Leonel Garay
CS-225: Discrete Structures in CS
Homework 9, Part 1

Exercise Set 1.4: Problem 9

Edges adjacent on V_1 - $e_1, e_2, e7$

Vertices adjacent to V_3 - V_1, V_2

All edges adjacent to e_1 - e_2, e_7

All loops e_1, e_3

Parallel edges $\{e_4, e_5\}$

Isolated vertices V_4

Degree of V_3 - 2

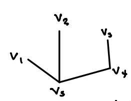
Degree of the graph $4+6+2+0+2=14$

Exercise Set 4.9: Problem 11

$n=5$

$\deg(v_1)=1$, $\deg(v_2)=1$, $\deg(v_3)=1$, $\deg(v_4)=2$, $\deg(v_5)=3$

$\sum_{i=1}^{5} \deg(v_i) = 1+2+3+3+5=14$



Total degree is even, none exceed 4, graph most likely exists

Exercise Set 4.9: Problem 21

C

By the pigeonhole principle there must be two vertices with same number of degree. In a simple graph with $n \geq 5$, n vertices, all cannot have different degrees

Exercise Set 4.9: Problem 23

E

All vectex in M have degree in n, and there are m vertices of degree n

$\deg(v_i) = n$ for $i = 1, 2, \ldots m$

$\deg(v_i) = m$ for $i = 1, 2, \ldots, n$

Total degree

$$\sum_{i=1}^{m} \deg(v_i) + \sum_{i=1}^{n} \deg(v_i) = \sum_{i=1}^{m} n + \sum_{i=1}^{n} m = mn + nm = 2mn$$

F

Handshake Theorem

$2e =$ Total degree $= 2mn$

$2e = 2mn$

$= e = mn$ $\quad K_{m,n}$ contains mn edges

Exercise Set 4.9: Problem 24

F

$v_1 \in V_1$ $\quad$ ANS: Not bipartite

$v_2 \in V_2$

$v_3 \in V_1$

$v_4 \in V_2$