$$L = \frac{1}{2N}\sum_{n=0}^{N}(y\_train - y_n)^2 \quad MSE$$

$$L = \frac{1}{2N}\sum_{n=0}^{N}\left[y\_train - \sigma(x_n)\right]^2$$

$$\sigma(x) = \frac{1}{1+e^{-x}} \quad \text{Sigmoid}$$

$$\frac{\partial\sigma}{\partial x} = \frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right)$$

$$= \frac{d}{dx}(1+e^{-x})^{-1}$$

$$= -(1+e^{-x})^{-2}(-e^{-x})$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \left[1 - \frac{1}{1+e^{-x}}\right]$$

$$\frac{\partial\sigma}{\partial x} = \sigma(x)\cdot(1-\sigma(x))$$

$$\because \; y = wx+b$$

$$\frac{\partial\sigma}{\partial w} = \frac{\partial\sigma}{\partial x}\cdot\frac{\partial x}{\partial w} = \frac{\partial\sigma}{\partial x}\cdot\frac{\partial(wx+b)}{\partial w}$$

$$= \sigma(x)\left[1-\sigma(x)\right]\cdot x$$

$$\frac{\partial\sigma}{\partial b} = \frac{\partial\sigma}{\partial x}\cdot\frac{\partial x}{\partial b} = \frac{\partial\sigma}{\partial x}\cdot\frac{\partial(wx+b)}{\partial b}$$

$$= \sigma(x)\left[1-\sigma(x)\right]\cdot 1$$

$$\therefore \frac{\partial L}{\partial w} = \frac{\partial L}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial w}$$

$$= \frac{\partial}{\partial \sigma}\left[\frac{1}{2N}\sum_{n=0}^{N}(y\_train - \sigma)^2\right] \cdot \frac{\partial \sigma}{\partial w}$$

$$= \frac{\partial}{\partial \sigma}\left[\frac{1}{2N}\sum_{n=0}^{N}(\sigma - y\_train)^2\right] \cdot \frac{\partial \sigma}{\partial w}$$

$$= \frac{1}{N}\sum_{n=0}^{N}(\sigma - y\_train) \cdot \sigma(x) \cdot (1-\sigma(x)) \cdot x$$

$$= \frac{1}{N}\sum_{n=0}^{N}(y_n - y\_train) \cdot y_n \cdot (1-y_n) \cdot x_n$$

$$\frac{\partial L}{\partial b} = \frac{1}{N}\sum_{n=0}^{N}(y_n - y\_train) \cdot y_n \cdot (1-y_n)$$