Supplementary Materials

I. Observability Analysis

A. IMU-MoCap Fusion

We rewrite the state variables of IMF as

$$\boldsymbol{x} \triangleq [\boldsymbol{p}^T, \boldsymbol{v}^T, \boldsymbol{s}^T, \boldsymbol{b_a}^T, \boldsymbol{b_g}^T]^T, \tag{1}$$

and organize the continuous-time true-state kinematics model of IMF into the following control-affine form, that is

$$\dot{\boldsymbol{x}} = f_0(\boldsymbol{x}) + f_a(\boldsymbol{x})\boldsymbol{a_m} + f_\omega(\boldsymbol{x})\boldsymbol{\omega_m}, \tag{2}$$

where all noises or perturbations are ignored, and

$$f_0(\boldsymbol{x}) = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \frac{\left(-s_x^2+s_y^2+s_z^2-1\right)b_{a_x}+\left(-2b_{a_y}s_y-2b_{a_z}s_z\right)s_x+2b_{a_z}s_y-2b_{a_y}s_z}{s_x^2+s_y^2+s_z^2+1} \\ \frac{\left(s_x^2-s_y^2+s_z^2-1\right)b_{a_y}+(-2b_{a_x}s_y-2b_{a_z})s_x-2s_z(b_{a_z}s_y-b_{a_x})}{s_x^2+s_y^2+s_z^2+1} \\ \frac{(g+b_{a_z})s_x^2+\left(-2b_{a_x}s_z+2b_{a_y}\right)s_x+(g+b_{a_z})s_y^2+\left(-2b_{a_y}s_z-2b_{a_x}\right)s_y+\left(s_z^2+1\right)(g-b_{a_z})}{s_x^2+s_y^2+s_z^2+1} \\ -\frac{b_{g_x}s_x^2}{2}+\frac{\left(-b_{g_y}s_y-b_{g_z}s_z\right)s_x}{2}+\frac{b_{g_z}s_y}{2}-\frac{b_{g_y}s_z}{2}-\frac{b_{g_x}}{2} \\ -\frac{b_{g_y}s_y^2}{2}+\frac{(-b_{g_x}s_x-b_{g_z}s_z)s_y}{2}-\frac{b_{g_z}s_x}{2}+\frac{b_{g_x}s_z}{2}-\frac{b_{g_y}}{2} \\ -\frac{b_{g_z}s_z^2}{2}+\frac{\left(-b_{g_x}s_x-b_{g_y}s_y\right)s_z}{2}+\frac{b_{g_y}s_x}{2}-\frac{b_{g_x}s_y}{2}-\frac{b_{g_z}}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

Then, taking the Lie derivatives yields

$$\boldsymbol{\mathcal{O}}_{IMF} \triangleq \begin{bmatrix} \mathcal{L}^0\boldsymbol{h} \\ \mathcal{L}^1_{f_0}\boldsymbol{h} \\ \mathcal{L}^2_{f_0f_0}\boldsymbol{h} \end{bmatrix} = \begin{bmatrix} h \\ \nabla_{\boldsymbol{x}}(\mathcal{L}^0\boldsymbol{h})\cdot f_0 \\ \nabla_{\boldsymbol{x}}(\mathcal{L}^1_{f_0}\boldsymbol{h})\cdot f_0 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ s_x \\ s_y \\ s_z \\ v_x \\ v_y \\ v_z \\ -\frac{b_{g_x}s_x^2}{2}+\frac{\left(-b_{g_y}s_y-b_{g_z}s_z\right)s_x}{2}+\frac{b_{g_z}s_y}{2}-\frac{b_{g_y}s_z}{2}-\frac{b_{g_x}}{2} \\ -\frac{b_{g_y}s_y^2}{2}+\frac{(-b_{g_x}s_x-b_{g_z}s_z)s_y}{2}-\frac{b_{g_z}s_x}{2}+\frac{b_{g_x}s_z}{2}-\frac{b_{g_y}}{2} \\ -\frac{b_{g_z}s_z^2}{2}+\frac{\left(-b_{g_x}s_x-b_{g_y}s_y\right)s_z}{2}+\frac{b_{g_y}s_x}{2}-\frac{b_{g_x}s_y}{2}-\frac{b_{g_z}}{2} \\ \frac{\left(-s_x^2+s_y^2+s_z^2-1\right)b_{a_x}+\left(-2b_{a_y}s_y-2b_{a_z}s_z\right)s_x+2b_{a_z}s_y-2b_{a_y}s_z}{s_x^2+s_y^2+s_z^2+1} \\ \frac{\left(s_x^2-s_y^2+s_z^2-1\right)b_{a_y}+(-2b_{a_x}s_y-2b_{a_z})s_x-2s_z(b_{a_z}s_y-b_{a_x})}{s_x^2+s_y^2+s_z^2+1} \\ \frac{(g+b_{a_z})s_x^2+\left(-2b_{a_x}s_z+2b_{a_y}\right)s_x+(g+b_{a_z})s_y^2+\left(-2b_{a_y}s_z-2b_{a_x}\right)s_y+\left(s_z^2+1\right)(g-b_{a_z})}{s_x^2+s_y^2+s_z^2+1} \\ \frac{\left(b_{g_x}s_x+b_{g_y}s_y+b_{g_z}s_z\right)\left(b_{g_x}s_x^2+\left(b_{g_y}s_y+b_{g_z}s_z\right)s_x-b_{g_z}s_y+b_{g_y}s_z+b_{g_x}\right)}{2} \\ \frac{\left(b_{g_x}s_x+b_{g_y}s_y+b_{g_z}s_z\right)\left(b_{g_y}s_y^2+(b_{g_x}s_x+b_{g_z}s_z)s_y+b_{g_z}s_x-b_{g_x}s_z+b_{g_y}\right)}{2} \\ \frac{\left(b_{g_x}s_x+b_{g_y}s_y+b_{g_z}s_z\right)\left(b_{g_z}s_z^2+\left(b_{g_x}s_x+b_{g_y}s_y\right)s_z-b_{g_y}s_x+b_{g_x}s_y+b_{g_z}\right)}{2} \end{bmatrix}. \tag{4}$$

If $\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}$ is of full column rank, the system is observable. The $\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}_{IMF}$ is given by

$$\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}_{IMF}=\begin{bmatrix}\boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3}\\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3}\\ \mathbf{0}^{3\times3} & \boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3}\\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{4,3}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{4,5}^{3\times3}\\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{5,3}^{3\times3} & \boldsymbol{M}_{5,4}^{3\times3} & \mathbf{0}^{3\times3}\\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{6,3}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{6,5}^{3\times3}\end{bmatrix}, \tag{5}$$

where all block terms are 3×3 matrices, $\boldsymbol{I}^{3\times3}$ is identity matrix, $\mathbf{0}^{3\times3}$ is zero matrix, and $\boldsymbol{M}_{i,j}^{3\times3}$ is the block term with index (i,j). By denoting the element at the x-th row and y-th column of $\boldsymbol{M}_{i,j}^{3\times3}$ as $\boldsymbol{M}_{i,j}^{3\times3}(x,y)$, we have

$$\begin{aligned}
\boldsymbol{M}_{4,3}^{3\times3}(1,1)&=-b_{g_x}s_x-\frac{b_{g_y}s_y}{2}-\frac{b_{g_z}s_z}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(1,2)&=-\frac{b_{g_y}s_x}{2}+\frac{b_{g_z}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(1,3)&=-\frac{b_{g_z}s_x}{2}-\frac{b_{g_y}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(2,1)&=-\frac{b_{g_x}s_y}{2}-\frac{b_{g_z}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(2,2)&=-b_{g_y}s_y-\frac{b_{g_x}s_x}{2}-\frac{b_{g_z}s_z}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(2,3)&=-\frac{b_{g_z}s_y}{2}+\frac{b_{g_x}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(3,1)&=-\frac{b_{g_x}s_z}{2}+\frac{b_{g_y}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(3,2)&=-\frac{b_{g_y}s_z}{2}-\frac{b_{g_x}}{2},\\
\boldsymbol{M}_{4,3}^{3\times3}(3,3)&=-b_{g_z}s_z-\frac{b_{g_x}s_x}{2}-\frac{b_{g_y}s_y}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(1,1)&=-\frac{1}{2}-\frac{s_x^2}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(1,2)&=-\frac{s_xs_y}{2}-\frac{s_z}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(1,3)&=-\frac{s_xs_z}{2}+\frac{s_y}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(2,1)&=\frac{s_z}{2}-\frac{s_xs_y}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(2,2)&=-\frac{s_y^2}{2}-\frac{1}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(2,3)&=-\frac{s_ys_z}{2}-\frac{s_x}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(3,1)&=-\frac{s_y}{2}-\frac{s_xs_z}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(3,2)&=-\frac{s_ys_z}{2}+\frac{s_x}{2},\\
\boldsymbol{M}_{4,5}^{3\times3}(3,3)&=-\frac{s_z^2}{2}-\frac{1}{2},\\
\boldsymbol{M}_{5,3}^{3\times3}(1,1)&=\frac{-2b_{a_y}s_y^3+(-4b_{a_x}s_x-2b_{a_z}s_z)\,s_y^2+\left(2b_{a_y}s_x^2-2b_{a_y}s_z^2-4b_{a_z}s_x-2b_{a_y}\right)s_y}{\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&\quad+\frac{2s_z\left(-2b_{a_x}s_xs_z+s_x^2b_{a_z}-b_{a_z}s_z^2+2b_{a_y}s_x-b_{a_z}\right)}{\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{5,3}^{3\times3}(1,2)&=\frac{-2b_{a_y}s_x^3+(4b_{a_x}s_y+2b_{a_z})\,s_x^2+\left(2b_{a_y}s_y^2+4b_{a_z}s_ys_z+\left(-2s_z^2-2\right)b_{a_y}\right)s_x}{\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&\quad+\frac{-2s_y^2b_{a_z}+\left(4b_{a_y}s_z+4b_{a_x}\right)s_y+\left(2s_z^2+2\right)b_{a_z}}{\left(s_x^2+s_y^2+s_z^2+1\right)^2},
\end{aligned}$$

$$\boldsymbol{M}_{5,3}^{3\times3}(1,3) = \frac{-2b_{a_z}s_x^3 + \left(4b_{a_x}s_z - 2b_{a_y}\right)s_x^2 + \left(2b_{a_z}s_z^2 + 4b_{a_y}s_ys_z + \left(-2s_y^2 - 2\right)b_{a_z}\right)s_x}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{2b_{a_y}s_z^2 + \left(-4b_{a_z}s_y + 4b_{a_x}\right)s_z + \left(-2s_y^2 - 2\right)b_{a_y}}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(2,1) = \frac{-2b_{a_x}s_y^3 + \left(4b_{a_y}s_x - 2b_{a_z}\right)s_y^2 + \left(2b_{a_x}s_x^2 + 4b_{a_z}s_xs_z + \left(-2s_z^2 - 2\right)b_{a_x}\right)s_y}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{2s_x^2b_{a_z} + \left(-4b_{a_x}s_z + 4b_{a_y}\right)s_x + \left(-2s_z^2 - 2\right)b_{a_z}}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(2,2) = \frac{-2b_{a_x}s_x^3 + \left(-4b_{a_y}s_y - 2b_{a_z}s_z\right)s_x^2 + \left(2b_{a_x}s_y^2 - 2b_{a_x}s_z^2 + 4b_{a_z}s_y - 2b_{a_x}\right)s_x}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{2s_z\left(-2b_{a_y}s_ys_z + s_y^2b_{a_z} - b_{a_z}s_z^2 - 2b_{a_x}s_y - b_{a_z}\right)}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(2,3) = \frac{-2b_{a_z}s_y^3 + \left(4b_{a_y}s_z + 2b_{a_x}\right)s_y^2 + \left(2b_{a_z}s_z^2 + 4b_{a_x}s_xs_z + \left(-2s_x^2 - 2\right)b_{a_z}\right)s_y}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{-2b_{a_x}s_z^2 + \left(4b_{a_z}s_x + 4b_{a_y}\right)s_z + \left(2s_x^2 + 2\right)b_{a_x}}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(3,1) = \frac{-2b_{a_x}s_z^3 + \left(4b_{a_z}s_x + 2b_{a_y}\right)s_z^2 + \left(2b_{a_x}s_x^2 + 4b_{a_y}s_xs_y + \left(-2s_y^2 - 2\right)b_{a_x}\right)s_z}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{-2b_{a_y}s_x^2 + \left(4b_{a_x}s_y + 4b_{a_z}\right)s_x + \left(2s_y^2 + 2\right)b_{a_y}}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(3,2) = \frac{-2b_{a_y}s_z^3 + \left(4b_{a_z}s_y - 2b_{a_x}\right)s_z^2 + \left(2b_{a_y}s_y^2 + 4b_{a_x}s_xs_y + \left(-2s_x^2 - 2\right)b_{a_y}\right)s_z}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{2b_{a_x}s_y^2 + \left(-4b_{a_y}s_x + 4b_{a_z}\right)s_y + \left(-2s_x^2 - 2\right)b_{a_x}}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,3}^{3\times3}(3,3) = \frac{-2b_{a_x}s_x^3 + \left(-2b_{a_y}s_y - 4b_{a_z}s_z\right)s_x^2 + \left(-2b_{a_x}s_y^2 + 2b_{a_x}s_z^2 - 4b_{a_y}s_z - 2b_{a_x}\right)s_x}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2} + \frac{-2s_y\left(b_{a_y}s_y^2 - b_{a_y}s_z^2 + 2b_{a_z}s_ys_z - 2b_{a_x}s_z + b_{a_y}\right)}{\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(1,1) = \frac{-s_x^2 + s_y^2 + s_z^2 - 1}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(1,2) = \frac{-2s_xs_y - 2s_z}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(1,3) = \frac{-2s_xs_z + 2s_y}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(2,1) = \frac{-2s_xs_y + 2s_z}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(2,2) = \frac{s_x^2 - s_y^2 + s_z^2 - 1}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(2,3) = \frac{-2s_ys_z - 2s_x}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(3,1) = \frac{-2s_xs_z - 2s_y}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(3,2) = \frac{-2s_ys_z + 2s_x}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{5,4}^{3\times3}(3,3) = \frac{s_x^2 + s_y^2 - s_z^2 - 1}{s_x^2 + s_y^2 + s_z^2 + 1},$$

$$\boldsymbol{M}_{6,3}^{3\times3}(1,1)=\frac{\left(3s_x^2+1\right)b_{g_x}^2}{2}+\frac{\left(\left(4s_xs_y+s_z\right)b_{g_y}+4\left(s_xs_z-\frac{s_y}{4}\right)b_{g_z}\right)b_{g_x}}{2}+\frac{\left(b_{g_y}s_y+b_{g_z}s_z\right)^2}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(1,2)=\frac{\left(2s_xs_y+s_z\right)b_{g_y}^2}{2}+\frac{\left(\left(2s_xs_z-2s_y\right)b_{g_z}+2b_{g_x}s_x^2+b_{g_x}\right)b_{g_y}}{2}-\frac{b_{g_x}b_{g_z}s_x}{2}-\frac{b_{g_z}^2s_z}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(1,3)=\frac{\left(2s_xs_z-s_y\right)b_{g_z}^2}{2}+\frac{\left(\left(2s_xs_y+2s_z\right)b_{g_y}+2b_{g_x}s_x^2+b_{g_x}\right)b_{g_z}}{2}+\frac{b_{g_x}b_{g_y}s_x}{2}+\frac{b_{g_y}^2s_y}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(2,1)=\frac{\left(2s_xs_y-s_z\right)b_{g_x}^2}{2}+\frac{\left(\left(2s_ys_z+2s_x\right)b_{g_z}+2b_{g_y}s_y^2+b_{g_y}\right)b_{g_x}}{2}+\frac{b_{g_y}b_{g_z}s_y}{2}+\frac{b_{g_z}^2s_z}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(2,2)=\frac{\left(3s_y^2+1\right)b_{g_y}^2}{2}+\frac{\left(\left(4s_xs_y-s_z\right)b_{g_x}+b_{g_z}\left(4s_ys_z+s_x\right)\right)b_{g_y}}{2}+\frac{\left(b_{g_x}s_x+b_{g_z}s_z\right)^2}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(2,3)=\frac{\left(2s_ys_z+s_x\right)b_{g_z}^2}{2}+\frac{\left(\left(2s_xs_y-2s_z\right)b_{g_x}+2b_{g_y}s_y^2+b_{g_y}\right)b_{g_z}}{2}-\frac{b_{g_x}^2s_x}{2}-\frac{b_{g_x}b_{g_y}s_y}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(3,1)=\frac{\left(2s_xs_z+s_y\right)b_{g_x}^2}{2}+\frac{\left(\left(2s_ys_z-2s_x\right)b_{g_y}+2b_{g_z}s_z^2+b_{g_z}\right)b_{g_x}}{2}-\frac{b_{g_y}^2s_y}{2}-\frac{b_{g_y}b_{g_z}s_z}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(3,2)=\frac{\left(2s_ys_z-s_x\right)b_{g_y}^2}{2}+\frac{\left(\left(2s_xs_z+2s_y\right)b_{g_x}+2b_{g_z}s_z^2+b_{g_z}\right)b_{g_y}}{2}+\frac{b_{g_x}^2s_x}{2}+\frac{b_{g_x}b_{g_z}s_z}{2},$$
$$\boldsymbol{M}_{6,3}^{3\times3}(3,3)=\frac{\left(3s_z^2+1\right)b_{g_z}^2}{2}+\frac{\left(\left(4s_xs_z+s_y\right)b_{g_x}-b_{g_y}\left(-4s_ys_z+s_x\right)\right)b_{g_z}}{2}+\frac{\left(b_{g_x}s_x+b_{g_y}s_y\right)^2}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(1,1)=b_{g_x}s_x^3+\frac{\left(2b_{g_y}s_y+2b_{g_z}s_z\right)s_x^2}{2}+\frac{\left(b_{g_y}s_z-b_{g_z}s_y+2b_{g_x}\right)s_x}{2}+\frac{b_{g_y}s_y}{2}+\frac{b_{g_z}s_z}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(1,2)=\frac{\left(2b_{g_y}s_x-b_{g_z}\right)s_y^2}{2}+\frac{\left(2b_{g_x}s_x^2+2b_{g_z}s_xs_z+2b_{g_y}s_z+b_{g_x}\right)s_y}{2}+\frac{b_{g_x}s_xs_z}{2}+\frac{b_{g_z}s_z^2}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(1,3)=\frac{\left(2b_{g_z}s_x+b_{g_y}\right)s_z^2}{2}+\frac{\left(2b_{g_x}s_x^2+2b_{g_y}s_xs_y-2b_{g_z}s_y+b_{g_x}\right)s_z}{2}-\frac{b_{g_x}s_xs_y}{2}-\frac{b_{g_y}s_y^2}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(2,1)=\frac{\left(2b_{g_x}s_y+b_{g_z}\right)s_x^2}{2}+\frac{\left(2b_{g_y}s_y^2+2b_{g_z}s_ys_z-2b_{g_x}s_z+b_{g_y}\right)s_x}{2}-\frac{b_{g_y}s_ys_z}{2}-\frac{b_{g_z}s_z^2}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(2,2)=b_{g_y}s_y^3+\frac{\left(2b_{g_x}s_x+2b_{g_z}s_z\right)s_y^2}{2}+\frac{\left(-b_{g_x}s_z+b_{g_z}s_x+2b_{g_y}\right)s_y}{2}+\frac{b_{g_x}s_x}{2}+\frac{b_{g_z}s_z}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(2,3)=\frac{\left(2b_{g_z}s_y-b_{g_x}\right)s_z^2}{2}+\frac{\left(\left(2b_{g_x}s_y+2b_{g_z}\right)s_x+2b_{g_y}s_y^2+b_{g_y}\right)s_z}{2}+\frac{b_{g_x}s_x^2}{2}+\frac{b_{g_y}s_xs_y}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(3,1)=\frac{\left(2b_{g_x}s_z-b_{g_y}\right)s_x^2}{2}+\frac{\left(\left(2b_{g_y}s_z+2b_{g_x}\right)s_y+2b_{g_z}s_z^2+b_{g_z}\right)s_x}{2}+\frac{b_{g_y}s_y^2}{2}+\frac{b_{g_z}s_ys_z}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(3,2)=\frac{\left(2b_{g_y}s_z+b_{g_x}\right)s_y^2}{2}+\frac{\left(\left(2b_{g_x}s_z-2b_{g_y}\right)s_x+2b_{g_z}s_z^2+b_{g_z}\right)s_y}{2}-\frac{b_{g_x}s_x^2}{2}-\frac{b_{g_z}s_xs_z}{2},$$
$$\boldsymbol{M}_{6,5}^{3\times3}(3,3)=b_{g_z}s_z^3+\frac{\left(2b_{g_x}s_x+2b_{g_y}s_y\right)s_z^2}{2}+\frac{\left(b_{g_x}s_y-b_{g_y}s_x+2b_{g_z}\right)s_z}{2}+\frac{b_{g_x}s_x}{2}+\frac{b_{g_y}s_y}{2}.$$

B. External Wrench Estimation

We rewrite the state variables of EWE as

$$\boldsymbol{x}\triangleq[\boldsymbol{v}^T,\boldsymbol{s}^T,\boldsymbol{\omega}^T,\boldsymbol{F_e}^T,\boldsymbol{M_e}^T]^T, \tag{6}$$

and organize the continuous-time true-state kinematics model of EWE into the following control-affine form, that is

$$\dot{\boldsymbol{x}}=f_0(\boldsymbol{x})+f_F(\boldsymbol{x})\boldsymbol{f_c}+f_M(\boldsymbol{x})\boldsymbol{M_c}, \tag{7}$$

where all noises or perturbations are ignored, and

$$f_0(\boldsymbol{x}) = \begin{bmatrix} \frac{\left(s_x^2-s_y^2-s_z^2+1\right)F_{e_x}+\left(2F_{e_y}s_y+2F_{e_z}s_z\right)s_x-2F_{e_z}s_y+2F_{e_y}s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\left(-s_x^2+s_y^2-s_z^2+1\right)F_{e_y}+(2F_{e_x}s_y+2F_{e_z})s_x+2s_z(F_{e_z}s_y-F_{e_x})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{(gm-F_{e_z})s_x^2+\left(2F_{e_x}s_z-2F_{e_y}\right)s_x+(gm-F_{e_z})s_y^2+\left(2F_{e_y}s_z+2F_{e_x}\right)s_y+\left(s_z^2+1\right)(gm+F_{e_z})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\omega_x s_x^2}{2}+\frac{(s_y\omega_y+s_z\omega_z)s_x}{2}-\frac{\omega_z s_y}{2}+\frac{\omega_y s_z}{2}+\frac{\omega_x}{2} \\ \frac{\omega_y s_y^2}{2}+\frac{(s_x\omega_x+s_z\omega_z)s_y}{2}+\frac{\omega_z s_x}{2}-\frac{\omega_x s_z}{2}+\frac{\omega_y}{2} \\ \frac{\omega_z s_z^2}{2}+\frac{(s_x\omega_x+s_y\omega_y)s_z}{2}-\frac{\omega_y s_x}{2}+\frac{\omega_x s_y}{2}+\frac{\omega_z}{2} \\ \frac{\omega_z(I_{yy}-I_{zz})\omega_y+M_{e_x}}{I_{xx}} \\ \frac{-\omega_z(I_{xx}-I_{zz})\omega_x+M_{e_y}}{I_{yy}} \\ \frac{\omega_y(I_{xx}-I_{yy})\omega_x+M_{e_z}}{I_{zz}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Then, taking the Lie derivatives yields

$$\boldsymbol{\mathcal{O}}_{EWE} \triangleq \begin{bmatrix} \mathcal{L}^0\boldsymbol{h} \\ \mathcal{L}^1_{f_0}\boldsymbol{h} \end{bmatrix} = \begin{bmatrix} h \\ \nabla_{\boldsymbol{x}}(\mathcal{L}^0\boldsymbol{h})\cdot f_0 \end{bmatrix} = \begin{bmatrix} \frac{F_{e_x}}{m} \\ \frac{F_{e_y}}{m} \\ \frac{f_c+F_{e_z}}{m} \\ \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \\ s_x \\ s_y \\ s_z \\ 0 \\ 0 \\ 0 \\ \frac{\omega_z(I_{yy}-I_{zz})\omega_y+M_{e_x}}{I_{xx}} \\ \frac{-\omega_z(I_{xx}-I_{zz})\omega_x+M_{e_y}}{I_{yy}} \\ \frac{\omega_y(I_{xx}-I_{yy})\omega_x+M_{e_z}}{I_{zz}} \\ \frac{\left(s_x^2-s_y^2-s_z^2+1\right)F_{e_x}+\left(2F_{e_y}s_y+2F_{e_z}s_z\right)s_x-2F_{e_z}s_y+2F_{e_y}s_z}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\left(-s_x^2+s_y^2-s_z^2+1\right)F_{e_y}+(2F_{e_x}s_y+2F_{e_z})s_x+2s_z(F_{e_z}s_y-F_{e_x})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{(gm-F_{e_z})s_x^2+\left(2F_{e_x}s_z-2F_{e_y}\right)s_x+(gm-F_{e_z})s_y^2+\left(2F_{e_y}s_z+2F_{e_x}\right)s_y+\left(s_z^2+1\right)(gm+F_{e_z})}{m\left(s_x^2+s_y^2+s_z^2+1\right)} \\ \frac{\omega_x s_x^2}{2}+\frac{(s_y\omega_y+s_z\omega_z)s_x}{2}-\frac{\omega_z s_y}{2}+\frac{\omega_y s_z}{2}+\frac{\omega_x}{2} \\ \frac{\omega_y s_y^2}{2}+\frac{(s_x\omega_x+s_z\omega_z)s_y}{2}+\frac{\omega_z s_x}{2}-\frac{\omega_x s_z}{2}+\frac{\omega_y}{2} \\ \frac{\omega_z s_z^2}{2}+\frac{(s_x\omega_x+s_y\omega_y)s_z}{2}-\frac{\omega_y s_x}{2}+\frac{\omega_x s_y}{2}+\frac{\omega_z}{2} \end{bmatrix}. \tag{9}$$

If $\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}$ is of full column rank, the system is observable. The $\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}_{EWE}$ is given by

$$\nabla_{\boldsymbol{x}}\boldsymbol{\mathcal{O}}_{EWE} = \begin{bmatrix} \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \frac{1}{m}\boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} \\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \\ \boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \\ \mathbf{0}^{3\times3} & \boldsymbol{I}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \\ \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{6,3}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{6,5}^{3\times3} \\ \mathbf{0}^{3\times3} & \boldsymbol{M}_{7,2}^{3\times3} & \mathbf{0}^{3\times3} & \boldsymbol{M}_{7,4}^{3\times3} & \mathbf{0}^{3\times3} \\ \mathbf{0}^{3\times3} & \boldsymbol{M}_{8,2}^{3\times3} & \boldsymbol{M}_{8,3}^{3\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \end{bmatrix}, \tag{10}$$

where all block terms are 3×3 matrices, $\boldsymbol{I}^{3\times3}$ is identity matrix, $\mathbf{0}^{3\times3}$ is zero matrix, and $\boldsymbol{M}_{i,j}^{3\times3}$ is the block term with index (i,j). By denoting the element at the x-th row and y-th column of $\boldsymbol{M}_{i,j}^{3\times3}$ as $\boldsymbol{M}_{i,j}^{3\times3}(x,y)$, we have

$$\begin{aligned}
\boldsymbol{M}_{6,3}^{3\times3}(1,1) &= 0,\\
\boldsymbol{M}_{6,3}^{3\times3}(1,2) &= \frac{\omega_z\left(I_{yy}-I_{zz}\right)}{I_{xx}},\\
\boldsymbol{M}_{6,3}^{3\times3}(1,3) &= \frac{\left(I_{yy}-I_{zz}\right)\omega_y}{I_{xx}},\\
\boldsymbol{M}_{6,3}^{3\times3}(2,1) &= -\frac{\omega_z\left(I_{xx}-I_{zz}\right)}{I_{yy}},\\
\boldsymbol{M}_{6,3}^{3\times3}(2,2) &= 0,\\
\boldsymbol{M}_{6,3}^{3\times3}(2,3) &= -\frac{\left(I_{xx}-I_{zz}\right)\omega_x}{I_{yy}},\\
\boldsymbol{M}_{6,3}^{3\times3}(3,1) &= \frac{\omega_y\left(I_{xx}-I_{yy}\right)}{I_{zz}},\\
\boldsymbol{M}_{6,3}^{3\times3}(3,2) &= \frac{\left(I_{xx}-I_{yy}\right)\omega_x}{I_{zz}},\\
\boldsymbol{M}_{6,3}^{3\times3}(3,3) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(1,1) &= \frac{1}{I_{xx}},\\
\boldsymbol{M}_{6,5}^{3\times3}(1,2) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(1,3) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(2,1) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(2,2) &= \frac{1}{I_{yy}},\\
\boldsymbol{M}_{6,5}^{3\times3}(2,3) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(3,1) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(3,2) &= 0,\\
\boldsymbol{M}_{6,5}^{3\times3}(3,3) &= \frac{1}{I_{zz}},\\
\boldsymbol{M}_{7,2}^{3\times3}(1,1) &= \frac{2F_{e_y}s_y^3+\left(4F_{e_x}s_x+2F_{e_z}s_z\right)s_y^2+\left(-2F_{e_y}s_x^2+2s_z^2F_{e_y}+4F_{e_z}s_x+2F_{e_y}\right)s_y}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&\quad+\frac{-2s_z\left(-2F_{e_x}s_xs_z+F_{e_z}s_x^2-F_{e_z}s_z^2+2F_{e_y}s_x-F_{e_z}\right)}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{7,2}^{3\times3}(1,2) &= \frac{2F_{e_y}s_x^3+\left(-4F_{e_x}s_y-2F_{e_z}\right)s_x^2+\left(-2s_y^2F_{e_y}-4s_ys_zF_{e_z}+\left(2s_z^2+2\right)F_{e_y}\right)s_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}\\
&\quad+\frac{2F_{e_z}s_y^2+\left(-4F_{e_y}s_z-4F_{e_x}\right)s_y+\left(-2s_z^2-2\right)F_{e_z}}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2},\\
\boldsymbol{M}_{7,2}^{3\times3}(1,3) &= \frac{2s_x^3F_{e_z}+\left(-4F_{e_x}s_z+2F_{e_y}\right)s_x^2+\left(-2F_{e_z}s_z^2-4s_ys_zF_{e_y}+\left(2s_y^2+2\right)F_{e_z}\right)s_x}{m\left(s_x^2+s_y^2+s_z^2+1\right)^2}
\end{aligned}$$

$$+\frac{-2s_z^2 F_{e_y} + (4F_{e_z} s_y - 4F_{e_x}) s_z + \left(2s_y^2 + 2\right) F_{e_y}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(2,1) = \frac{2F_{e_x} s_y^3 + \left(-4F_{e_y} s_x + 2F_{e_z}\right) s_y^2 + \left(-2F_{e_x} s_x^2 - 4s_z F_{e_z} s_x + \left(2s_z^2 + 2\right) F_{e_x}\right) s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{-2F_{e_z} s_x^2 + \left(4F_{e_x} s_z - 4F_{e_y}\right) s_x + \left(2s_z^2 + 2\right) F_{e_z}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(2,2) = \frac{2F_{e_x} s_x^3 + \left(4F_{e_y} s_y + 2F_{e_z} s_z\right) s_x^2 + \left(-2F_{e_x} s_y^2 + 2F_{e_x} s_z^2 - 4F_{e_z} s_y + 2F_{e_x}\right) s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{-2s_z\left(-2s_y s_z F_{e_y} + F_{e_z} s_y^2 - F_{e_z} s_z^2 - 2F_{e_x} s_y - F_{e_z}\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(2,3) = \frac{2F_{e_z} s_y^3 + \left(-4F_{e_y} s_z - 2F_{e_x}\right) s_y^2 + \left(-2F_{e_z} s_z^2 - 4F_{e_x} s_x s_z + \left(2s_x^2 + 2\right) F_{e_z}\right) s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{2F_{e_x} s_z^2 + \left(-4F_{e_z} s_x - 4F_{e_y}\right) s_z + \left(-2s_x^2 - 2\right) F_{e_x}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(3,1) = \frac{2F_{e_x} s_z^3 + \left(-4F_{e_z} s_x - 2F_{e_y}\right) s_z^2 + \left(-2F_{e_x} s_x^2 - 4F_{e_y} s_x s_y + \left(2s_y^2 + 2\right) F_{e_x}\right) s_z}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{+2F_{e_y} s_x^2 + (-4F_{e_x} s_y - 4F_{e_z}) s_x + \left(-2s_y^2 - 2\right) F_{e_y}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(3,2) = \frac{2F_{e_y} s_z^3 + (-4F_{e_z} s_y + 2F_{e_x}) s_z^2 + \left(-2s_y^2 F_{e_y} - 4s_x F_{e_x} s_y + \left(2s_x^2 + 2\right) F_{e_y}\right) s_z}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{-2F_{e_x} s_y^2 + \left(4F_{e_y} s_x - 4F_{e_z}\right) s_y + \left(2s_x^2 + 2\right) F_{e_x}}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,2}^{3\times3}(3,3) = \frac{2F_{e_x} s_x^3 + \left(2F_{e_y} s_y + 4F_{e_z} s_z\right) s_x^2 + \left(2F_{e_x} s_y^2 - 2F_{e_x} s_z^2 + 4F_{e_y} s_z + 2F_{e_x}\right) s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2}$$

$$+\frac{2s_y\left(s_y^2 F_{e_y} - s_z^2 F_{e_y} + 2s_y s_z F_{e_z} - 2F_{e_x} s_z + F_{e_y}\right)}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)^2},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(1,1) = \frac{s_x^2 - s_y^2 - s_z^2 + 1}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(1,2) = \frac{2s_x s_y + 2s_z}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(1,3) = \frac{2s_x s_z - 2s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(2,1) = \frac{2s_x s_y - 2s_z}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(2,2) = \frac{-s_x^2 + s_y^2 - s_z^2 + 1}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(2,3) = \frac{2s_y s_z + 2s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(3,1) = \frac{2s_x s_z + 2s_y}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(3,2) = \frac{2s_y s_z - 2s_x}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{7,4}^{3\times3}(3,3) = \frac{-s_x^2 - s_y^2 + s_z^2 + 1}{m\left(s_x^2 + s_y^2 + s_z^2 + 1\right)},$$

$$\boldsymbol{M}_{8,2}^{3\times3}(1,1) = s_x\omega_x + \frac{s_y\omega_y}{2} + \frac{s_z\omega_z}{2},$$

$$
\begin{aligned}
&\boldsymbol{M}_{8,2}^{3\times 3}(1,2) = \frac{\omega_y s_x}{2} - \frac{\omega_z}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(1,3) = \frac{\omega_z s_x}{2} + \frac{\omega_y}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(2,1) = \frac{\omega_x s_y}{2} + \frac{\omega_z}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(2,2) = s_y\omega_y + \frac{s_x\omega_x}{2} + \frac{s_z\omega_z}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(2,3) = \frac{\omega_z s_y}{2} - \frac{\omega_x}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(3,1) = \frac{\omega_x s_z}{2} - \frac{\omega_y}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(3,2) = \frac{\omega_y s_z}{2} + \frac{\omega_x}{2},\\
&\boldsymbol{M}_{8,2}^{3\times 3}(3,3) = s_z\omega_z + \frac{s_x\omega_x}{2} + \frac{s_y\omega_y}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(1,1) = \frac{1}{2} + \frac{s_x^2}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(1,2) = \frac{s_z}{2} + \frac{s_x s_y}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(1,3) = -\frac{s_y}{2} + \frac{s_x s_z}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(2,1) = -\frac{s_z}{2} + \frac{s_x s_y}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(2,2) = \frac{1}{2} + \frac{s_y^2}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(2,3) = \frac{s_x}{2} + \frac{s_y s_z}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(3,1) = \frac{s_y}{2} + \frac{s_x s_z}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(3,2) = -\frac{s_x}{2} + \frac{s_y s_z}{2},\\
&\boldsymbol{M}_{8,3}^{3\times 3}(3,3) = \frac{1}{2} + \frac{s_z^2}{2}.
\end{aligned}
$$