Leonel Garay
CS-225: Discrete Structures in CS
Homework 2, Part 1

Exercise Set 3.1: Problem #16

b. $\forall$ real number x, x is positive, negative, or zero.
d. $\forall$ logician x, x is not lazy.
f. $\forall$ real number x, x2 is not -1

Exercise Set 3.1: Problem #17

b. $\exists$ real number x, such that x is rational.

Exercise Set 3.1: Problem #18

a. $\exists s \in D$, such that $M(s) \wedge E(s)$
b. $\forall s \in D$, $C(s) \rightarrow E(s)$
c. $\forall s \in D$, $C(s) \rightarrow \neg E(s)$
d. $\exists s \in D$, such that $C(s) \vee M(s)$
e. ($\exists s \in D$, such that $C(s) \wedge E(s)$) $\wedge$ ($\exists s \in D$, such that $C(s) \wedge \neg E(s)$)

Exercise Set 3.1: Problem #22

b. $\forall$ argument x, if x with true premise then x has true conclusion.

Exercise Set 3.1: Problem #26

a. $\forall$x, if x integer then x is a rational number, but $\exists$ rational number x, such that x is not an integer.
b. $(\forall x \in D, Int(x) \rightarrow Ralt(x)) \wedge (\exists s \in D, Ralt(x) \wedge \neg Int(x))$

Exercise Set 3.2: Problem #2

All Dogs are loyal
≡ $\forall d \in D$, d is loyal
≡ $\neg(\forall d \in D$, d is loyal$)$
≡ $\exists d \in D$, d is disloyal
≡ Some Dogs are disloyal or There is a dog that is disloyal

Answer: c and f

Exercise Set 3.2: Problem #4

b. Some graphs are disconnected
d. All estimates are inaccurate

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Exercise Set 3.2: Problem #21

A(x) = divisible by 6
B(x) = divisible by 2
C(x) = divisible by 3

$\neg(\forall x \in D$, if $A(x)$ then $B(x) \wedge C(x))$
$\equiv \exists x \in D$, such that $A(x)$ then $\neg(B(x) \wedge C(x))$
$\equiv \exists x \in D$, such that $A(x)$ then $\neg B(x) \vee \neg C(x)$

Answer: ∃ integers n, if n is divisible by 6 and n is not divisible by 2 or n is not divisible by 3.

Exercise Set 3.2: Problem #29

$\forall x \in Z$, if n is prime then n is odd or n = 2
P(x) = n is prime
Q(x) = n is odd
R(x) = n = 2
$\forall x \in Z, P(x) \rightarrow (Q(x) \vee R(x))$

<u>**Converse**</u>
$\equiv \forall x \in Z, (Q(x) \vee R(x)) \rightarrow P(x)$
$\equiv \forall x \in Z$, if n is odd or n = 2 then n is prime
False.
Counterexample: 49 is odd and divisible by 7.

<u>**Inverse**</u>
$\equiv \forall x \in Z, \neg P(x) \rightarrow \neg(Q(x) \vee R(x))$
$\equiv \forall x \in Z, \neg P(x) \rightarrow \neg Q(x) \wedge \neg R(x))$
$\equiv \forall x \in Z$, if n is not prime then n is not odd and $n \neq 2$
False.
Counterexample: 49 is not prime, and 49 is odd.

<u>**Contrapositive**</u>
$\equiv \forall x \in Z, \neg(Q(x) \vee R(x)) \rightarrow \neg P(x)$
$\equiv \forall x \in Z, \neg Q(x) \wedge \neg R(x) \rightarrow \neg P(x)$
$\equiv \forall x \in Z$, if n is even and $n \neq 2$, then n is not prime
True

Exercise Set 3.2: Problem #40

If a number is divisible by 8 then that number is divisible by 4

Exercise Set 3.2: Problem #44

If a polygon is square then it has four sides.