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CS-225: Discrete Structures in CS
Homework 6, Part 1

Exercise 1

$d_k = 3d_{k-1} + 5 \qquad k \geq 2$

$d_1 = 2 \qquad (1, 38, 119$

$= 3d_{k-1} + 5$

$= 3(3d_{k-1} + 5) + 5 = 3^2 d_{k-1} + 3 \cdot 5 + 5$

$= 3^2(3d_{k-1} + 5) + 3 \cdot 5 + 5 = 3^3 d_{k-1} + 3 \cdot 5 + 5 + 5$

$= 2 \cdot 3^{k-1} + 5 \cdot \frac{3^{k-1} - 1}{2}$

$= \frac{9 \cdot 3^{k-1} - 5}{2} = \frac{1}{2} \cdot 3^{k+1} - \frac{5}{2}$

$n = \frac{1}{2} \cdot 3^{n+1} - \frac{5}{2}$

Exercise 2

$d_1 = 2 = \frac{1}{2} \cdot 3^2 - \frac{5}{2} = \frac{1}{2} \cdot 9 - \frac{5}{2} = \frac{9}{2} - \frac{5}{2} = \frac{4}{2} = \boxed{2}$

The formula is valid for $k \geq 1$, $k = n$

$d_2 = \frac{1}{2} \cdot 3^{2+1} - \frac{5}{2} = \frac{1}{2} \cdot 3^3 - \frac{5}{2} = \frac{1}{2} \cdot \frac{27}{1} - \frac{5}{2}$

$= \frac{27}{2} - \frac{5}{2} = \frac{27-5}{2} = \frac{22}{2} = \boxed{11}$

Then $k = n+1$

Exercise 3

$t_k = t_{k-1} + 3k + 1 \qquad k \geq 1, \ t_0 = 0$

$t_{k-1} = t_{k-2} + 3(k-1) + 1 - 2$

$t_{k-2} = t_{k-3} + 3(k-2) + 1 - 3$

$n_k = \frac{3n(n+1)}{2} + n$

$t_1 = \frac{3(1)(2)}{2} + 1 = 4$

$t_2 = \frac{3(2)(3)}{2} + 2 = 11$

Exercise 4

1: $0 \in S$

2: If $x, y \in S$, then $x + y \in S$

3: Nothing is in S other than those obtained from 1 and 2

Exercise 5

1: $S \to \varepsilon$ (aa, ab, ba, bb are in S)

2: If ε, then aas, bbs, abs, bas, bb are also in ε

3: Nothing in S other than those in 1 and 2