Final - Part IV

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August 13, 2018

Problem 3

We are given the following system of equations:

$$\begin{aligned} (.2324)x_1 + x_2 + 2x_3 &= -1 \\ (.4314)x_1 + 3x_2 + 4x_3 &= r_2 \\ \gamma x_1 + 5x_2 + 6x + 3 &= -3 \end{aligned}$$

For the values $\gamma_0 = 5$ and $r_2 = -1.8564$, this system of equations is consistent but not linearly independent.

Problem 4

We are asked to perform LU-decomposition on the given matrix A with $S = (1, 2, 3, 4)$, an array of swap indices.

$$A = \begin{bmatrix} 1 & -2 & 1 & -2 \\ 1 & -2 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

1. Compute the scaled values to determine the initial pivot.

$$\frac{|a_{11}|}{\hat{a}_1} = 1, \qquad \frac{|a_{21}|}{\hat{a}_2} = 1, \qquad \frac{|a_{31}|}{\hat{a}_3} = 1, \qquad \frac{|a_{41}|}{\hat{a}_4} = 1.$$

   Since all of the pivots are equal, no swaps are necessary. However, by inspection, we can see that x_1 has already been isolated in r_4, so we swap r_1 and r_4 so that:

$$S_1 = (4, 2, 3, 1)$$

2. We use Gaussian elimination to solve for the remaining variables. The multiplier entries for the matrix L are bolded.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & -2 & 1 & -2 \end{bmatrix}$$

Eliminate x_1 from the other rows subtracting r_1 through.

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{-1} & -2 & 1 & 0 \\ \mathbf{-1} & -2 & 0 & 0 \\ \mathbf{-1} & -2 & 1 & -2 \end{bmatrix}$$

Swap r_2 and r_3 such that $S = (4, 3, 2, 1)$.

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{1} & -2 & 0 & 0 \\ \mathbf{1} & -2 & 1 & 0 \\ \mathbf{1} & -2 & 1 & -2 \end{bmatrix}$$

Subtract r_2 from r_3 and r_4.

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{1} & -2 & 0 & 0 \\ \mathbf{1} & \mathbf{1} & 1 & 0 \\ \mathbf{1} & \mathbf{1} & 1 & -2 \end{bmatrix}$$

Subtract r_3 from r_4.

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{1} & -2 & 0 & 0 \\ \mathbf{1} & \mathbf{1} & 1 & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & -2 \end{bmatrix}$$

The final array of swap indices $S = (4, 3, 2, 1)$.

$det(A) = 4$.

To find the matrix X which is the solution of the following equation:

$$AX = B \qquad \text{where} \qquad B = \frac{1}{2}(A + A^T)$$

We find the inverse of A and multiply each side by A^{-1}.

$$\begin{aligned} A^{-1}(AX) &= A^{-1}B \\ IX &= A^{-1}B \end{aligned}$$

$$X = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ -0.75 & 0.25 & 0 & 0 \\ -1.5 & -1.5 & -0.5 & 0 \\ -0.75 & -0.75 & -0.75 & 0.25 \end{bmatrix}$$

Problem 5

We are given the following tridiagonal matrix:

$$T = \begin{bmatrix} 3+\mu^2 & 1+\mu & 0 & 0 & 0 \\ 1+\mu & 3+\mu^2 & 1+\mu & 0 & 0 \\ 0 & 1+\mu & 3+\mu^2 & 1+\mu & 0 \\ 0 & 0 & 1+\mu & 3+\mu^2 & 1+\mu \\ 0 & 0 & 0 & 1+\mu & 3+\mu^2 \end{bmatrix}$$

To find the values of μ such that T is diagonally dominant, we find:

$$\begin{aligned} |3+\mu^2| =& |2+2u| \\ |3+\mu^2-(2+2u)| >& 0 \\ |\mu^2-2u+1| =& 0 \\ |(\mu-1)(\mu-1)| =& 0 \\ |\mu| &= 1 \\ \mu &= \pm 1 \end{aligned}$$

When $\mu = 1$, the matrix is weakly diagonally dominant.

For all other values $\in \mathbb{R}$, the matrix is strongly diagonally dominant.

There are no values for μ which would produce a non-diagonally dominant matrix.

The solution to the system

$$T_{\mu=0}X = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \quad \text{is} \quad X = \begin{bmatrix} 0.2083 \\ 0.3750 \\ 0.6667 \\ 0.6250 \\ 0.8333 \end{bmatrix}$$