DEPT. OF COMPUTER SCIENCE, UNIVERSITY OF CALIFORNIA, DAVIS
ECS122A INSTRUCTOR: QIRUN ZHANG SEPTEMBER 6TH, 2017

Homework #4: Due 9/12/17 by 11:55pm*

1. Give a recursive algorithm MATRIX-CHAIN-MULTIPLY(A, s, i, j) that actually performs the optimal matrix-chain multiplication for a chain of matrices $A_i \times A_{i+1} \times \ldots \times A_j$. Suppose that we are given a sequence of matrices $\langle A_1, A_2, \ldots, A_n \rangle$ and the s table has already been computed by MATRIX-CHAIN-ORDER. Note that the initial call would be MATRIX-CHAIN-MULTIPLY$(A, s, 1, n)$.

   Solution. Your solution here.

2. Give pseudocode to reconstruct an LCS from the completed c table and the original sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$ in $O(m+n)$ time, without using the b table.

   Solution. Your solution here.

3. There are two teams A and B playing against each other to see who will win n games first. A and B are equally competent. Therefore, each team has a 50% chance of winning any particular game. Suppose they have already played $x + y$ games, of which A has won x and B has won y games, respectively. Give an algorithm to compute the probability that A will go on to win the match. What's the worst-case time complexity of your algorithm? For instance,if $x = n - 1$ and $y = n - 3$ then the probability that A will win the match is 7/8, since it must win any of the next three games.

   Solution. Your solution here.

4. Let $G = (V, E)$ be a weighted, directed graph with weight function $w \colon E \to \{-1, +1\}$. The weight $w(p)$ of a path p is the sum of all edge weights in the path. We say a path p is a *Z-path* iff

   - its path weight $w(p)$ is 0, *and*
   - any positive edge (*i.e.,* the edge with $+1$ weight) appears after all negative edges (*i.e.,* the edges with -1 weights) in path p.

*Last update September 6, 2017

For example, the path "$-1, -1, +1, +1$" is a Z-path but "$-1, +1, -1, +1$" is not. A node v in V is *Z-path-reachable* from node u iff there is a Z-path p from u to v in G. Give an $O(EV)$-time algorithm compute the *all* Z-path-reachable node pairs in the digraph G. Note that the graph can contain negative cycles.

Solution. Your solution here.