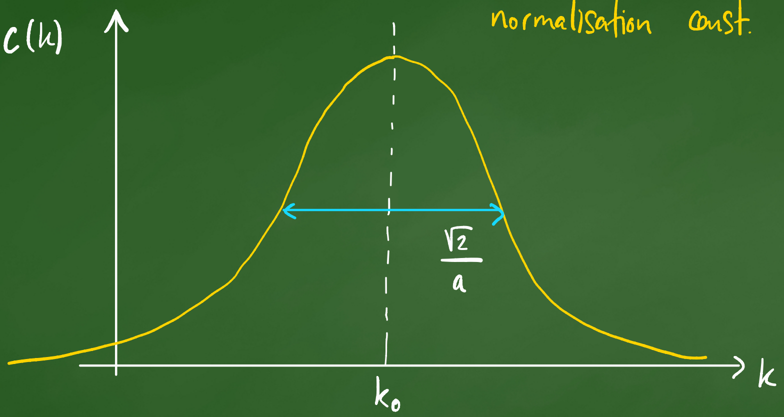
Example: Gaussian Wavepacket

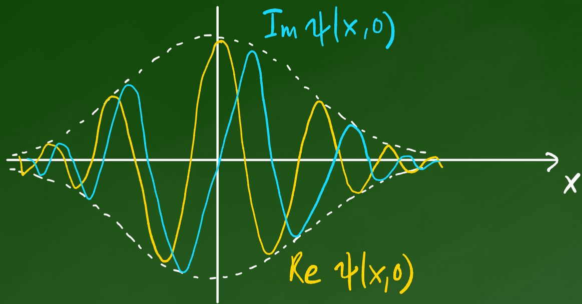
Consider $$c(k) = \underbrace{\left(\frac{2a^2}{\pi}\right)^{1/4}}_{\text{normalisation const.}} e^{-a^2(k-k_0)^2}$$

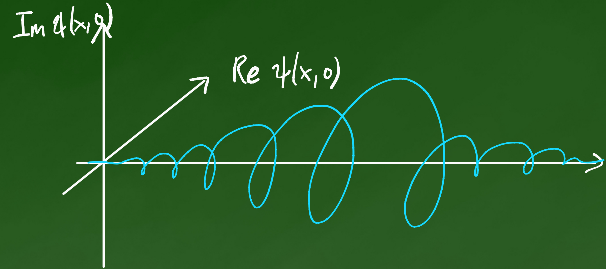
$a > 0$, k_0 constants



Q: What is $$\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\frac{2a^2}{\pi}\right)^{1/4} e^{-a^2(k-k_0)^2} e^{ikx}\, dk$$

Problem Sheet: $$= \left(\frac{1}{a\sqrt{2\pi}}\right)^{1/2} \underbrace{e^{-x^2/4a^2}}_{\text{Gaussian}} \underbrace{e^{ik_0 x}}_{\text{phase}}$$





+ 3rd method: use colour to denote phase.

$$P(x,0) = |\psi(x,0)|^2 = \frac{1}{a\sqrt{2\pi}}\, e^{-x^2/2a^2}$$

Normal distribution!

$$e^{-i\theta} \cdot e^{i\theta} = 1$$

$$\langle x \rangle = 0$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = a$$

Q: what about later times?

possible to solve, but long + tedious calculation

$$\psi(x,t) = \left(\frac{1}{a(1+i\gamma t)\sqrt{2\pi}} \right)^{1/2} \exp\left(-\frac{(x - \hbar k_0 t/M)^2}{4a^2(1+i\gamma t)} + ik_0\left(x - \frac{\hbar k_0 t}{2M}\right)\right)$$

$$\gamma = \frac{\hbar}{2Ma^2}$$

Check: $t=0 \rightarrow \psi(x,0)$ ✓

From video: - wavepacket moves to the right
- it spreads out in time as it moves.

$$P(x,t) = |\psi(x,t)|^2 = \frac{1}{a\sqrt{2\pi(1+\gamma^2 t^2)}} \exp\left(- \frac{(x-\hbar k_0 t/M)^2}{2a^2(1+\gamma^2 t^2)} \right)$$

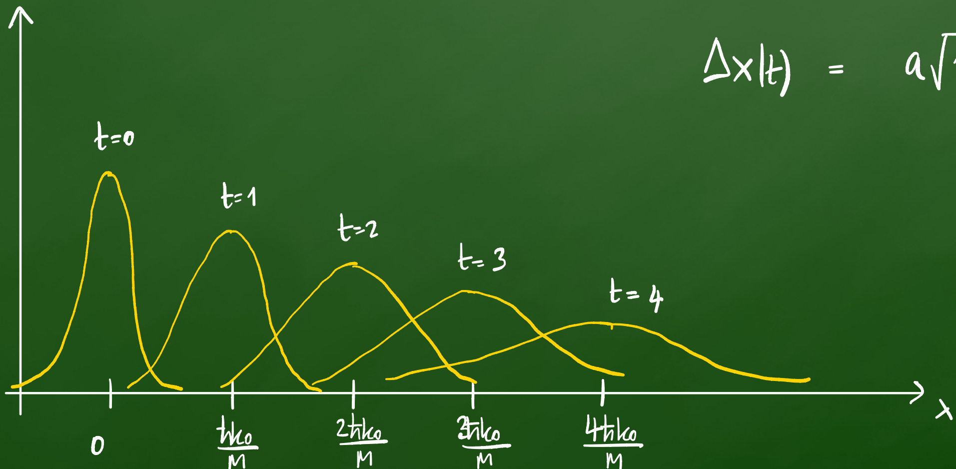
normal distribution for all t

$$\langle x(t) \rangle = \frac{\hbar k_0}{M} t$$

const. speed.

$$\Delta x(t) = a\sqrt{1+\gamma^2 t^2}$$

t large
$\Delta x(t) \approx a\gamma t$



Particle has momentum.

Finally: look at parameters:

- Rate of spreading is independent of speed (k_0)

$$\gamma = \frac{\hbar}{2Ma^2}$$

- Heavier particles spread slower

- Particles that are initial more confined spread faster

particles don't like to stay confined.