Supplementary Materials

I. Observability Analysis

We rewrite the state variables of the contact wrench estimation system as

$$\boldsymbol{x} \triangleq [\boldsymbol{v}^T, \boldsymbol{s}^T, \boldsymbol{\omega}^T, \boldsymbol{F_c}^T, \boldsymbol{M_c}^T]^T,$$

and organize the continuous-time true-state kinematics model of the contact wrench estimation system into the following control-affine form, that is

$$\dot{\boldsymbol{x}} = f_0(\boldsymbol{x}) + f_{u_{ss}}(\boldsymbol{x})u_{ss} + f_{u_{\epsilon ss}}(\boldsymbol{x})u_{\epsilon ss} + f_{u_s}(\boldsymbol{x})u_s + \boldsymbol{f}_{\boldsymbol{u_{ssr}}}(\boldsymbol{x})\boldsymbol{u_{ssr}} + \boldsymbol{f}_{\boldsymbol{u_{sr}}}(\boldsymbol{x})\boldsymbol{u_{sr}},$$

where all noises or perturbations are ignored, and

$$f_0 = \begin{bmatrix} \frac{\left(s_x^2-s_y^2-s_z^2+1\right)F_{c_x}+\left(2F_{c_y}s_y+2F_{c_z}s_z\right)s_x-2F_{c_z}s_y+2F_{c_y}s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\left(-s_x^2+s_y^2-s_z^2+1\right)F_{c_y}+(2F_{c_x}s_y+2F_{c_z})s_x+2s_z(F_{c_z}s_y-F_{c_x})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{(gm-F_{c_z})s_x^2+\left(2F_{c_x}s_z-2F_{c_y}\right)s_x+(gm-F_{c_z})s_y^2+\left(2F_{c_y}s_z+2F_{c_x}\right)s_y+\left(s_z^2+1\right)(gm+F_{c_z})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\omega_x s_x^2}{2} + \frac{(s_y\omega_y+s_z\omega_z)s_x}{2} - \frac{\omega_z s_y}{2} + \frac{\omega_y s_z}{2} + \frac{\omega_x}{2} \\ \frac{\omega_y s_y^2}{2} + \frac{(s_x\omega_x+s_z\omega_z)s_y}{2} + \frac{\omega_z s_x}{2} - \frac{\omega_x s_z}{2} + \frac{\omega_y}{2} \\ \frac{\omega_z s_z^2}{2} + \frac{(s_x\omega_x+s_y\omega_y)s_z}{2} - \frac{\omega_y s_x}{2} + \frac{\omega_x s_y}{2} + \frac{\omega_z}{2} \\ \frac{\omega_y\omega_z(J_{yy}-J_{zz})J_{xx}+M_{c_x}}{J_{xx}} \\ \frac{-\omega_x\omega_z(J_{xx}-J_{zz})J_{yy}+M_{c_y}}{J_{yy}} \\ \frac{\omega_x\omega_y(J_{xx}-J_{yy})J_{zz}+M_{c_z}}{J_{zz}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The measurement model of the contact wrench estimation system can be rewrite as

$$\boldsymbol{h} = \begin{bmatrix} \frac{-\frac{u_s d_x\left(s_x^2-s_y^2-s_z^2+1\right)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_x(2s_xs_y-2s_z)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_x(2s_xs_z+2s_y)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_x}}{m} \\ \frac{-\frac{u_s d_y(2s_xs_y+2s_z)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_y\left(-s_x^2+s_y^2-s_z^2+1\right)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_y(2s_ys_z-2s_x)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_y}}{m} \\ \frac{k_f u_{ss}-\frac{u_s d_z(2s_xs_z-2s_y)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_z(2s_ys_z+2s_x)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_z\left(-s_x^2-s_y^2+s_z^2+1\right)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_z}}{m} \\ \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \\ s_x \\ s_y \\ s_z \end{bmatrix}.$$

Then, taking the Lie derivatives yields

$$
\mathcal{O} \triangleq \begin{bmatrix} \mathcal{L}^0 \boldsymbol{h} \\ \mathcal{L}^1_{f_0} \boldsymbol{h} \end{bmatrix} = \begin{bmatrix} h \\ \nabla_{\boldsymbol{x}} (\mathcal{L}^0 \boldsymbol{h}) \cdot f_0 \end{bmatrix} = \begin{bmatrix}
\frac{-\frac{u_s d_x\left(s_x^2-s_y^2-s_z^2+1\right)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_x\left(2s_x s_y-2s_z\right)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_x\left(2s_x s_z+2s_y\right)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_x}}{m} \\
\frac{-\frac{u_s d_y\left(2s_x s_y+2s_z\right)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_y\left(-s_x^2+s_y^2-s_z^2+1\right)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_y\left(2s_y s_z-2s_x\right)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_y}}{m} \\
\frac{k_f u_{ss}-\frac{u_s d_z\left(2s_x s_z-2s_y\right)v_x}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_z\left(2s_y s_z+2s_x\right)v_y}{s_x^2+s_y^2+s_z^2+1}-\frac{u_s d_z\left(-s_x^2-s_y^2+s_z^2+1\right)v_z}{s_x^2+s_y^2+s_z^2+1}+F_{c_z}}{m} \\
\omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \\ s_x \\ s_y \\ s_z \\ o_x \\ o_y \\ o_z \\
\frac{\omega_y\omega_z(J_{yy}-J_{zz})J_{xx}+M_{c_x}}{J_{xx}} \\
\frac{-\omega_x\omega_z(J_{xx}-J_{zz})J_{yy}+M_{c_y}}{J_{yy}} \\
\frac{\omega_x\omega_y(J_{xx}-J_{yy})J_{zz}+M_{c_z}}{J_{zz}} \\
\frac{\left(s_x^2-s_y^2-s_z^2+1\right)F_{c_x}+\left(2F_{c_y}s_y+2F_{c_z}s_z\right)s_x-2F_{c_z}s_y+2F_{c_y}s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\
\frac{\left(-s_x^2+s_y^2-s_z^2+1\right)F_{c_y}+(2F_{c_x}s_y+2F_{c_z})s_x+2s_z(F_{c_z}s_y-F_{c_x})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\
\frac{(gm-F_{c_z})s_x^2+\left(2F_{c_x}s_z-2F_{c_y}\right)s_x+(gm-F_{c_z})s_y^2+\left(2F_{c_y}s_z+2F_{c_x}\right)s_y+\left(s_z^2+1\right)(gm+F_{c_z})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\
\frac{\omega_x s_x^2}{2}+\frac{(s_y\omega_y+s_z\omega_z)s_x}{2}-\frac{\omega_z s_y}{2}+\frac{\omega_y s_z}{2}+\frac{\omega_x}{2} \\
\frac{\omega_y s_y^2}{2}+\frac{(s_x\omega_x+s_z\omega_z)s_y}{2}+\frac{\omega_z s_x}{2}-\frac{\omega_x s_z}{2}+\frac{\omega_y}{2} \\
\frac{\omega_z s_z^2}{2}+\frac{(s_x\omega_x+s_y\omega_y)s_z}{2}-\frac{\omega_y s_x}{2}+\frac{\omega_x s_y}{2}+\frac{\omega_z}{2}
\end{bmatrix},
$$

where

$$
\begin{aligned}
o_x = &-\frac{2d_x\left(\left(\left(-\frac{v_z\omega_y}{2}+\frac{\omega_z v_y}{2}\right)s_x^2+\left(-\omega_z s_y v_x+\left(\omega_y v_x+g\right)s_z+v_z\omega_z+\omega_y v_y\right)s_x\right)m\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2} \\
&-\frac{2d_x\left(\left(\left(-\frac{v_z\omega_y}{2}-\frac{\omega_z v_y}{2}\right)s_y^2+\left(\left(\omega_y v_y-v_z\omega_z\right)s_z-\omega_y v_x+g\right)s_y\right)m\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2} \\
&-\frac{2d_x\left(\left(\left(\frac{v_z\omega_y}{2}+\frac{\omega_z v_y}{2}\right)s_z^2-\omega_z s_z v_x+\frac{v_z\omega_y}{2}-\frac{\omega_z v_y}{2}\right)m+\frac{F_{c_x}\left(s_x^2+s_y^2+s_z^2+1\right)}{2}\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2}, \\
o_y = &\frac{2d_y u_s\left(\left(\left(-\frac{v_z\omega_x}{2}-\frac{\omega_z v_x}{2}\right)s_x^2+\left(-\omega_z s_y v_y+\left(\omega_x v_x-v_z\omega_z\right)s_z+\omega_x v_y+g\right)s_x\right)m\right)}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2} \\
&+\frac{2d_y u_s\left(\left(\left(-\frac{v_z\omega_x}{2}+\frac{\omega_z v_x}{2}\right)s_y^2+\left(\left(\omega_x v_y-g\right)s_z-v_z\omega_z-\omega_x v_x\right)s_y\right)m\right)}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2} \\
&+\frac{2d_y u_s\left(\left(\left(\frac{v_z\omega_x}{2}+\frac{\omega_z v_x}{2}\right)s_z^2+\omega_z s_z v_y+\frac{v_z\omega_x}{2}-\frac{\omega_z v_x}{2}\right)m-\frac{F_{c_y}\left(s_x^2+s_y^2+s_z^2+1\right)}{2}\right)}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2}, \\
o_z = &\frac{d_z\left(\left(\left(\omega_y v_x+\omega_x v_y+g\right)s_x^2+\left(\left(-2\omega_x v_x+2\omega_y v_y\right)s_y+2v_z\left(\omega_y s_z+\omega_x\right)\right)s_x\right)m\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2} \\
&+\frac{d_z\left(\left(\left(-\omega_y v_x-\omega_x v_y+g\right)s_y^2-2v_z\left(\omega_x s_z-\omega_y\right)s_y\right)m\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2}
\end{aligned}
$$

$$+\frac{d_z\left(\left(\left(-\omega_y v_x+\omega_x v_y-g\right)s_z^2+\left(-2\omega_x v_x-2\omega_y v_y\right)s_z-\omega_x v_y+\omega_y v_x-g\right)m-F_{c_z}\left(s_x^2+s_y^2+s_z^2+1\right)\right)u_s}{\left(s_x^2+s_y^2+s_z^2+1\right)m^2}.$$

If $\nabla_{\boldsymbol{x}}\mathcal{O}$ is of full column rank, the system is observable. The $\nabla_{\boldsymbol{x}}\mathcal{O}$ is given by

$$\nabla_{\boldsymbol{x}}\mathcal{O}=\begin{bmatrix}\boldsymbol{M}_{1,1} & \boldsymbol{M}_{1,2} & \mathbf{0} & c_{1,4}\boldsymbol{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0}\\ \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \boldsymbol{M}_{5,1} & \boldsymbol{M}_{5,2} & \boldsymbol{M}_{5,3} & \boldsymbol{M}_{5,4} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{M}_{6,3} & \mathbf{0} & \boldsymbol{M}_{6,5}\\ \mathbf{0} & \boldsymbol{M}_{7,2} & \mathbf{0} & \boldsymbol{M}_{7,4} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{M}_{8,2} & \boldsymbol{M}_{8,3} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$

where all block terms are 3×3 matrices, $\boldsymbol{I}$ is identity matrix, $\mathbf{0}$ is zero matrix, and $\boldsymbol{M}_{i,j}$ is the block term with index (i,j). By denoting the element at the x-th row and y-th column of $\boldsymbol{M}_{i,j}$ as $\boldsymbol{M}_{i,j}(x,y)$, we have

$$\begin{aligned}
\boldsymbol{M}_{1,1}(1,1)&=\frac{2u_s d_x\left(-s_y^3 v_y+\left(-2s_x v_x-s_z v_z\right)s_y^2+\left(s_x^2 v_y-s_z^2 v_y+2s_x v_z-v_y\right)s_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&+\frac{2u_s d_x\left(s_z\left(v_z s_x^2-2s_x v_x s_z-v_z s_z^2-2s_x v_y-v_z\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(1,2)&=-\frac{2u_s d_x\left(s_x^3 v_y+\left(-2s_y v_x+v_z\right)s_x^2+\left(-s_y^2 v_y-2s_y s_z v_z+v_y\left(s_z^2+1\right)\right)s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&-\frac{2u_s d_x\left(-v_z s_y^2+\left(2v_y s_z-2v_x\right)s_y+v_z\left(s_z^2+1\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(1,3)&=-\frac{2\left(s_x^3 v_z+\left(-2s_z v_x-v_y\right)s_x^2+\left(-v_z s_z^2-2s_y v_y s_z+v_z\left(s_y^2+1\right)\right)s_x\right)u_s d_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&-\frac{2\left(s_z^2 v_y+\left(-2v_z s_y-2v_x\right)s_z-s_y^2 v_y-v_y\right)u_s d_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(2,1)&=\frac{2u_s\left(-s_y^3 v_x+\left(2s_x v_y+v_z\right)s_y^2+\left(s_x^2 v_x+2s_x s_z v_z-s_z^2 v_x-v_x\right)s_y\right)d_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&+\frac{2u_s\left(-v_z s_x^2+\left(2s_z v_x+2v_y\right)s_x+v_z\left(s_z^2+1\right)\right)d_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(2,2)&=-\frac{2u_s d_y\left(s_x^3 v_x+\left(2v_y s_y+s_z v_z\right)s_x^2+\left(-s_y^2 v_x+s_z^2 v_x+2v_z s_y+v_x\right)s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&-\frac{2u_s d_y\left(-s_z\left(v_z s_y^2-2s_y v_y s_z-v_z s_z^2+2s_y v_x-v_z\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(2,3)&=-\frac{2u_s d_y\left(-s_z^2 v_x+\left(2s_x v_z-2v_y\right)s_z+v_x\left(s_x^2+1\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&-\frac{2u_s d_y\left(s_y^3 v_z+\left(-2v_y s_z+v_x\right)s_y^2+\left(-v_z s_z^2-2s_x v_x s_z+v_z\left(s_x^2+1\right)\right)s_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(3,1)&=\frac{2u_s d_z\left(-s_z^3 v_x+\left(2s_x v_z-v_y\right)s_z^2+\left(s_x^2 v_x+2s_x s_y v_y-s_y^2 v_x-v_x\right)s_z\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&+\frac{2u_s d_z\left(s_x^2 v_y+\left(-2s_y v_x+2v_z\right)s_x-s_y^2 v_y-v_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{1,1}(3,2)&=-\frac{2u_s d_z\left(s_z^3 v_y+\left(-2v_z s_y-v_x\right)s_z^2+\left(-s_y^2 v_y-2s_y v_x s_x+v_y\left(s_x^2+1\right)\right)s_z\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&-\frac{2u_s d_z\left(s_y^2 v_x+\left(-2s_x v_y-2v_z\right)s_y-s_x^2 v_x-v_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{M}_{1,1}(3,3) = & -\frac{2u_s d_z \left(s_x^3 v_x + \left(v_y s_y + 2 s_z v_z\right) s_x^2 + \left(s_y^2 v_x - s_z^2 v_x - 2 v_y s_z + v_x\right) s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_z \left(s_y \left(s_y^2 v_y + 2 s_y s_z v_z - s_z^2 v_y + 2 s_z v_x + v_y\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(1,1) = & \frac{2u_s d_x \left(-s_y^3 v_y + \left(-2 s_x v_x - s_z v_z\right) s_y^2 + \left(s_x^2 v_y - s_z^2 v_y + 2 s_x v_z - v_y\right) s_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& +\frac{2u_s d_x \left(s_z \left(v_z s_x^2 - 2 s_x v_x s_z - v_z s_z^2 - 2 s_x v_y - v_z\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(1,2) = & -\frac{2u_s d_x \left(s_x^3 v_y + \left(-2 s_y v_x + v_z\right) s_x^2 + \left(-s_y^2 v_y - 2 s_y s_z v_z + v_y\left(s_z^2+1\right)\right) s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_x \left(-v_z s_y^2 + \left(2 v_y s_z - 2 v_x\right) s_y + v_z\left(s_z^2+1\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(1,3) = & -\frac{2\left(s_x^3 v_z + \left(-2 s_z v_x - v_y\right) s_x^2 + \left(-v_z s_z^2 - 2 s_y v_y s_z + v_z\left(s_y^2+1\right)\right) s_x\right) u_s d_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2\left(s_z^2 v_y + \left(-2 v_z s_y - 2 v_x\right) s_z - s_y^2 v_y - v_y\right) u_s d_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(2,1) = & \frac{2u_s \left(-s_y^3 v_x + \left(2 s_x v_y + v_z\right) s_y^2 + \left(s_x^2 v_x + 2 s_x s_z v_z - s_z^2 v_x - v_x\right) s_y\right) d_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& +\frac{2u_s \left(-v_z s_x^2 + \left(2 s_z v_x + 2 v_y\right) s_x + v_z\left(s_z^2+1\right)\right) d_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(2,2) = & -\frac{2u_s d_y \left(s_x^3 v_x + \left(2 v_y s_y + s_z v_z\right) s_x^2 + \left(-s_y^2 v_x + s_z^2 v_x + 2 v_z s_y + v_x\right) s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_y \left(-s_z \left(v_z s_y^2 - 2 s_y v_y s_z - v_z s_z^2 + 2 s_y v_x - v_z\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(2,3) = & -\frac{2u_s d_y \left(s_y^3 v_z + \left(-2 v_y s_z + v_x\right) s_y^2 + \left(-v_z s_z^2 - 2 s_x v_x s_z + v_z\left(s_x^2+1\right)\right) s_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_y \left(-s_z^2 v_x + \left(2 s_x v_z - 2 v_y\right) s_z + v_x\left(s_x^2+1\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(3,1) = & \frac{2u_s d_z \left(-s_z^3 v_x + \left(2 s_x v_z - v_y\right) s_z^2 + \left(s_x^2 v_x + 2 s_x s_y v_y - s_y^2 v_x - v_x\right) s_z\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& \frac{2u_s d_z \left(s_x^2 v_y + \left(-2 s_y v_x + 2 v_z\right) s_x - s_y^2 v_y - v_y\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(3,2) = & -\frac{2u_s d_z \left(s_z^3 v_y + \left(-2 v_z s_y - v_x\right) s_z^2 + \left(-s_y^2 v_y - 2 s_y v_x s_x + v_y\left(s_x^2+1\right)\right) s_z\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_z \left(s_y^2 v_x + \left(-2 s_x v_y - 2 v_z\right) s_y - s_x^2 v_x - v_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{1,2}(3,3) = & -\frac{2u_s d_z \left(s_x^3 v_x + \left(v_y s_y + 2 s_z v_z\right) s_x^2 + \left(s_y^2 v_x - s_z^2 v_x - 2 v_y s_z + v_x\right) s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
& -\frac{2u_s d_z \left(s_y \left(s_y^2 v_y + 2 s_y s_z v_z - s_z^2 v_y + 2 s_z v_x + v_y\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
c_{1,4} = & \frac{1}{m} \\
\boldsymbol{M}_{5,1}(1,1) = & \frac{2 d_x \left(s_x s_y \omega_z - s_x s_z \omega_y + s_y \omega_y + s_z \omega_z\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)},
\end{aligned}
$$

$$\boldsymbol{M}_{5,1}(1,2) = -\frac{u_s\left(\left(s_x^2 - s_y^2 + s_z^2 - 1\right)\omega_z + 2\omega_y\left(s_y s_z + s_x\right)\right)d_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(1,3) = \frac{u_s d_x\left(\left(s_x^2 + s_y^2 - s_z^2 - 1\right)\omega_y - 2\omega_z\left(-s_y s_z + s_x\right)\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(2,1) = -\frac{\left(\left(s_x^2 - s_y^2 - s_z^2 + 1\right)\omega_z - 2\omega_x\left(s_x s_z - s_y\right)\right)u_s d_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(2,2) = -\frac{2\left(\left(\omega_z s_y - \omega_x\right)s_x - s_z\left(\omega_x s_y + \omega_z\right)\right)u_s d_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(2,3) = -\frac{u_s\left(\left(s_x^2 + s_y^2 - s_z^2 - 1\right)\omega_x + 2\omega_z\left(s_x s_z + s_y\right)\right)d_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(3,1) = \frac{u_s d_z\left(\left(s_x^2 - s_y^2 - s_z^2 + 1\right)\omega_y - 2\omega_x\left(s_x s_y + s_z\right)\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(3,2) = \frac{u_s\left(\left(s_x^2 - s_y^2 + s_z^2 - 1\right)\omega_x + 2\omega_y\left(s_x s_y - s_z\right)\right)d_z}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,1}(3,3) = \frac{2u_s d_z\left(\left(\omega_y s_z + \omega_x\right)s_x - \left(\omega_x s_z - \omega_y\right)s_y\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\begin{aligned}
\boldsymbol{M}_{5,2}(1,1) = {} & \frac{2d_x u_s\left(\left(-\omega_y v_x - g\right)s_z^3 + \left(2v_z\omega_y s_x + \omega_z s_y v_x - \omega_y v_y - v_z\omega_z\right)s_z^2\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& + \frac{2d_x u_s\left(\left(\left(\omega_y v_x + g\right)s_x^2 + \left(\left(2\omega_y v_y - 2v_z\omega_z\right)s_y - 2\omega_z v_x\right)s_x - \left(s_y^2 + 1\right)\left(\omega_y v_x + g\right)\right)s_z\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& + \frac{2d_x u_s\left(\left(-\omega_z s_y v_x + \omega_y v_y + v_z\omega_z\right)s_x^2\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& + \frac{2d_x u_s\left(\left(-2s_y^2 v_y\omega_z + \left(-2\omega_y v_x + 2g\right)s_y + 2v_z\omega_y - 2\omega_z v_y\right)s_x\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& + \frac{2d_x u_s\left(-\left(s_y^2 + 1\right)\left(-\omega_z s_y v_x + \omega_y v_y + v_z\omega_z\right)\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{M}_{5,2}(1,2) = {} & -\frac{2d_x\left(\left(\omega_y v_y - v_z\omega_z\right)s_z^3 + \left(\left(-2\omega_z v_y - 2v_z\omega_y\right)s_y - \omega_z v_x s_x - \omega_y v_x + g\right)s_z^2\right)u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2d_x\left(\left(\left(-\omega_y v_y + v_z\omega_z\right)s_y^2 + \left(\left(-2\omega_y v_x - 2g\right)s_x + 2\omega_z v_x\right)s_y - \left(s_x^2 + 1\right)\left(-\omega_y v_y + v_z\omega_z\right)\right)s_z\right)u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2d_x\left(\left(\omega_z v_x s_x + \omega_y v_x - g\right)s_y^2 - 2\left(\omega_z s_x + \omega_y\right)\left(s_x v_y + v_z\right)s_y + \left(s_x^2 + 1\right)\left(-\omega_z v_x s_x - \omega_y v_x + g\right)\right)u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{M}_{5,2}(1,3) = {} & -\frac{2\left(\left(\omega_y v_y - v_z\omega_z\right)s_y^3 + \left(\left(\omega_y v_x + g\right)s_x + \left(2\omega_z v_y + 2v_z\omega_y\right)s_z - \omega_z v_x\right)s_y^2\right)d_x u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2\left(\left(\left(\omega_y v_y - v_z\omega_z\right)s_x^2 + 2\omega_z s_x s_z v_x + \left(-\omega_y v_y + v_z\omega_z\right)s_z^2 + \left(2\omega_y v_x - 2g\right)s_z - v_z\omega_z + \omega_y v_y\right)s_y\right)d_x u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2\left(\left(\omega_y v_x + g\right)s_x^3 + \left(2s_z v_z\omega_y - \omega_z v_x\right)s_x^2\right)d_x u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2\left(\left(\left(-\omega_y v_x - g\right)s_z^2 + \left(-2\omega_y v_y - 2v_z\omega_z\right)s_z + \omega_y v_x + g\right)s_x + \omega_z\left(s_z^2 v_x + 2v_y s_z - v_x\right)\right)d_x u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{M}_{5,2}(2,1) = {} & -\frac{2\left(\left(-\omega_x v_x + v_z\omega_z\right)s_z^3 + \left(\left(2\omega_z v_x + 2v_z\omega_x\right)s_x + \omega_z s_y v_y - \omega_x v_y - g\right)s_z^2\right)d_y u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\
& - \frac{2\left(\left(\left(\omega_x v_x - v_z\omega_z\right)s_x^2 + \left(\left(2\omega_x v_y - 2g\right)s_y + 2\omega_z v_y\right)s_x + \left(s_y^2 + 1\right)\left(-\omega_x v_x + v_z\omega_z\right)\right)s_z\right)d_y u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}
\end{aligned}$$

$$
- \frac{2\left(\left(-\omega_z s_y v_y + \omega_x v_y + g\right) s_x^2 + 2\left(-\omega_z s_y + \omega_x\right)\left(-s_y v_x + v_z\right) s_x - \left(s_y^2+1\right)\left(-\omega_z s_y v_y + \omega_x v_y + g\right)\right) d_y u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
$$

$$
\begin{aligned}
\boldsymbol{M}_{5,2}(2,2) = &- \frac{2d_y\left(\left(-\omega_x v_y + g\right) s_z^3 + \left(s_x v_y \omega_z + 2v_z\omega_x s_y + \omega_x v_x + v_z\omega_z\right) s_z^2\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y\left(\left(\left(\omega_x v_y - g\right) s_y^2 + \left(\left(2\omega_x v_x - 2v_z\omega_z\right) s_x + 2\omega_z v_y\right) s_y + \left(s_x^2+1\right)\left(-\omega_x v_y + g\right)\right) s_z\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y\left(\left(-s_x v_y \omega_z - \omega_x v_x - v_z\omega_z\right) s_y^2 + \left(-2s_x^2 v_x \omega_z + \left(2\omega_x v_y + 2g\right) s_x + 2v_z\omega_x - 2\omega_z v_x\right) s_y\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y\left(\left(s_x^2+1\right)\left(s_x v_y \omega_z + \omega_x v_x + v_z\omega_z\right)\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{M}_{5,2}(2,3) = &- \frac{2d_y u_s\left(\left(-\omega_x v_x + v_z\omega_z\right) s_x^3 + \left(\left(-\omega_x v_y + g\right) s_y + \left(-2\omega_z v_x - 2v_z\omega_x\right) s_z - \omega_z v_y\right) s_x^2\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y u_s\left(\left(\left(-\omega_x v_x + v_z\omega_z\right) s_y^2 - 2\omega_z s_y s_z v_y + \left(\omega_x v_x - v_z\omega_z\right) s_z^2 + \left(2\omega_x v_y + 2g\right) s_z + v_z\omega_z - \omega_x v_x\right) s_x\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y u_s\left(\left(-\omega_x v_y + g\right) s_y^3 + \left(-2s_z v_z \omega_x - \omega_z v_y\right) s_y^2\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{2d_y u_s\left(\left(\left(\omega_x v_y - g\right) s_z^2 + \left(-2\omega_x v_x - 2v_z\omega_z\right) s_z - \omega_x v_y + g\right) s_y + \omega_z\left(s_z^2 v_y - 2s_z v_x - v_y\right)\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{M}_{5,2}(3,1) = &\frac{4d_z\left(\left(\left(\frac{\omega_x v_x}{2} - \frac{\omega_y v_y}{2}\right) s_y - \frac{v_z\left(\omega_y s_z + \omega_x\right)}{2}\right) s_x^2\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{4d_z\left(\left(\left(\omega_y v_x + \omega_x v_y\right) s_y^2 + v_z\left(\omega_x s_z - \omega_y\right) s_y + \left(\omega_y v_x + g\right) s_z^2 + \left(\omega_x v_x + \omega_y v_y\right) s_z + \omega_x v_y + g\right) s_x\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{4d_z\left(\frac{\left(s_y^2+s_z^2+1\right)\left(\left(-\omega_x v_x + \omega_y v_y\right) s_y + v_z\left(\omega_y s_z + \omega_x\right)\right)}{2}\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{M}_{5,2}(3,2) = &\frac{4d_z\left(\left(\frac{\omega_y v_y}{2} - \frac{\omega_x v_x}{2}\right) s_x^3 + \left(\left(-\omega_y v_x - \omega_x v_y\right) s_y - \frac{v_z\left(\omega_x s_z - \omega_y\right)}{2}\right) s_x^2\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{4d_z\left(\left(\left(\frac{\omega_x v_x}{2} - \frac{\omega_y v_y}{2}\right) s_y^2 - v_z\left(\omega_y s_z + \omega_x\right) s_y - \frac{\left(s_z^2+1\right)\left(\omega_x v_x - \omega_y v_y\right)}{2}\right) s_x\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{4d_z\left(\frac{v_z\left(\omega_x s_z - \omega_y\right) s_y^2}{2} + \left(\left(-\omega_x v_y + g\right) s_z^2 + \left(\omega_x v_x + \omega_y v_y\right) s_z - \omega_y v_x + g\right) s_y - \frac{v_z\left(s_z^2+1\right)\left(\omega_x s_z - \omega_y\right)}{2}\right) u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{M}_{5,2}(3,3) = &- \frac{4\left(\left(-\frac{1}{2}v_z\omega_x s_y + \frac{1}{2}v_z\omega_y s_x - \frac{1}{2}\omega_x v_x - \frac{1}{2}\omega_y v_y\right) s_z^2\right) d_z u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{4\left(\left(\left(\omega_y v_x + g\right) s_x^2 + \left(\left(-\omega_x v_x + \omega_y v_y\right) s_y + v_z\omega_x\right) s_x + \left(-\omega_x v_y + g\right) s_y^2 + v_z\omega_y s_y - \omega_x v_y + \omega_y v_x\right) s_z\right) d_z u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&- \frac{4\left(\frac{\left(s_x^2+s_y^2+1\right)\left(-v_z\omega_y s_x + v_z\omega_x s_y + \omega_x v_x + \omega_y v_y\right)}{2}\right) d_z u_s}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\boldsymbol{M}_{5,3}(1,1) = 0,
$$

$$
\boldsymbol{M}_{5,3}(1,2) = \frac{\left(v_z s_x^2 + \left(-2s_z v_x - 2v_y\right) s_x + v_z s_y^2 + \left(-2v_y s_z + 2v_x\right) s_y - v_z s_z^2 - v_z\right) u_s d_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)},
$$

$$\boldsymbol{M}_{5,3}(1,3) = -\frac{\left(s_x^2 v_y + \left(-2 s_y v_x + 2 v_z\right) s_x - s_y^2 v_y - 2 s_y s_z v_z + s_z^2 v_y - 2 s_z v_x - v_y\right) u_s d_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,3}(2,1) = -\frac{\left(v_z s_x^2 + \left(-2 s_z v_x - 2 v_y\right) s_x + v_z s_y^2 + \left(-2 v_y s_z + 2 v_x\right) s_y - v_z s_z^2 - v_z\right) u_s d_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,3}(2,2) = 0,$$

$$\boldsymbol{M}_{5,3}(2,3) = -\frac{d_y u_s\left(s_x^2 v_x + 2 s_x s_y v_y + 2 s_x s_z v_z - s_y^2 v_x - s_z^2 v_x + 2 v_z s_y - 2 v_y s_z + v_x\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,3}(3,1) = \frac{d_z\left(s_x^2 v_y + \left(-2 s_y v_x + 2 v_z\right) s_x - s_y^2 v_y - 2 s_y s_z v_z + s_z^2 v_y - 2 s_z v_x - v_y\right) u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,3}(3,2) = \frac{d_z\left(s_x^2 v_x + 2 s_x s_y v_y + 2 s_x s_z v_z - s_y^2 v_x - s_z^2 v_x + 2 v_z s_y - 2 v_y s_z + v_x\right) u_s}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{5,3}(3,3) = 0,$$

$$\boldsymbol{M}_{5,4} = \begin{bmatrix} -\frac{u_s d_x}{m^2} & 0 & 0 \\ 0 & -\frac{u_s d_y}{m^2} & 0 \\ 0 & 0 & -\frac{u_s d_z}{m^2} \end{bmatrix},$$

$$\boldsymbol{M}_{6,3} = \begin{bmatrix} 0 & \omega_z\left(J_{yy} - J_{zz}\right) & \omega_y\left(J_{yy} - J_{zz}\right) \\ -\omega_z\left(J_{xx} - J_{zz}\right) & 0 & -\omega_x\left(J_{xx} - J_{zz}\right) \\ \omega_y\left(J_{xx} - J_{yy}\right) & \omega_x\left(J_{xx} - J_{yy}\right) & 0 \end{bmatrix},$$

$$\boldsymbol{M}_{6,5} = \begin{bmatrix} \frac{1}{J_{xx}} & 0 & 0 \\ 0 & \frac{1}{J_{yy}} & 0 \\ 0 & 0 & \frac{1}{J_{zz}} \end{bmatrix},$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(1,1) = {} & \frac{2 F_{c_y} s_y^3 + \left(4 F_{c_x} s_x + 2 F_{c_z} s_z\right) s_y^2 + \left(-2 F_{c_y} s_x^2 + 2 F_{c_y} s_z^2 + 4 F_{c_z} s_x + 2 F_{c_y}\right) s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{-2 s_z\left(-2 s_x F_{c_x} s_z + F_{c_z} s_x^2 - F_{c_z} s_z^2 + 2 F_{c_y} s_x - F_{c_z}\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(1,2) = {} & \frac{2 F_{c_y} s_x^3 + \left(-4 F_{c_x} s_y - 2 F_{c_z}\right) s_x^2 + \left(-2 F_{c_y} s_y^2 - 4 F_{c_z} s_y s_z + \left(2 s_z^2 + 2\right) F_{c_y}\right) s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{2 F_{c_z} s_y^2 + \left(-4 F_{c_y} s_z - 4 F_{c_x}\right) s_y + \left(-2 s_z^2 - 2\right) F_{c_z}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(1,3) = {} & \frac{2 F_{c_z} s_x^3 + \left(-4 F_{c_x} s_z + 2 F_{c_y}\right) s_x^2 + \left(-2 F_{c_z} s_z^2 - 4 s_y F_{c_y} s_z + \left(2 s_y^2 + 2\right) F_{c_z}\right) s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{-2 F_{c_y} s_z^2 + \left(4 F_{c_z} s_y - 4 F_{c_x}\right) s_z + \left(2 s_y^2 + 2\right) F_{c_y}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(2,1) = {} & \frac{2 F_{c_x} s_y^3 + \left(-4 F_{c_y} s_x + 2 F_{c_z}\right) s_y^2 + \left(-2 F_{c_x} s_x^2 - 4 F_{c_z} s_x s_z + \left(2 s_z^2 + 2\right) F_{c_x}\right) s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{-2 F_{c_z} s_x^2 + \left(4 F_{c_x} s_z - 4 F_{c_y}\right) s_x + \left(2 s_z^2 + 2\right) F_{c_z}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(2,2) = {} & \frac{2 F_{c_x} s_x^3 + \left(4 F_{c_y} s_y + 2 F_{c_z} s_z\right) s_x^2 + \left(-2 F_{c_x} s_y^2 + 2 F_{c_x} s_z^2 - 4 F_{c_z} s_y + 2 F_{c_x}\right) s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{-2 s_z\left(-2 s_y F_{c_y} s_z + F_{c_z} s_y^2 - F_{c_z} s_z^2 - 2 F_{c_x} s_y - F_{c_z}\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$\begin{aligned}\boldsymbol{M}_{7,2}(2,3) = {} & \frac{2 F_{c_z} s_y^3 + \left(-4 F_{c_y} s_z - 2 F_{c_x}\right) s_y^2 + \left(-2 F_{c_z} s_z^2 - 4 s_x F_{c_x} s_z + \left(2 s_x^2 + 2\right) F_{c_z}\right) s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} \\ & + \frac{2 F_{c_x} s_z^2 + \left(-4 F_{c_z} s_x - 4 F_{c_y}\right) s_z + \left(-2 s_x^2 - 2\right) F_{c_x}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{M}_{7,2}(3,1) &= \frac{2F_{c_x}s_z^3 + \left(-4F_{c_z}s_x - 2F_{c_y}\right)s_z^2 + \left(-2F_{c_x}s_x^2 - 4F_{c_y}s_xs_y + \left(2s_y^2+2\right)F_{c_x}\right)s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{2F_{c_y}s_x^2 + \left(-4F_{c_x}s_y - 4F_{c_z}\right)s_x + \left(-2s_y^2-2\right)F_{c_y}}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{7,2}(3,2) &= \frac{2F_{c_y}s_z^3 + \left(-4F_{c_z}s_y + 2F_{c_x}\right)s_z^2 + \left(-2F_{c_y}s_y^2 - 4s_xF_{c_x}s_y + \left(2s_x^2+2\right)F_{c_y}\right)s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&= \frac{-2F_{c_x}s_y^2 + \left(4F_{c_y}s_x - 4F_{c_z}\right)s_y + \left(2s_x^2+2\right)F_{c_x}}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}, \\
\boldsymbol{M}_{7,2}(3,3) &= \frac{2F_{c_x}s_x^3 + \left(2F_{c_y}s_y + 4F_{c_z}s_z\right)s_x^2 + \left(2F_{c_x}s_y^2 - 2F_{c_x}s_z^2 + 4F_{c_y}s_z + 2F_{c_x}\right)s_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2} \\
&+ \frac{2s_y\left(F_{c_y}s_y^2 - F_{c_y}s_z^2 + 2F_{c_z}s_ys_z - 2F_{c_x}s_z + F_{c_y}\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}
$$

$$
\boldsymbol{M}_{7,4} = \begin{bmatrix}
\frac{s_x^2-s_y^2-s_z^2+1}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{2s_xs_y+2s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{2s_xs_z-2s_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\
\frac{2s_xs_y-2s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{-s_x^2+s_y^2-s_z^2+1}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{2s_ys_z+2s_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\
\frac{2s_xs_z+2s_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{2s_ys_z-2s_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)} & \frac{-s_x^2-s_y^2+s_z^2+1}{m\left(s_x^2+s_y^2+s_z^2+1\right)}
\end{bmatrix},
$$

$$
\boldsymbol{M}_{8,2} = \begin{bmatrix}
s_x\omega_x + \frac{s_y\omega_y}{2} + \frac{s_z\omega_z}{2} & \frac{\omega_ys_x}{2} - \frac{\omega_z}{2} & \frac{\omega_zs_x}{2} + \frac{\omega_y}{2} \\
\frac{\omega_xs_y}{2} + \frac{\omega_z}{2} & s_y\omega_y + \frac{s_x\omega_x}{2} + \frac{s_z\omega_z}{2} & \frac{\omega_zs_y}{2} - \frac{\omega_x}{2} \\
\frac{\omega_xs_z}{2} - \frac{\omega_y}{2} & \frac{\omega_ys_z}{2} + \frac{\omega_x}{2} & s_z\omega_z + \frac{s_x\omega_x}{2} + \frac{s_y\omega_y}{2}
\end{bmatrix},
$$

$$
\boldsymbol{M}_{8,3} = \begin{bmatrix}
\frac{1}{2} + \frac{s_x^2}{2} & \frac{s_z}{2} + \frac{s_xs_y}{2} & -\frac{s_y}{2} + \frac{s_xs_z}{2} \\
-\frac{s_z}{2} + \frac{s_xs_y}{2} & \frac{1}{2} + \frac{s_y^2}{2} & \frac{s_x}{2} + \frac{s_ys_z}{2} \\
\frac{s_y}{2} + \frac{s_xs_z}{2} & -\frac{s_x}{2} + \frac{s_ys_z}{2} & \frac{1}{2} + \frac{s_z^2}{2}
\end{bmatrix}.
$$

II. Implementation Details

A. Parameters

TABLE I: Parameters in Simulation

Parameter	Value	Unit
mass	950	g
torque of inertia	$[3.5, 4.0, 5.0]$	$g.m^2$
thrust coefficient	0.02	$-$
torque coefficient	0.0015	$-$
drag coefficient	$[0.05, 0.05, 0.1]$	$-$
flap torque coefficient	5.0×10^{-5}	$-$
1σ of velocity measurement noise	0.05	m/s
1σ of accelerometer measurement noise	0.01	m/s^2
1σ of gyroscope measurement noise	0.004	rad/s
1σ of attitude measurement noise	0.02	rad
1σ of rotor speed measurement noise	0.05	rad/s