Homework 6 - Numerical Integration

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Problem 6.1

6.1.1

This integral is improper because its domain is unbounded/infinite.

6.1.2

This integral is proper.

6.1.3

This integral is proper.

6.1.4

This integral is proper.

6.1.5

This integral is improper because the function is not defined at $x = 1$.

6.1.6

This integral is improper because the function is not defined at $x \geq 1$ and because the upper bound is infinite.

6.1.7

This integral is improper because the function is discontinuous at $x = 0$.

6.1.8

This integral is improper because it evaluates to $\frac{0}{0}$ at $x = 1$.

6.1.9

This integral is improper because it is not defined at the lower bound $x = 0$.

6.1.10

This integral is proper.

6.2

6.2.1

This integral is proper when $\alpha \neq 1, -1$.

6.2.2

This integral is proper when $\beta \neq 1$.

6.3

n	midpoint	trapezoid	Simpson
2	1	1	4/3
4	1	1	0.8333
6	1	1	0.8519
8	1	1	0.9167
10	1	1	0.9067
12	1	1	0.9444
14	1	1	0.9320
16	1	1	0.9583
18	1	1	0.9465
20	1	1	0.9667

n	Simpson - 1.0	$\frac{4}{3n^2}$
2	1/3	1/3
6		1/27
10		1/75
14		1/147
18		1/243

6.4

n	midpoint	trapezoid	Simpson
2	1.4613	1.6253	1.5290
4	1.5001	1.5433	1.560
6	1.5079	1.5273	1.5147
8	1.5107	1.5217	1.5145
10	1.5120	1.5190	1.5144
12	1.5127	1.5176	1.5144
14	1.5131	1.5167	1.5144
16	1.5134	1.5162	1.5143
18	1.5136	1.5158	1.5143
20	1.5138	1.5155	1.5143
22	1.5139	1.5153	1.5143
24	1.5139	1.5152	1.5143
26	1.5140	1.5150	1.5143
28	1.5140	1.5149	1.5143
30	1.5140	1.5149	1.5143
40	1.5142	1.5146	1.5143
50	1.5142	1.5145	1.5143
100	1.5143	1.5144	1.5143
200	1.5143	1.5144	1.5143
400	1.5143	1.5143	1.5143

The value of the integrand converges to 1.5143.

6.5

γ	$I_{Simpson}$
0.1	1.3360
0.2	1.3442
0.3	1.3584
0.4	1.3800
0.5	1.4100
0.6	1.4510
0.7	1.5094
0.8	1.5962
0.9	1.7451

6.6

n	J_{mid}	J_{trap}	J_{Simp}
1	-8.6191	0	0
2	-0.346615	-4.30955	-5.74607
4	4.20584	-2.32808	-1.66759
8	2.01429	0.938878	2.02786
16	0.27037	1.47658	1.65582
32	-1.72424	0.873477	0.672442
64	-0.646546	-0.425383	-0.858336
128	-0.535964	-0.535964	-0.52825
256	-0.535964	-0.535964	-0.535964