Leonel Garay
CS-225: Discrete Structures in CS
Homework 4, Part 2

Exercise Set 4.7: Problem 13

$\forall$ r and s, if r is irrational and s is any non-zero rational number, then r + s is irrational

Negation

$\exists$ r and s, if r is rational and s is irrational, then r+s is rational

$$r = \frac{a}{b}$$

$$r + s = \frac{c}{d}$$

$$= \frac{a}{b} + r = \frac{c}{d}$$

$$r = \frac{c}{d} - \frac{a}{b}$$

$$r = \frac{bc - ad}{bd}$$

Exercise Set 4.7: Problem 24

Contraposition

$$\frac{1}{x} = \frac{a}{b}$$

$$\frac{b}{x} = a$$

$$b = ax$$

$$\frac{b}{a} = x$$

Contradition

$$\frac{1}{x} = \frac{a}{b}$$

$$\frac{b}{x} = a$$

$$b = ax$$

$$\frac{b}{a} = x$$

If x is irrational, then the reciprocal 1/x is irrational

Exercise Set 4.7: Problem 29

Contradition

$m = 2r + 1$, $n = 2s$

$m + n = 2r + 1 + 2s$

$= 2(r+s) + 1$

m+n is even

Exercise Set 4.8: Problem 18

Prove that for every integer a, if a^3 is even then a is even

a is odd

$a^3 = (2k+1)^3$

$= (2k)^3 + 3(2k)^2 \cdot 1 + 3(2k) \cdot 1 + 1$

$= 8k^3 + 12k^2 + 6k + 1$

$= 2(4k^3 + 6k^2 + 3k) + 1$

$= 2r + 1 \longrightarrow a^3$ is odd

Prove that $\sqrt[3]{2}$ is irrational

$a = \sqrt[3]{2}$ $\quad a = r/s$

$a^3 = 2$

$(r/s)^3 = 2$

$\frac{r^3}{s^3} = 2$

$r^3 = 2s^3$

r^3 is even, p is even