Table .1
RESULTS OF THREE PREPROCESSING ALGORITHMS FOR THE INSTANCES IN CLASS 2.[a]

K	$\|V\|$	$\|A\|$	q_1	No. solved			No. bounds			Node (arc) reductions (%)			Average process time (s)		
				Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2
2	100	1128	0.8	917	999	1000	72/83	1/1	—	98.08 (99.73)	97.96 (99.65)	—	0.01	0.01	0.01
2	100	1128	0.6	868	995	997	122/132	4/5	2/3	97.67 (99.64)	90.82 (98.12)	86.73 (97.22)	0.01	0.01	0.01
2	100	1128	0.4	748	976	991	232/252	20/24	7/9	97.00 (99.37)	90.86 (96.58)	83.67 (91.99)	0.01	0.01	0.01
2	100	1128	0.2	556	897	941	427/444	101/103	57/59	96.16 (99.27)	91.68 (98.02)	88.93 (97.01)	0.01	0.01	0.01
2	100	1128	0.1	346	699	804	619/654	289/301	187/196	92.60 (97.77)	87.26 (95.63)	83.49 (93.70)	0.01	0.01	0.02
2	100	2250	0.8	977	999	999	21/23	1/1	1/1	97.29 (99.79)	78.57 (97.76)	78.57 (97.76)	0.01	0.01	0.01
2	100	2250	0.6	952	997	999	46/48	2/3	1/1	98.11 (99.87)	94.56 (99.62)	95.92 (99.65)	0.01	0.01	0.01
2	100	2250	0.4	888	991	994	108/112	9/9	6/6	95.66 (98.87)	69.05 (87.33)	55.44 (81.11)	0.01	0.01	0.01
2	100	2250	0.2	664	933	964	325/336	63/67	34/36	94.86 (98.51)	83.40 (93.09)	73.21 (87.40)	0.02	0.02	0.02
2	100	2250	0.1	422	756	845	574/578	243/244	155/155	89.55 (95.70)	79.01 (90.04)	70.13 (84.51)	0.03	0.03	0.03
2	100	4758	0.8	991	998	998	9/9	2/2	2/2	83.90 (94.06)	33.16 (73.50)	33.16 (73.50)	0.02	0.01	0.02
2	100	4758	0.6	968	997	997	31/32	3/3	3/3	93.62 (99.32)	46.94 (93.35)	46.94 (93.35)	0.02	0.02	0.02
2	100	4758	0.4	897	991	994	102/103	9/9	6/6	92.82 (98.58)	36.17 (84.44)	6.63 (76.73)	0.02	0.02	0.02
2	100	4758	0.2	680	946	969	313/320	53/54	30/31	93.70 (98.80)	74.40 (93.29)	58.72 (88.42)	0.03	0.03	0.03
2	100	4758	0.1	435	774	864	563/565	226/226	136/136	89.66 (97.27)	78.10 (93.31)	67.25 (89.00)	0.04	0.04	0.04
2	500	11 712	0.8	988	998	998	11/12	2/2	2/2	85.07 (93.34)	12.55 (60.19)	12.55 (60.19)	0.05	0.06	0.06
2	500	11 712	0.6	955	998	998	44/45	2/2	2/2	98.89 (99.86)	84.64 (97.46)	84.64 (97.46)	0.06	0.06	0.06
2	500	11 712	0.4	908	993	994	91/92	7/7	6/6	95.58 (97.75)	47.50 (70.86)	38.92 (66.01)	0.07	0.07	0.07
2	500	11 712	0.2	697	941	970	300/303	59/59	30/30	96.09 (97.71)	82.34 (88.39)	66.24 (77.23)	0.10	0.10	0.10
2	500	11 712	0.1	455	792	863	543/545	208/208	137/137	93.64 (96.16)	84.39 (90.01)	76.97 (84.87)	0.13	0.14	0.15
2	500	37 638	0.8	992	995	995	8/8	5/5	5/5	49.85 (80.79)	19.96 (69.27)	19.96 (69.27)	0.11	0.11	0.11
2	500	37 638	0.6	983	997	997	17/17	3/3	3/3	88.63 (96.85)	37.48 (82.17)	37.48 (82.17)	0.11	0.11	0.12
2	500	37 638	0.4	944	993	995	56/56	7/7	5/5	91.52 (97.35)	35.23 (78.88)	9.72 (70.44)	0.14	0.14	0.15
2	500	37 638	0.2	768	944	960	232/232	56/56	40/40	88.55 (95.17)	54.31 (80.02)	36.42 (72.04)	0.23	0.23	0.24
2	500	37 638	0.1	515	823	880	485/485	177/177	120/120	88.41 (95.44)	69.40 (87.53)	55.50 (81.62)	0.29	0.30	0.32
2	500	102 528	0.8	995	997	997	5/5	3/3	3/3	39.92 (89.26)	0.00 (82.11)	0.00 (82.11)	0.20	0.20	0.20
2	500	102 528	0.6	983	994	994	17/17	6/6	6/6	65.98 (94.13)	4.42 (83.37)	4.42 (83.37)	0.23	0.23	0.24
2	500	102 528	0.4	942	989	991	58/58	11/11	9/9	87.50 (97.46)	36.09 (86.62)	22.04 (83.64)	0.30	0.30	0.30
2	500	102 528	0.2	776	947	958	224/224	53/53	42/42	88.83 (97.91)	54.61 (91.20)	43.00 (88.90)	0.42	0.42	0.43
2	500	102 528	0.1	539	817	867	461/461	183/183	133/133	86.08 (96.81)	65.95 (91.98)	53.66 (88.97)	0.56	0.56	0.58
2	1000	29 550	0.8	992	998	998	7/8	2/2	2/2	94.66 (99.23)	79.51 (96.96)	79.51 (96.96)	0.17	0.17	0.17
2	1000	29 550	0.6	971	998	998	28/29	2/2	2/2	93.63 (97.37)	10.92 (62.10)	10.92 (62.10)	0.17	0.17	0.18
2	1000	29 550	0.4	916	988	991	83/84	12/12	9/9	95.31 (97.19)	68.59 (80.43)	58.31 (73.92)	0.21	0.21	0.23
2	1000	29 550	0.2	729	940	956	268/271	60/60	44/44	94.51 (96.65)	76.25 (84.91)	67.82 (79.43)	0.28	0.29	0.39
2	1000	29 550	0.1	533	842	895	464/467	157/158	104/105	92.67 (96.34)	78.98 (89.21)	68.70 (83.78)	0.38	0.40	0.67
2	1000	118 062	0.8	994	996	997	5/6	3/4	3/3	55.54 (85.60)	33.44 (78.40)	11.66 (71.20)	0.30	0.30	0.31
2	1000	118 062	0.6	989	998	998	11/11	2/2	2/2	81.76 (95.21)	0.55 (73.66)	0.55 (73.66)	0.36	0.36	0.37
2	1000	118 062	0.4	945	989	989	55/55	11/11	11/11	83.37 (95.27)	17.84 (76.36)	17.84 (76.36)	0.45	0.45	0.49
2	1000	118 062	0.2	750	941	951	250/250	59/59	49/49	86.14 (94.79)	42.25 (77.95)	30.61 (73.45)	0.65	0.65	0.82
2	1000	118 062	0.1	559	823	874	441/441	177/177	126/126	86.98 (94.63)	68.09 (86.63)	55.43 (81.22)	0.80	0.81	1.12
2	1000	375 998	0.8	996	997	997	4/4	3/3	3/3	24.97 (91.94)	0.03 (89.26)	0.03 (89.26)	0.63	0.63	0.65
2	1000	375 998	0.6	986	995	995	14/14	5/5	5/5	64.37 (95.06)	0.60 (86.17)	0.60 (86.17)	0.78	0.77	0.80
2	1000	375 998	0.4	945	990	991	55/55	10/10	9/9	83.47 (98.55)	10.26 (92.01)	0.33 (91.12)	0.89	0.89	0.93
2	1000	375 998	0.2	795	957	967	205/205	43/43	33/33	90.78 (98.52)	57.26 (92.96)	44.49 (90.83)	1.17	1.16	1.24
2	1000	375 998	0.1	544	808	865	456/456	192/192	135/135	82.72 (97.01)	59.46 (92.89)	42.67 (89.89)	1.83	1.81	2.16

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Table .1
RESULTS OF THREE PREPROCESSING ALGORITHMS FOR THE INSTANCES IN CLASS 2.[a] (CONTINUED)

K	$\|V\|$	$\|A\|$	q_1	No. solved			No. bounds			Node (arc) reductions (%)			Average process time (s)		
				Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2
5	100	1128	0.8	817	982	988	144/183	13/18	9/12	98.25 (99.73)	96.09 (99.37)	95.75 (99.29)	0.02	0.02	0.02
5	100	1128	0.6	717	957	963	221/283	38/43	33/37	97.78 (99.67)	95.75 (99.35)	95.70 (99.34)	0.02	0.02	0.02
5	100	1128	0.4	567	861	878	288/433	119/139	107/122	96.23 (99.33)	92.71 (98.58)	92.31 (98.47)	0.03	0.03	0.03
5	100	1128	0.2	511	721	766	242/489	197/279	173/234	87.24 (95.18)	79.33 (91.82)	76.41 (90.39)	0.05	0.05	0.05
5	100	1128	0.1	528	734	780	79/472	72/266	68/220	79.37 (89.55)	65.09 (81.71)	59.03 (78.06)	0.06	0.06	0.06
5	100	2250	0.8	909	998	998	77/91	2/2	2/2	98.59 (99.90)	96.43 (99.78)	96.43 (99.78)	0.04	0.04	0.04
5	100	2250	0.6	742	966	969	230/258	33/34	31/31	97.89 (99.84)	94.18 (99.45)	93.94 (99.42)	0.05	0.05	0.05
5	100	2250	0.4	546	832	860	394/454	152/168	126/140	96.43 (99.72)	93.63 (99.47)	93.19 (99.41)	0.08	0.07	0.08
5	100	2250	0.2	368	588	641	454/632	353/412	309/359	80.43 (92.65)	71.11 (88.80)	67.72 (87.20)	0.15	0.15	0.16
5	100	2250	0.1	395	566	631	246/605	234/434	218/369	58.70 (77.92)	43.29 (69.28)	34.36 (63.94)	0.18	0.18	0.19
5	100	4758	0.8	932	998	999	63/68	2/2	1/1	98.72 (99.95)	96.43 (99.88)	96.94 (99.89)	0.06	0.06	0.06
5	100	4758	0.6	798	974	975	193/202	25/26	24/25	98.27 (99.94)	96.86 (99.87)	96.90 (99.87)	0.08	0.08	0.08
5	100	4758	0.4	567	873	888	393/433	120/127	107/112	96.35 (99.73)	92.20 (99.23)	91.70 (99.14)	0.13	0.13	0.13
5	100	4758	0.2	333	527	576	538/667	420/473	380/424	78.81 (94.37)	70.97 (92.10)	68.25 (91.20)	0.28	0.28	0.29
5	100	4758	0.1	272	414	464	402/728	390/586	373/536	46.67 (71.10)	34.24 (64.11)	28.76 (60.79)	0.39	0.38	0.40
5	500	11 712	0.8	912	997	998	78/88	3/3	2/2	99.69 (99.98)	99.13 (99.94)	99.00 (99.93)	0.21	0.22	0.22
5	500	11 712	0.6	780	975	975	206/220	23/25	23/25	99.55 (99.97)	98.96 (99.92)	98.96 (99.92)	0.27	0.28	0.28
5	500	11 712	0.4	536	843	866	428/464	148/157	125/134	99.18 (99.93)	98.50 (99.84)	98.39 (99.83)	0.37	0.39	0.39
5	500	11 712	0.2	295	498	563	586/705	448/502	387/437	91.58 (97.02)	88.38 (95.83)	86.84 (95.22)	0.78	0.81	0.88
5	500	11 712	0.1	306	458	513	414/694	389/542	367/487	67.77 (82.47)	58.87 (77.56)	54.39 (75.04)	1.09	1.11	1.52
5	500	37 638	0.8	973	1000	1000	27/27	—	—	99.73 (99.99)	—	—	0.40	0.39	0.40
5	500	37 638	0.6	892	994	994	105/108	6/6	6/6	99.62 (99.99)	99.13 (99.98)	99.13 (99.98)	0.56	0.56	0.57
5	500	37 638	0.4	609	889	911	383/391	109/111	88/89	99.15 (99.97)	98.12 (99.92)	97.92 (99.91)	0.92	0.92	0.94
5	500	37 638	0.2	260	502	575	677/740	477/498	407/425	91.03 (97.71)	86.92 (96.61)	84.91 (96.03)	2.30	2.30	2.41
5	500	37 638	0.1	144	242	280	701/856	674/758	648/720	41.98 (66.46)	34.54 (62.13)	31.18 (60.13)	4.37	4.33	5.29
5	500	102 528	0.8	980	999	999	18/20	1/1	1/1	99.74 (100.00)	99.60 (100.00)	99.60 (100.00)	0.84	0.83	0.84
5	500	102 528	0.6	879	994	994	120/121	6/6	6/6	99.64 (100.00)	99.20 (99.99)	99.20 (99.99)	1.01	1.00	1.02
5	500	102 528	0.4	592	897	918	402/408	103/103	82/82	99.20 (99.98)	98.12 (99.93)	97.85 (99.91)	2.06	2.03	2.07
5	500	102 528	0.2	227	452	530	751/773	541/548	463/470	91.38 (98.69)	88.05 (98.15)	86.30 (97.84)	5.05	4.99	5.16
5	500	102 528	0.1	100	177	219	797/900	752/823	718/781	34.40 (67.16)	28.31 (64.09)	24.55 (62.16)	11.31	11.11	12.29
5	1000	29 550	0.8	952	995	995	43/48	5/5	5/5	98.39 (99.61)	85.89 (96.35)	85.89 (96.35)	0.67	0.75	0.77
5	1000	29 550	0.6	852	985	987	136/148	15/15	13/13	99.68 (99.97)	98.47 (99.83)	98.29 (99.81)	0.80	0.89	0.89
5	1000	29 550	0.4	588	892	915	380/412	103/108	81/85	99.64 (99.98)	99.29 (99.96)	99.23 (99.95)	1.14	1.27	1.29
5	1000	29 550	0.2	276	507	590	626/724	459/493	385/410	94.42 (98.12)	91.94 (97.25)	90.44 (96.70)	2.31	2.60	3.60
5	1000	29 550	0.1	206	327	377	584/794	553/673	527/623	65.14 (81.08)	58.92 (77.68)	55.68 (75.89)	3.66	3.93	17.11
5	1000	118 062	0.8	978	1000	1000	22/22	—	—	99.88 (100.00)	—	—	1.28	1.31	1.33
5	1000	118 062	0.6	901	994	994	99/99	6/6	6/6	99.80 (100.00)	99.53 (99.99)	99.53 (99.99)	1.67	1.70	1.72
5	1000	118 062	0.4	621	912	931	374/379	87/88	68/69	99.44 (99.93)	98.29 (99.70)	97.96 (99.61)	2.88	2.90	2.97
5	1000	118 062	0.2	230	474	562	744/770	519/526	434/438	94.48 (98.70)	92.05 (98.10)	90.58 (97.72)	7.08	7.18	7.95
5	1000	118 062	0.1	103	180	223	808/897	766/820	732/777	41.65 (70.08)	36.19 (67.27)	32.71 (65.46)	14.70	14.68	38.35
5	1000	375 998	0.8	985	1000	1000	15/15	—	—	99.86 (100.00)	—	—	2.54	2.54	2.57
5	1000	375 998	0.6	922	996	996	78/78	4/4	4/4	99.81 (100.00)	99.57 (100.00)	99.57 (100.00)	3.74	3.72	3.77
5	1000	375 998	0.4	675	926	948	321/325	74/74	52/52	99.68 (100.00)	99.39 (100.00)	99.38 (100.00)	6.01	5.96	6.06
5	1000	375 998	0.2	220	467	558	773/780	530/533	440/442	96.33 (99.20)	94.77 (98.84)	93.82 (98.60)	16.69	16.54	17.21
5	1000	375 998	0.1	54	109	166	873/946	833/891	780/834	39.71 (71.51)	36.00 (69.75)	31.69 (67.69)	40.22	39.58	54.80

[a] K, number of features on each arc; $|V|$, number of nodes; $|A|$, number of arcs; q_1, the control parameter about constraint tightness; No. solved, number of instances solved to optimality in preprocessing; No. bounds, number of instances with initial solution (cost upper bounds) provided for the second stage; Node (arc) reductions, percentage of node (arc) elimination for those instances not solved by preprocessing. Each parameter combination (instance type) had 1000 randomly generated instances with different random seeds, leading to a total of 90 000 instances, and the seeds were set from 1 to 90 000.

Table .2
RESULTS OF THREE PREPROCESSING ALGORITHMS FOR THE INSTANCES IN CLASS 2.[a]

K	$\|V\|$	$\|A\|$	q_2	No. solved			No. bounds			Node (arc) reductions (%)			Average process time (s)		
				Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2
2	100	2550	-0.2	276	574	683	716/724	419/426	312/317	88.52 (97.37)	82.87 (95.68)	79.44 (94.33)	0.02	0.02	0.02
2	100	2550	-0.1	233	504	626	763/767	495/496	373/374	87.66 (97.60)	82.81 (96.40)	79.51 (95.36)	0.02	0.02	0.02
2	100	2550	0.0	18	243	417	980/982	755/757	581/583	87.14 (97.88)	84.39 (97.32)	81.59 (96.62)	0.02	0.02	0.02
2	100	2550	0.1	19	253	382	972/981	742/747	613/618	85.16 (97.13)	81.61 (96.30)	79.06 (95.60)	0.02	0.02	0.02
2	100	3750	-0.2	258	573	713	738/742	426/427	286/287	89.28 (98.09)	83.91 (96.78)	79.34 (95.33)	0.03	0.03	0.03
2	100	3750	-0.1	239	518	643	755/761	480/482	355/357	86.90 (97.60)	81.42 (96.29)	77.45 (95.09)	0.03	0.03	0.03
2	100	3750	0.0	14	303	459	983/986	695/697	539/541	86.73 (97.82)	82.67 (96.97)	79.50 (96.16)	0.03	0.03	0.03
2	100	3750	0.1	11	236	375	986/989	761/764	622/625	85.72 (97.89)	82.59 (97.31)	80.15 (96.77)	0.02	0.02	0.02
2	100	4758	-0.2	262	545	685	738/738	455/455	315/315	87.51 (97.12)	81.81 (95.40)	76.69 (93.44)	0.03	0.03	0.03
2	100	4758	-0.1	243	507	627	755/757	492/493	372/373	86.76 (97.60)	81.51 (96.38)	77.83 (95.28)	0.03	0.03	0.03
2	100	4758	0.0	20	285	463	974/980	714/715	536/537	87.42 (98.12)	84.07 (97.46)	80.90 (96.69)	0.03	0.03	0.03
2	100	4758	0.1	9	232	395	991/991	768/768	605/605	85.97 (98.30)	82.97 (97.84)	80.12 (97.31)	0.03	0.03	0.03
2	500	37 638	-0.2	314	575	678	686/686	425/425	322/322	94.55 (98.44)	91.73 (97.49)	89.58 (96.70)	0.18	0.18	0.19
2	500	37 638	-0.1	267	537	658	733/733	463/463	342/342	95.34 (98.82)	93.16 (98.15)	91.35 (97.50)	0.16	0.17	0.17
2	500	37 638	0.0	3	255	381	996/997	744/745	618/619	94.93 (98.82)	93.51 (98.42)	92.50 (98.11)	0.16	0.17	0.18
2	500	37 638	0.1	4	222	338	996/996	778/778	662/662	95.46 (99.29)	94.43 (99.09)	93.73 (98.94)	0.14	0.16	0.16
2	500	65 142	-0.2	283	569	682	717/717	431/431	318/318	94.94 (98.88)	92.16 (98.14)	90.00 (97.49)	0.23	0.24	0.25
2	500	65 142	-0.1	258	539	657	741/742	460/461	342/343	94.84 (99.01)	92.24 (98.41)	90.16 (97.86)	0.22	0.23	0.23
2	500	65 142	0.0	1	281	407	998/999	718/719	592/593	96.12 (99.50)	94.97 (99.30)	94.24 (99.16)	0.18	0.20	0.20
2	500	65 142	0.1	3	231	346	996/997	768/769	653/654	93.98 (98.94)	92.47 (98.63)	91.42 (98.39)	0.20	0.22	0.23
2	500	102 528	-0.2	334	627	725	666/666	373/373	275/275	94.58 (98.82)	91.02 (97.89)	88.41 (97.14)	0.34	0.34	0.35
2	500	102 528	-0.1	274	532	648	726/726	468/468	352/352	94.34 (99.17)	91.74 (98.72)	89.59 (98.31)	0.29	0.30	0.30
2	500	102 528	0.0	2	239	369	998/998	761/761	631/631	95.45 (99.40)	94.31 (99.21)	93.46 (99.05)	0.26	0.27	0.28
2	500	102 528	0.1	1	227	333	999/999	773/773	667/667	95.92 (99.80)	94.99 (99.74)	94.46 (99.70)	0.23	0.24	0.24
5	100	2550	-0.2	182	253	281	132/818	127/747	124/719	22.71 (40.49)	15.61 (34.85)	12.62 (32.34)	0.28	0.29	0.30
5	100	2550	-0.1	142	220	249	101/858	97/780	92/751	21.98 (38.50)	14.42 (32.36)	11.43 (29.77)	0.29	0.30	0.31
5	100	2550	0.0	53	159	209	81/947	80/841	79/791	24.50 (38.07)	15.23 (30.28)	10.28 (25.90)	0.28	0.29	0.30
5	100	2550	0.1	43	150	182	80/957	79/850	78/818	21.20 (34.14)	11.51 (25.87)	8.30 (22.98)	0.28	0.29	0.30
5	100	3750	-0.2	134	198	227	141/866	141/802	140/773	19.92 (37.67)	13.72 (32.71)	10.78 (30.20)	0.41	0.42	0.42
5	100	3750	-0.1	112	159	189	158/888	156/841	151/811	14.86 (33.26)	10.22 (29.54)	7.20 (26.94)	0.41	0.41	0.43
5	100	3750	0.0	35	132	169	125/965	125/868	125/831	19.81 (35.26)	11.06 (28.03)	7.39 (24.84)	0.39	0.40	0.41
5	100	3750	0.1	27	97	127	122/973	122/903	120/873	15.66 (30.31)	9.25 (24.92)	6.38 (22.35)	0.40	0.41	0.42
5	100	4758	-0.2	118	166	197	193/882	192/834	190/803	16.14 (35.50)	11.44 (31.80)	8.32 (29.18)	0.50	0.51	0.52
5	100	4758	-0.1	97	138	160	175/903	175/862	174/840	14.13 (31.95)	10.16 (28.72)	8.02 (26.86)	0.50	0.50	0.52
5	100	4758	0.0	37	107	134	139/963	137/893	135/866	14.35 (29.50)	7.79 (23.98)	5.15 (21.62)	0.49	0.50	0.51
5	100	4758	0.1	31	86	122	141/969	139/914	137/878	13.42 (28.45)	8.32 (24.15)	4.86 (21.05)	0.49	0.50	0.51
5	500	37 638	-0.2	42	62	78	567/958	563/938	553/922	12.44 (37.51)	10.58 (36.18)	9.07 (35.07)	4.03	4.11	6.44
5	500	37 638	-0.1	43	60	75	469/957	466/940	462/925	11.18 (34.68)	9.58 (33.50)	8.15 (32.42)	3.93	4.02	6.29
5	500	37 638	0.0	9	29	51	472/991	471/971	464/949	12.04 (37.64)	10.24 (36.36)	8.19 (34.88)	3.59	3.68	6.37
5	500	37 638	0.1	5	27	34	454/995	453/973	451/966	9.03 (35.60)	6.99 (34.15)	6.33 (33.67)	3.43	3.51	6.45
5	500	65 142	-0.2	25	39	51	583/975	580/961	577/949	9.34 (39.30)	8.03 (38.42)	6.89 (37.64)	6.07	6.20	7.97
5	500	65 142	-0.1	16	31	46	583/984	580/969	570/954	10.45 (41.28)	9.08 (40.37)	7.68 (39.43)	5.79	5.91	7.77
5	500	65 142	0.0	8	29	37	540/992	537/971	534/963	8.51 (39.72)	6.54 (38.42)	5.77 (37.91)	5.55	5.67	7.67
5	500	65 142	0.1	6	21	29	527/994	525/979	524/971	8.67 (41.38)	7.27 (40.48)	6.52 (39.99)	5.25	5.37	7.47
5	500	102 528	-0.2	25	39	47	628/975	622/961	620/953	8.48 (43.59)	7.16 (42.77)	6.39 (42.29)	8.70	8.87	10.43

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Table .2
RESULTS OF THREE PREPROCESSING ALGORITHMS FOR THE INSTANCES IN CLASS 2.[a] (CONTINUED)

K	$\|V\|$	$\|A\|$	q_2	No. solved			No. bounds			Node (arc) reductions (%)			Average process time (s)		
				Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2	Est0	Est1	Est2
5	500	102 528	-0.1	13	20	31	597/987	596/980	590/969	7.79 (42.85)	7.14 (42.45)	6.11 (41.79)	8.23	8.39	10.02
5	500	102 528	0.0	10	19	27	602/990	601/981	597/973	8.12 (44.79)	7.28 (44.28)	6.54 (43.83)	8.05	8.21	9.81
5	500	102 528	0.1	7	15	17	582/993	581/985	580/983	6.61 (46.08)	5.85 (45.65)	5.66 (45.53)	7.42	7.56	9.27

[a] The random seeds for a total of 48 000 instances were set from 90 001 to 138 000.

Table .3
RESULTS OF THE LB-FIRST ALGORITHM AND TWO-STAGE METHOD FOR THE INSTANCES IN CLASS 2.[a]

K	$\|V\|$	$\|A\|$	q_2	Advantages of Stage 1			No. with better solutions	Average process time (s)					Average speedup	
								Two-stage method			LB-first Pulse	Pulse	Pulse/ Two-stage method	Pulse/ LB-first Pulse
				No. solved	No. bounds	Node (arc) reductions (%)		Stage 1	Stage 2	Sum				
2	100	2550	-0.2	63	37/37	79.04 (94.70)	7/37	0.02	0.001	0.02	0.03	0.05	2.12	1.85
2	100	2550	-0.1	55	45/45	78.16 (93.57)	6/45	0.02	0.004	0.02	0.04	0.08	3.21	1.99
2	100	2550	0	42	58/58	83.57 (98.16)	12/58	0.02	0.001	0.02	0.03	0.08	3.54	2.65
2	100	2550	0.1	36	64/64	80.87 (96.48)	10/64	0.02	0.001	0.02	0.06	0.09	4.51	1.64
2	100	3750	-0.2	70	30/30	69.80 (89.46)	9/30	0.03	0.003	0.03	0.04	0.11	3.57	2.66
2	100	3750	-0.1	59	41/41	80.66 (95.32)	6/41	0.02	0.001	0.02	0.05	0.11	4.63	2.41
2	100	3750	0	44	56/56	82.42 (98.15)	6/56	0.02	0.001	0.02	0.05	0.14	6.33	2.87
2	100	3750	0.1	38	61/62	71.20 (90.56)	15/62	0.03	0.005	0.04	0.05	0.14	3.85	2.61
2	100	4758	-0.2	70	30/30	72.72 (90.54)	6/30	0.03	0.004	0.03	0.05	0.17	5.08	3.06
2	100	4758	-0.1	67	33/33	74.46 (93.98)	8/33	0.03	0.001	0.03	0.11	0.16	4.78	1.52
2	100	4758	0	45	54/55	84.27 (99.31)	5/55	0.02	0.001	0.02	0.06	0.19	8.26	3.00
2	100	4758	0.1	45	55/55	82.52 (97.57)	7/55	0.03	0.002	0.03	0.06	0.19	5.84	3.10
2	500	37 638	-0.2	72	28/28	90.23 (97.39)	5/28	0.18	0.012	0.19	0.64	2.03	10.80	3.16
2	500	37 638	-0.1	69	31/31	92.86 (98.09)	5/31	0.14	0.012	0.14	0.67	2.97	20.72	4.42
2	500	37 638	0	39	61/61	96.75 (99.93)	1/61	0.13	0.001	0.14	0.97	10.28	76.06	10.65
2	500	37 638	0.1	34	66/66	94.19 (99.52)	3/66	0.14	0.002	0.14	0.85	2.97	21.63	3.51
2	500	65 142	-0.2	68	32/32	97.20 (99.97)	0/32	0.21	0.000	0.21	1.49	5.87	28.53	3.95
2	500	65 142	-0.1	61	38/39	87.38 (99.02)	4/39	0.20	0.004	0.20	1.46	6.08	30.38	4.15
2	500	65 142	0	35	65/65	94.61 (99.09)	2/65	0.20	0.009	0.20	1.37	7.23	35.51	5.26
2	500	65 142	0.1	32	68/68	94.33 (99.48)	2/68	0.19	0.002	0.19	1.30	6.28	32.51	4.83
2	500	102 528	-0.2	75	25/25	89.25 (96.18)	2/25	0.34	0.096	0.36	1.65	10.27	28.44	6.21
2	500	102 528	-0.1	68	32/32	84.45 (96.76)	4/32	0.31	0.026	0.32	2.25	12.04	37.44	5.35
2	500	102 528	0	39	61/61	91.93 (98.84)	3/61	0.30	0.008	0.30	2.08	13.33	44.46	6.42
2	500	102 528	0.1	35	65/65	95.25 (99.96)	6/65	0.24	0.001	0.24	2.34	13.17	53.76	5.62
5	100	2550	-0.2	21	16/79	17.24 (38.91)	52/79	0.31	0.389	0.62	0.37	1.06	1.72	2.89
5	100	2550	-0.1	26	15/74	13.97 (34.77)	48/74	0.29	0.683	0.80	0.58	1.36	1.71	2.34
5	100	2550	0	17	9/83	8.15 (26.99)	65/83	0.33	0.868	1.05	0.79	3.08	2.93	3.91
5	100	2550	0.1	10	11/90	7.55 (27.39)	69/90	0.33	1.152	1.37	1.16	3.67	2.68	3.17
5	100	3750	-0.2	27	21/73	11.11 (32.63)	58/73	0.40	0.539	0.79	0.51	1.10	1.38	2.13
5	100	3750	-0.1	19	11/81	11.29 (32.66)	59/81	0.40	1.042	1.25	0.97	2.21	1.77	2.28
5	100	3750	0	17	15/83	9.56 (23.99)	68/83	0.38	1.514	1.64	1.51	5.99	3.65	3.97
5	100	3750	0.1	16	8/84	4.87 (20.95)	70/84	0.40	1.972	2.05	1.83	6.49	3.16	3.56

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Table .3
RESULTS OF THE LB-FIRST ALGORITHM AND TWO-STAGE METHOD FOR THE INSTANCES IN CLASS 2.[a] (CONTINUED)

K	$\|V\|$	$\|A\|$	q_2	Advantages of Stage 1			No. with better solutions	Average process time (s)					Average speedup	
								Two-stage method			LB-first Pulse	Pulse	Pulse/ Two-stage method	Pulse/ LB-first Pulse
				No. solved	No. bounds	Node (arc) reductions (%)		Stage 1	Stage 2	Sum				
5	100	4758	-0.2	20	22/80	5.88 (31.78)	63/80	0.52	0.747	1.12	0.75	1.75	1.56	2.35
5	100	4758	-0.1	15	21/85	2.29 (16.21)	78/85	0.51	1.320	1.63	1.29	5.28	3.24	4.10
5	100	4758	0	9	12/91	5.49 (22.36)	74/91	0.52	1.820	2.17	1.81	6.13	2.82	3.38
5	100	4758	0.1	16	8/84	6.49 (21.30)	71/84	0.48	2.429	2.52	2.25	11.61	4.61	5.15
5	500	37 638	-0.2	10	57/90	10.37 (37.75)	78/90	5.12	4.793	9.44	6.50	24.78	2.63	3.81
5	500	37 638	-0.1	5	50/95	7.78 (31.02)	84/95	4.78	7.873	12.26	10.18	48.43	3.95	4.76
5	500	37 638	0	3	42/97	12.81 (34.23)	81/97	5.06	9.492	14.26	12.59	75.78	5.31	6.02
5	500	37 638	0.1	3	54/97	8.95 (41.52)	84/97	4.26	12.456	16.34	17.08	150.68	9.22	8.82
5	500	65 142	-0.2	3	61/97	5.78 (29.16)	87/97	8.33	8.789	16.85	11.30	55.78	3.31	4.94
5	500	65 142	-0.1	6	60/94	10.18 (42.67)	83/94	6.62	10.928	16.89	17.20	132.35	7.83	7.70
5	500	65 142	0	6	46/94	6.42 (37.41)	86/94	6.71	15.191	20.99	20.00	178.29	8.50	8.91
5	500	65 142	0.1	0	61/100	6.14 (40.54)	91/100	6.49	16.145	22.64	23.80	379.12	16.75	15.93
5	500	102 528	-0.2	0	68/100	9.74 (47.01)	87/100	9.67	10.580	20.25	18.19	148.45	7.33	8.16
5	500	102 528	-0.1	4	57/96	6.52 (44.07)	88/96	9.09	16.794	25.21	24.71	238.39	9.46	9.65
5	500	102 528	0	2	62/98	4.29 (44.37)	94/98	9.11	22.256	30.92	32.55	348.11	11.26	10.69
5	500	102 528	0.1	5	55/95	5.82 (41.56)	88/95	9.27	29.691	37.47	40.35	602.92	16.09	14.94

[a] No. with better solutions, the number of instances where a better solution exists for those instances not solved in stage 1; Average speedup, the ratio of the average process time of two algorithms. Note that "Stage 2" represents the average time in stage 2 for only unsolved instances in Stage 1, leading to "Stage 1"+"Stage 2"≥"Sum". Only 100 test instances were randomly generated for each instance type for the sake of time and a total of 4800 random seeds were set from 138 001 to 142 800.