Example: Gaussian wavepacket

Previously:
$$c(k) = \left(\frac{2a^2}{\pi}\right)^{1/4} e^{-a^2(k-k_0)^2}$$

$$\rightarrow \quad \tilde{\Psi}(p,0) = \frac{1}{\sqrt{\hbar}}\, c(p/\hbar)$$

$p_0 = \hbar k_0$

$$= \left(\frac{2a^2}{\pi\hbar^2}\right)^{1/4} e^{-a^2(p-p_0)^2/\hbar^2}$$

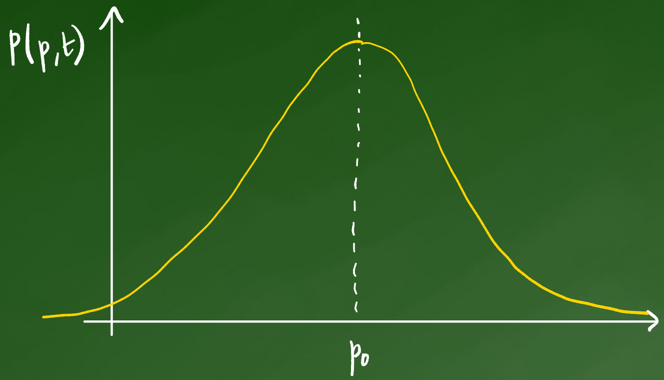
Can also show
$$\tilde{\Psi}(p,t) \quad \left(\frac{2a^2}{\pi\hbar^2}\right)^{1/4} e^{-a^2(p-p_0)^2/\hbar^2} \underbrace{e^{-ip^2t/2m\hbar}}_{\text{time-dependent phase factor.}}$$

$$P(p,t) = |\tilde{\Psi}(p,t)|^2 = \left(\frac{2a^2}{\pi\hbar^2}\right)^{1/2} e^{-2a^2(p-p_0)^2/\hbar^2}$$

$e^{i\theta}e^{-i\theta} = 1$

Independent of time because free particle.

this is conservation of momentum when there are no forces in QM.



$$\langle p \rangle = p_0$$

normal distribution

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$= \frac{\hbar}{2a}$$

→ particle has a range of momenta

→ this explains why wavepacket spreads