High-Risk vs. Low-Risk Portfolio Analysis

Mini Project 1 Executive Summary, Guoqin Liu

Objective. Construct and compare two equal-weight portfolios—one "defensive" and one "aggressive"—using publicly–available market data from January 2023 to the present. The goal was to illustrate how systematic risk (beta) and recent momentum translate into very different risk-return profiles.

Methodology Highlights.

- *Data pipeline.* Daily adjusted closes were pulled via `yfinance`, then log-returns computed in `pandas/numpy` for both individual tickers and the S&P 500 benchmark.
- *Systematic-risk quantification.* A five-year beta for each security was calculated as the slope of the return covariance with the market, fully vectorised for efficiency.
- *Momentum screen.* A simple 52-week price-change filter ranked candidates for each basket.
- *Portfolio construction.* Equal 20 % weights ensured direct, transparent translation of asset betas into a portfolio-level beta.

Key Results.

- **Low-Risk basket** ($\bar{\beta} = 0.23$): consumer-staples blue-chips and a bond ETF delivered a respectable 7.3% trailing return while halving market volatility.
- **High-Risk basket** ($\bar{\beta} = 2.11$): AI, EV, crypto and biotech names produced a spectacular 118% weighted return, at the cost of far larger draw-downs.
- The side-by-side contrast crystallises the classic volatility–upside trade-off and the importance of diversification and periodic rebalancing.

Skill-building & Accomplishments.

1. Deepened practical understanding of beta estimation and market-model assumptions, including nuances of covariance stability with rolling windows.
2. Self-taught the `yfinance` API and automated data cleaning to create a fully reproducible notebook pipeline.
3. Implemented vectorised NumPy operations that cut runtime from minutes to seconds for large ticker sets.
4. Experimented with visual storytelling in Beamer slides, refining the ability to distill quantitative insights for non-technical audiences.

Future Directions. Extend the framework to multi-factor models (Fama–French, momentum, quality) and stress-test portfolios under historical shock scenarios to assess tail-risk resilience.

Hypothesis Testing of Normality in ETF Returns
Mini Project 2 Executive Summary, Guoqin Liu

Objective. Evaluate whether daily log-returns for five flagship ETFs obey the Normal distribution often assumed in textbook portfolio theory, and determine when (if ever) that assumption is tenable.

Methodology Highlights.

- *Dataset.* SPY, QQQ, IWM, TLT, GLD; 3 900 daily observations from 2010-01-01 to 2025-06-26.
- *Battery of normality tests:* Shapiro-Wilk, Anderson-Darling, Jarque-Bera, Kolmogorov-Smirnov. All four must retain H_0 to "accept" normality.
- *Extensions.* 252-day rolling windows, 1 % Winsorisation of tails, and an equal-weight five-asset portfolio.

Key Results.

- **Whole-period data:** Every ETF decisively rejects $N(\mu, \sigma^2)$; heavy tails and volatility clustering are the main culprits.
- **Rolling windows:** Only ∼1–2 % of 1-year windows pass all four tests (typically during very calm market regimes).
- **Tail-trimming:** Winsorising reduces excess kurtosis but still fails AD/KS, showing that non-Gaussian mass extends beyond the outermost 1 %.
- **Portfolio aggregation:** A 20 % equal-weight basket still rejects normality, indicating that diversification alone cannot eliminate fat-tail risk.

Skill-Building & Accomplishments.

1. Mastered four distinct goodness-of-fit tests in `scipy.stats`, including their differing sensitivities to skew, kurtosis and CDF deviations.
2. Automated a rolling-window framework that processes 5 × 3 650 subsets in seconds via vectorised NumPy operations.
3. Self-taught efficient Winsorisation techniques and built re-usable plotting utilities for p-value heat-maps and distribution diagnostics.

Future Directions. Incorporate GARCH-scaled returns, fit heavy-tailed (Student-t, skew-t) distributions, and re-estimate VaR/ES under non-Gaussian regimes. Extend the analysis to global assets and risk-parity weightings.

Visualising Black–Scholes Option Sensitivities

Mini Project 3 Executive Summary, Guoqin Liu

Objective. Illustrate how Black–Scholes call/put prices and their first-order Greek (Δ) respond to *time to expiry* and *spot price* for an out-of-the-money call / in-the-money put ($S_0 = 100,\ K = 110,\ \sigma = 30\%,\ r = 0$).

Methodology Highlights.

- Analytical formulas for C, P, and Δ coded in vectorised Python; helper functions leverage `scipy.stats.norm`.
- Generated 70-point grids for $T \in [1/12, 5]$ yrs and $S_0 \in [50, 160]$ to build smooth plots of price– and delta–surfaces.
- Adopted clear styling and captioned slides to emphasise intuition behind curvature and "S-shape" Delta behaviour.

Key Results.

- *Time decay:* Call price rises rapidly for long maturities, then flattens as expiry approaches; put shows the same curvature, converging to intrinsic value $K - S_0$.
- *Spot sensitivity:* Call Δ moves smoothly from 0 (deep OTM) to 1 (deep ITM); put Δ is its mirror, from –1 to 0. Steepest slope (highest Γ) occurs near-ATM.
- *Trading intuition:* Deep-ITM options mimic the underlying, whereas deep-OTM behave like lottery tickets with negligible Delta.

Skill-Building & Accomplishments.

1. Reinforced theoretical grasp of Black–Scholes Greeks and their economic meaning.
2. Self-taught efficient vectorisation and plot aesthetics to convey continuous sensitivities.
3. Developed re-usable snippets for option pricing that will feed later Monte-Carlo and hedging projects.

Future Directions. Extend to higher-order Greeks (Γ, θ, ρ), implied-vol surfaces, and stress-test hedging error under stochastic volatility.

Δ– and σ–Hedging under Stochastic Volatility

Mini Project 4 Executive Summary, Guoqin Liu

Objective. Quantify the residual P&L that survives daily delta-hedging a short European call when volatility is *stochastic*, and test whether adding a vega-neutral "σ-hedge" (a second option) can materially suppress that risk.

Methodology Highlights.

- Simulated price paths under three volatility engines: a toy three-state σ_t, the Heston stochastic-vol model, and a GARCH(1,1) process.
- Re-hedged Δ each trading day for a one-year option with $S_0 = K = 100$, $r = 2\%$; ran 1 000 paths for rapid diagnostics and 30 000 for production statistics to stabilise tail estimates.

Key Results.

- **Heston (high vol-of-vol $\xi = 0.35$):** Fat left tail down to –6 % and $\sigma_{\text{P\&L}} \approx 2.0$ even after delta hedging.
- **GARCH(1,1):** Much tighter distribution ($\sigma_{\text{P\&L}} \approx 0.19$) with only mild skew because variance shocks decay quickly.
- Residual risk scales with Vega×vol-of-vol; the sign of mean P&L flips with the gap between realised and implied volatility.
- A rolling vega-neutral σ-hedge cuts P&L volatility by roughly 70–80 %.

Skill-Building & Accomplishments.

1. Implemented Monte-Carlo engines for Heston and GARCH with antithetic variance-reduction and exact/OU discretisations.
2. Automated dynamic hedging loops with vectorised `NumPy/Pandas`, shrinking runtime to seconds for 30 k paths.
3. Deepened intuition for vega risk and the economics of "vol-trading" via option overlays.

Future Directions. Sweep κ, ξ in Heston and α, β in GARCH; add leverage (EGARCH) effects, transaction-cost modelling, and intraday hedging schedules.