JHU Computer Organization Module 2 Hint

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1 Converting Decimal Fractions into Binary

To convert a fraction to binary, we think of each successive binary digit that comes after the "radix" (decimal point) as being 2 raised to a negative power. For example, $1.5_{10} = 1.1_2$ because the 1 to the left of the radix $= 1 \times 2^0 = 1$ and the 1 after the decimal point $= 1 \times 2^{-1} = 0.5$. Combining those two binary values together gives us $1.5_{10} = 1.1_2$. Let's do another simple example. Let's convert 0.75_{10} into binary. We know, in decimal, $0.75_{10} = \frac{1}{2} + \frac{1}{4}$. In binary this becomes $1 \times 2^{-1} + 1 \times 2^{-2} = 0.11_2$, and we're done.

This works well if we know which fractions we need to add together to get our decimal number, but what about an "uglier" number like 0.703125? Is there a general process for converting such numbers from decimal to binary? Yes! To convert a fraction to binary, we will successively multiply the fraction portion by 2 and taking the resulting leading bits in reverse order:

$0.703125 = 1.40625 \rightarrow 1$ (keep track of the bit to the left of the radix)
$0.40625 \times 2 = 0.8125 \rightarrow 0$
$0.8125 \times 2 = 1.625 \rightarrow 1$
$0.625 \times 2 = 1.25 \rightarrow 1$
$0.25 \times 2 = 0.50 \rightarrow 0$
$0.50 \times 2 = 1.0 \rightarrow 1$ (stop when the decimal portion equals 0)

We now take each of the resulting bits in reverse order to get: 0.101101_2. We can verify this equals 0.703125 by performing the binary math: $0.101101_2 = 1 \times 2^{-1} + 1 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-6} = \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} = \frac{32}{64} + \frac{8}{64} + \frac{4}{64} + \frac{1}{64} = \frac{45}{64} = 0.703125$. Boom, done!