JHU Computer Organization Module 2 Examples

jkovba

September 2020

1 Introduction

To convert a binary number to decimal, multiply each bit by 2 (the base) to the power of the current column. The powers for each column begin at the far right (starting at 0) and increase by one as you move to the left:

$10101 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 21$

To convert a number from decimal to binary, we continually divide by two (until we get to 0), and we take the remainders in reverse order:

$16/2 = 8R0$
$8/2 = 4R0$
$4/2 = 2R0$
$2/2 = 1R0$
$1/2 = 0R1$

Taking all of the remainders in reverse order (from the bottom up) we are left with: $10000_2 = 16_{10}$

To flip the sign of a number using sign and magnitude format, we simply flip the first bit. Remember that signed numbers that begin with a 0 are positive while signed numbers that begin with a 1 are negative.

$+7 = 0111 \rightarrow -7 = 1111$
$-13 = 11101 \rightarrow +13 = 01101$

To flip the sign of a number using one's complement you flip all of the bits; the 0's become 1's and the 1's becomes 0's. Again, signed numbers that begin with a 0 are positive while signed numbers that begin with a 1 are negative.

$+20 = 010100 \rightarrow -20 = 101011$
$-17 = 101110 \rightarrow +17 = 010001$

To flip the sign of a number using one's complement you flip all of the bits;

the 0's become 1's and the 1's becomes 0's. Again, signed numbers that begin with a 0 are positive while signed numbers that begin with a 1 are negative.

$+20 = 010100 \rightarrow -20 = 101011$
$-17 = 101110 \rightarrow +17 = 010001$

To flip the sign of a number using two's complement you flip all of the bits then add 1 to the result. One last time, signed numbers that begin with a 0 are positive while signed numbers that begin with a 1 are negative.

$+15 = 01111 \rightarrow -15 = 10000 + 1 = 10001$
$-12 = 10100 \rightarrow +12 = 01011 + 1 = 01100$

To flip the sign of a number using two's complement you flip all of the bits then add 1 to the result. One last time, signed numbers that begin with a 0 are positive while signed numbers that begin with a 1 are negative.

To convert a decimal (floating-point) number from decimal to binary, we continually multiply the decimal portion by 2 (until we get 0), and we take the whole number portion of the result of the multiplication:

$0.875 \times 2 = 1.75$
$0.75 \times 2 = 1.5$
$0.5 \times 2 = 1.0$

Finally, we'll take a look at converting numbers in a single signed representation to different bases. Let's start with the number C8, which is given in 1's complement. If we convert this to binary we get: 11000000. Since the leading bit is a 1 we know this is a negative number. Since we're given that this is a 1's complement number, we can find the positive value of the number by flipping all of the bits: $00111111 = 63_{10}$. Therefore we know $C8 = -63_{10}$.

Here's another example. Let's start with -17_8 and convert it to binary, hex, and decimal. We can rewrite this by pulling out the negative sign: $-(17_8)$. Ignoring the negative sign for a moment we know that $17_8 = 1 \times 8^1 + 7 \times 8^0 = 15_{10}$. This means, if we reattach the negative sign, $-17_8 = -15_{10}$. We can do something similar to get the binary representation: $-17_8 = -(17_8) = -(001111_2)$. The value inside the parenthesis is positive, but there is a negative sign out front. Since we know this is a 1's complement number, it's very simple to take the positive number inside the parenthesis and make it negative by applying the negative sign out front: just flip all the bits: $-(001111_2) = 110000_2$. And, finally, we can convert this binary representation to hex very simply by chunking up the binary number into groups of 4 bits. You might be saying: "But the binary number doesn't have a 8 bits, so how can I make two groups of 4?". We can think of the leading bit, 1 in this case, as being repeated an infinite number of times: $110000_2 = 11.....110000_2$. For our case we only need two show 2 of

these additional leading 1's to get a nice, round byte that can be represented using precisely 2 hex digits: $11110000_2 = F0_{16}$. Working backwards to check our work we see: $F0_{16} = 11110000_2 = -(00001111) by 1's complement = -(15_{10})$.