

Module: Derivation of Algorithms

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Year: 4

$$3+4+5+3+2+5+5+4+8+6 = 45/50$$

90/100

J.g. work.

FORMAL SPECIFICATION

1.

$\boxed{[\text{CON } N: \text{int } \{N > 0\}]}$

Var

$f: \text{array } [0..N] \text{ of } \text{int};$ char \times

$\{ \forall j: 0 \leq j < N: f.j = \neg \vee f.j = \wedge \vee f.j = \vee \}$

characters

$k: \text{int};$

$t: \text{int};$

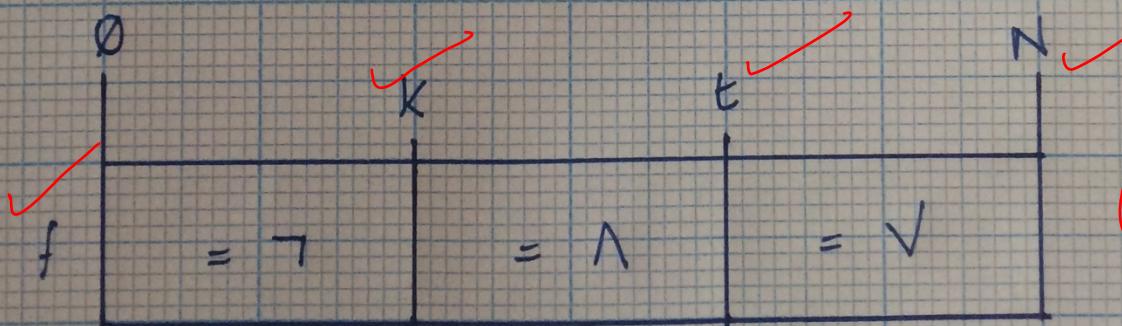
s

3/5

$\{ \exists k, t: 0 \leq k \leq t \leq N: \forall j: 0 \leq j < k: f.j = \neg \wedge$
 $\vee j: k \leq j < t: f.j = \wedge \wedge$
 $\forall j: t \leq j < N: f.j = \vee \}$

]

2. POST CONDITION DIAGRAM

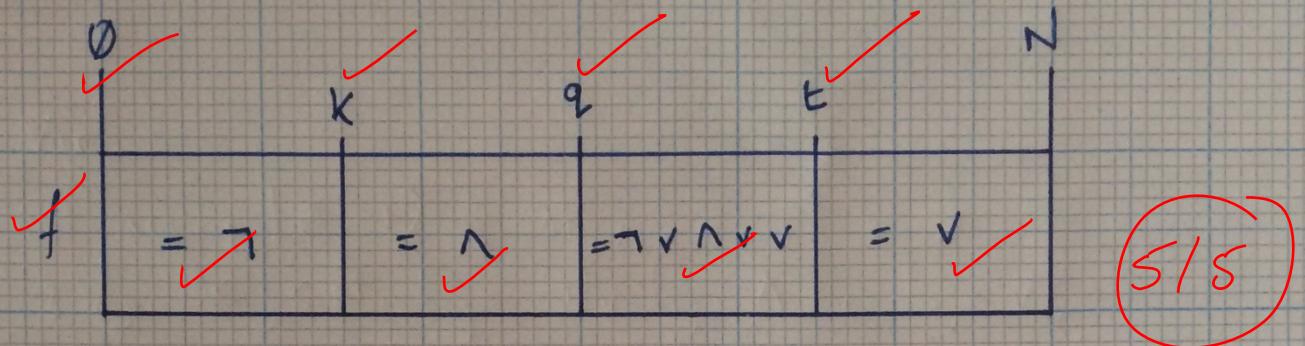


4/4

$\{ \exists k, t: 0 \leq k \leq t \leq N: \forall j: 0 \leq j < k: f.j = \neg \wedge \forall j: k \leq j < t: f.j = \wedge$
 $\wedge \forall j: t \leq j < N: f.j = \vee \}$

SNAPSHOT DURING PROCESSING

3.

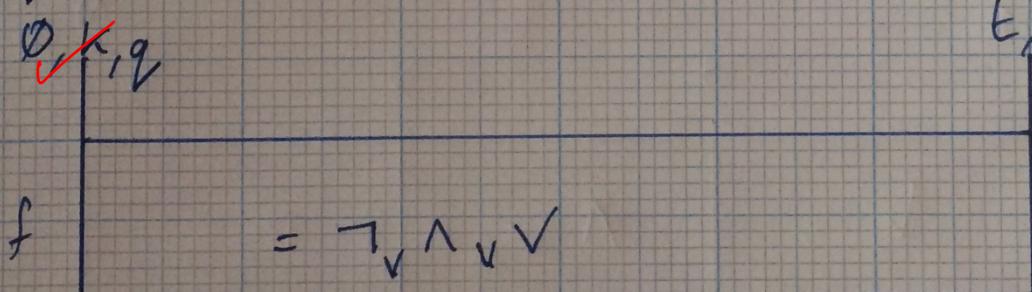


$$\{ \exists k, t, q : 0 \leq k \leq q \leq t \leq N : \forall j : 0 \leq j < k : f_{\cdot j} = \top \wedge \forall j : k \leq j < q : f_{\cdot j} = \top \wedge \forall j : q \leq j < t : f_{\cdot j} = \top \vee f_{\cdot j} = \perp \wedge \forall j : t \leq j < N : f_{\cdot j} = \top \}$$

4. VALUES OF VARIABLES AT BEGINNING

BECAUSE AT THE BEGINNING WE CONSIDER THE ENTIRE ARRAY TO BE UN SORTED, THE SEGMENT $q \rightarrow t$ MUST BE CONSIDERED FROM $\emptyset \rightarrow N$ ∴

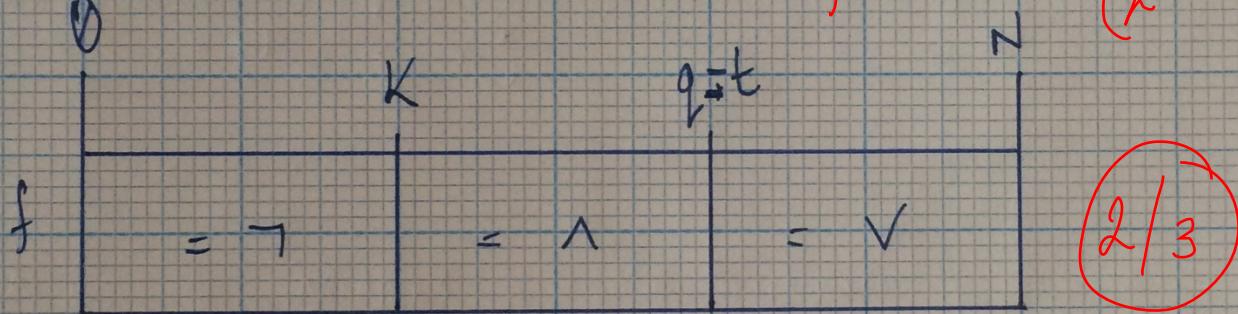
$$k, q, t = \emptyset, \emptyset, N \checkmark$$



5. VALUES OF VARIABLES AT END \Rightarrow determine end state & guard

WE DO NOT KNOW HOW MANY CHARACTERS ARE IN THE ARRAY SO WE DON'T KNOW THE FINAL VALUES FOR K, q, t . WHAT WE DO KNOW IS THAT WHEN THE PROCESS STOPS, THE ARRAY MUST BE SORTED. THIS MEANS THE NUMBER OF CHARACTERS IN SEGMENT $q \rightarrow t = 0$

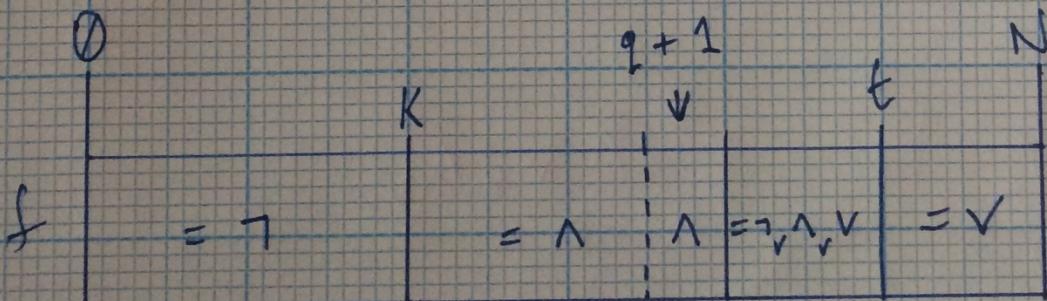
$\therefore q = t$. ✓ guard or loop can be determined from this ($q < t$)



6. ASSIGNMENTS OF VARIABLES DURING THE MIDDLE

$f.q = \wedge =$ INCREASE q by 1
 $q := q + 1$ ✓

5/5

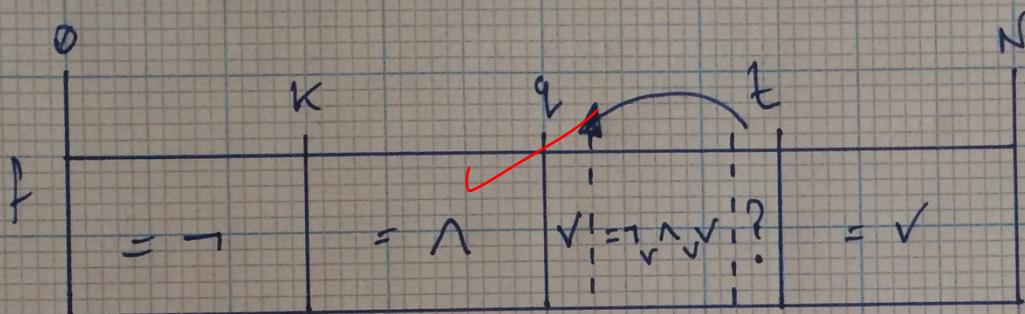


2

$f.g = v = \text{Swap } f.g \text{ with } f:t-1 \text{ AND } t = t-1$

$$f.g, f.e - 1 := f.e - 1, f.g, \\ e := e - 1;$$

5/5



iii

f,g = \Rightarrow SWAP f,g WITH f,k AND $k := k + 1, q := q + 1$

$$\begin{array}{l} f \cdot g := f \cdot k \\ f \cdot k := f \cdot g \end{array}$$

$$f \cdot k := f \circ$$

$$g, \mathcal{K} := g,$$

$$q_1, K := q+1, K+1$$

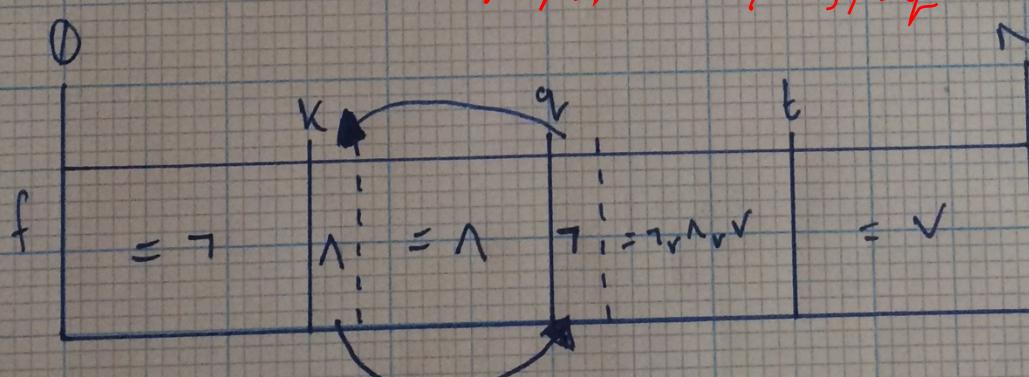
null loose value of f.g

need to do concurrent assignment

assignment

Only small
thing but
important

$$f \circ g, f \circ k := f \circ k, f \circ g$$



45

7 TERMINATION PROOF

INITIALISATION

$$\frac{(t - q > 0) \quad (q, t := 0, N)}{\in \{ \text{SUBSTITUTION} \}}$$

$$N - 0 > 0 \quad \checkmark$$

$$\in \{ \text{ARITHMETIC} \}$$

$$N > 0 \quad \checkmark$$

$$\in \{ \text{PROBLEM DECLares THAT } N > 0 \}$$

$$\{ \Leftarrow \text{GIVEN } N > 0 \}$$

TRUE \checkmark (2)

$$\frac{q := q + 1}{(t - q)}$$

$$(t - q)(q := q + 1)$$

$$\in \{ \text{SUBSTITUTION} \}$$

$$t - (q + 1)$$

$$\in \{ \text{ARITHMETIC} \}$$

$$t - q - 1$$

<

$$t - q$$

TRUE (2)

(8/8)

// $\left\{ \begin{array}{l} q := q + 1 \\ \text{HAPPENS TWICE, PROOF SHOULD} \end{array} \right.$

ONCE (2) \checkmark

// $\left\{ \begin{array}{l} k := k + 1 \\ \text{HAPPENS ONCE BUT HAS A} \\ \text{BEARING ON THE GUARD} \end{array} \right.$

$$\frac{t := t - 1}{(t - q)(t := t - 1)}$$

$$\in \{ \text{SUBSTITUTION} \}$$

$$(t - 1) - q$$

$$\in \{ \text{ARITHMETIC} \}$$

$$t - q - 1$$

<

$$t - q$$

TRUE (2)

8 COMPLETE SOLUTION

1 [CON N : int {N > 0}] ✓

VAR

f : array [0..N) of int, char X

{ $\forall j : 0 \leq j \leq N : f.j = 'A' \vee f.j = 'V' \vee f.j = 'N'$ }

q : int;

K : int; ✓

t : int;

q, K, t = 0, 0, N; ✓

do q < t →

if f.q = 'A' →

f.q, f.K = f.K, f.q; ✓

q, k = q + 1, k + 1;

[] f.q = 'N' →

q := q + 1; ✓

[] f.q = 'V' →

f.q, f.t - 1 := f.t - 1, f.q; ✓

t := t - 1;

fi

od

{ $\exists K, t : 0 \leq k \leq t \leq N : \forall j : 0 \leq j \leq K : f.j = 'A' \vee f.j = 'V'$ } ✓

$\forall j : K \leq j < t : f.j = 'N'$

$\forall j : t \leq j \leq N : f.j = 'V'$