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CS-225: Discrete Structures in CS
Homework 5, Part 1

Exercise Set 5.2: Problem 11

$1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$n = 1$

$= \left[\frac{1(1+1)}{2}\right]^2$

$= \left[\frac{1 \cdot 2}{2}\right]^2$

$= 1^2$

$= 1$

$1^3 + 2^3 + \dots n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\left[\frac{(k+1)[(k+1)+1)}{2}\right]^2$

$\frac{(k+1)^2(k+2)^2}{4}$

$\left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$

$\frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4}$

$\frac{k^2(k+1)^2 + 4(k+1)^2(k+1)}{4}$

$\frac{(k+1)^2(k^2+4(k+1)}{4}$

$\frac{(k+1)^2(k^2+4k+4)}{4}$

$\frac{(k+1)^2(k+2)^2}{4}$

Exercise Set 5.2: Problem 15

$(1+1)! - 1 = 2! - 1$

$= 2 - 1$

$= 1$

$1 = 1$

Hypothesis

$= ((k+1)+1)! - 1$

$= (k+2)! - 1$

Right

$(k+1)! - 1 + (k+1)((k+1)!)$

$(k+1)![1+(k+1)] - 1$

$(k+1)!(k+2) - 1$

$(k+2)! - 1$

Exercise Set 5.3: Problem 9

$7^k - 1 = 6r$

$7^{k+1} - 1$

$7^{k+1} - 1 = 7^k(7-1)$

$= 7(7^k) - 1$

$= 7(7^k - 1 + 1) - 1$

$7(6r+1) - 1$

$42r + 7 - 1$

$42r + 6$

$6(7r+1)$

$n = 0$

$7^0 - 1 = 1 - 1 = 0$

Exercise Set 5.3: Problem 18

$5^n + 9 < 6^n$, for each integer $n \geq 2$

$5 + 9 < 6$

$5^2 + 9 < 6^2$

$25 + 9 < 36$

$34 < 36$

$5^k + 9 < 6^k$

$5^k < 6^k - 9$

$5^{k+1} + 9 = 5 \cdot 5^k + 9$

$= 5 \cdot (6^k - 9) + 9$

$= 5 \cdot 6^k - 45 + 9$

$= 5 \cdot 6^k - 36$

$5 \cdot 6^k < 6 \cdot 6^k \quad -36 < 0 \quad \therefore 5^{k+1} + 9 < 6^{k+1}$

Exercise Set 5.3: Problem 26

$C_0, C_1, C_2 \ldots$

$C_0 = 3 \quad C_k = (C_{k-1})^2$

$C_n = 3^{2^n}$, for each integer $n \geq 0$

$n = 0$

$C_0 = 3^{2^0} = 3^1 = 3 \quad , \quad 3 \geq 0$

$C_k = 3^{2^k}$

$$
\begin{aligned}
C_{k+1} &= (C_k)^2 \\
&= 3^{2^k} \cdot 3^{2^k} \\
&= 3^{(2^k + 2^k)} \\
&= 3^{2 \cdot 2^k} \\
&= 3^{2^{k+1}}
\end{aligned}
$$