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Impact Angle Control Based on Feedback Linearization

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The nonlinear impact angle control problem against stationary targets is investigated. The guidance laws utilized in this paper are derived via feedback linearization where the line of sight rate and the line of sight angle are controlled. First, the line of sight rate is driven to zero to ensure the intercept. Second, an outer loop is integrated for impact angle control. After extracting the guidance commands, the guidance gains are made to vary throughout flight in order to avoid singularity of the guidance command and increase the impact angle zone. The varying gain guidance configuration turns the proposed guidance scheme into a well-known linear optimal guidance law structure. The performance of the proposed guidance algorithm is demonstrated through simulations. In addition, a comparison is held against optimal impact angle guidance laws. The results imply that feedback linearization is a feasible design domain for guidance problems.

Nomenclature

v	=	pursuer velocity
r	=	range
λ	=	line-of-sight-angle
γ	=	path angle
ε	=	look angle
t_{go}	=	time to go
a	=	acceleration
N	=	navigation gain
K_1	=	inner loop guidance gain
K_2	=	outer loop guidance gain
υ	=	pseudo control input
u	=	control input
x	=	state vector
t	=	time

I. Introduction

Intercepting the target may not always be sufficient as the only objective of guidance design; some other requirements could necessitate other objectives such as impact time, impact angle or both. These altervatives provide different oppurtunities. Main advantages of impact angle control are exploiting the weak points of the target, increasing the warhead effectiveness, avoiding directional defense mechanisms, etc.

There are quite a number of studies on impact angle control in the literature which can be categorized into two design domains: optimal control theory and proportional navigation (PN). For example, the "trajectory shaping guidance" law in Ref. 1 solves the linear optimal control problem using the Schwarz inequality. Ref. 2 and Ref. 3

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have similar optimality concerns. Without claiming optimality, the PN guidance law can be performed for impact angle control as expressed in Ref. 4-6. In addition to stationary targets, Ref. 7 and 8 use the PN domain for moving targets. Moreover, impact angle guidance algorithms under physical constraints have also been studied under the optimal control framework and PN based approaches in Ref. 9-10.

Although optimal control theory and proportional navigation are feasible design domains, nonlinear control theory can also be applied to guidance problems. In this paper, feedback linearization (FL) or as sometimes called the nonlinear geometric method is employed to solve the impact angle control problem. Although there exists much research on feedback linearization and nonlinear inverse dynamics on control problems, the guidance designs based on FL is rare. The early study on guidance law design via FL is Ref. 11 to the knowledge of the authors. In the study, FL is used for command to line of sight (LOS) guidance in a way that it transforms a three-dimensional guidance problem into a nonlinear tracking problem. In Ref. 12, FL is employed to direct the missile towards the predicted intercept point. Because the resulting guidance law is singular when the missile velocity vector is on the LOS, the algorithm switches to PN as soon as the difference between the missile heading and the predicted intercept heading angle becomes smaller than a threshold. Ref. 13 applies the FL technique on the relative heading error, which solves the singularity problem in Ref. 12 by changing the controlled parameter. A recent work, Ref. 14 presents relative heading error control for three dimensional engagements with an emphasis on control allocation and acceleration limits through a similar idea in Ref. 13. In addition to capturing the target, FL can also be an effective design domain for other guidance purposes such as impact time as studied in Ref. 15. Although the mentioned FL based guidance algorithms are valuable in their own context, none of these studies address the impact angle control problem directly.

This paper introduces an application of FL for impact angle control. The main aim is to demonstrate that FL can be a powerful tool in guidance design and that it does not require more parameters than the optimal guidance laws. In addition, it has an ease of application and a clear basis of understanding. Another important aspect is that the studied approach in this paper can be applied to moving targets or maneuvering targets as well. In addition, with control structure change, more robust guidance algorithms can also be designed as presented in Ref. 15. But, for the sake of simplicity, stationary target engagement and proportional control is preferred to show the effectiveness of FL for guidance algorithm design and the relation of the method with optimal guidance strategies.

The paper is organized as follows: Section II describes the nonlinear engagement geometry between a pursuer and a stationary target. In Sec. III, a brief information about FL is given and the guidance command is derived for capturing the target while nulling the line of sight rate. Afterwards, the FL method is applied for LOS angle control since this enables impact angle control. The target is assumed to be stationary; thus, it can be deduced that the look angle should vanishes at the instant of impact. This leads to the equality of final values of the path and LOS angles and. Along with the prescribed constant guidance gain approach, in order to increase the achievable impact angle zone, avoiding high look angles and singularity, guidance gains are varied. This approach lead the design with the same structure of the optimal impact angle control guidance laws. In Sec. V, simulation results of various engagement geometries for a ground target are presented. Comparative runs against optimal solutions are used as to be helpful in performance evaluation.

II. Engagement Kinematics

Fig. 1 describes the engagement between a stationary target *T* and its pursuer *P*, where *r* is the range. The speed of the pursuer is *v* and assumed to be constant. The magnitude of its acceleration component perpendicular to the velocity vector is *a* , γ being the path angle. The angle between the velocity vector and the LOS is the look angle ε and λ denotes the LOS angle. The look angle, which is the angle between the velocity vector and the (LOS), approximates the seeker angle when the angle-of-attack is negligible and can be treated as a constraint.

The differential equations governing the engagement are

$$\dot{r} = -v\cos\varepsilon \tag{1a}$$

$$r\dot{\lambda} = -v\sin\varepsilon \tag{1b}$$

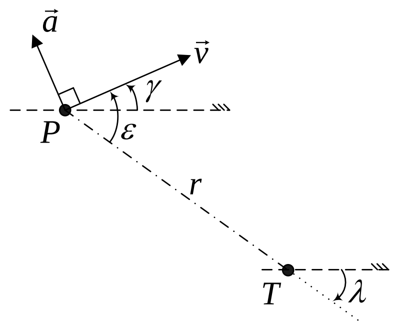


Figure 1. Engagement geometry.

Also, the complementary equation defining the relationship between the angles is

$$\varepsilon = \gamma - \lambda \tag{2}$$

and all angles are positive in the counter-clockwise direction.

III. Impact Angle Control via Feedback Linearization

The guidance strategy presented in this study is based on the application of FL for LOS angle control in the outer loop and LOS rate control in the inner loop basically. Controlling LOS rate alone results in capture. However, in addition to capture, impact angle is demanded as an additional intercept goal. LOS angle control results impact angle control when look angle is zero at the intercept. This, according to (2), leads to the equality of final values of the flight path and LOS angles. Moreover, this assumption is expected to be realized against stationary target engagements from application driven point of view, too. If the look angle is not zero at the final time of flight, then the required acceleration at the final time tends to infinity which is not desired as the physical systems have limited maneuver capacity.

First, a brief information is given for feedback linearization. It has been introduced for nulling the LOS rate. Then, FL is applied for LOS rate and angle control. Afterwards, the guidance commands are extracted from the pseudo controllers for LOS rate and angle control separately. The physical guidance command for LOS angle control requires two gains for the inner and the outer loops. When the guidance command is extracted from pseudo-controller, the singularity in guidance command is observed depending on the look angle which must be avoided. This issue is discussed in Sec. IV.

A. Feedback Linearization

A single input-output nonlinear system is affine if it can be described in the state space form

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \tag{3}$$

where the states are $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$ and $f: D \rightarrow \Re^n$ and $g: D \rightarrow \Re^{n \times p}$ are defined on a domain $D \subset \Re^n$. The functions f, g and h are smooth mappings defined on D and have partial derivatives of any order. It is known that if a nonlinear system is in the form which is shown in the Eq. (3), it can be transformed to a system whose input-output relation is linear within nonlinear control theory. For this purpose, some fundamental theory is given.

The time derivative $\dot{y}$ is given by

$$\dot{y} = \frac{\partial h}{\partial x}\left[f(x) + g(x)u \right] = L_f h(x) + L_g h(x)u \tag{4}$$

where

$$\frac{\partial h}{\partial x} f(x) = L_f h(x), \qquad \frac{\partial h}{\partial x} g(x) = L_g h(x) \tag{5}$$

are called the Lie Derivatives of h with respect to f and g. If $L_g h(x)$ is equal to zero and the first derivative of the output is independent of the input, this process repeats until input appears in r^{th} derivative of y ($y^{(r)}$) explicitly with a non-zero coefficient:

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x) u \tag{6}$$

The following control input shows the system is input-output linearizable with the given input

$$u = \frac{\left(-L_f^r h(x) + \upsilon\right)}{L_g L_f^{r-1} h(x)} \tag{7}$$

where υ is called as pseudo control input. The control input reduces the input output map to $y^{(r)} = \upsilon$ which is a chain of r integrators. The linearizing feedback forces the system to be equivalent to a chain of r integrators where r is called the equivalent relative degree of the system. If the system has a relative degree $r < n$, the linearizing feedback will produce a system which is only partially linear and the stability of the internal dynamics should be observed. The detailed theory can be found in Ref. 16 and Ref. 17.

If FL is applied to the guidance problem, the output should be defined in a sense that it could result in target capture. LOS rate is one of the most important parameters in guidance which can serve for this purpose if it is controlled. The control input holds $a = v\dot{\gamma}$ equality where a is the commanded acceleration to guide the missile as illustrated in Fig.1. Thus, it can be said that Eq. (1) represent the engagement in terms of a set of nonlinear differential equations which is affine in the control input where the output is $\dot{\lambda}$ and the control input is $\dot{\gamma}$. Concluding, the first time derivative of LOS rate includes control input where the system can be represented in the form of Eq. (3). After several mathematical modifications the first time derivative of $\dot{\lambda}$ can be expressed as

$$\ddot{\lambda} = \left(-\frac{v}{r}\cos\varepsilon\right)\dot{\gamma} - \frac{v^2}{r^2}\sin\left(2\varepsilon\right). \tag{8}$$

Here, with the analogy of Eq. (3) the f and g functions become:

$$g(x) = -\frac{v}{r}\cos\varepsilon, \; f(x) = -\frac{v^2}{r^2}\sin\left(2\varepsilon\right) \tag{9}$$

Thus, we can apply nonlinear control theory to find the linearizing feedback as follows

$$\dot{\gamma} = -\frac{r}{v\cos\varepsilon}\left(\upsilon + \frac{v^2}{r^2}\sin\left(2\varepsilon\right)\right) \tag{10}$$

Finally, the resulting feedback linearized input output system is

$$\upsilon = \ddot{\lambda}. \tag{11}$$

Before proceeding further, it needs to be mentioned that look angle should not be equal to $\pm\pi/2$ to avoid singularity of the guidance command which is given in Eq. (10). Fortunately, this requirement is in parallel with the physical limitations of many applications.

If a proportional control strategy is selected with the gain K, the input of the linearized system can be written as

$$\upsilon = K(\dot{\lambda}_d - \dot{\lambda}) \tag{12}$$

where $\dot{\lambda}_d$ depicts the desired LOS rate which is zero according to the parallel navigation rule to capture the target as described in Ref. 18. Assuming an ideal autopilot, the closed loop solution of LOS rate becomes a first order system which is

$$\dot{\lambda}(t) = \dot{\lambda}_0 e^{-Kt} . \tag{13}$$

The time domain information of any parameter is precious in guidance problems. Guidance algorithm design via FL presents time domain solution for the controlled parameter including some of the other guidance parameters. Hence, guidance design via FL can also be considered for designing different guidance strategies which can serve for the other guidance goals. For instance, impact time control can be designed in a switched guidance scheme, as in Ref. 15 while controlling the look angle. In addition to these advantages, it is also possible to have other control strategies instead of proportional controller like proportional integral control which can provide better performance under system lag or have a better response for different type of inputs whilst high gain control is also being avoided.

B. Extracting the Guidance Commands from the Pseudocontrollers

In the following section, first LOS rate control is described and the guidance command is derived for capturing the target. Then, LOS angle control is studied in a cascaded structure where LOS rate control is the inner loop and the LOS angle is the outer loop as illustrated in Fig. 2. In this section, the controller gains are constant to get the generalized physical guidance commands.

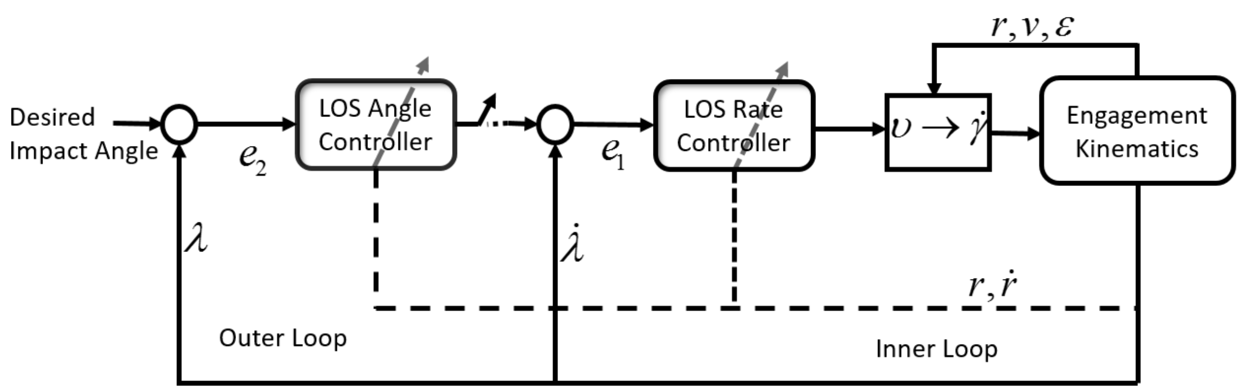


Figure 2. LOS rate and angle control structure.

1) LOS Rate Control Command

In order to extract the guidance command, first the function *f(x)* is divided by *g(x)* where the modified result is

$$\frac{f(x)}{g(x)} = \frac{-\frac{v}{r^2} v \sin(2\varepsilon)}{-\frac{v}{r}\cos\varepsilon} = \frac{2v\sin(\varepsilon)}{r} = -2\dot{\lambda} . \tag{14}$$

After that, if *g(x)* is written in another form using Eq. (1b), it turns out to be

$$g(x) = -\frac{v}{r}\cos\varepsilon = \frac{\dot{r}}{r} . \tag{15}$$

To have the physical guidance command, Eq. (14) and Eq. (15) are replaced in Eq. (10) where the pseudo control input is $k_1(\dot{\lambda}_d - \dot{\lambda})$. Finally, the guidance command becomes

$$\dot{\gamma} = k_1 \frac{r}{\dot{r}}(\dot{\lambda}_d - \dot{\lambda}) + 2\dot{\lambda} \tag{16}$$

Here, k_1 represents the inner loop guidance gain. If LOS rate is demanded to be zero for a successful intercept, then the physical guidance command becomes

$$\dot{\gamma} = \left(2 - k_1 \frac{r}{\dot{r}}\right)\dot{\lambda} . \tag{17}$$

Here, for a successful intercept k_1 must be equal or greater than zero assuming that $|\varepsilon| < pi/2$ and so $\dot{r}$ remains less than zero. If k_1 is zero, the guidance command becomes the well-known PN law with N=2 where N is the PN constant. On the contrary, the controller gain and the initial engagement geometry are factors that will determine the maximum acceleration; which happens to be at the initial time. Hence, k_1 can be chosen with respect to the physical limits systematically during flight. The maximum value of the look angle also depends on the initial condition of the engagement geometry for stationary targets. Although, the look angle time domain solution is not available; it has a tendency of behaving like a first order system thanks to the geometric relations. Thus, if look angle is less than $\pi/2$ at the beginning of the flight, there won't be a singularity in the guidance command for LOS rate control as the look angle will be decreasing function. However, constant guidance gain approach is not feasible since it requires high acceleration at the initial time of flight such as in Ref. 14. This issue will be addressed in the Section IV for impact angle control.

2)LOS Angle Control Command

LOS angle control requires another definition for the desired LOS rate which was zero for interception in the previous subsection. For LOS angle control, the desired LOS rate is extracted from the outer loop for LOS angle control as illustrated in Fig. 2. Thus, the desired LOS rate is defined as

$$\dot{\lambda}_d = k_2\left(\lambda_d - \lambda\right). \tag{18}$$

Here, k_2 represents the outer loop guidance gain and λ_d as the desired LOS angle which is the impact angle. Then the pseudo control input is obtained with the help of Eq. (18) as

$$\upsilon = k_1(k_2\left(\lambda_d - \lambda\right) - \dot{\lambda}) . \tag{19}$$

Finally, replacing Eq. (19) into Eq. (16), the guidance command for impact angle control can be presented as follows

$$\dot{\gamma} = -\frac{r}{\dot{r}} k_1 k_2 \left(\lambda_d - \lambda\right) + \left(2 - \frac{r}{\dot{r}} k_1\right)\dot{\lambda} . \tag{20}$$

For a successful impact angle control, k_1 and k_2 must be greater than zero. In addition, time scale separation must be ensured by the selection of guidance gains for the inner and the outer loops. So far, the extraction of physical guidance commands are described within the constant guidance concept. Depending on the desired impact angle, constant gain approach might not be efficient. The reason behind this problem is that constant gain results high acceleration demand at the initial time of the flight which drives the guidance command to the singular point very quickly. Hence, varying guidance gain is examined in following section to solve this problem and meanwhile to increase the impact angle zone.

IV. Varying Guidance Gains for Impact Angle Control

There are two guidance gains for the impact angle control which provide freedom to the designer. It can be inferred that an infinite number of gain couples lead to the same impact angle. For instance, both of them can be preferred to be constant. It is also possible that both of them can be varying with respect to the flight parameters or one of them might remain constant. Nonetheless, time scale separation should be verified between the inner and the outer loops. In addition, varying guidance gains should accomplish several design purposes; one is avoiding singularity of the guidance command, the other is the application based concerns such as acceleration limit and robustness under noise.

In this study, in order to widen the impact angle envelope and avoid high acceleration demands, both of the guidance gains are varied with respect to $g(x)$, (Eq. (15)). First, the inner loop gain is varied without the outer loop where the only goal is to hit the target with zero LOS rate demand. The inner loop guidance gain is not a constant value but a varying one with respect to $g(x)$ which is defined to be as

$$k_1 = -\dot{r} K_1 / r \tag{22}$$

where now K_1 is constant. Using Eq. (22) in Eq. (17), the guidance command becomes the well known PN command

$$\dot{\gamma} = \dot{\lambda}\left(2 + K_1\right). \tag{23}$$

Afterwards, the outer loop is included for impact angle control and the related guidance gain for the outer loop is also chosen to be varying such as in Eq. 22 which is

$$k_2 = -\dot{r} K_2 / r \tag{24}$$

where K_2 is also constant. When Eq. (24) is inserted into Eq. (20) with Eq. (22), the guidance command to control impact angle turns out to be

$$\dot{\gamma}_c = -\frac{\dot{r}}{r} K_1 K_2 \left(\lambda_d - \lambda\right) + \left(2 + K_1\right)\dot{\lambda}. \tag{25}$$

It is also possible to represent Eq. (25) in another form with the following definition for $g(x)$

$$t_{go} \simeq -\frac{r}{\dot{r}} \tag{26}$$

which is time to go approximation. Then, the guidance command gets the following form

$$\dot{\gamma}_c = \frac{K_1 K_2 \left(\lambda - \lambda_d\right)}{t_{go}} + \left(2 + K_1\right)\dot{\lambda}. \tag{27}$$

In order to impose an impact angle at intercept, K_1 cannot be chosen to be zero as it is also effective in shaping the trajectory for impact angle control. Designer must be aware of that any combination of the gain set will not result capture or impact angle control. It is important to choose the guidance gains and know the effect of the gain selections to the flight parameters. For instance, increasing the inner loop gain while the outer loop gain is constant will cause reaching to the maximum look angle faster. On the other hand the acceleration demand will increase. Increase in the outer gain while the inner loop gain is remaining constant results settling to the desired LOS angle before intercept. These details can be varied regarding to different analyses.

It should finally be noted that Eq. (27) which is obtained via the nonlinear control theory is in the form of a well-known trajectory shaping guidance command. The guidance command of energy optimal trajectory shaping guidance expressed in linear domain using Schwarz inequality in Ref. 1 is:

$$\dot{\gamma}_c = 4\dot{\lambda} + \frac{2(\lambda - \lambda_d)}{t_{go}} \tag{28}$$

If the guidance gains of Eq. (27) are chosen as $K_1 = 2$ and $K_2 = 1$, then Eq. (28) is obtained.

V. Example Scenarios

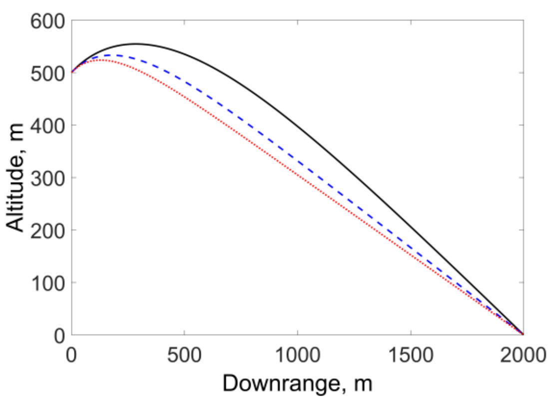
The proposed guidance approach is demonstrated through several scenarios under ideal conditions for stationary targets. Before impact angle control, nonlinear simulations are performed to investigate the basic properties of the proposed method for LOS rate control on various constant gains where the guidace command is given in Eq. (17). Secondly, simulations are carried out for various impact angles with varying guidance gain as given in Eq. (25). The simulations performed in the Simulink environment are ideal; so the impact objective is perfectly achieved in each case. Lastly, a comparison is held between two optimal guidance laws and the one proposed here.

A. LOS Rate Control

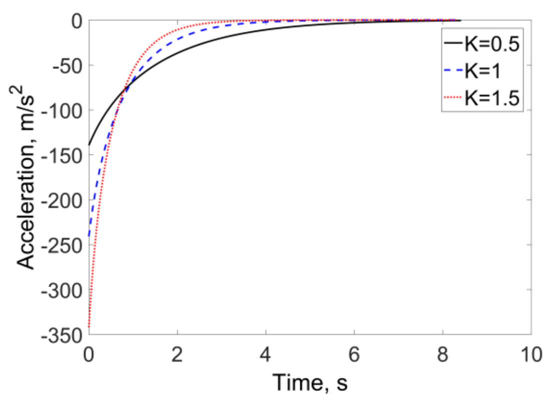
The initial parameters of the scenario are given in Table 1. The constant gain simulations are run in order to show the direct relation of the maximum acceleration with the guidance gain. In this scenario, as explained before, the guidance algorithm drives the engagement into collision course as soon as possible as which is the main characteristic of FL based guidance approaches.

Table 1. Scenario Parameters

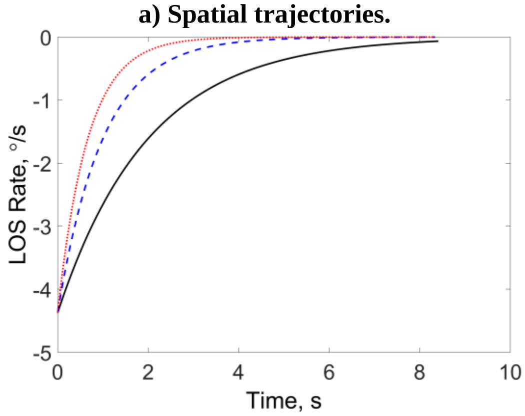
	X_0,m	Z_0,m	γ,°	V_0,m/s
Target	2000	000	-	-
Missile	0000	500	25	250



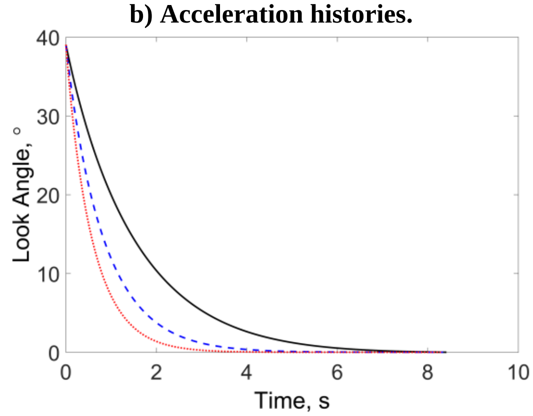
a) Spatial trajectories.



b) Acceleration histories.



c) LOS rate variations.



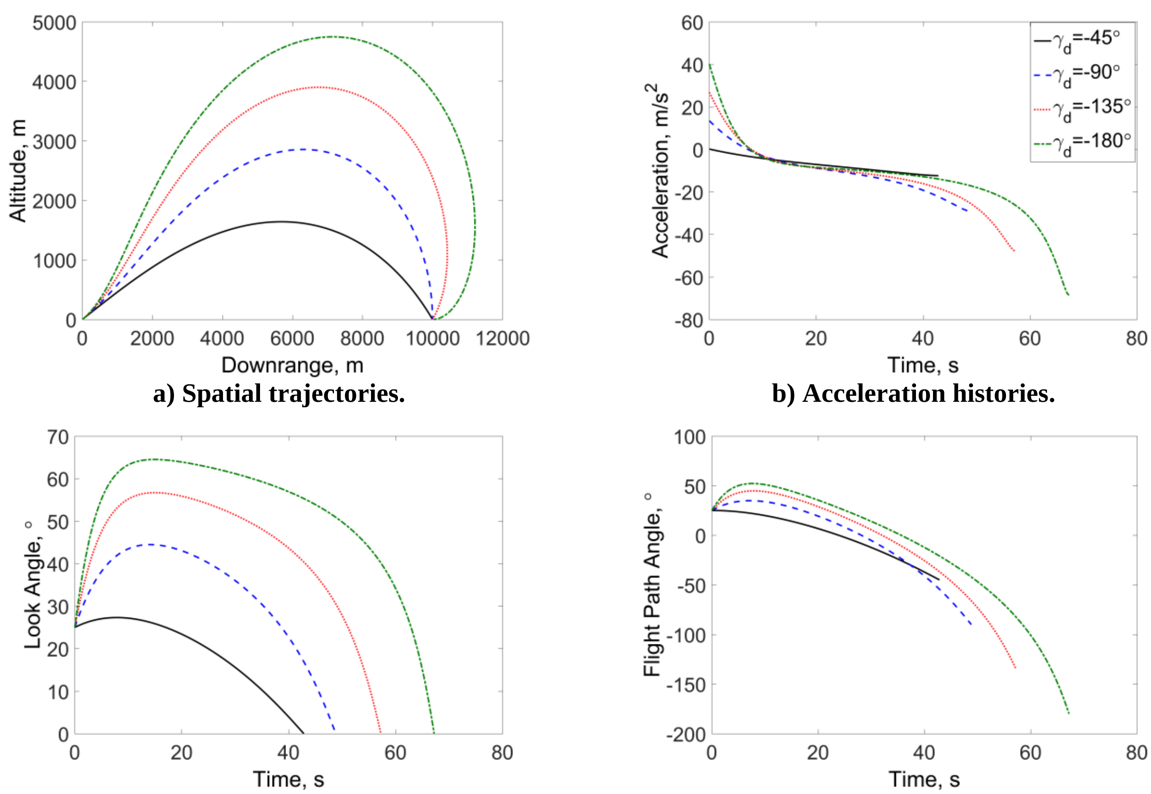
d) Look angle variations.

Figure 3. Results for LOS rate control.

Figure 3a represents the trajectory of the engagement showing that increase in the guidance gain results in lower altitude flight. The corresponding acceleration profiles are illustrated in Fig. 3b. The increase in the guidance gain leads to an increase in the maximum acceleration which is observed at the initial time. Fig. 3c presents the LOS rate which has a similar behavior like Lyapunov based guidance laws as in Ref. 19 and Ref. 20. In Fig. 3d, how the look angles change can be observed. It could be said that the time domain tendencies of the LOS rate and the look angle are similar, unfortunately the domain solution of look angle is not analytically available. Thus, it could only be said that the look angle will decrease from its initial point to zero. It has to be noted that as the first order response of a system reaches to its final value at infinity; however, guidance is a finite time problem. Thus, guidance gain must be chosen carefully to fulfill the performance demands in finite time. Instead of using a proportional controller, a proportional integral control could also be chosen to guarantee that LOS rate becomes zero before intercept. It will also increase the robustness of the guidance under uncertainties and the ignored system lag as demonstrated in Ref. 15. Nevertheless, it is not presented in this paper since it is not the main objective of the study.

B. Impact Angle Control

A ground-to-ground engagement geometry is considered. The pursuer with a constant velocity of 250 m/s attacks a stationary target 10 km away. The initial path angle is 25° and various impact angles are desired. The guidance gains are used as $k_2 = 3/t_{go}$ and $k_1 = 1/t_{go}$. Thus, the singularity in the guidance command is avoided whereas the look angle is less than $\pi/2$. Figs. 4 present the results of the simulations. It is seen in Fig. 4a, as the desired impact angle increases, the pursuer climbs to a higher altitude. Fig. 4b shows that the acceleration demand of the scenarios are also in correlation with the desired impact angles. It requires more acceleration for the increased impact angle demand. Fig. 4c and Fig. 4d present the look angle and flight path angle histories throughout the flight. It is seen that, the look angle is less than $\pi/2$ for the given gain set, thus singularity of the guidance command is avoided.



a) Spatial trajectories. **b) Acceleration histories.**

c) Look angle variations. **d) Path angle variations.**

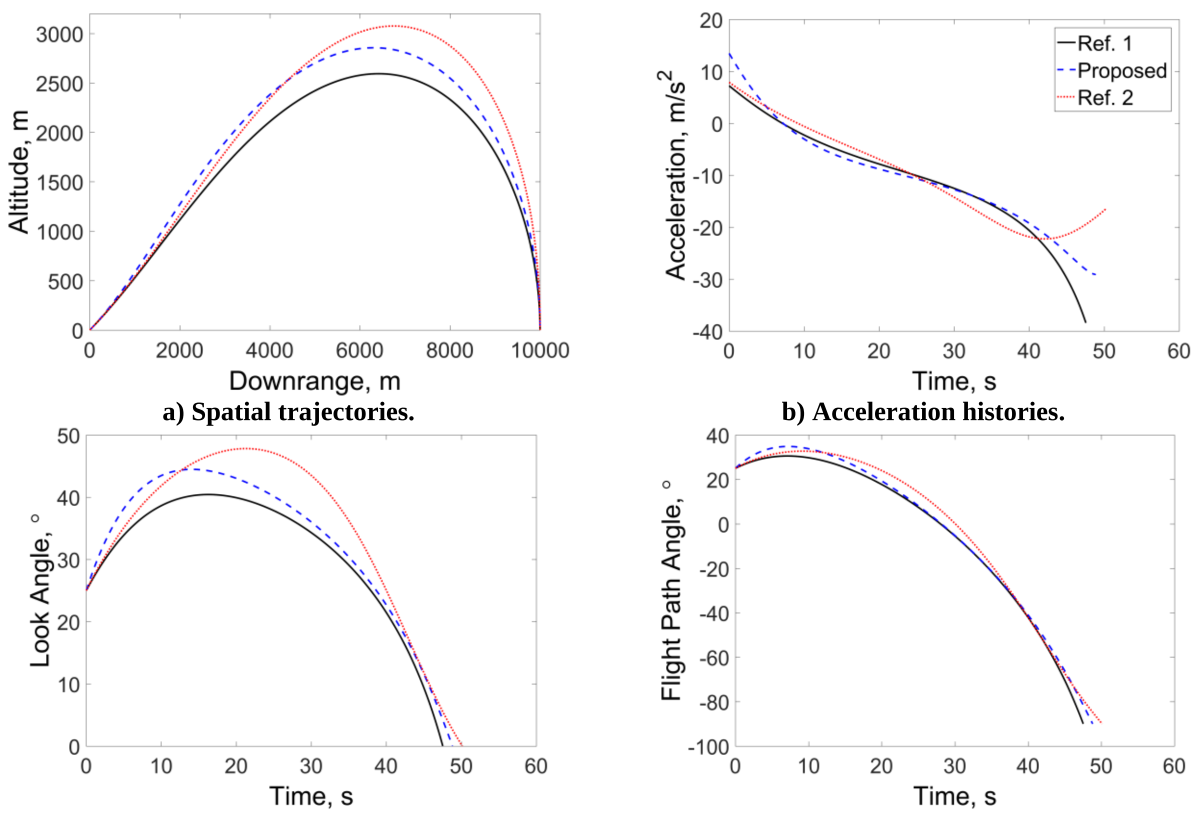
Figure 4. Results for impact angle control.

C. Comparison with Optimal Impact Angle Control Algorithms

The initial engagement geometry and the guidance gains are the same as the previous impact angle control scenarios. A vertical impact angle comparison is performed. Trajectory shaping guidance algorithms presented in

Ref. 1 and Ref. 2 are chosen for the comparison purposes. While Ref. 1 is minimizing the control effort which is the integral of the commanded acceleration squared in the linear domain, Ref. 2 minimizes the miss distance and the same control effort as in Ref. 1. They also require time to go for implementation; thus, time to go is approximated with the function given in Eq. (26).

The visual summary of the simulations is presented in Fig. 5. It is seen in Fig. 5a that the trajectory belonging to Ref. 1 has the lowest maximum altitude. The price paid for this lower trajectory is apparent in Fig. 5b, where it is seen that Ref. 1 requires more acceleration at the end of the simulation although it has the minimum acceleration at the beginning of the flight. Ref. 2 has a different tendency of acceleration history with respect to the other two algorithms. The proposed guidance method has the maximum acceleration at the beginning of the flight. In Fig. 5c, it is seen that Ref. 2 requires the maximum look angle and Ref. 1 has the lowest look angle requirement in correlation with its altitude history. The proposed method with the chosen gain set is in between of the Ref. 1 and Ref. 2 results. In comparison to the other flight results, flight path angle histories have the least variance that is examined in Fig. 5d. In addition to the simulation results, the cost which is the integral of the commanded acceleration squared is compared. The results are in correlation with the order in Fig. 5 which are [10050 9623 9269] s^{-1} , respectively. It shows that Ref. 2 has the minimum cost, while the proposed guidance algorithm with the selected gains performs better than Ref. 1. It is apparent that even though, both of the reference algorithms are energy optimal in linear domain; in nonlinear environment their performances differ.



a) Spatial trajectories. **b) Acceleration histories.**

c) Look angle variations. **d) Path angle variations.**

Figure 5. Comparison of the optimal and proposed impact angle guidance algorithms.

VI. Conclusion

The guidance algorithm introduced in this paper is motivated by the fact that feedback linearization provides a valuable design framework for guidance requirements and provides flexibility for different guidance goals. Hence, it has been applied to nonlinear two-dimensional guidance problem against stationary targets. The controlled parameters are either LOS rate for only capturing the target or LOS angle control for both capturing the target with a specified impact angle where LOS angle is the impact angle under the defined conditions. For LOS rate control, one guidance gain and for impact angle control two guidance gains are needed to be selected. In addition, the guidance command involves singularity which must be avoided. For this purpose and additionally to expand the impact angle envelope, the guidance gains are formed to be varying. After manipulating the guidance command, the varying gain guidance structure is ended up with a linear optimal impact angle control law under the appropriate gain selection. The performance of the proposed method is illustrated through some simulations and they verify that the methods work as intended.

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