

Simulate Asset Price Evolutions and Reprice Risky up-and-out Call Option

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Abstract

This report presents the results of the simulation of a European up-and-out call option over twelve months, but the difference between this submission and submission 1 is the implementation of a non-constant interest rate and local volatility. We use LIBOR forward rate model to simulate interest rates. We also use discount factor to value the option without default risk and then the value of the option with counterparty default risk.

1 Introduction

In this paper we go beyond the constant risk-free continuously compounded rate. There are several models to implement stochastic interest rates. The short rate models which describe instantaneous continuously-compounded interest rate at time t, r_t include the Vasicek model introduced in [1], Mamon in [2] advances this model by presenting approaches in obtaining the closed-form solution using the Vasicek model. Under the Vasicek model the short rate dynamics is given by,

$$dr_t = \alpha(b - r_t)dt + \sigma dW_t$$

Where W_t is Brownian motion, b is the level to which the short rate will tend in the long run, α is the rate at which the short rate will tend towards b and σ is the volatility of the short rate. The stochastic differential equation can be solved to give r_t

$$r_t = e^{-\alpha t} [r_0 + b(e^{\alpha t} - 1) + \int_0^t \sigma e^{\alpha s} dW_s]$$

Cox, Ingersoll and Ross also presented a stochastic differential equation for short term rates in [4] given by

$$dr_t = \alpha(b - r_t)dt + \sigma\sqrt{r_t}dW_t$$

This model prevents the short from becoming 0 but offers no closed form solution

The Hull-White model described in [3] also presents stochastic differential equation for the short term rate,

$$dr_t = (\theta(t) - \alpha(t)r_t)dt + \sigma(t)dW_t$$

where $\theta(t) = \alpha b$ which is a non constant term, allowing the mean reversion level to vary.

The model we implement in this assignment is the LIBOR forward rate model to simulate interest rates. The initial values for the LIBOR forward rates need to be calibrated to the market forward rates which can be deduced through the market zero-coupon bond prices. This continuously compounded interest rate is given by,

$$e^{r_{ti}(t_{i+1}-t_i)} = 1 + L(t_i, t_{i+1})(t_{i+1} - t_i)$$

Most of code implemented in this submission is derived from Module 6 [8] and Module 7 [9] of the course.

We initialize most variables as given by the question.

- Option maturity is one year
- The option is struck at-the-money
- The current share price is \$100
- The up-and-out barrier for the option is \$150
- The risk-free continuously compounded interest rate is 8%
- The volatility for the underlying share is 30%
- The volatility for counterparty's firm value is 25%
- The counterparty's debt, due in one year, is \$175
- The current firm value for the counterparty is \$200
- The correlation between the counterparty and the stock is constant at 0.2
- The recovery rate with the counterparty is 25%

- 2 LIBOR forward rates, stock paths, and counterparty firm values.
- 3 Discount factor and Value of the Option
- 4 Conclusion

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