\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2 - 3\cos\left(\lambda\sqrt{3}\right)\lambda^3\right.\right. \\
& \left.\left. - 5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right) + 15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^7 t^4\right) \Big/ \left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8\right. \\
& + 54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6 - 18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7 + 27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8 \\
& - 2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4 + 900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5 - 270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6 \\
& + 3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2 - 4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3 + 2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4 \\
& \left. - 1575\sin\left(\lambda\sqrt{3}\right)^2 + 3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda - 4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right) \\
& - \cos\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^6 t^4 + \left(\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\left(15\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^4\right.\right. \\
& \left. - 3\cos\left(\lambda\sqrt{3}\right)\lambda^5 - 140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2 + 105\cos\left(\lambda\sqrt{3}\right)\lambda^3 + 105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\right. \\
& \left.\left. - 315\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^5 t^4\right) \Big/ \left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8 + 54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6\right. \\
& - 18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7 + 27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8 - 2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4 \\
& + 900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5 - 270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6 + 3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2 \\
& - 4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3 + 2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4 - 1575\sin\left(\lambda\sqrt{3}\right)^2 \\
& \left. + 3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda - 4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right) \\
& + 3\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^5 t^3 - \cos\left(\lambda\sqrt{3}\,t\right)\_C1\,\lambda^4 t^4
\end{aligned}
$$

$$-\left(9\cos\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)+15\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^6 t^3\right.$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$\left.-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)$$

$$+\left(13\sqrt{3}\sin\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\right.\right.$$

$$\left.\left.+15\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^5 t^2\right)\Big/\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6\right.$$

$$-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$

$$-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$

$$\left.+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)$$

$$+13\cos\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^4 t^2+\left(5\sqrt{3}\sin\left(\lambda\sqrt{3}\,t\right)\left(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4\right.\right.$$

$$\left.-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\right.$$

$$\left.\left.-315\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^3 t^2\right)\Big/\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6\right.$$

$$-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$

$$-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$

$$+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)$$

$$-10\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^3\,t-5\cos\left(\lambda\sqrt{3}\,t\right)\_C1\,\lambda^2\,t^2$$

$$+\Big(30\cos\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^4$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)$$

$$+5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\_C1\,\lambda\,t-\Big(10\sqrt{3}\,\sin\left(\lambda\sqrt{3}\,t\right)\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2$$

$$-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^3\Big)\Big/\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8$$

$$+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8$$

$$-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)$$

$$+\Big(15\cos\left(\lambda\sqrt{3}\,t\right)\left(15\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2$$

$$+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)-315\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda^2\,t\Big)\Big/$$

$$\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7\right.$$

$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3$$

$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$

$$\left.-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)-10\cos\left(\lambda\sqrt{3}\,t\right)\_C3\,\lambda^2$$

$$-\left(5\sqrt{3}\sin\left(\lambda\sqrt{3}\,t\right)\left(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2+105\cos(\lambda\sqrt{}\right.$$
$$-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$
$$+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$
$$-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$
$$\left.+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)+5\cos\left(\lambda\sqrt{3}\,t\right)\_C1\Big)$$

>

By series expansion of F and G, to avoid the singularity of F(t) and G(t) at the origin t=0, we know that _C1 = _C3 = 0

```
> F2 := t->eval(F(t), sol);
  series(F2(t), t=0, 10) ;
  G2 := t->eval(G(t), sol);
  series(G2(t), t=0, 10)
```

$$F2 := t \mapsto eval(F(t), sol)$$

$$105\,\_C3\,t^{-8}+\left(\frac{35\,\_C3\,\lambda^2}{2}-5\,\_C1\right)t^{-6}+\left(\frac{15}{8}\,\_C3\,\lambda^4-\frac{3}{2}\,\lambda^2\,\_C1\right)t^{-4}$$
$$+\left(\frac{3}{16}\,\_C3\,\lambda^6-\frac{3}{8}\,\lambda^4\,\_C1\right)t^{-2}+\frac{3}{128}\,\_C3\,\lambda^8-\frac{3}{16}\,\lambda^6\,\_C1$$
$$+\frac{9}{35}\left(\lambda^8\left(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2\right.\right.$$
$$\left.\left.+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\sin\left(\lambda\sqrt{3}\right)-315\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\right)\Big/\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8\right.$$
$$+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8$$
$$-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6$$

$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$
$$-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)t+$$
$$O(t^2)$$

$$G2:=t\mapsto eval(G(t),sol)$$

$$\frac{-10\_C3\lambda^2+5\_C1}{\lambda}t^{-7}+\frac{-2\_C3\lambda^4+\frac{5}{2}\lambda^2\_C1}{\lambda}t^{-5}+\frac{-\frac{1}{4}\_C3\lambda^6+\frac{7}{8}\lambda^4\_C1}{\lambda}t^{-3}$$
$$+\frac{9}{16}\lambda^5\_C1\,t^{-1}+\frac{1}{\lambda}\Bigg(\Big(3\lambda^{10}\Big(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3$$
$$-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)+15\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\Big)\Big/\Big(35\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8$$
$$+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8$$
$$-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6$$
$$+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4$$
$$-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)\Big)$$
$$-\Big(6\lambda^8\Big(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2$$
$$+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\sin\left(\lambda\sqrt{3}\right)-315\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\Big)\Big/$$
$$\Big(7\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7$$
$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$
$$-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3$$
$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$
$$-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)\Big)+\frac{-\frac{3}{64}\_C3\lambda^{10}-\frac{33}{128}\lambda^8\_C1}{\lambda}t+\frac{1}{\lambda}\Bigg($$
$$-\Big(3\lambda^{12}\Big(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)$$

$$+15\,\lambda\cos(\lambda\sqrt{3})))\Big/(70\,(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6$$

$$-18\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4$$

$$+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2$$

$$-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2$$

$$+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2))$$

$$+(6\lambda^{10}(15\sqrt{3}\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2+105\cos(\lambda\sqrt{3})\lambda^3+105$$
$$+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5$$
$$-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$
$$+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$
$$-4725\cos(\lambda\sqrt{3})^2\lambda^2)))t^2+O(t^3)$$

Hence we set _C3 =_C1=0 and obtain a simplified form of F1 and G1 (denoted by F1_new, G1_new) as follows:

```
> F1_new := simplify(subs(_C3 = 0, _C1 = 0, F1));
  G1_new := simplify(subs(_C3 = 0, _C1 = 0, G1))
```

$$F1\_new := \left(\lambda\left(\lambda t\left(\left(10\lambda^5t^4-10\lambda^5t^2-35\lambda^3t^4+35\lambda^3+175\lambda t^2-175\lambda\right)\cos(\lambda\sqrt{3})\right.\right.\right.$$
$$+\sqrt{3}\sin(\lambda\sqrt{3})\left(t^4\lambda^6+\left(-15t^4+15t^2\right)\lambda^4+\left(\frac{35}{3}t^4+\frac{175}{3}t^2-70\right)\lambda^2-\frac{175t^2}{3}\right.$$
$$\left.\left.+\frac{175}{3}\right)\right)\cos(\lambda\sqrt{3}\,t)-\left(\lambda\sqrt{3}\left(t^4\lambda^6+\left(15t^4-15t^2\right)\lambda^4+\left(-70t^4+\frac{175}{3}t^2\right.\right.\right.$$
$$\left.\left.+\frac{35}{3}\right)\lambda^2+\frac{175t^2}{3}-\frac{175}{3}\right)\cos(\lambda\sqrt{3})-85\,(t+1)\left(\frac{35}{51}+\lambda^4t^2+\left(-\frac{14t^2}{17}\right.\right.$$
$$\left.\left.\left.-\frac{14}{17}\right)\lambda^2\right)(t-1)\sin(\lambda\sqrt{3})\right)\sin(\lambda\sqrt{3}\,t)\Big)\Big)\Big/\left(t^8\left(\left(-12\lambda^6+\frac{490}{3}\lambda^4\right.\right.\right.$$
$$\left.\left.-\frac{950}{3}\lambda^2+\frac{175}{3}\right)\cos(\lambda\sqrt{3})^2$$
$$-\frac{2\sin(\lambda\sqrt{3})\sqrt{3}\,\lambda\left(\lambda^6-50\lambda^4+250\lambda^2-175\right)\cos(\lambda\sqrt{3})}{3}+\lambda^8+2\lambda^6-\frac{265\lambda^4}{3}$$

$$+\frac{425\,\lambda^2}{3}-\frac{175}{3}\Bigg)\Bigg)$$

$$G1\_new := \Bigg(\Bigg(10\,\lambda\sqrt{3}\left(\frac{35}{2}+\left(t^4-\frac{3}{5}\,t^2\right)\lambda^6+\left(-\frac{7}{2}\,t^4+8\,t^2+\frac{1}{2}\right)\lambda^4+\left(-\frac{35\,t^2}{2}-\frac{15}{2}\right)\lambda^2\right)\cos\left(\lambda\sqrt{3}\right)+3\left(-\frac{175}{3}+\lambda^8\,t^4+\left(-15\,t^4+17\,t^2\right)\lambda^6+\left(\frac{35}{3}\,t^4-85\,t^2-15\right)\lambda^4+\left(\frac{175\,t^2}{3}+\frac{250}{3}\right)\lambda^2\right)\sin\left(\lambda\sqrt{3}\right)\Bigg)\sin\left(\lambda\sqrt{3}\,t\right)+3\Bigg(\left(\lambda^7\,t^2+\left(-5\,t^2-5\right)\lambda^5+75\,\lambda^3-175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)-2\left(-\frac{175}{6}+\lambda^6\,t^2+\left(-\frac{5\,t^2}{6}-\frac{15}{2}\right)\lambda^4+\frac{125\,\lambda^2}{3}\right)\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\Bigg)\lambda\cos\left(\lambda\sqrt{3}\,t\right)t\Bigg)\Big/\Bigg(3\,t^7\Bigg(\left(-12\,\lambda^6+\frac{490}{3}\,\lambda^4-\frac{950}{3}\,\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\,\lambda^4+250\,\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}+\lambda^8+2\,\lambda^6-\frac{265\,\lambda^4}{3}+\frac{425\,\lambda^2}{3}-\frac{175}{3}\Bigg)\Bigg) \tag{7}$$

Double check when F1_new and G1_new is not singular at t=0.

```
> F2_new := t→ simplify(subs(_C3 = 0, _C1 = 0, F1));
  series(F2_new(t), t = 0, 10)
```

$$F2\_new := t \mapsto simplify(subs(\_C3 = 0, \_C1 = 0, F1))$$

$$\Bigg(\lambda\Bigg(\lambda\left(\left(35\,\lambda^3-175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(-70\,\lambda^2+\frac{175}{3}\right)\right)-\left(\lambda\sqrt{3}\left(\frac{35\,\lambda^2}{3}-\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(-\frac{35}{51}+\frac{14\,\lambda^2}{17}\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda\sqrt{3}\Bigg)\Big/\Bigg(\left(-12\,\lambda^6+\frac{490}{3}\,\lambda^4-\frac{950}{3}\,\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\,\lambda^4+250\,\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}+\lambda^8+2\,\lambda^6-\frac{265\,\lambda^4}{3}+\frac{425\,\lambda^2}{3}-\frac{175}{3}\Bigg)\,t^{-7}+\Bigg(\lambda\Bigg(-\frac{3\,\lambda^3\left(\left(35\,\lambda^3-175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(-70\,\lambda^2+\frac{175}{3}\right)\right)}{2}+\lambda\left(\left(\tag{8}$$

$$\left(-10\,\lambda^5+175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(15\,\lambda^4+\frac{175}{3}\,\lambda^2-\frac{175}{3}\right)\Bigg)$$

$$+\frac{\left(\lambda\sqrt{3}\left(\frac{35\,\lambda^2}{3}-\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(-\frac{35}{51}+\frac{14\,\lambda^2}{17}\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^3\sqrt{3}}{2}$$

$$-\Bigg(\lambda\sqrt{3}\left(-15\,\lambda^4+\frac{175}{3}\,\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\frac{35}{51}$$

$$-\lambda^4\right)\sin\left(\lambda\sqrt{3}\right)\Bigg)\lambda\sqrt{3}\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(\left(-12\,\lambda^6+\frac{490}{3}\,\lambda^4-\frac{950}{3}\,\lambda^2\right.$$

$$\left.+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\,\lambda^4+250\,\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}$$

$$+\lambda^8+2\,\lambda^6-\frac{265\,\lambda^4}{3}+\frac{425\,\lambda^2}{3}-\frac{175}{3}\Bigg)t^{-5}$$

$$+\Bigg(\lambda\Bigg(\frac{3\,\lambda^5\left(\left(35\,\lambda^3-175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(-70\,\lambda^2+\frac{175}{3}\right)\right)}{8}$$

$$-\frac{3\,\lambda^3\left(\left(-10\,\lambda^5+175\,\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(15\,\lambda^4+\frac{175}{3}\,\lambda^2-\frac{175}{3}\right)\right)}{2}$$

$$+\lambda\left(\left(10\,\lambda^5-35\,\lambda^3\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(\lambda^6-15\,\lambda^4+\frac{35}{3}\,\lambda^2\right)\right)$$

$$-\frac{1}{40}\Bigg(3\left(\lambda\sqrt{3}\left(\frac{35\,\lambda^2}{3}-\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(-\frac{35}{51}\right.\right.$$

$$\left.\left.+\frac{14\,\lambda^2}{17}\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^5\sqrt{3}\Bigg)$$

$$+\frac{1}{2}\Bigg(\Bigg(\lambda\sqrt{3}\left(-15\,\lambda^4+\frac{175}{3}\,\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\frac{35}{51}\right.$$

$$\left.-\lambda^4\right)\sin\left(\lambda\sqrt{3}\right)\Bigg)\lambda^3\sqrt{3}\Bigg)-\Bigg(\lambda\sqrt{3}\left(\lambda^6+15\,\lambda^4-70\,\lambda^2\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\lambda^4\right.$$

$$\left.-\frac{14}{17}\,\lambda^2\right)\sin\left(\lambda\sqrt{3}\right)\Bigg)\lambda\sqrt{3}\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(\left(-12\,\lambda^6+\frac{490}{3}\,\lambda^4-\frac{950}{3}\,\lambda^2\right.$$

$$\left.+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\,\lambda^4+250\,\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}$$

$$+\lambda^8+2\lambda^6-\frac{265\lambda^4}{3}+\frac{425\lambda^2}{3}-\frac{175}{3}\Bigg)t^{-3}+\Bigg(\lambda\Bigg($$

$$-\frac{3\lambda^7\left(\left(35\lambda^3-175\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(-70\lambda^2+\frac{175}{3}\right)\right)}{80}$$

$$+\frac{3\lambda^5\left(\left(-10\lambda^5+175\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(15\lambda^4+\frac{175}{3}\lambda^2-\frac{175}{3}\right)\right)}{8}$$

$$-\frac{3\lambda^3\left(\left(10\lambda^5-35\lambda^3\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(\lambda^6-15\lambda^4+\frac{35}{3}\lambda^2\right)\right)}{2}$$

$$+\frac{1}{560}\Bigg(3\left(\lambda\sqrt{3}\left(\frac{35\lambda^2}{3}-\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(-\frac{35}{51}\right.\right.$$

$$\left.\left.+\frac{14\lambda^2}{17}\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^7\sqrt{3}\Bigg)$$

$$-\frac{1}{40}\Bigg(3\left(\lambda\sqrt{3}\left(-15\lambda^4+\frac{175}{3}\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\frac{35}{51}\right.\right.$$

$$\left.\left.-\lambda^4\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^5\sqrt{3}\Bigg)$$

$$+\frac{1}{2}\Bigg(\Bigg(\lambda\sqrt{3}\left(\lambda^6+15\lambda^4-70\lambda^2\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\lambda^4\right.$$

$$\left.\left.-\frac{14}{17}\lambda^2\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^3\sqrt{3}\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(-12\lambda^6+\frac{490}{3}\lambda^4-\frac{950}{3}\lambda^2$$

$$+\frac{175}{3}\Bigg)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\lambda^4+250\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}$$

$$+\lambda^8+2\lambda^6-\frac{265\lambda^4}{3}+\frac{425\lambda^2}{3}-\frac{175}{3}\Bigg)t^{-1}$$

$$+\Bigg(\lambda\Bigg(\frac{9\lambda^9\left(\left(35\lambda^3-175\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(-70\lambda^2+\frac{175}{3}\right)\right)}{4480}$$

$$-\frac{3\lambda^7\left(\left(-10\lambda^5+175\lambda\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\left(15\lambda^4+\frac{175}{3}\lambda^2-\frac{175}{3}\right)\right)}{80}$$

$$+\frac{3\lambda^5\left(\left(10\lambda^5-35\lambda^3\right)\cos\left(\lambda\sqrt{3}\right)+\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\left(\lambda^6-15\lambda^4+\frac{35}{3}\lambda^2\right)\right)}{8}$$

$$-\frac{\left(\lambda\sqrt{3}\left(\frac{35\lambda^2}{3}-\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(-\frac{35}{51}+\frac{14\lambda^2}{17}\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^9\sqrt{3}}{4480}$$

$$+\frac{1}{560}\left(3\left(\lambda\sqrt{3}\left(-15\lambda^4+\frac{175}{3}\lambda^2+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\frac{35}{51}\right.\right.\right.$$

$$\left.\left.\left.-\lambda^4\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^7\sqrt{3}\right)$$

$$-\frac{1}{40}\left(3\left(\lambda\sqrt{3}\left(\lambda^6+15\lambda^4-70\lambda^2\right)\cos\left(\lambda\sqrt{3}\right)-85\left(\lambda^4\right.\right.\right.$$

$$\left.\left.\left.-\frac{14}{17}\lambda^2\right)\sin\left(\lambda\sqrt{3}\right)\right)\lambda^5\sqrt{3}\right)\right)\Bigg/\left(\left(-12\lambda^6+\frac{490}{3}\lambda^4-\frac{950}{3}\lambda^2\right.\right.$$

$$\left.\left.+\frac{175}{3}\right)\cos\left(\lambda\sqrt{3}\right)^2-\frac{2\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6-50\lambda^4+250\lambda^2-175\right)\cos\left(\lambda\sqrt{3}\right)}{3}\right.$$

$$\left.+\lambda^8+2\lambda^6-\frac{265\lambda^4}{3}+\frac{425\lambda^2}{3}-\frac{175}{3}\right)t+\mathrm{O}\left(t^3\right)$$

Solve _C1 and _C2 by matching the boundary condition F1_new(1) = 0, G1_new(1) =1.

```
> boundary1 := simplify(subs(t = 1, F1_new));
  boundary2 := simplify(subs(t = 1, G1_new)) - 1
```

$$boundary1 := 0$$

$$boundary2 := 0 \qquad (9)$$

Obtain sol2 = (_C2 , _C4) as follows.

$$Obtain\left[\left[-\left(\left(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2\right.\right.\right. \qquad (10)$$

$$\left.\left.\left.+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\sin\left(\lambda\sqrt{3}\right)-315\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda\right)\Big/$$

$$\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7\right.$$

$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$\left.-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3\right.$$

$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$

$$-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)=-\Big(\Big(15\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$-140\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\sin\left(\lambda\sqrt{3}\right)$$

$$-315\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\lambda\Big)\Big/\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6$$

$$-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$

$$-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$

$$+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big),$$

$$\Big(\Big(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)+15\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\lambda\Big)\Big/$$

$$\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7$$

$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3$$

$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$

$$-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\Big)=\Big(\Big(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3$$
$$-5\sqrt{3}\sin\left(\lambda\sqrt{3}\right)+15\lambda\cos\left(\lambda\sqrt{3}\right)\Big)\lambda\Big)\Big/\Big(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6$$
$$-18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^5-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2$$
$$-4500\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda^3+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2$$
$$+3150\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\,\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2)]]=($$

$$-((15\sqrt{3}\,\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\,\sin(\lambda\sqrt{3})\lambda^2$$

$$+105\cos(\lambda\sqrt{3})\lambda^3+105\sqrt{3}\,\sin(\lambda\sqrt{3})-315\lambda\cos(\lambda\sqrt{3}))\lambda)/$$

$$(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6-18\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7$$

$$+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4+900\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5$$

$$-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2-4500\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$

$$+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$

$$-4725\cos(\lambda\sqrt{3})^2\lambda^2),((6\sqrt{3}\,\sin(\lambda\sqrt{3})\lambda^2-3\cos(\lambda\sqrt{3})\lambda^3$$
$$-5\sqrt{3}\,\sin(\lambda\sqrt{3})+15\lambda\cos(\lambda\sqrt{3}))\lambda)/(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6$$
$$-18\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4$$
$$+900\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2$$
$$-4500\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2$$
$$+3150\sqrt{3}\,\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2))\ \textit{as follows}$$

```
> sol2 := solve({boundary1 = 0, boundary2 = 0}, [_C2, _C4])
Warning, solving for expressions other than names or functions
is not recommended.
```

$$sol2:=[[-((15\sqrt{3}\,\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\,\sin(\lambda\sqrt{3})\lambda^2 \tag{11}$$

$$+105\cos(\lambda\sqrt{3})\lambda^3+105\sqrt{3}\,\sin(\lambda\sqrt{3})-315\lambda\cos(\lambda\sqrt{3}))\lambda)/$$

$$\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7\right.$$

$$+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5$$

$$-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3$$

$$+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda$$

$$\left.-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right)=-\left(\left(15\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^4-3\cos\left(\lambda\sqrt{3}\right)\lambda^5\right.\right.$$

$$-140\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2+105\cos\left(\lambda\sqrt{3}\right)\lambda^3+105\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)$$

$$\left.\left.-315\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda\right)\Big/\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6\right.$$

$$-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4$$

$$+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5-270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6+3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2$$

$$-4500\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3+2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4-1575\sin\left(\lambda\sqrt{3}\right)^2$$

$$\left.+3150\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda-4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right),$$

$$\left(\left(6\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\lambda^2-3\cos\left(\lambda\sqrt{3}\right)\lambda^3-5\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)+15\,\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda\right)\Big/$$

$$\left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8+54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6-18\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7\right.$$

$$\left.+27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8-2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4+900\sqrt{3}\,\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5\right.$$

$$-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$

$$+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$

$$-4725\cos(\lambda\sqrt{3})^2\lambda^2)=((6\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2-3\cos(\lambda\sqrt{3})\lambda^3$$
$$-5\sqrt{3}\sin(\lambda\sqrt{3})+15\lambda\cos(\lambda\sqrt{3}))\lambda)/(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6$$
$$-18\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4$$
$$+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2$$
$$-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2$$
$$+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda-4725\cos(\lambda\sqrt{3})^2\lambda^2)]]$$

>

> $$\_C2:=-((15\sqrt{3}\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2$$
$$+105\cos(\lambda\sqrt{3})\lambda^3+105\sqrt{3}\sin(\lambda\sqrt{3})-315\cos(\lambda\sqrt{3})\lambda)\lambda)/$$
$$(3(9\sin(\lambda\sqrt{3})^2\lambda^8+18\sin(\lambda\sqrt{3})^2\lambda^6-6\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7$$
$$+9\cos(\lambda\sqrt{3})^2\lambda^8-795\sin(\lambda\sqrt{3})^2\lambda^4+300\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5$$
$$-90\cos(\lambda\sqrt{3})^2\lambda^6+1275\sin(\lambda\sqrt{3})^2\lambda^2-1500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$
$$+675\cos(\lambda\sqrt{3})^2\lambda^4-525\sin(\lambda\sqrt{3})^2+1050\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$
$$-1575\cos(\lambda\sqrt{3})^2\lambda^2))$$

$$\_C2:=-((15\sqrt{3}\sin(\lambda\sqrt{3})\lambda^4-3\cos(\lambda\sqrt{3})\lambda^5-140\sqrt{3}\sin(\lambda\sqrt{3})\lambda^2 \tag{12}$$
$$+105\cos(\lambda\sqrt{3})\lambda^3+105\sqrt{3}\sin(\lambda\sqrt{3})-315\lambda\cos(\lambda\sqrt{3}))\lambda)/$$
$$(27\sin(\lambda\sqrt{3})^2\lambda^8+54\sin(\lambda\sqrt{3})^2\lambda^6-18\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^7$$
$$+27\cos(\lambda\sqrt{3})^2\lambda^8-2385\sin(\lambda\sqrt{3})^2\lambda^4+900\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^5$$
$$-270\cos(\lambda\sqrt{3})^2\lambda^6+3825\sin(\lambda\sqrt{3})^2\lambda^2-4500\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda^3$$
$$+2025\cos(\lambda\sqrt{3})^2\lambda^4-1575\sin(\lambda\sqrt{3})^2+3150\sqrt{3}\sin(\lambda\sqrt{3})\cos(\lambda\sqrt{3})\lambda$$
$$-4725\cos(\lambda\sqrt{3})^2\lambda^2)$$

> *simplify*(*_C2*)

$$(3((\lambda^5-35\lambda^3+105\lambda)\cos(\lambda\sqrt{3})-5(\lambda^4-\frac{28}{3}\lambda^2+7)\sin(\lambda\sqrt{3})\sqrt{3})\lambda)/((-324\lambda^6+4410\lambda^4-8550\lambda^2+1575)\cos(\lambda\sqrt{3})^2-18\sin(\lambda\sqrt{3})\sqrt{3}\lambda(\lambda^6-50\lambda^4+250\lambda^2-175)\cos(\lambda\sqrt{3})+27\lambda^8+54\lambda^6-2385\lambda^4+3825\lambda^2-1575) \tag{13}$$

```
> _C4 := ((6*sqrt(3)*sin(λ*sqrt(3))*λ^2 - 3*cos(λ*sqrt(3))*λ^3 - 5*sqrt(3)*sin(λ*sqrt(3))
      + 15*cos(λ*sqrt(3))*λ)*λ)/(3*(9*sin(λ*sqrt(3))^2*λ^8 + 18*sin(λ*sqrt(3))^2*λ^6
      - 6*sqrt(3)*sin(λ*sqrt(3))*cos(λ*sqrt(3))*λ^7 + 9*cos(λ*sqrt(3))^2*λ^8 - 795*sin(λ*sqrt(3))^2*λ^4
      + 300*sqrt(3)*sin(λ*sqrt(3))*cos(λ*sqrt(3))*λ^5 - 90*cos(λ*sqrt(3))^2*λ^6 + 1275*sin(λ*sqrt(3))^2*λ^2
      - 1500*sqrt(3)*sin(λ*sqrt(3))*cos(λ*sqrt(3))*λ^3 + 675*cos(λ*sqrt(3))^2*λ^4 - 525*sin(λ*sqrt(3))^2
      + 1050*sqrt(3)*sin(λ*sqrt(3))*cos(λ*sqrt(3))*λ - 1575*cos(λ*sqrt(3))^2*λ^2))
```

$$\_C4 := \left(\left(6\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\lambda^2 - 3\cos\left(\lambda\sqrt{3}\right)\lambda^3 - 5\sqrt{3}\sin\left(\lambda\sqrt{3}\right) + 15\lambda\cos\left(\lambda\sqrt{3}\right)\right)\lambda\right) \Big/ \left(27\sin\left(\lambda\sqrt{3}\right)^2\lambda^8 + 54\sin\left(\lambda\sqrt{3}\right)^2\lambda^6 - 18\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^7 + 27\cos\left(\lambda\sqrt{3}\right)^2\lambda^8 - 2385\sin\left(\lambda\sqrt{3}\right)^2\lambda^4 + 900\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^5 - 270\cos\left(\lambda\sqrt{3}\right)^2\lambda^6 + 3825\sin\left(\lambda\sqrt{3}\right)^2\lambda^2 - 4500\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda^3 + 2025\cos\left(\lambda\sqrt{3}\right)^2\lambda^4 - 1575\sin\left(\lambda\sqrt{3}\right)^2 + 3150\sqrt{3}\sin\left(\lambda\sqrt{3}\right)\cos\left(\lambda\sqrt{3}\right)\lambda - 4725\cos\left(\lambda\sqrt{3}\right)^2\lambda^2\right) \tag{14}$$

```
> simplify(_C4)
```

$$-\left(3\lambda\left(\left(\lambda^3 - 5\lambda\right)\cos\left(\lambda\sqrt{3}\right) - 2\sin\left(\lambda\sqrt{3}\right)\left(\lambda^2 - \frac{5}{6}\right)\sqrt{3}\right)\right) \Big/ \left(\left(-324\lambda^6 + 4410\lambda^4 - 8550\lambda^2 + 1575\right)\cos\left(\lambda\sqrt{3}\right)^2 - 18\sin\left(\lambda\sqrt{3}\right)\sqrt{3}\,\lambda\left(\lambda^6 - 50\lambda^4 + 250\lambda^2 - 175\right)\cos\left(\lambda\sqrt{3}\right) + 27\lambda^8 + 54\lambda^6 - 2385\lambda^4 + 3825\lambda^2 - 1575\right) \tag{15}$$

```
>
```