Homework 4 - Forwards and Futures

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Problem 4.1

Suppose the price of a stock is $S_0 = 100.5$ at time $t_0 = 0$. The stock pays no dividends. Suppose also the interest rate is $r = 5.5\%$. Recall $F = S_0 e^{r(T-t_0)}$.

The fair value of the forward price at the time $T = 0.75$ years is ≈ 104.7323.

Suppose the price of a stock is $S_0 = 95.5$ at time $t_0 = 0$ and it pays a continuous dividend yield$q = 1.1\%$. Suppose also the interest rate is $r = 5.1\%$. Recall $F = S_0 e^{(r-q)(T-t_0)}$.

The fair value of the forward price at the time $T = 0.65$ years is ≈ 98.0156.

Problem 4.2

The price of a stock is $S_0 = 100.0$ at time $t_0 = 0$. The stock pays no dividends. The interest rate is $r = 5.0\%$. The expiration time of the forward contract is $T = 1.0$ years.

The future value of the stock, F, is ≈ 105.1271.

For each case below, formulate an arbitrage strategy and state how much profit your strategy yields at time T.

Case 1

The forward price is $F_1 = 105.0$ so that $F > F_1$.

1. We enter into a forward contract to sell the stock at the forward price F.
2. We borrow money, B_0 to buy one share of stock today at the price S_0.
3. At time T, we sell the stock at price F and repay our loan, B_0, which has compounded to $B(T) = B_0 e^{r(T-T_0)}$. Since $F > B(T)$, we end up with a profit of $F - B(T) = F - e^{r(T-t_0)} > 0$.

Case 2

The forward price is $F_2 = 106.0$ so that $F < F_2$.

1. We enter into a forward contract to buy the stock at the forward price F.
2. We sell one share of stock today, worth S_0, and put the money, B_0 in a bank where it accumulates interest.
3. At a future time T, the contract expires and we buy the stock at price F. Meanwhile, the money in the bank has compounded to $B(T) = B_0 e^{r(T-T_0)}$.
4. Since $B(T)$ is greater than F, we have enough money to buy back the stock we sold at t_0, as well as additional money in the form of interest.
5. Therefore, our net profit is $B(T) - F = e^{r(T-t_0)} - F > 0$.

Problem 4.3

Suppose the stock price today is $S_0 = 100.0$. We have a futures contract which will expire in 5 days. The futures price today is $F_0 = 105.5$. Today is $t_0 = 0$ and the expiration date is $t_5 = 5$ (measured in days, not years). On the expiration day, $F_5 = S_5$.

t_i	S_i	F_i
1	S_1	F_1
2	S_2	F_2
3	S_3	F_3
4	S_4	F_4
5	S_5	F_5

Every day the futures contract is marked to market. This means that if the futures price on day i is F_i, (for $i = 1, 2, 3, 4, 5$), the following happens:

1. If $F_i > F_i - 1$, the investor receives a cash amount $F_i - F_i - 1$ in the mark to market account.
2. If $F_i < F_i - 1$, the investor pays a cash amount $F_i - 1 - Fi$ into the mark to market account.
3. If $F_i = F_i - 1$, nothing happens.

Calculate the money paid or received in the mark to market account for the prices listed below.

t_i	S_i	F_i	paid	received
1	99.5	103.3	2.2	
2	101.3	104.1		.8
3	101.3	102.1	2	
4	100.2	101.3	1	
5	99.3	99.3	2	

On the final day, the investor pays 105.7.

Calculate the money paid or received in the mark to market account for the prices listed below.

t_i	S_i	F_i	paid	received
1	100.9	106.3		.8
2	103.8	108.7		2.4
3	106.1	109.2		.5
4	107.5	108.3	.9	
5	108.3	108.3		

On the final day, the investor pays 107.1.

The random walk of the futures prices affects the total amount paid by the investor, but the random walk of the stock prices does not.

Consider the case in which the investor does not hold the futures contract to expiration, but instead sells it.

t_i	S_i	F_i	paid	received
1	99.5	106.3		.8
2	101.3	105.1	1.2	
3	101.3	105.8		.7
4	100.1	104.2	1.6	
5	102.3	102.3	1.9	

If she sells it on day 2, the total loss is .8. If she sells it on day 3, the total loss is .1.

t_i	S_i	F_i	paid	received
1	99.7	106.3		.8
2	101.2	105.1	1.2	
3	101.5	105.8		.7
4	102.8	104.2	1.6	
5	102.3	102.3	1.9	

If she sells it on day 2, the total loss is .8. If she sells it on day 3, the total loss is .1. The random walk of the stock prices do not affect the net profit/loss of the investor.