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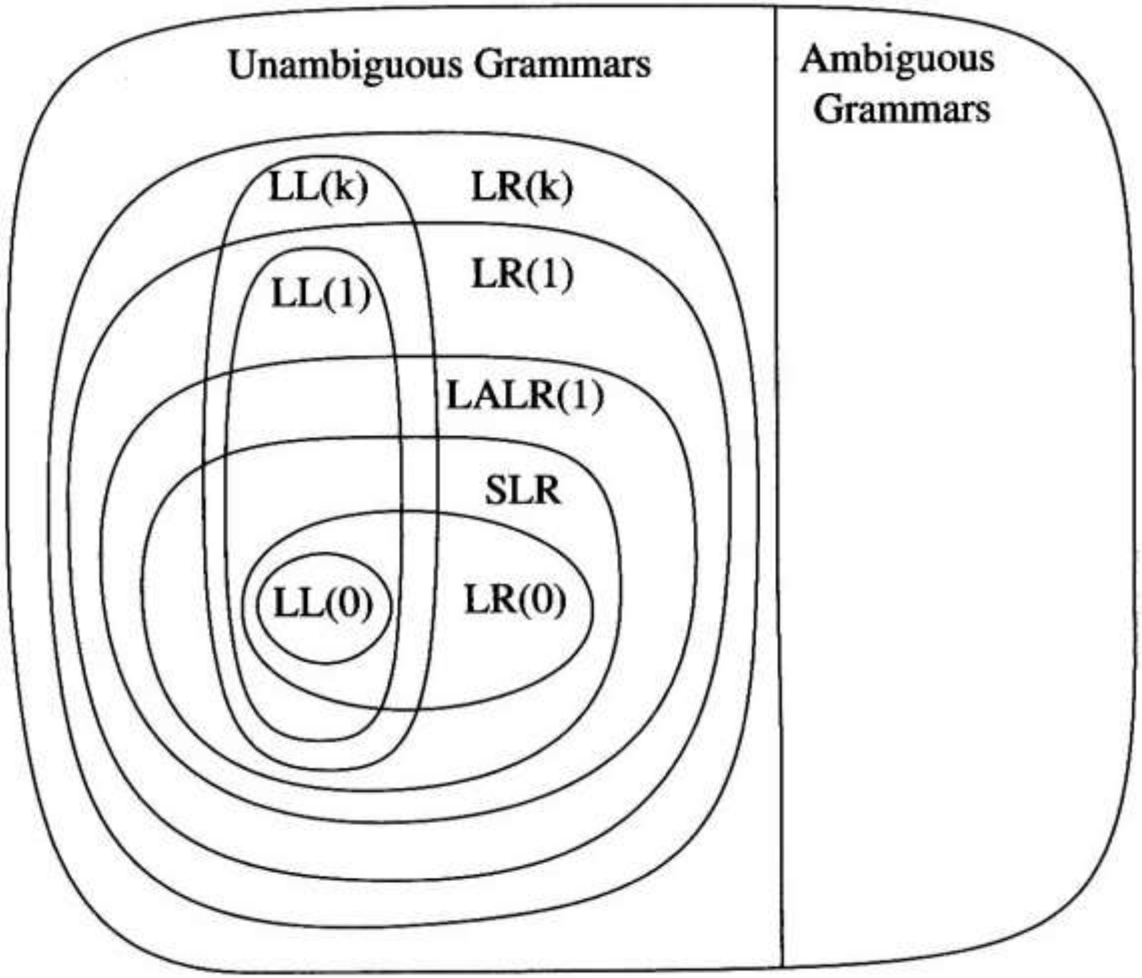
Unit 8.
LL(k) grammars

Hierarchy of grammar classes



LL(k) grammar

- What is LL(k)?
  - The first *L* stands for scanning the input from **l**eft to right,
  - the second *L* stands for producing a **l**eftmost derivation,
  - And k stands for using **k** input symbols of lookahead at each step to make parsing action decision.

LL(k) Grammar

- Subset of CFG's
- Permits deterministic left-to-right recognition with a look ahead of k symbols
- Builds the parse tree top-down
- If the correct production can be deduced from the partially constructed tree and the next k symbols in the unscanned string, for every possible step, then the grammar is said to be LL(k)
- If a parse table can be constructed for the grammar, then it is LL(k), if it can't, it is not LL(k)

LL(k) Grammars

- An LL(k) grammar has the property that a parser can be constructed to scan an input string from left to right and build a leftmost derivation by examining next k input symbols to determine the unique production for each derivation step.
- If a language has an LL(k) grammar, it is called an LL(k) language.
- LL(k) languages are deterministic context-free languages, but there are deterministic context-free languages that are not LL(k)

How to Build Parse Tables? FIRST and FOLLOW Sets

For a string of grammar symbols α define FIRST(α) as

- The set of tokens that appear as the first symbol in some string that derives from α
- If $\alpha \Rightarrow^* \varepsilon$, then ε is in FIRST(α)

For a non-terminal symbol *A*, define FOLLOW(*A*) as

The set of terminal symbols that can appear immediately to the right of *A* in some sentential form

FIRST Set Construction

To construct FIRST(X) for a grammar symbol X, apply the following rules until no more symbols can be added to FIRST(X)

- If X is a terminal FIRST(X) is $\{X\}$
- If $X \to \varepsilon$ is a production then ε is in FIRST(X)
- If X is a nonterminal and $X \to Y_1 Y_2 \ldots Y_k$ is a production then put every symbol in FIRST(Y_1) other than ε to FIRST(X)
- If X is a nonterminal and $X \to Y_1 Y_2 \ldots Y_k$ is a production, then put terminal a in FIRST(X) if a is in FIRST(Y_i) and ε is in FIRST(Y_j) for all $1 \le j < i$
- If X is a nonterminal and $X \to Y_1 Y_2 \ldots Y_k$ is a production, then put ε in FIRST(X) if ε is in FIRST(Y_i) for all $1 \le i \le k$

Computing FIRST Sets for Strings of Symbols

To construct the FIRST set for any string of grammar symbols $X_1X_2 \ldots X_k$ (given the FIRST sets for symbols X_1 , X_2 , ... X_k) apply the following rules.

FIRST($X_1X_2 \ldots X_k$) contains:

- Any symbol in FIRST(X_1) other than ε
- Any symbol in FIRST(X_i) other than ε, if ε is in FIRST(X_j) for all $1 \le j < i$
- ε , if ε is in FIRST(X_j) for all $1 \le i \le n$

FIRST$_k$(α)

Description: Given a CFG, k is a natural number, α contains both terminals and nonterminals.

FIRST$_k$(α)consists of all terminal prefixes of length k (or less if α derives a terminal string of lengh less than k) of terminal strings that can be derived from α

Definition: Given grammar G= (Σ, Δ, P, S), natural number k , $\alpha \in V^*$

FIRSTk(α) =

$\{ x \in \Sigma^* \mid \alpha \Rightarrow x\beta \text{ and } |x| = k \text{ or } \alpha \Rightarrow x \text{ and } |x| < k\}$

$FOLLOW_k(\alpha)$

includes the set of terminal strings that can occur immediately to the right of α in any sentential form

Especially, if α is nonterminal A and βA is a sentential form then ε is in $FOLLOW_1(A)$.

$\text{FOLLOW}_k(\alpha)$

$\text{FOLLOW}_k(\alpha) =$

$\{x \in \Sigma^* \mid S \Rightarrow^* \beta\alpha\delta \text{ and } x \in \text{FIRST}_k(\delta)\}$

Especially, if $\alpha = A \in \Delta^*$, $S \Rightarrow^* \beta A$ then $\text{FOLLOW}_1(A) = \{\varepsilon\}$ or $\text{FOLLOW}_1(A) = \{\$\}$ (EOF)

FOLLOW(A) is $\text{FOLLOW}_1(A)$

LL(k) Grammars

Definition Let $G = (\Sigma, \Delta, P, S)$ is a CFG and $k \in N$. G is LL(k) if for any two leftmost derivations

$$S \Rightarrow xA\alpha \Rightarrow x\beta_1\alpha \Rightarrow xZ_1$$

$$S \Rightarrow xA\alpha \Rightarrow x\beta_2\alpha \Rightarrow xZ_2$$

if $FIRST_k(Z_1) = FIRST_k(Z_2)$ then $\beta_1 = \beta_2$

It can be shown that *LL*(k) grammars are not ambiguous and not left-recursive.

Example

Grammar G :

$$S \rightarrow aSb \mid ab$$

is not LL(1), but is LL(2)

Simple LL(1) Grammars

For simple LL(1) grammars all rules have the form

$$A \rightarrow a_1\alpha_1 \mid a_2\alpha_2 \mid \ldots \mid a_n\alpha_n$$

where

- a_i is a terminal, $1 \leq i \leq n$
- $a_i \neq a_j$ for $i \neq j$ and
- α_i is a sequence of terminals and non-terminal or is empty, $1 \leq i \leq n$

How to recognize a LL(1) grammar?

Theorem A context-free grammar $G = (\Sigma, \Delta, P, S)$ is LL(1) if and if only if for every nonterminal *A* and every strings of symbols

$$A \to \alpha_1 \mid \alpha_2 \mid . . . . \mid \alpha_n , n \geq 2 \text{ we have}$$

$$FIRST_1(\alpha_i) \cap FIRST_1(\alpha_j) = \emptyset, \ i \neq j$$

If $\alpha_i \Rightarrow * \varepsilon$ then

$$FIRST_1(\alpha_i) \cap FOLLOW_1(A) = \emptyset , i \neq j$$

KPL is LL(1)?– FIRST & FOLLOW

A	FIRST(A)	FOLLOW(A)
Block	KW_CONST, KW_VAR, KW_TYPE, KW_FUNCTION, KW_PROCEDURE, KW_BEGIN	SB_PERIOD, SB_SEMICOLON
Unsignedconst	TK_IDENT, TK_NUMBER, TK_CHAR.	
Constant	SB_PLUS,SB_MINUS, TK_IDENT TK_NUMBER	
Type	TK_IDENT, TK_NUMBER, TK_CHAR, KW_ARRAY	
Statement	TK_IDENT, KW_CALL, KW_BEGIN, KW_IF, KW_WHILE, KW_FOR	KW_ELSE,SB_SEMICOLON, KW_END
Expression3	SB_PLUS,SB_MINUS, ε	SB_COMMA,SB_SEMICOLON, KW_END, KW_TO, KW_THEN, KW_DO, SB_RPAR, SB_RSEL, SB_EQ, SB_NEQ, SB_LT, SB_LE, SB_GT, SB_GE, KW_ELSE

KPL is LL(1)?

Consider the following set of production with Statement on LHS

```
Statement ::= AssignSt
Statement ::= CallSt
Statement ::= GroupSt
Statement ::= IfSt
Statement ::= WhileSt
Statement ::= ForSt
Statement ::= ε
```

FIRST (RHS1)={TK_IDEN}
FIRST (RHS2)={KW_CALL}
FIRST (RHS3)={KW_BEGIN}
FIRST (RHS4)={KW_IF}
FIRST (RHS5)={KW_WHILE}
FIRST (RHS6)={KW_FOR}
FIRST (RHS7)={ε}
FOLLOW(LHS)={SB_SEMICOLON, KW_END, KW_ELSE}

The set of productions of Statement satisfies LL(1) condition

KPL is LL(1)?

```
Factor ::= UnsignedConstant
Factor ::= Variable
Factor ::= FunctionApplication
Factor ::= SB_LPAR Expression SB_RPAR
```

FIRST(RHS1)={TK_IDENT, TK_NUMBER, TK_CHAR}
FIRST(RHS2)={TK_IDENT}
FIRST(RHS3)={TK_IDENT}
FIRST(RHS4)={SB_LPAR}
The set of productions of Factor does not satisfy LL(1) condition, but satisties LL(2) condition

KPL is LL(2)

```
Factor ::= TK_NUMBER
Factor ::= TK_CHAR
Factor ::= TK_IDENT
Factor ::= TK_IDENT Arguments
Factor ::= TK_IDENT Indexes
Factor ::= SB_LPAR Expression SB_RPAR
```

$FIRST_2$(RHS1)={TK_NUMBER}
$FIRST_2$ (RHS2)={TK_CHAR}
$FIRST_2$ (RHS3)={TK_IDENT}
$FIRST_2$ (RHS4)={TK_IDENT SB_LPAR}
$FIRST_2$ (RHS5)={TK_IDENT SB_LSEL}
$FIRST_2$ (RHS6) ={SB_LPAR TK_IDENT, SB_LPAR TK_NUMBER, SB_LPAR SB_LPAR , SB_LPAR TK_CHAR}

The set of productions satisfies LL(2) condition

Grammar Transformations

- Left factoring: Sometimes we can "left-factor" an LL(k) grammar to obtain an equivalent LL(n) grammar where n < k.
- Example. The grammar $S \to aaS \mid ab \mid b$ is LL(2) but not LL(1). But we can factor out the common prefix *a* from productions $S \to aaS \mid ab$ to obtain

  $S \to aT$

  $T \to aS \mid b.$

  This gives the new grammar:

  $S \to aT \mid b$

  $T \to aS \mid b.$

Left Factoring to obtain LL(1) grammars

When more than one production for nonterminal *A* starts with the same symbols, the FIRST sets are not disjoint

$$\begin{aligned} ifstmt &\rightarrow \textbf{if}\ cond\ \textbf{then}\ stmt \\ &\quad \mid \textbf{if}\ cond\ \textbf{then}\ stmt\ \textbf{else}\ stmt \end{aligned}$$

Use *left factoring* to fix the problem

$$\begin{aligned} ifstmt &\rightarrow \textbf{if}\ \mathrm{expr}\ \textbf{then}\ stmt\ elsestmt \\ elsestmt &\rightarrow \textbf{else}\ stmt \\ &\quad \mid \varepsilon \end{aligned}$$