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JHU EP 605.204 – Computer Organization

Module 13 Assignment: Quantum Computation

State Vectors

Given the following state vector, please calculate the following and show your work:

1. **Show that a state vector must be unitary, that is, that the sum of the squares of the probability amplitudes sum to 1.**
2. **How many classical bits of information will we obtain by performing a measurement on psi?**
3. **What is the probability that the result of a measurement on psi ends in a 0?**
4. **What is the probability that the result of a measurement on psi begins with a 1?**

$$|\psi\rangle = \sqrt{\frac{3}{8}}|10\rangle + \sqrt{\frac{3}{8}}|01\rangle - \frac{1}{2}|11\rangle$$

1) $0^2+(\sqrt{\frac{3}{8}})^2+(\sqrt{\frac{3}{8}})^2+(-\frac{1}{2})^2=\frac{3}{8}+\frac{3}{8}+\frac{2}{8}=1$

The sum of the squares of the probability amplitudes is equal to 1.

2) We will only receive 1 Classical bit of information when we measure ψ.

3) The probability of measurement on ψ ends in 0

$=$ Sum of corresponding probability amplitudes squared

$= 0^2+(\sqrt{\frac{3}{8}})^2 = \frac{3}{8} = 37.5\%$

4) Similar idea to Q3.

The probability of measurement on ψ begins with 1

$= (\sqrt{\frac{3}{8}})^2+(-\frac{1}{2})^2 = \frac{3}{8}+\frac{2}{8}=\frac{5}{8} = 62.5\%$

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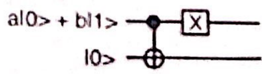
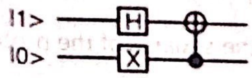
Quantum Circuits

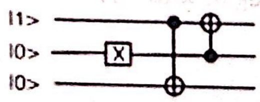
Calculate the outputs for each of the following 2-qubit (tensor) circuits. Please show your results as both a state vector and algebraically (matrix format):

1. 

$X(\alpha|0\rangle + b|1\rangle) = \alpha|1\rangle + b|0\rangle.$

$CNOT(\alpha|1\rangle + b|0\rangle, |0\rangle) =$

2. 

3. 

① $CNOT(a|0\rangle + b|1\rangle, |0\rangle) \longrightarrow a|0\rangle + b|1\rangle \otimes |0\rangle \longrightarrow \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} \otimes |0\rangle = \begin{bmatrix} a \\ 0 \\ b \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \\ b \end{bmatrix} = a|00\rangle + b|11\rangle.$

$a|0\rangle + b|1\rangle \xrightarrow{X} a|1\rangle + b|0\rangle.$

$a|0\rangle + b|1\rangle \oplus |0\rangle \longrightarrow a|00\rangle + b|11\rangle.$

② $|1\rangle \xrightarrow{H} H(|1\rangle) = \frac{|0\rangle - |1\rangle}{\sqrt{2}}. \quad |0\rangle \xrightarrow{X} |1\rangle \xrightarrow{CNOT(|1\rangle, \frac{|0\rangle-|1\rangle}{\sqrt{2}})}. \quad \frac{-|10\rangle + |11\rangle}{\sqrt{2}}.$

$\frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \quad |1\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \frac{-|10\rangle + |11\rangle}{\sqrt{2}}.$

③ $|0\rangle \xrightarrow{X} |1\rangle \xrightarrow{CNOT(|1\rangle, |1\rangle)} |1\rangle|0\rangle, \quad \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |1\rangle|0\rangle.$

$|0\rangle \xrightarrow{CNOT(|1\rangle|0\rangle)} |1\rangle|1\rangle, \quad \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle|1\rangle.$

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Quantum Swap Circuit

Start by devising a classical circuit that takes two bits as input and swaps their values. Remember the truth table for a classical XOR shown below. As a hint you can perform a classical swap with just 3 gates. Please provide a drawing of your circuit.

1

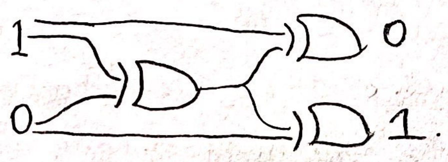
Bit 1	Bit 2	Output
0	0	0
0	1	1
1	0	1
1	1	0

Once you have your classical swap gate complete, devise a quantum swap gate that takes two qubits and swaps their values. Please provide a drawing of your quantum circuit and complete the following truth table:

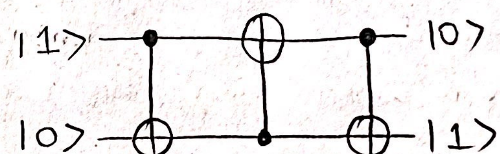
2

Qubit 1	Qubit 2	Output 1	Output 2
|0>	|0>	|0>	|0>
|0>	|1>	|0>	|1>
|1>	|0>	|1>	|1>
|1>	|1>	|1>	|0>

1



2



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Quantum Parallelism

1. How many basis states are there using 4 qubits? List them.
2. Given a single qubit that has been placed into a superposition, why can't we extract the values of alpha and beta directly? How many classical bits would we get when measuring such a qubit?
3. If we apply an X gate to a qubit in the 0 computational basis state N times, where N is even, what is the result.
4. If we apply an H gate to a qubit in the 1 computational basis state N times, where N is odd, what is the result? What are the possible values that could result from a measurement and how likely are each of the results to occur?
5. How do qubits allow us to store more information than classical bits? How can we leverage this to achieve "quantum advantage" (to solve problems using a quantum computer that are not possible classically)?

① 4 qubits will give $2^4 = 16$ basis states. They are:

$|0000\rangle$ $|0100\rangle$ $|1000\rangle$ $|1100\rangle$
$|0001\rangle$ $|0101\rangle$ $|1001\rangle$ $|1101\rangle$
$|0010\rangle$ $|0110\rangle$ $|1010\rangle$ $|1110\rangle$
$|0011\rangle$ $|0111\rangle$ $|1011\rangle$ $|1111\rangle$

② As the lecture states, when we measure a qubit that is in a superposition, we will only receive 1 classical bit of information. The superposition state is extremely fragile and is 'destroyed' when it's measured. So we can't extract the values of α & β directly.

③
$|0\rangle \xrightarrow{n=1} X|0\rangle = |1\rangle \xrightarrow{n=2} X|1\rangle = |0\rangle \ldots$

So if n is even, the result will be $|0\rangle$.

④ $|1\rangle \xrightarrow{n=1} H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \xrightarrow{n=2} H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = |1\rangle \xrightarrow{n=3} H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$.

So if n is odd, the result is $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$. possible values are $|0\rangle$ and $|1\rangle$. the probability of each is 50%.

⑤ Quantum advantage is about that quantum computing is able to offer a compelling solution to problems that are hard or intractable for classical bits. As the lecture states, the f(0), f(1) problem. we can use only 1 circuit, 2 qubits to solve both f(0) & f(1) simultaneously.