> $d := 2$

$$d := 2 \tag{1}$$

> $\text{zeta} := \text{sqrt}\left(\frac{-1 + I\cdot\text{sqrt}(7)}{2}\right)$

$$\zeta := \frac{\sqrt{-2 + 2\,\mathrm{I}\sqrt{7}}}{2} \tag{2}$$

> $\text{alpha} := \mathit{evalf}\left(\frac{1}{2}\cdot \text{zeta}^3 + \text{zeta}\right)$

$$\alpha := -0.1400242167 + 1.180934707\,\mathrm{I} \tag{3}$$

> $\mathit{alias}\left(\mathit{eta\_plus} = d + 3 + \text{sqrt}\left(d^2 - 10\cdot d + 9\right), \mathit{eta\_minus} = d + 3 - \text{sqrt}\left(d^2 - 10\cdot d + 9\right)\right)$

$$\mathit{eta\_plus}, \mathit{eta\_minus} \tag{4}$$

> $\mathit{alias}(\mathit{beta\_plus} = \text{sqrt}(2 * \text{sqrt}(d\hat{}2 - 10 * d + 9) + 2 * d - 6), \mathit{beta\_minus} = \text{sqrt}(-2 * \text{sqrt}(d\hat{}2 - 10 * d + 9) + 2 * d - 6))$

$$\mathit{eta\_plus}, \mathit{eta\_minus}, \mathit{beta\_plus}, \mathit{beta\_minus} \tag{5}$$

>

Verify alpha = -beta_minus*eta_minus*8*2 / (*eta_minus·eta_plus·beta_minus·beta_plus*). zeta = beta_plus/2.

$$\mathit{eta\_minus}\ \mathit{eta\_plus}\ \mathit{beta\_minus}\ \mathit{beta\_plus}$$

$$\frac{\mathit{beta\_plus}}{2} = 0.5000000000\ \mathit{beta\_plus} \tag{6}$$

> $\mathit{evalf}(\mathit{eta\_minus}\cdot\mathit{beta\_minus})$

$$1.584193165 - 13.36075103\,\mathrm{I} \tag{7}$$

> $\mathit{evalf}\left(\text{alpha} + \frac{\mathit{eta\_minus}\cdot\mathit{beta\_minus}\cdot 8\cdot 2}{\mathit{eta\_minus}\cdot\mathit{eta\_plus}\cdot\mathit{beta\_minus}\cdot\mathit{beta\_plus}}\right)$

$$-2.\ 10^{-10} + 0.\ \mathrm{I} \tag{8}$$

>

> $\mathit{evalf}\left(\frac{\mathit{beta\_plus}}{2} - \text{zeta}\right)$

$$0. \tag{9}$$

Define the polynomail approximation for the Bessel function (J_appx) and the error bounds (error_bar).

> $\mathit{J\_appx} := (v, x, n) \rightarrow \mathit{sum}\left(\frac{\left(\frac{x}{2}\right)^v\cdot(-1)^k}{k!\cdot\text{GAMMA}(k + v + 1)}\cdot\left(\frac{x^2}{4}\right)^k, k = 0..n\right)$

$$\mathit{J\_appx} := (v, x, n) \mapsto \sum_{k=0}^{n} \frac{\left(\frac{x}{2}\right)^v\cdot(-1)^k\cdot\left(\frac{x^2}{4}\right)^k}{k!\cdot\Gamma(k + v + 1)} \tag{10}$$

> $error\_bar := (x, n, v) \rightarrow \dfrac{\left|\frac{x}{2}\right|^{2\,n+v}}{\mathrm{factorial}(n)\cdot\mathrm{GAMMA}(n+v+1)\cdot\left(1-\frac{|x|^2}{(2\,n+2)^2}\right)}$

$$error\_bar := (x, n, v) \mapsto \frac{\left|\frac{x}{2}\right|^{2\cdot n+v}}{n!\cdot\Gamma(n+v+1)\cdot\left(1-\frac{|x|^2}{(2\cdot n+2)^2}\right)} \quad \mathbf{(11)}$$

> $lambda0 := 2.5$

$$\lambda 0 := 2.5 \quad \mathbf{(12)}$$

> $n := 6$

$$n := 6 \quad \mathbf{(13)}$$

> $num1 := conjugate(\mathrm{alpha})\cdot\mathrm{BesselJ}(1, \mathrm{lambda}\cdot conjugate(\mathrm{zeta}))$

$$num1 := (-0.1400242167 - 1.180934707\,\mathrm{I})\,\mathrm{BesselJ}\left(1, \frac{\lambda\, beta\_minus}{2}\right) \quad \mathbf{(14)}$$

> $num1\_appx := evalf(simplify(conjugate(\mathrm{alpha})\cdot J\_appx(1, \mathrm{lambda}\cdot conjugate(\mathrm{zeta}), n)))$

$$num1\_appx := (-2.102456606\ 10^{-11} - 1.112515464\ 10^{-11}\,\mathrm{I})\,\lambda\,\big(13.22875656\,\mathrm{I}\,\lambda^{12} - 444.4862202\,\mathrm{I}\,\lambda^{10} + 9.\,\lambda^{12} - 160015.0393\,\mathrm{I}\,\lambda^{8} + 1848.\,\lambda^{10} - 4.267067714\ 10^{6}\,\mathrm{I}\,\lambda^{6} + 20160.\,\lambda^{8} + 2.048192503\ 10^{8}\,\mathrm{I}\,\lambda^{4} - 8.064000\ 10^{6}\,\lambda^{6} + 4.915662007\ 10^{9}\,\mathrm{I}\,\lambda^{2} - 2.32243200\ 10^{8}\,\lambda^{4} + 1.857945600\ 10^{9}\,\lambda^{2} + 2.972712960\ 10^{10}\big) \quad \mathbf{(15)}$$

> $num2 := \mathrm{BesselJ}(0, \mathrm{lambda}\cdot conjugate(\mathrm{zeta}))$

$$num2 := \mathrm{BesselJ}\left(0, \frac{\lambda\, beta\_minus}{2}\right) \quad \mathbf{(16)}$$

> $num2\_appx := evalf(simplify(J\_appx(0, \mathrm{lambda}\cdot conjugate(\mathrm{zeta}), n)))$

$$num2\_appx := 3.115043299\ 10^{-9}\,\mathrm{I}\,\big(\lambda^{10} - 28.80000000\,\lambda^{8} - 8640.\,\lambda^{6} - 184320.\,\lambda^{4} + 6.635520\ 10^{6}\,\lambda^{2} + 1.06168320\ 10^{8}\big)\,\lambda^{2} + 2.119276259\ 10^{-9}\,\lambda^{12} + 3.729926215\ 10^{-7}\,\lambda^{10} + 3.390842014\ 10^{-6}\,\lambda^{8} - 0.001085069444\,\lambda^{6} - 0.02343750000\,\lambda^{4} + 0.1250000000\,\lambda^{2} + 1. \quad \mathbf{(17)}$$

> $num\_appx\_poly := evalf(mtaylor(num1\_appx + num2\_appx, \mathrm{lambda}, 2\cdot n + 2))$

$$num\_appx\_poly := 1. - (0.6250000000 + 0.3307189138\,\mathrm{I})\,\lambda + (0.1250000000 + 0.3307189138\,\mathrm{I})\,\lambda^{2} + (0.01562499998 - 0.1240195927\,\mathrm{I})\,\lambda^{3} + (-0.02343750000 + 0.02066993211\,\mathrm{I})\,\lambda^{4} + (0.007161458333 - 0.001722494344\,\mathrm{I})\,\lambda^{5} - (0.001085069444 + 0.0005741647809\,\mathrm{I})\,\lambda^{6} + (0.0001220703125 + 0.0001794264941\,\mathrm{I})\,\lambda^{7} + (3.390842014\ 10^{-6} - 0.00002691397410\,\mathrm{I})\,\lambda^{8} + (-2.204047309\ 10^{-6} + 3.139963647\ 10^{-6}\,\mathrm{I})\,\lambda^{9} + (3.729926215\ 10^{-7} - 8.971324701\ 10^{-8}\,\mathrm{I})\,\lambda^{10} \quad \mathbf{(18)}$$

$- (4.379837601\ 10^{-8} + 1.121415588\ 10^{-8}\ \mathrm{I})\ \lambda^{11} + (2.119276259\ 10^{-9} + 3.115043299\ 10^{-9}\ \mathrm{I})\ \lambda^{12} - (4.204913212\ 10^{-11} + 3.782552579\ 10^{-10}\ \mathrm{I})\ \lambda^{13}$

Rebase the Taylor approximation of the numerator at lambda = 2.5.

> $num\_appx\_poly := evalf(mtaylor(num1\_appx + num2\_appx, \text{lambda}, 2\cdot n + 2))$

$$num\_appx\_poly := 1. - (0.6250000000 + 0.3307189138\ \mathrm{I})\ \lambda + (0.1250000000 + 0.3307189138\ \mathrm{I})\ \lambda^2 + (0.01562499998 - 0.1240195927\ \mathrm{I})\ \lambda^3 + (-0.02343750000 + 0.02066993211\ \mathrm{I})\ \lambda^4 + (0.007161458333 - 0.001722494344\ \mathrm{I})\ \lambda^5 - (0.001085069444 + 0.0005741647809\ \mathrm{I})\ \lambda^6 + (0.0001220703125 + 0.0001794264941\ \mathrm{I})\ \lambda^7 + (3.390842014\ 10^{-6} - 0.00002691397410\ \mathrm{I})\ \lambda^8 + (-2.204047309\ 10^{-6} + 3.139963647\ 10^{-6}\ \mathrm{I})\ \lambda^9 + (3.729926215\ 10^{-7} - 8.971324701\ 10^{-8}\ \mathrm{I})\ \lambda^{10} - (4.379837601\ 10^{-8} + 1.121415588\ 10^{-8}\ \mathrm{I})\ \lambda^{11} + (2.119276259\ 10^{-9} + 3.115043299\ 10^{-9}\ \mathrm{I})\ \lambda^{12} - (4.204913212\ 10^{-11} + 3.782552579\ 10^{-10}\ \mathrm{I})\ \lambda^{13} \tag{19}$$

> $num\_appx\_poly := taylor(num\_appx\_poly, \text{lambda} = 2.5, 2\cdot n + 2)$

$$num\_appx\_poly := 0.05571991410 - 0.1191501194\ \mathrm{I} + (-0.2038227731 - 0.1691288541\ \mathrm{I})\ (\lambda - 2.5) + (0.08924438989 - 0.1841500928\ \mathrm{I})\ (\lambda - 2.5)^2 + (0.04794898389 - 0.04969664116\ \mathrm{I})\ (\lambda - 2.5)^3 + (0.02505187496 + 0.002940904440\ \mathrm{I})\ (\lambda - 2.5)^4 + (0.004206931529 + 0.002409864719\ \mathrm{I})\ (\lambda - 2.5)^5 + (0.0002694300435 + 0.001041714827\ \mathrm{I})\ (\lambda - 2.5)^6 + (-0.00002495428079 + 0.0001173298795\ \mathrm{I})\ (\lambda - 2.5)^7 + (-0.00001852042859 + 2.28366396\ 10^{-6}\ \mathrm{I})\ (\lambda - 2.5)^8 + (-1.824330501\ 10^{-6} - 2.814323422\ 10^{-6}\ \mathrm{I})\ (\lambda - 2.5)^9 + (-1.451683209\ 10^{-7} - 8.034753567\ 10^{-7}\ \mathrm{I})\ (\lambda - 2.5)^{10} + (-7.1904015\ 10^{-10} - 1.021622951\ 10^{-7}\ \mathrm{I})\ (\lambda - 2.5)^{11} + (7.52679465\ 10^{-10} - 9.178252581\ 10^{-9}\ \mathrm{I})\ (\lambda - 2.5)^{12} + (-4.204913212\ 10^{-11} - 3.782552579\ 10^{-10}\ \mathrm{I})\ (\lambda - 2.5)^{13} \tag{20}$$

>

> $\mathrm{Im}(evalf(subs(\text{lambda} = 2.5, num\_appx\_poly)))$

$$-0.1191501194 \tag{21}$$

> $err1 := evalf\left(error\_bar\left(3\cdot\text{zeta}, n, \frac{d}{2}\right)\right);$
> $error\_poly := err1\cdot|\text{alpha}| + err1;$

$$err1 := 0.0005456706415$$
$$error\_poly := 0.001194586051 \tag{22}$$

> $num\_appx\_poly\_truncated\_remainder := 0.003878485916 \cdot (\lambda - 2.5)^4 + 0.002911755581 \cdot (\lambda - 2.5)^5 + 0.001241126706 \cdot (\lambda - 2.5)^6 + 0.0001794264947 \cdot (\lambda - 2.5)^7 + 0.00001850335726 \cdot (\lambda - 2.5)^8 + 8.97132473\ 10^{-7} \cdot (\lambda - 2.5)^9 + 8.971324704\ 10^{-8} \cdot (\lambda - 2.5)^{10}$

$$num\_appx\_poly\_truncated\_remainder := 0.003878485916\ (\lambda - 2.5)^4 + 0.002911755581\ (\lambda - 2.5)^5 + 0.001241126706\ (\lambda - 2.5)^6 + 0.0001794264947\ (\lambda - 2.5)^7 + 0.00001850335726\ (\lambda - 2.5)^8 + 8.971324730\ 10^{-7}\ (\lambda - 2.5)^9 + 8.971324704\ 10^{-8}\ (\lambda - 2.5)^{10} \quad \textbf{(23)}$$

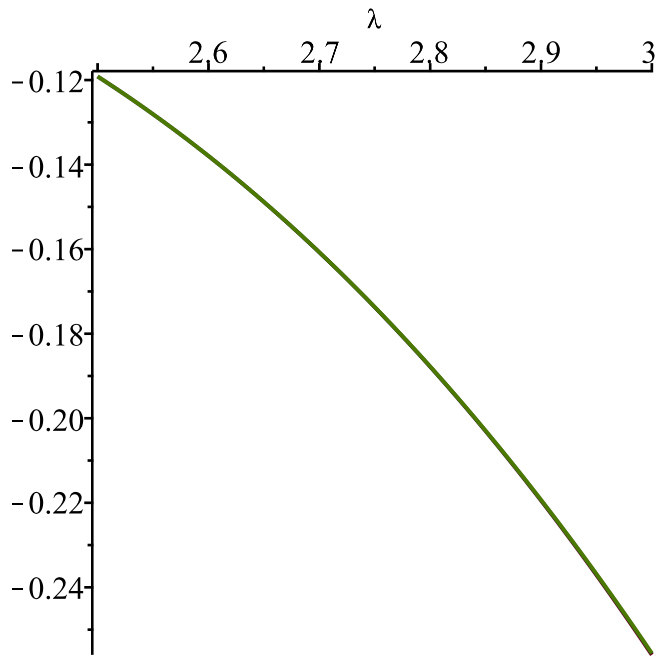
> $error\_num\_appx\_poly\_truncated := evalf(subs(\mathrm{lambda} = 3, num\_appx\_poly\_truncated\_remainder))$

$$error\_num\_appx\_poly\_truncated := 0.0003542662245 \quad \textbf{(24)}$$

> $T3 := mtaylor(num\_appx\_poly, \mathrm{lambda} = 2.5, 4)$

$$T3 := 0.05571991410 - 0.1191501194\,\mathrm{I} - (0.2038227731 + 0.1691288541\,\mathrm{I})\ (\lambda - 2.5) + (0.08924438989 - 0.1841500928\,\mathrm{I})\ (\lambda - 2.5)^2 + (0.04794898389 - 0.04969664116\,\mathrm{I})\ (\lambda - 2.5)^3 \quad \textbf{(25)}$$

> $plot(\{\mathrm{Im}(num1 + num2), \mathrm{Im}(num\_appx\_poly), \mathrm{Im}(T3)\}, \mathrm{lambda} = 2.5 ..3)$



> $evalf(\mathrm{Im}(subs(\mathrm{lambda} = 2.5, T3))) + error\_num\_appx\_poly\_truncated + error\_poly$

$$-0.1176012671 \quad \textbf{(26)}$$

>