Module: Derivation of Algorithms
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Year: 4

FORMAL SPECIFICATION

1.

|[CON N: int {N > 0}

Var

f: array [0..N) of int;

{∀j: 0 ≤ j < N: f.j = ¬ ∨ f.j = ∧ ∨ f.j = ∨}

K: int;

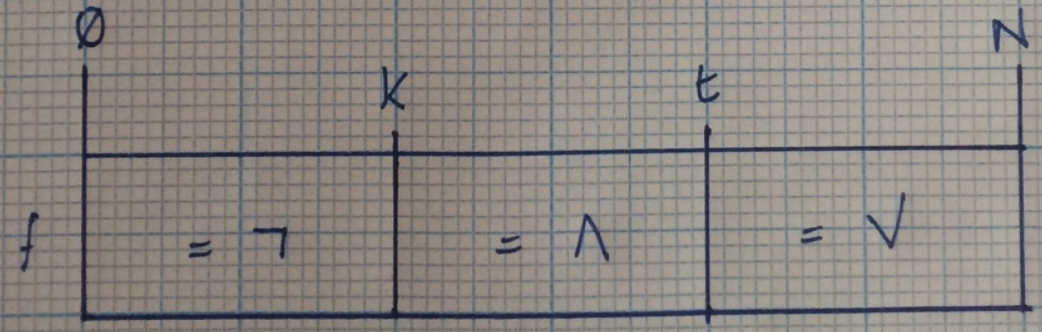
t: int;

S

{∃k,t: 0 ≤ K ≤ t ≤ N: ∀j: 0 ≤ j < k: f.j = ¬ ∧
∀j: k ≤ j < t: f.j = ∧ ∧
∀j: t ≤ j < N: f.j = ∨}

]|

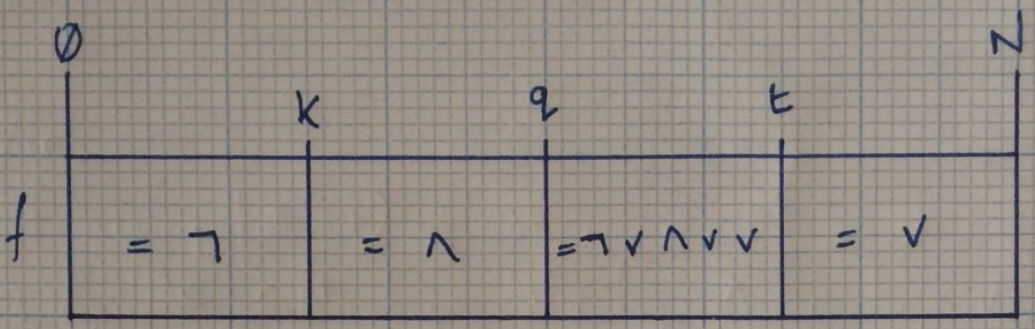
2. POST CONDITION DIAGRAM



{∃K,t: 0 ≤ K ≤ t ≤ N: ∀j: 0 ≤ j < K: f.j = ¬ ∧ ∀j: K ≤ j < t: f.j = ∧

SNAPSHOT DURING PROCESSING

3.

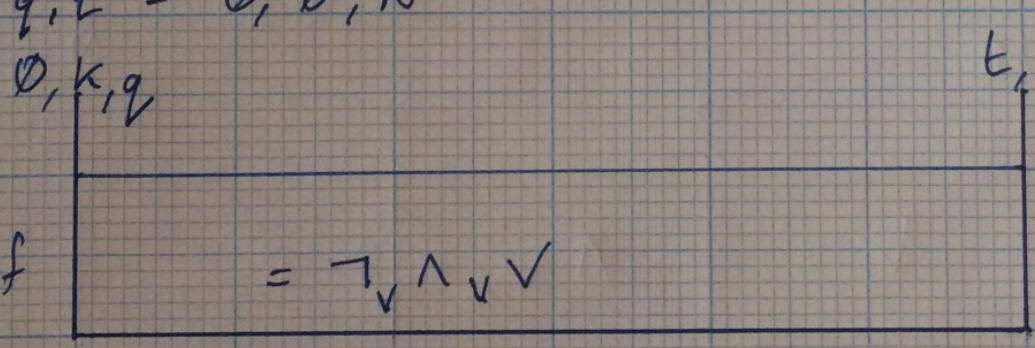


$\{\exists k,t,q : \emptyset \le k \le q \le t \le N : \forall j : \emptyset \le j < k : f.j = \neg \wedge \forall j : k \le j < q : f.j = \wedge \wedge \forall j : q \le j < t : f.j = \neg \vee f.j = \wedge \vee f.j = \vee \wedge \forall j : t \le j < N : f.j = \vee \}$

4. VALUES OF VARIABLES AT BEGINNING

BECAUSE AT THE BEGINNING WE CONSIDER THE ENTIRE ARRAY TO BE UNSORTED, THE SEGMENT $q \rightarrow t$ MUST BE CONSIDERED FROM $\emptyset \rightarrow N$ ∴

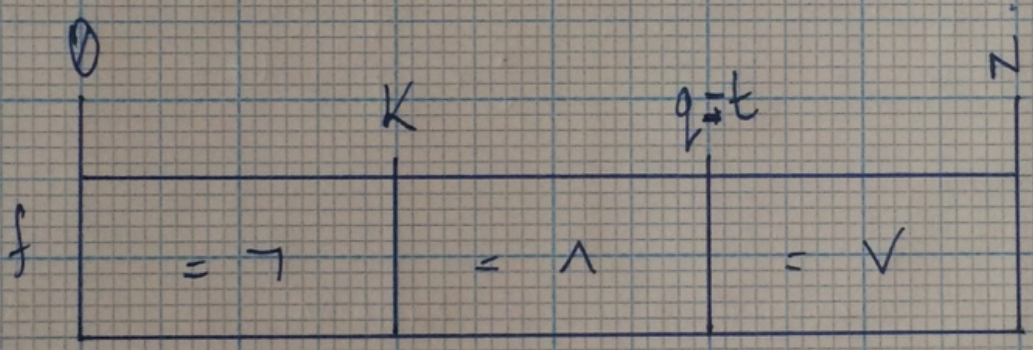
$K, q, t = \emptyset, \emptyset, N$



5. VALUES OF VARIABLES AT END

WE DO NOT KNOW HOW MANY CHARACTERS ARE IN THE ARRAY SO WE DONT KNOW THE FINAL VALUES FOR K, q, t. WHAT WE DO KNOW IS THAT WHEN THE PROCESS STOPS, THE ARRAY MUST BE SORTED. THIS MEA THE NUMBER OF CHARACTERS IN SEGMENT $q \to t = 0$

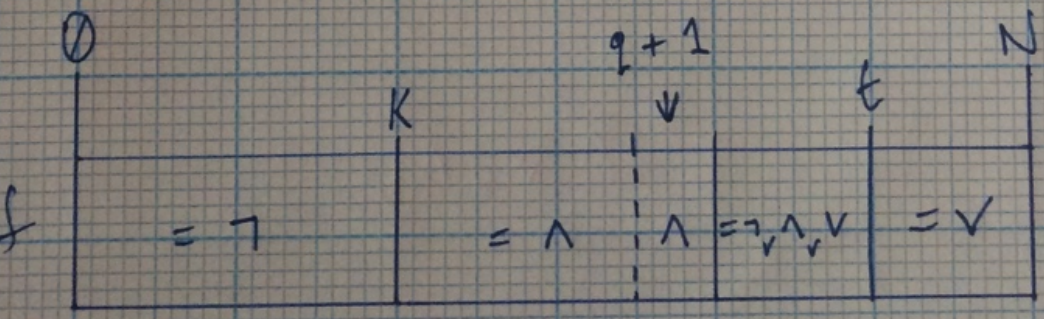
$\therefore \quad q = t.$



6. ASSIGNMENTS OF VARIABLES DURING THE MIDDLE

$f.q = \wedge$ = INCREASE q by 1

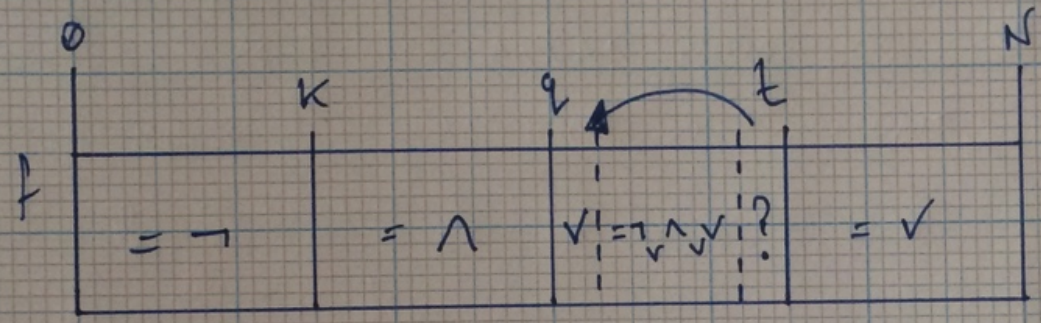
$q := q + 1$



ii)

f.q = V = SWAP f.q WITH f.t-1 AND t := t-1

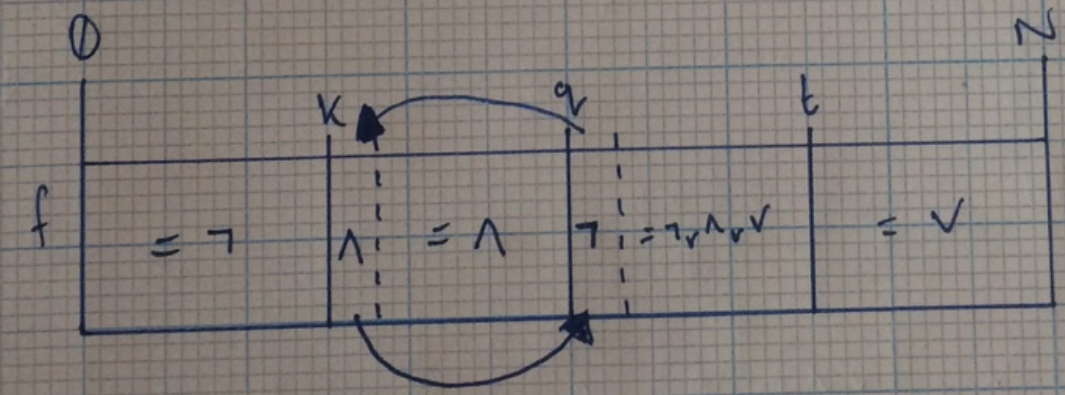
f.q , f.t-1 := f.t-1 , f.q ;
t := t-1 ;



iii)

f.q = ¬ SWAP f.q WITH f.k AND k := k+1 , q := q+1

f.q := f.k
f.k := f.q
q,k := q+1 , k+1



7 TERMINATION PROOF

INITIALISATION

$(t - q > 0)\,(q, t := 0, N)$

$\equiv$ {SUBSTITUTION}

$N - 0 > 0$

$=$ {ARITHMETIC}

$N > 0$

$\Leftarrow$ {PROBLEM DECLARES THAT $N > 0$}

{$\Leftarrow$ GIVEN $N > 0$}

TRUE

$q := q + 1$

$(t - q)\,(q := q + 1)$

$\equiv$ {SUBSTITUTION}

$t - (q + 1)$

$=$ {ARITHMETIC}

$t - q - 1$

$<$

$t - q$

TRUE

$t := t - 1$

$(t - q)\,(t := t - 1)$

$\equiv$ {SUBSTITUTION}

$(t - 1) - q$

$=$ {ARITHMETIC}

$t - q - 1$

$<$

$t - q$

TRUE

// { $q := q + 1$ HAPPEN
TWICE, PROOF SHOU
ONCE

// { $K := K + 1$ HAPPENS
ONCE BUT HAS N
BEARING ON THE
GUARD

8 COMPLETE SOLUTION

|[CON N : int {N > 0}

VAR
f : array [0..N) of int ;
{∀j : 0 ≤ j < N : f.j = ¬ ∨ f.j = ∧ ∨ f.j = ∨}
q : int ;
K : int ;
t : int ;
q, K, t := 0, 0, N ;

do q < t →
if f.q = ¬ →
f.q, f.k = f.k, f.q ;
q, k = q+1, k+1 ;
[] f.q = ∧ →
q := q+1 ;
[] f.q = ∨ →
f.q, f.t-1 := f.t-1, f.q ;
t := t-1 ;
fi
od

{∃K, t : 0 ≤ k ≤ t ≤ N : ∀j : 0 ≤ j < K : f.j = ¬ ∧
∀j : K ≤ j < t : f.j = ∧ ∧
∀j : t ≤ j < N : f.j = ∨}