Visualising Black–Scholes Option Sensitivities

Erdös Quantitative Finance – Mini Project 3

Guoqin Liu

June 26, 2025

Road-map

1. Model & Method
2. Results – Call
3. Results – Put
4. Take-aways

Black–Scholes set-up

- Underlying spot price: $S_0 = 100$
- Strike price: $K = 110$ (out-of-the-money call, in-the-money put)
- Volatility: $\sigma = 30\%$
- Risk-free rate: $r = 0$ (for clarity)
- Time grid: $T \in [1/12,\ 5]$ years (70 points)

Key Greeks

Call price
$$C(S_0, T) = S_0\, N(d_1) - Ke^{-rT} N(d_2)$$

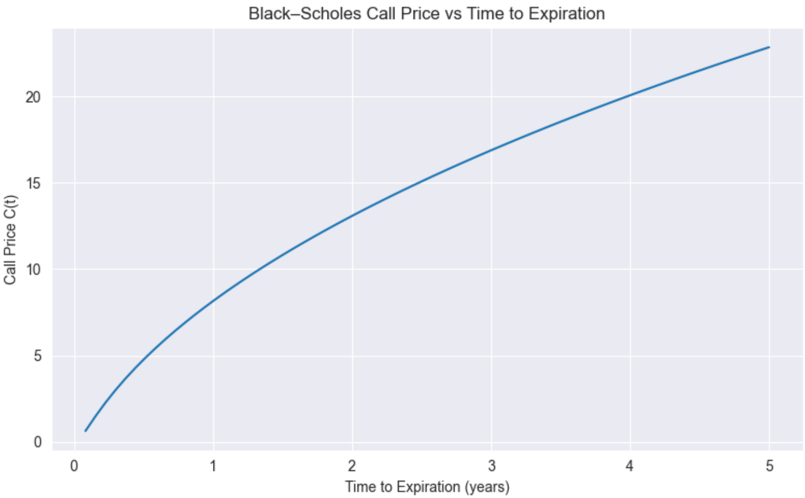
Put price
$$P(S_0, T) = -S_0\, N(-d_1) + Ke^{-rT} N(-d_2)$$

where $d_1 = \dfrac{\ln \frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Python helper snip

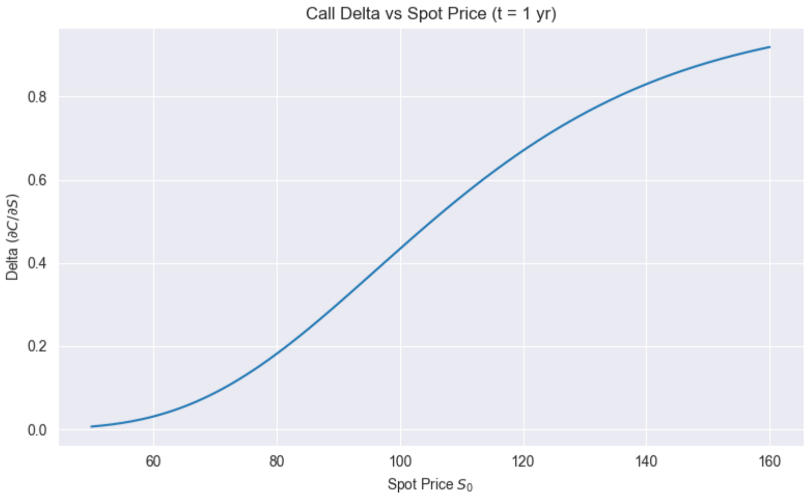
```
from scipy.stats import norm

def bs_call(S0,K,sigma,t,r=0):
    d1=(np.log(S0/K)+(r+.5*sigma**2)*t)/(sigma*np.sqrt(t))
    d2=d1-sigma*np.sqrt(t)
    return S0*norm.cdf(d1)-K*np.exp(-r*t)*norm.cdf(d2)

def call_delta(S0,K,sigma,t,r=0):
    d1=(np.log(S0/K)+(r+.5*sigma**2)*t)/(sigma*np.sqrt(t))
    return norm.cdf(d1)
# put: similar, delta = call_delta - 1
```

Call price vs time



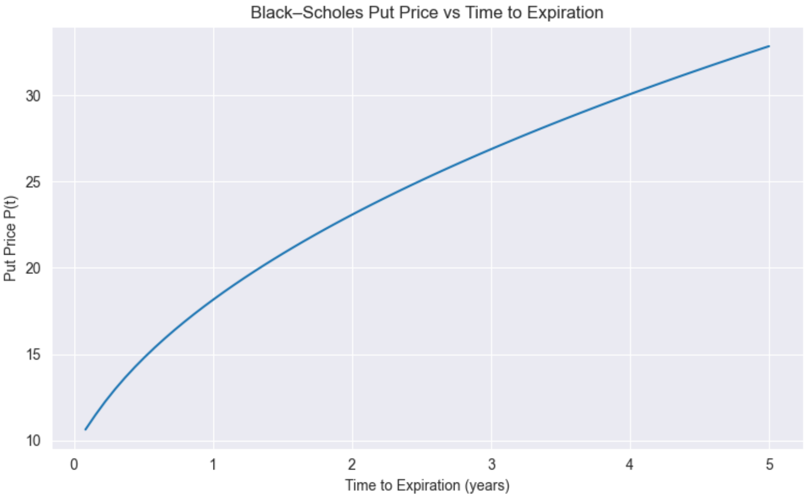
Price rises quickly when T is large (high time-value) $\rightarrow$ flattens as expiry approaches.

Call Δ vs spot price



Δ transitions $0 \to 1$: minimal sensitivity deep OTM, maximum slope (highest Γ) near ATM, stock-like deep ITM.

Put price vs time



Similar curvature: high time-value far from expiry, converging to intrinsic value $K - S_0$.

Put Δ vs spot price

Mirror image of call Δ: starts near -1 deep ITM and approaches 0 deep OTM.

Key observations

- **Time dependence** : Both call and put exhibit diminishing marginal time-value as $T \to 0$ (*theta* becomes less negative).
- **Spot dependence** : S-shaped Δ curves – maximal Γ at-the-money.
- **Trading intuition** : Deep ITM options mimic the underlying (call $\Delta \approx 1$, put $\Delta \approx -1$); deep OTM behave like lottery tickets ($|\Delta| \ll 1$).