# CS 6310 Project 1: Integer programming example

Prof. Eric Feron

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Here is a worked out example of course assignment. This is a very simple example that aims at guiding you through encoding logical constraints.

We consider $\{s_1, s_2\}$ a set of two students. They need to take *two* courses during two semesters. We assume the courses are the same (there are only two courses $\{c_1, c_2\}$ to be taken). We also assume that the students cannot take more than one class per semester, and we'd like to determine the minimum capacity $X$ that must be offered by the classroom, assuming they all offer the *same* capacity. The answer is clearly $X = 1$ (the students just take their classes with opposite schedules).

The mathematical formulation proceeds as follow: We introduce the variables

- $y_{111}$ a binary unknown that indicates whether student 1 will take course 1 during semester 1;
- $y_{112}$ is a binary unknown that indicates whether student 1 will take course 1 during semester 2;
- $y_{121}$ is a binary unknown that indicates whether student 1 will take course 2 during semester 1;
- $y_{122}$ is a binary unknown that indicates whether student 1 will take course 2 during semester 2;
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- $y_{211}$ a binary unknown that indicates whether student 2 will take course 1 during semester 1;
- $y_{212}$ is a binary unknown that indicates whether student 2 will take course 1 during semester 2;
- $y_{221}$ is a binary unknown that indicates whether student 2 will take course 2 during semester 1;
- $y_{222}$ is a binary unknown that indicates whether student 2 will take course 2 during semester 2;

Important constraints can be written as follows:

1. A course can be taken only once:

$$\begin{aligned} y_{111} + y_{112} &= 1 \\ y_{121} + y_{122} &= 1 \\ y_{211} + y_{212} &= 1 \\ y_{221} + y_{222} &= 1 \end{aligned} \tag{1}$$

2. The number of courses a student can take during a semester is less than or equal to 1:

$$\begin{aligned} y_{111} + y_{121} &\leq 1 \\ y_{112} + y_{122} &\leq 1 \\ y_{211} + y_{221} &\leq 1 \\ y_{212} + y_{222} &\leq 1 \end{aligned} \tag{2}$$

3. the number of students in any course at any time is less than or equal to $X$:

$$\begin{aligned} y_{111} + y_{211} &\leq X \\ y_{112} + y_{212} &\leq X \\ y_{121} + y_{221} &\leq X \\ y_{122} + y_{222} &\leq X \end{aligned} \tag{3}$$

So the binary integer program is to minimze $X$ subject to the constraints (1), (2) and (3).