Homework 1

Michelle Chung

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Problem 1.3

Let today's time be $t_0 = 0$.
Consider a newly issued bond with a maturity of two years.
Suppose the bond pays semiannual coupons (two coupons per year).
Let the face be F and the annualized coupon rates be $c_1 \dots c_4$ and the yield be y.
The formula relating the bond price, B and yield is described by:

$$B = \frac{\frac{1}{2}c_1}{1+\frac{1}{2}y} + \frac{\frac{1}{2}c_2}{(1+\frac{1}{2}y)^2} + \frac{\frac{1}{2}c_3}{(1+\frac{1}{2}y)^3} + \frac{\frac{1}{2}c_4 + F}{(1+\frac{1}{2}y)^4}$$

1.3.1

Let $F = 100$ and $c_1 \dots c_4 = 4$. Then, for the following values of y, $B(y)$ would be:

y	B(y)
0	102.50
2	98.05
4	94.29
6	90.71
8	87.29

Let the market price of the bond be $B_{market} = 100.5$.
The lower and upper bound for the true yield is $(0, 2)$, so that

$$y_{low} = 0, \quad y_{high} = 2, \quad y_{mid} = 1, \quad B(y_{mid}) = 100.00$$

For the next iteration,

$$y_{low} = 0, \quad y_{high} = 1, \quad y_{mid} = .5, \quad B(y_{mid}) = 100.99$$

1.3.2

Let $F = 100$ and $c_1 = 1, c_2 = 3, c_3 = 5, c_4 = 7$. Then, for the following values of y, $B(y)$ would be:

y	B(y)
1	105.90
3	98.97
5	43.37
7	32.01
9	24.16

Let the market price of the bond be $B_{market} = 100$.
The lower and upper bound for the true yield is $(0, 2)$, so that

$$y_{low} = 1, \quad y_{high} = 3, \quad y_{mid} = 2, \quad B(y_{mid}) = 103.85$$

For the next iteration,

$$y_{low} = 2, \quad y_{high} = 3, \quad y_{mid} = 1.5, \quad B(y_{mid}) = 102.85$$

Problem 1.4

Consider only bonds with semiannual coupons so that $c = 2$. The bonds all have face $F = 100$. Let us have three newly issued par bonds, with maturities of 0.5, 1.0, 1.5 years. We are given the following values for the yields and asked to find the discount factors and interest rates of each bond:

$$y_{0.5} = 4.0, \quad y_{1.0} = 4.2, \quad y_{1.5} = 4.1$$

The first bond, $B_{0.5}$, dispenses only one coupon at maturity ($n = 1$). Its value can be calculated by the following formula:

$$\begin{aligned} B_{0.5} &= \frac{\frac{1}{2}c_1 + F}{1 + \frac{1}{2}y} \\ &= \frac{1 + 100}{1 + .02} \\ &= 99.0196 \end{aligned} \tag{1}$$

The first bond, $B_{1.0}$, dispenses two coupons ($n = 2$). Its value can be calculated by the following formula:

$$\begin{aligned} B_{1.0} &= \frac{\frac{1}{2}c_1}{1 + \frac{1}{2}y} + \frac{\frac{1}{2}c_2 + F}{(1 + \frac{1}{2}y)^2} \\ &= \frac{1}{1 + .021} + \frac{1 + 100}{(1 + .021)^2} \\ &= 99.9020 \end{aligned} \tag{2}$$

The third bond, $B_{1.5}$, dispenses three coupons ($n = 3$). Its value can be calculated by the following formula:

$$\begin{aligned} B_{1.0} &= \frac{\frac{1}{2}c_1}{1 + \frac{1}{2}y} + \frac{\frac{1}{2}c_2 + F}{(1 + \frac{1}{2}y)^2} + \frac{\frac{1}{2}c_3 + F}{(1 + \frac{1}{2}y)^3} \\ &= \frac{1}{1 + .0205} + \frac{1}{(1 + .0205)^2} + \frac{1 + 100}{(1 + .0205)^3} \\ &= 96.9749 \end{aligned} \tag{3}$$

Therefore, the discount values of each bond, given by the formula $df = \frac{PV}{FV}$, are:

$$d_{0.5} = .9910, \quad d_{1.0} = .9990, \quad d_{1.5} = .9697$$

And the continuously compounded interest rates, given by the formula $r = -\frac{ln(d)}{t - t_0}$, are:

$$r_{0.5} = .0180, \quad r_{1.0} = .0010, \quad r_{1.5} = .0205$$