Leonel Garay
CS-225: Discrete Structures in CS
Homework 2, Part 2

Exercise Set 5.1: Problem #13

k	a_k	$\frac{1}{k} - \frac{1}{1+k}$
1	$1 - \frac{1}{2}$	$\frac{1}{1} - \frac{1}{1+1}$
2	$\frac{1}{2} - \frac{1}{3}$	$\frac{1}{2} - \frac{1}{1+2}$
3	$\frac{1}{3} - \frac{1}{4}$	$\frac{1}{3} - \frac{1}{1+3}$
4	$\frac{1}{4} - \frac{1}{5}$	$\frac{1}{4} - \frac{1}{1+4}$
5	$\frac{1}{5} - \frac{1}{6}$	$\frac{1}{5} - \frac{1}{1+5}$
6	$\frac{1}{6} - \frac{1}{7}$	$\frac{1}{6} - \frac{1}{1+6}$

Answer: $\boxed{a_k = \frac{1}{k} - \frac{1}{1+k}}$

Exercise Set 5.1: Problem #15

k	a_k	$\frac{k-1}{k} \cdot (-1)^{k-1}$
1	0	$\frac{1-1}{1} \cdot (-1)^{1-1}$
2	$-\frac{1}{2}$	$\frac{2-1}{2} \cdot (-1)^{2-1}$
3	$\frac{2}{3}$	$\frac{3-1}{3} \cdot (-1)^{3-1}$
4	$-\frac{3}{4}$	$\frac{4-1}{4} \cdot (-1)^{4-1}$
5	$\frac{4}{5}$	$\frac{5-1}{5} \cdot (-1)^{5-1}$
6	$-\frac{5}{6}$	$\frac{6-1}{6} \cdot (-1)^{6-1}$

Answer: $\boxed{\frac{k-1}{k} \cdot (-1)^{k-1}}$

Exercise Set 5.1: Problem #57

$$\sum_{i=1}^{n-1} \frac{i}{(n-i)^2}$$

$j = i - 1 \qquad j = 1 - 1 \qquad (n-1) - 1$

$i = j + 1 \qquad j = 0 \qquad n - 2$

$$\boxed{\sum_{j=0}^{n-2} \frac{j+1}{(n-j-1)^2}}$$

$$\frac{i}{(n-i)^2} = \frac{j+1}{(n-(j+1))^2} = \frac{j+1}{(n-j-1)^2}$$

Answer

Exercise Set 5.1: Problem #60

$$2 \cdot \sum_{k=1}^{n} (3k^2 + 4) + 5 \cdot \sum_{k=1}^{n} (2k^2 - 1)$$

$$2(3k^2 + 4) + 5 \cdot (2k^2 - 1)$$

$$(6k^2 + 8) + (10k^2 - 5)$$

$$6k^2 + 10k^2 + 8 - 5$$

$$16k^2 + 3$$

$$\boxed{\sum_{k=1}^{n} (16k^2 + 3)} \quad \text{Answer}$$

Exercise #5

$$\sum_{i=4}^{50} (4i - 3) = \boxed{4935}$$

$$\sum_{i=4}^{50} 4i + \sum_{i=4}^{50} (-3)$$

$$4\sum_{i=1}^{50} i - 4\sum_{i=1}^{3} i + \sum_{i=1}^{50} (-3) - \sum_{i=1}^{3} (-3)$$

$$4\left(\frac{n(n+1)}{2}\right) \qquad -150 - (-9)$$

$$4 \cdot \frac{50(51)}{2} - 4 \cdot \frac{3(4)}{2} \quad -150 + 9$$

$$(4 \cdot 1275) - (4 \cdot 6)$$

$$5,100 \; -24 \; -150 + 9$$

$$\boxed{4935}$$

Exercise #6

$$\sum_{j=0}^{36} (j^2 - (-1)^j) = \boxed{16243}$$

$$\sum_{j=0}^{36} j^2 + \sum_{j=0}^{36} 1^j$$

$$\frac{(n+1)(2n+1)}{6} + 37$$

$$\frac{6(37)(2 \cdot 36 + 1)}{6} + 37$$

$$\frac{1332 \cdot 73}{6} + 37$$

$$16,206 + 37$$

$$\boxed{16243}$$

Exercise #7

$$\sum_{j=2}^{70} (2^j + 4^j)$$

$$\sum_{j=2}^{70} 2^j + \sum_{j=2}^{70} 4^j$$

$$\sum_{j=0}^{70} 2^j - \sum_{j=0}^{68} 2^j + \sum_{j=0}^{70} 4^j - \sum_{j=0}^{68} 4^j$$

$$\frac{ar^{n+1} - a}{r - 1}$$

$$\left(\frac{2^{70+1} - 1}{2 - 1} - \frac{2^{68+1} - 1}{2 - 1}\right) + \left(\frac{4^{70+1} - 1}{4 - 1} - \frac{4^{68+1} - 1}{4 - 1}\right)$$