Support Vector Machine

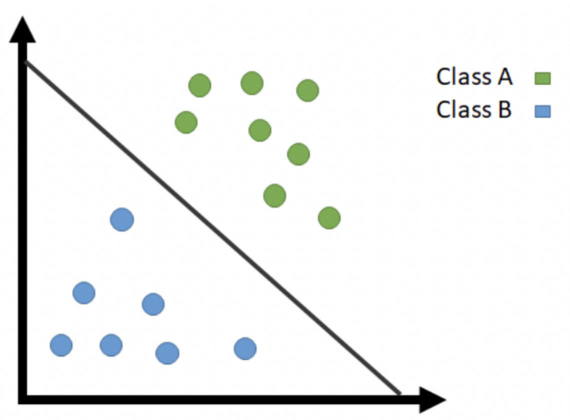
SVM help us find Non-Linear Decision boundaries.
SVM is a Supervised Machine Learning Algorithm which can be used for both Classification and Regression.

📌 **Terminology**
HyperPlane
A decision boundary used to separate and classify Data. It is of (n-1) dimensions where n is number of features.
Support Vectors
Vectors\Points that are closest to HyperPlane in each class
Margin
The distance between Hyperplane and Supporting Vector

> Here the target DataSet is Labelled as -1 and 1 as in Logistic Regression Labelled as 0 and 1

Linear SVM (Optimal Margin Classifier)

📌 Here it’s assumed that the Data Points are Linearly Separable



(Source — Medium.com)Class A and B are divided using Linear Decision Boundary

Functional Margin :

Geometric Margin :

$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$ and $W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}$

Here $g(X)$ represents the Hyperplane,

$g(X) = W^T X + b$

where,
b — Bias/Intercept
n — Number of Features
W — Weight/Regression Coefficients
Y' — Predicated Class

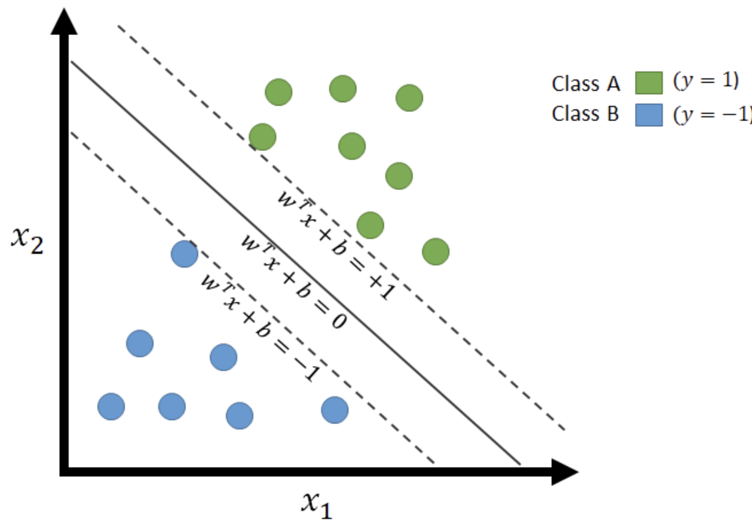
Equation of Hyperplane is $W^T X + b = 0$ and Class A is labelled as 1 ($Y' = 1$) while that of Class B is -1 ($Y' = -1$),

$W^T X + b = 1$ for $Y' = 1$

$W^T X + b = -1$ for $Y' = -1$

On combining the above 2 equations,

$(W^T X + b)Y' \geq -1$



Source: Medium.com

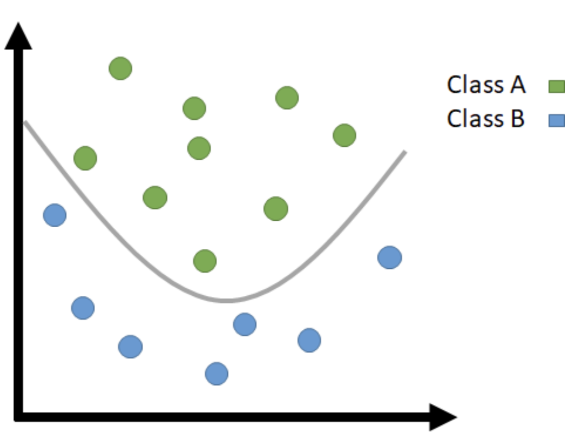
📌 **Calculating the Margin**
In this case (Considering 2 Features),
Distance between HyperPlane and the Supporting Vector is
$\frac{|W^T X + b|}{||W||} = \frac{1}{||W||}$
The Value of Margin is $\frac{2}{||W||}$ (Since 2 Features)

Here our aim is to Maximise the Margin which is to reduce the value of $||W||$

The objective function of Linear SVM,

$\min[\frac{||W||^2}{2}]$

Kernel



(Source: Medium.com)Class A and B are separated using Non-Linear Function

For the Data Sets which can be separated better using Polynomial Function, **Kernel Function** is used. Kernel Function transforms data, most likely to be Linearly Separable.

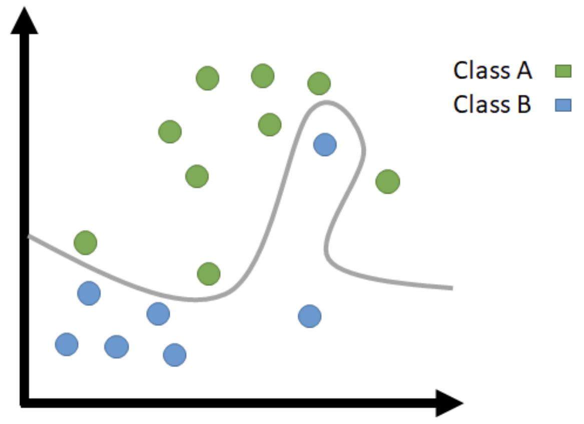
$K(X_a, X_b) = (\gamma(X_a.X_b) + r)^d$

γ – Kernel Coefficient
(X_a, X_b) – Are two different features

📌 **Gaussian Radial Bias Function (RBF)**
This Kernel works when there is no prior knowledge about the data.
$K(X_a, X_b) = e^{-\gamma||X_a - X_b||^2}$

Non-Linear SVM

Linearly Non-separable Data points are handled using **Non-Linear SVM**.



(Source — Medium.com)Class A and B are divided using Non-Linear Decision Boundary

Here Linear SVM is used but it is allowed to make some mistakes, Which is called as Soft-Margin SVM.

$$\min[\frac{||W||^2}{2} + C\sum_{i=1}^{m}\xi_i]$$

where, $(W^T X + b)Y' \geq -1 - \xi \quad (\xi \geq 0)$

ξ — Slack Variable/Penalty which tells the distance between misclassified Data Point and Class Margin.
C — Regularisation Parameter

Questions

1) How can Kernel be used for Linear SVM ?

> Kernel is used for Linearly Separable data to speed up the calculation of SVM Classifier,
> $K(X_a, X_b) = X_a.X_b$

2) How does "C" help in Non-Linear SVM ?

> C is multiplied so that SVM doesn't try hard to separate the data (For lower values of C), which produces generalised model.