

Homework #2: Due 8/23/17 by 11:55pm*

1. a) Show that an n -element heap has height $\lfloor \log_2 n \rfloor$.
Solution. Your solution here.
b) Show that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any n -element heap.
Solution. Your solution here.
2. a) Using Figure 7.1 in CLRS as a model, illustrate the operation of PARTITION on the array $A = \langle 9, 5, 2, 6, 3, 4 \rangle$.
Solution. Your solution here.
b) Use Strassen's algorithm to compute the matrix product $\begin{pmatrix} 1 & 2 \\ 6 & 9 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 4 & 8 \end{pmatrix}$. Give the intermediate steps with respect to each \pm and \times operation. How many \pm operations are there? How many \times operations are there?
Solution. Your solution here.
3. Design a divide-and-conquer algorithm to solve the following problem in $O(n \log n)$ time: Given an integer array A of size n , compute the number $|S|$, where

$$S = \{(i, j) \mid i < j \text{ and } A[i] > A[j]\}.$$

Solution. Your solution here.

4. In equation (4.10) of CLRS, we see that the product of two matrices can be expressed in terms of the blocks. In this problem, we will prove this property. Formally, given two $n \times n$ matrices $X = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ and $Y = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$, where A_{11} , A_{12} , A_{21} , A_{22} , B_{11} , B_{12} , B_{21} , and B_{22} are $n/2 \times n/2$ submatrices, prove that the product XY can be expressed in terms of these blocks, *i.e.*,

$$XY = \begin{pmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{pmatrix}.$$

Solution. Your solution here.

*Last update August 17, 2017

5. Try to analyze the following algorithm.

```
Function Foo( $n$ )  
  if  $n > 1$  then  
    Print 'A'  
    Foo( $n/3$ )  
    for  $i$  from 1 to  $n$  do  
      | Print 'B'  
    end  
    Foo( $n/3$ )  
  end
```

- a) What is the running time of the function **Foo**(n)? Give detailed steps to obtain the asymptotic running time in big- Θ notation.
- b) How many 'A's will be printed by **Foo**(n)? Give detailed steps to obtain your solution in big- Θ notation (in terms of n).

Solution. Your solution here.