1) Assign the value of beta_plus, beta_minus, eta_plus and eta_minus

$$beta\_plus, beta\_minus, eta\_plus \textbf{ and } eta\_minus \quad (1)$$

> alias(beta_plus = sqrt(2 * sqrt(d^2 - 10 * d + 9) + 2 * d - 6), beta_minus = sqrt(- 2 * sqrt(d^2 - 10 * d + 9) + 2 * d - 6))

$$beta\_plus, beta\_minus \quad (2)$$

> alias(eta_plus = d + 3 + sqrt(d^2 - 10·d + 9), eta_minus = d + 3 - sqrt(d^2 - 10·d + 9))

$$beta\_plus, beta\_minus, eta\_plus, eta\_minus \quad (3)$$

2) Check (A1, C1) and (A2, C2) are the basis of the solution to (ode1, ode2).

2.1 Set up the ode system (ode1, ode2) = 0;

> ode1 := simplify(t^2·diff(F(t), t, t) + (d - 1)·t·diff(F(t), t) - (d - 1)·F(t) + 2· lambda·t^2 · diff(G(t), t) + lambda^2· t^2· F(t)) assuming t > 0

$$ode1 := t^2 \left(\frac{d^2}{dt^2} F(t)\right) + (d-1)\, t \left(\frac{d}{dt} F(t)\right) + 2\,\lambda\, t^2 \left(\frac{d}{dt} G(t)\right) - F(t) \left(-\lambda^2 t^2 + d - 1\right) \quad (4)$$

> ode2 := - d·lambda^2· t·G(t) - 2· lambda·t·diff(F(t), t) - 2· lambda·(d - 1)·F(t) - (d - 1)·diff(G(t), t) - t·diff(G(t), t, t)

$$ode2 := -d\,\lambda^2\, t\, G(t) - 2\,\lambda\, t \left(\frac{d}{dt} F(t)\right) - 2\,\lambda\,(d-1)\, F(t) - (d-1)\left(\frac{d}{dt} G(t)\right) - t \left(\frac{d^2}{dt^2} G(t)\right) \quad (5)$$

Check the fundemental solution to ode1 = 0 and ode2=0.

> dsolve({ode1, ode2}, {F(t), G(t)})

$$\left\{ F(t) = t^{-\frac{d}{2}+1} \left(\mathrm{BesselY}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, beta\_minus\, t}{2}\right) \_C4 + \mathrm{BesselY}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, beta\_plus}{2}\right) \_C3 + \mathrm{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, beta\_minus\, t}{2}\right) \_C2 + \mathrm{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \right.\right.\right. \quad (6)$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \_C1\Big),\ G(t) = -\frac{1}{16\, d\, \lambda^{3}}\Big(-12\, t^{-\frac{d}{2}+1}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}+1,$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, \mathit{beta\_minus}\, \_C2\, \lambda^{3} - 12\, t^{-\frac{d}{2}+1}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}+1,$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \mathit{beta\_plus}\, \_C1\, \lambda^{3} - 12\, t^{-\frac{d}{2}+1}\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2}+1,$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \mathit{beta\_plus}\, \_C3\, \lambda^{3} - 12\, t^{-\frac{d}{2}+1}\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2}+1,$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, \mathit{beta\_minus}\, \_C4\, \lambda^{3} + 6\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2},$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \sqrt{d^{2}}\, t^{-\frac{d}{2}}\, \_C3\, \lambda^{2} + 6\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, t^{-\frac{d}{2}}\, \_C3\, d\, \lambda^{2}$$

$$+ \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \sqrt{d^{2}}\, t^{-2-\frac{d}{2}}\, d^{2}\, \_C3 + 6\, \sqrt{d^{2}}\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, t^{-\frac{d}{2}}\, \_C4\, \lambda^{2} + 6\, \_C2\, \sqrt{d^{2}}\, t^{-\frac{d}{2}}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, \lambda^{2}$$

$$+ 6\, \_C1\, \sqrt{d^{2}}\, t^{-\frac{d}{2}}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \lambda^{2} + 6\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, t^{-\frac{d}{2}}\, \_C4\, d\, \lambda^{2} + 6\, \_C2\, t^{-\frac{d}{2}}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, d\, \lambda^{2}$$

$$+ 6\, \_C1\, t^{-\frac{d}{2}}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, d\, \lambda^{2} + \sqrt{d^{2}}\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, t^{-2-\frac{d}{2}}\, d^{2}\, \_C4 + \_C2\, \sqrt{d^{2}}\, t^{-2-\frac{d}{2}}\, d^{2}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big) + \_C1\, \sqrt{d^{2}}\, t^{-2-\frac{d}{2}}\, d^{2}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)$$

$$- t^{-\frac{d}{2}+1}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}+1, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Big)\, \big(2\sqrt{d^{2}-10\, d+9} + 2\, d - 6\big)^{3/2}$$

$$\_C1\, \lambda^{3} - t^{-\frac{d}{2}+1}\, \mathrm{BesselJ}\Big(\frac{\sqrt{d^{2}}}{2}+1, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Big)\, \big(-2\sqrt{d^{2}-10\, d+9}$$

$$+ 2\, d - 6\big)^{3/2}\, \_C2\, \lambda^{3} - t^{-\frac{d}{2}+1}\, \mathrm{BesselY}\Big(\frac{\sqrt{d^{2}}}{2}+1,$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Bigg) \left(2 \sqrt{d^2 - 10\, d + 9} + 2\, d - 6\right)^{3/2} \_C3\, \lambda^3 - t^{-\frac{d}{2} + 1} \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2}$$

$$+ 1, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Bigg) \left(-2 \sqrt{d^2 - 10\, d + 9} + 2\, d - 6\right)^{3/2} \_C4\, \lambda^3$$

$$+ 2 \operatorname{BesselY}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\right) t^{-\frac{d}{2}}\, d^2\, \_C3\, \lambda^2 + 2 \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Bigg) t^{-\frac{d}{2}}\, d^2\, \_C4\, \lambda^2 + 2\, \_C2\, t^{-\frac{d}{2}}\, d^2 \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\right) \lambda^2$$

$$+ 2\, \_C1\, t^{-\frac{d}{2}}\, d^2 \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\right) \lambda^2 - \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Bigg) \left(d^2\right)^{3/2} t^{-2 - \frac{d}{2}}\, \_C3 - \left(d^2\right)^{3/2} \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, \mathit{beta\_minus}\, t}{2}\Bigg) t^{-2 - \frac{d}{2}}\, \_C4 - \_C2 \left(d^2\right)^{3/2} t^{-2 - \frac{d}{2}} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\right)$$

$$- \_C1 \left(d^2\right)^{3/2} t^{-2 - \frac{d}{2}} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\right)$$

$$- 2\, \_C2\, t^{-\frac{d}{2}} \sqrt{d^2 - 10\, d + 9} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\right) d\, \lambda^2$$

$$+ 2\, \_C1\, t^{-\frac{d}{2}} \sqrt{d^2 - 10\, d + 9} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\right) d\, \lambda^2$$

$$+ 2 \sqrt{d^2} \operatorname{BesselY}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\right) t^{-\frac{d}{2}}\, \_C4\, d\, \lambda^2$$

$$+ 2\, \_C2 \sqrt{d^2}\, t^{-\frac{d}{2}} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, \mathit{beta\_minus}\, t}{2}\right) d\, \lambda^2$$

$$+ 2\, \_C1 \sqrt{d^2}\, t^{-\frac{d}{2}} \operatorname{BesselJ}\left(\frac{\sqrt{d^2}}{2}, \frac{\lambda\, t\, \mathit{beta\_plus}}{2}\right) d\, \lambda^2 + 2 \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Bigg) \sqrt{d^2}\, t^{-\frac{d}{2}} \sqrt{d^2 - 10\, d + 9}\, \_C3\, \lambda^2 + 2 \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Bigg) t^{-\frac{d}{2}} \sqrt{d^2 - 10\, d + 9}\, \_C3\, d\, \lambda^2 + 2 \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\frac{\lambda\, t\, \mathit{beta\_plus}}{2}\Bigg) \sqrt{d^2}\, t^{-\frac{d}{2}}\, \_C3\, d\, \lambda^2 - 2 \sqrt{d^2}\, t^{-\frac{d}{2}} \sqrt{d^2 - 10\, d + 9}\, \_C4 \operatorname{BesselY}\Bigg(\frac{\sqrt{d^2}}{2},$$

$$\left.\frac{\lambda\, beta\_minus\, t}{2}\right)\lambda^2 - 2\, \_C2\, \sqrt{d^2}\, t^{-\frac{d}{2}}\, \sqrt{d^2-10\,d+9}\ \mathrm{BesselJ}\left(\frac{\sqrt{d^2}}{2},\right.$$

$$\left.\frac{\lambda\, beta\_minus\, t}{2}\right)\lambda^2 + 2\, \_C1\, \sqrt{d^2}\, t^{-\frac{d}{2}}\, \sqrt{d^2-10\,d+9}\ \mathrm{BesselJ}\left(\frac{\sqrt{d^2}}{2},\right.$$

$$\left.\frac{\lambda\, t\, beta\_plus}{2}\right)\lambda^2 - 2\ \mathrm{BesselY}\left(\frac{\sqrt{d^2}}{2},\right.$$

$$\left.\left.\frac{\lambda\, beta\_minus\, t}{2}\right) t^{-\frac{d}{2}}\, \sqrt{d^2-10\,d+9}\ \_C4\, d\, \lambda^2\right)\right\}$$

2.2 Check (A1, C1) satisfies (ode1, ode2)

$$> A1 := t \rightarrow \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\mathrm{lambda}}{2}\cdot beta\_plus\cdot t\right)\cdot t^{-\frac{d}{2}+1}$$

$$A1 := t \mapsto \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\cdot beta\_plus\cdot t}{2}\right)\cdot t^{-\frac{d}{2}+1} \tag{7}$$

>

$$> C1 := t \rightarrow -\frac{1}{8\,d}\cdot t^{-\frac{d}{2}+1}\cdot beta\_plus\cdot eta\_plus\cdot \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\mathrm{lambda}\cdot t\cdot beta\_plus}{2}\right)$$

$$C1 := t \mapsto -\frac{t^{-\frac{d}{2}+1}\cdot beta\_plus\cdot eta\_plus\cdot \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\cdot beta\_plus\cdot t}{2}\right)}{8\cdot d} \tag{8}$$

$$> ode1\_flag1 := simplify(eval(subs(\{F=A1, G=C1\}, ode1)))$$

$$ode1\_flag1 := 0 \tag{9}$$

$$> ode2\_flag1 := simplify(eval(subs(\{F=A1, G=C1\}, ode2)))$$

$$ode2\_flag1 := 0 \tag{10}$$

2.2 Check (A1, C1) satisfies (ode1, ode2)

$$> A2 := t \rightarrow \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\mathrm{lambda}}{2}\cdot beta\_minus\cdot t\right)\cdot t^{-\frac{d}{2}+1}$$

$$A2 := t \mapsto \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\cdot beta\_minus\cdot t}{2}\right)\cdot t^{-\frac{d}{2}+1} \tag{11}$$

$$> C2 := t \rightarrow -\frac{1}{8\,d}\cdot t^{-\frac{d}{2}+1}\cdot beta\_minus\cdot eta\_minus\cdot \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\mathrm{lambda}\cdot t\cdot beta\_minus}{2}\right)$$

$$C2 := t \mapsto -\frac{t^{-\frac{d}{2}+1}\cdot beta\_minus\cdot eta\_minus\cdot \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\cdot beta\_minus\cdot t}{2}\right)}{8\cdot d} \tag{12}$$

$$> ode1\_flag2 := simplify(eval(subs(\{F=A2, G=C2\}, ode1)))$$

$$ode1\_flag2 := 0 \tag{13}$$

> ode2_flag2 := simplify(eval(subs({F = A2, G = C2}, ode2)))

$$ode2\_flag2 := 0 \tag{14}$$

3. Matching the boundary condition

> deno2 := C2(1)·A1(1) − C1(1)·A2(1)

$$deno2 := -\frac{beta\_minus\ eta\_minus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\ beta\_minus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\ beta\_plus}{2}\right)}{8\ d} + \frac{beta\_plus\ eta\_plus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\ beta\_plus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\ beta\_minus}{2}\right)}{8\ d} \tag{15}$$

>

> deno := Im(eta_minus·beta_minus·BesselJ(d/2 − 1, beta_minus·lambda/2)·BesselJ(d/2, beta_plus·lambda/2))

$$deno := \Im\left(eta\_minus\ beta\_minus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\ beta\_minus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\ beta\_plus}{2}\right)\right) \tag{16}$$

> simplify(deno·lambda/(8·d))

$$\frac{1}{4\ d}\left(\Im\left(\frac{1}{\lambda}\left(\mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\ beta\_plus}{2}\right)\left(-\frac{\mathrm{BesselJ}\left(\frac{d}{2}+1, \frac{\lambda\ beta\_minus}{2}\right)\lambda\ beta\_minus}{2} + d\ \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\ beta\_minus}{2}\right)\right) eta\_minus\right)\right)\lambda\right) \tag{17}$$

> b

$$b \tag{18}$$

> q

$$q \tag{19}$$

> _coef1 := (C2(1)·b − A2(1)·q)/deno2

$$\_coef1 := \left(-\frac{beta\_minus\ eta\_minus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\ beta\_minus}{2}\right) b}{8\ d} - \mathrm{BesselJ}\left(\frac{d}{2}, \right.\right. \tag{20}$$

$$\left.\left.\frac{\lambda\, beta\_minus}{2}\right) q\right) \Bigg/ \left(-\frac{beta\_minus\ eta\_minus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\, beta\_minus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\, beta\_plus}{2}\right)}{8\, d} + \frac{beta\_plus\ eta\_plus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\, beta\_plus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\, beta\_minus}{2}\right)}{8\, d}\right)$$

> $\_coef2 := \frac{(-C1(1)\cdot b + A1(1)\cdot q)}{deno2}$

$$\_coef2 := \left(\frac{beta\_plus\ eta\_plus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\, beta\_plus}{2}\right) b}{8\, d} + \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\, beta\_plus}{2}\right) q\right) \Bigg/ \left(-\frac{beta\_minus\ eta\_minus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\, beta\_minus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\, beta\_plus}{2}\right)}{8\, d} + \frac{beta\_plus\ eta\_plus\ \mathrm{BesselJ}\left(\frac{d}{2}-1, \frac{\lambda\, beta\_plus}{2}\right) \mathrm{BesselJ}\left(\frac{d}{2}, \frac{\lambda\, beta\_minus}{2}\right)}{8\, d}\right) \quad \textbf{(21)}$$

>

> $simplify(\_coef1\cdot A1(1) + \_coef2\cdot A2(1))$

$$b \quad \textbf{(22)}$$

> $simplify(\_coef1\cdot C1(1) + \_coef2\cdot C2(1))$

$$q \quad \textbf{(23)}$$

> $A := t \to \_coef1\cdot A1(t) + \_coef2\cdot A2(t)$

$$A := t \mapsto \_coef1\cdot A1(t) + \_coef2\cdot A2(t) \quad \textbf{(24)}$$

> $C := t \to \_coef1\cdot C1(t) + \_coef2\cdot C2(t)$

$$C := t \mapsto \_coef1\cdot C1(t) + \_coef2\cdot C2(t) \quad \textbf{(25)}$$

> $simplify(A(1))$

$$b \quad \textbf{(26)}$$

> $simplify(C(1))$

$$q \quad \textbf{(27)}$$

> $ode1\_flag3 := simplify(eval(subs(\{F = A, G = C\}, ode1)))$

$$ode1\_flag3 := 0 \quad \textbf{(28)}$$

> $ode1\_flag3 := simplify(ode1\_flag3)$

$$ode1\_flag3 := 0 \quad \textbf{(29)}$$

>