Team13 Ex01

Q1: Why would a SLAM system need a map?

An: the map is often required to support other tasks; the map allows limiting the error committed in estimating the state of the robot

Q2: How can we apply SLAM technology into real-world application

An: SLAM can be applied mostly in emergence of indoor applications and provide an alternative to user-built maps, showing that robot operation is possible in the absence of an ad hoc localization infrastructure.
First we should have sensors , which collect data for extraction of relevant features(For instance, in vision-based slam, the front-end extracts the pixel location of few distinguishable points in the environment). The data would be used for association module in front-end part of SLAM. The front-end abstracts sensor data into models that are amenable for estimation. And then the back-end usually feeds back information to the front-end, e.g., to support loop closure detection and validation.
Additionally should we also pay attention to robustness of system and curse of computational load caused by extended period of time over larger areas.

Q3: Describe History of SLAM

An:The SLAM problem was given by Durrant-Whyte and Bailey, when we call this age as "Classical age" (1986-2004). At that time could we see the main probabilistic formulations for SLAM (such as EKF, Particle Filter, maximum likelihood Estimation) . And then the subsequent period is called as Algorithmic-Analysis-Age(2004-2015), in this age the sparsity towards efficient SLAM is looked very crucial. And many SLAM open-source libraries are developed. Now we are entering a third era for SLAM, the robust-perception age(2015-now).

Proving the following equation holds,

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\hat{\underline{w}})^n = \underline{\underline{I}} + \frac{1-\cos(\theta)}{\theta^2} \hat{\underline{w}} + \frac{\theta - \sin\theta}{\theta^3} (\hat{\underline{w}})^2 = J$$

J is the left Jacobian Matrix of $SO(3)$

proof:

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\hat{\underline{w}})^n = \frac{1}{(n+1)!} (\hat{v} t)^n, \quad \text{setting } t = |w| = \theta$$

Taylor expansion $\Rightarrow I + \frac{1}{2!} \hat{v} t + \frac{1}{3!} t^2 \hat{v}^2 - \frac{1}{4!} t^3 \hat{v} - \frac{t^4}{5!} \hat{v}^2 \quad v = \frac{w}{|w|} = \frac{w}{\theta}$.

due to: $\hat{v}^2 = vv^T - I$.

$\hat{v}^3 = -\hat{v}$

$$\Rightarrow I + \left(\frac{t}{2!} - \frac{t^3}{4!} + \frac{t^5}{6!}\right)\hat{v} + \left(\frac{t^2}{3!} - \frac{t^4}{5!} + \frac{t^6}{7!}\right)\hat{v}^2$$

$$\Rightarrow I + \frac{1}{t}\left(\frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} \cdots\right)\hat{v} + \frac{1}{t}\left(\frac{t^3}{3!} - \frac{t^5}{5!} \cdots + \frac{t^7}{7!}\right)$$

and $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \cdots$

$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \cdots$

$\therefore$ proto $= I + \frac{1-\cos\theta}{\theta^2} \hat{w} + \frac{\theta - \sin(\theta)}{\theta^3} (\hat{w})^2 = J$, equation holds.

Subsequently: prove the equation $J = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\hat{w})^n$

here we set $a = \frac{w}{|w|}$ $|w| = \theta$

from definition $\xi = \begin{bmatrix} v \\ w \end{bmatrix}$ $\hat{\xi} = \begin{bmatrix} \hat{w} & v \\ 0 & 0 \end{bmatrix}$ and $SE(3) = \exp(\hat{\xi})$
$= \sum_{n=0}^{\infty} \frac{1}{n!} (\hat{\xi})^n.$

$$\hat{\xi} = \begin{bmatrix} \hat{w} & v \\ 0 & 0 \end{bmatrix} = \sum_{n=0}^{\infty} \frac{1}{n!} (\hat{\xi})^n = I + \sum_{n=1}^{\infty} \frac{1}{n!} \begin{bmatrix} \hat{w} & v \\ 0 & 0 \end{bmatrix}^n$$

due to $\begin{bmatrix} \theta\hat{a} & v \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta\hat{a} & v \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (\theta\hat{a})^2 & \theta\hat{a} v \\ 0 & 0 \end{bmatrix}$

$$\text{proto} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} + \sum_{n=1}^{\infty} \frac{1}{n!} \begin{bmatrix} (\theta\hat{a})^n & (\theta\hat{a})^{n-1} v \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{n=0}^{\infty} \frac{1}{n!} (\theta\hat{a})^n & \sum_{n=1}^{\infty} (\hat{a}\theta)^{n-1} v \frac{1}{n!} \\ 0 & 1 \end{bmatrix}$$

from $SE(3) = \begin{bmatrix} \exp(\hat{w}) & J \cdot v \\ 0 & 1 \end{bmatrix}$

$$\therefore J = \sum_{n=1}^{\infty} (\hat{a}\theta)^{n-1} \frac{1}{n!} = \sum_{n=0}^{\infty} (\hat{a}\theta)^n \cdot \frac{1}{(n+1)!}$$

so the above equation holds.