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CS-225: Discrete Structures in CS
Homework 5, Part 2

Exercise Set 5.4: Problem 3

$C_0, C_1, C_2 \ldots$

$C_0 = 2, \; C_1 = 2, \; C_2 = 6$

$C_k = 3C_{k-3}$, for every integer $k \geq 3$

Prove that $C_n = 2n$ for integer $n \geq 0$

$C_0 = 2$

$C_1 = 2$

$C_2 = 6$

$C_3 = 3C_{3-3} = 3C_0 = 3 \cdot 2 = 6$

$C_4 = 3C_{4-3} = 3C_1 = 3 \cdot 2 = 6$

$0 \leq i \leq k$

Assome that C_i is even

$C_{k+1} = 3C_{k+1-3}$

$= 3C_{k-2}$

$k \geq 2$

$k - 2 \geq 0$

$k = 2$

$= 3C_0$

$= 3 \cdot 2$

$= 6$

Exercise Set 5.4: Problem 6

$f_0 = 3 \cdot 2^0 + 2 \cdot 5^0$

$= 3 + 2$

$= 5$

$n = 1$

$f_1 = 3 \cdot 2 + 2 \cdot 5$

$= 6 + 10$

$= 16$

$f_{k+1} = 7f_k - 10f_{k-1}$

$= 7(3 \cdot 2^k + 2 \cdot 5^k) - 10(3 \cdot 2^{k-1} + 2 \cdot 5^{k-1})$ Ind. hypo

$= 21 \cdot 2^k + 14 \cdot 5^k - 15 \cdot 2 \cdot 2^{k-1} - 4 \cdot 5 \cdot 5^{k-1}$

$= 21 \cdot 2^k + 14 \cdot 5^k - 15 \cdot 2^k - 4 \cdot 5^k$

$= 3 \cdot 2 \cdot 2^k + 2 \cdot 5 \cdot 5^k$

$= 3 \cdot 2^{k+1} + 2 \cdot 5^{k+1}$

Let P(n) be the statement that a postage of n cents can be formed using just 3-cent and 5-cent stamps." Use strong mathematical induction to prove that P(n) is true for n ≥ 8. Answer the following questions to show a complete proof.

1:

$P(8) = 3 + 5 = 8$

$P(9) = 3 + 3 + 3 = 9$

$P(10) = 5 + 5 = 10$

2:

$P(n)$ true for $8 \leq n \leq k$ where $k \geq 10$

3:

$P(k+1)$ is true

4:

$k \geq 10$, then $k + 1 = (k-2) + 3$

$k - 2 \geq 8$ then $P(k-2)$ is true

$P(k+1)$ is true