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Sec:

$$\bar{A} = 2x^2y\hat{x} - y^2\hat{y} + 4xz^2\hat{z}$$

$$y^2+z^2=9, \quad x=2$$

We know,

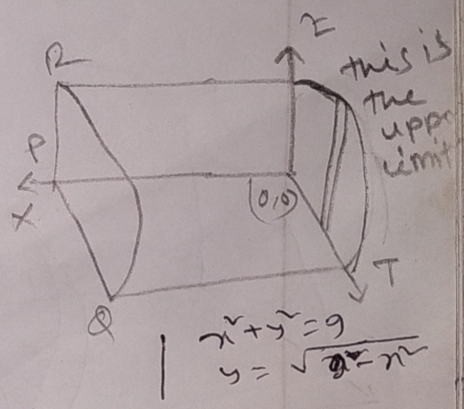
the divergence theorem

$$\int_S \bar{A}\cdot\hat{N}\,ds = \int_E \operatorname{div}\bar{A}\,dv$$

The divergence of $\bar{A}$ is

$$\operatorname{div}\bar{A} = \frac{\partial}{\partial x}(2x^2y) - \frac{\partial}{\partial y}y^2 + \frac{\partial}{\partial z}4xz^2$$

$$= 4xy - 2y + 8xz$$



Let E be the region $\{0 \le z \le 3,\ 0 \le y \le \sqrt{9-x^2} \text{ and } 0 \le x \le z\}$

By the divergence theorem we have

$$\iiint_E \operatorname{div}\bar{A}\,dv = \iiint_E (4xy - 2y + 8xz)\,dv$$

$$= \int_{z=0}^{3}\int_{y=0}^{\sqrt{9-z^2}}\int_{x=0}^{z} (4xy - 2y + 8xy)\,dx\,dy\,dz$$

$$= \int_0^3 dz \int_0^{\sqrt{9-x^2}} (4y + 16z)\,dy\,\cancel{dz}$$

$$= \int_0^3 \left[2y^2 - 16yz\right]_0^{\sqrt{9-x^2}}$$

$$= \int_0^2 2(9-z^2) + 16\sqrt{9-x^2}\, dz$$

$$= \left[18z - \frac{2z^3}{3} + 16(9-z^2)^{3/2}\left(\frac{2}{3}\times\frac{-1}{2}\right)\right]_0^3$$

$$= (18\times3) - 2\,18 + 0 - 16(9)^{3/2}\times\left(-\frac{1}{3}\right)$$

$$= 36 + 16\times9$$

$$= 180 \quad \boxed{\text{Ans:-}}$$