PHYS22040 Problem Set I

7. The HERA (Hadron-Elektron-Ring-Anlage) electron/positron proton collider in Hamburg collided beams of 820 GeV protons with 27.5 GeV electrons. What was the centre of mass energy of this accelerator?

10. Particle A with total energy E_A and mass m_A is incident on a stationary particle of mass m_B. The interaction produces a resonant state with mass m_C. Show that the mass of the produced resonance is given by

$$m_C = \sqrt{2E_a m_B + m_B^2 + m_A^2}$$

35. A pion beam is incident on stationary protons. In the process a Δ^{++} and a π^0 is produced. Calculate the minimum energy of the pions for this reaction to happen.

36. A Σ^+ with an energy of 1500 MeV decays into a n and a π^+. The π^+ emerges under 90^o with respect to the direction of the Σ^+. The mass of a Σ^+ is 1189.4 MeV. Calculate the energy of the π^+.

55. In the BaBar detector 9.0 GeV electrons collide head-on with 3.1 GeV positrons to produce an Υ(4s) which has a mass of 10.58 GeV. The electrons move in the $+z$-direction. The rest mass of the electron may be neglected. The Υ(4s) decays immediately into a B^0-$\overline{B}^0$ pair (or a B^+ and B^- which we will not use in this question). The mass of the B^0 and $\overline{B}^0$ is 5.29 GeV. In an event the B^0 decays into $J/\psi K^+\pi^-$ and the $\overline{B}^0$ into $K^-\pi^+$. Subsequently, the J/ψ decays into a $\mu^+\mu^-$ pair. Six particles are measured by the detector. In GeV, their momenta are

$$p_1 = \begin{pmatrix} 0.810 \\ -0.676 \\ 0.300 \end{pmatrix} \qquad p_2 = \begin{pmatrix} -1.717 \\ -1.940 \\ 1.519 \end{pmatrix} \qquad p_3 = \begin{pmatrix} -0.094 \\ -0.076 \\ 1.303 \end{pmatrix} \tag{1}$$

$$p_4 = \begin{pmatrix} 2.000 \\ 1.723 \\ 1.4203 \end{pmatrix} \qquad p_5 = \begin{pmatrix} 0.685 \\ -0.427 \\ -0.177 \end{pmatrix} \qquad p_6 = \begin{pmatrix} -1.684 \\ 1.396 \\ 1.534 \end{pmatrix} \tag{2}$$

and their energies are

$$E_1 = 1.102 \qquad E_2 = 3.044 \qquad E_3 = 1.316 \tag{3}$$

$$E_4 = 3.000 \qquad E_5 = 0.963 \qquad E_6 = 2.674 \tag{4}$$

Particle 1 and particle 6 are muons.

(a) Calculate the energy and momentum of the Υ(4s) in the lab frame.

(b) Calculate the invariant mass of the J/ψ.

(c) Which two of these particles are the particles originating from the $\overline{B}^0$ decay? Explain your answer.

(d) Calculate the energy and momentum of the B^0.

(e) Demonstrate that the B^0 and the $\overline{B}^0$ indeed originate from the Υ(4s).

The Fundamental Forces

Force	Gauge Boson	Mass (GeV/c^2)
Strong	Gluon	0
Weak	W, Z	80, 91
Electromagnetic	Photon	0
Gravity	Graviton	0

The Leptons / **The Quarks**

Flavour	Charge	Mass (GeV/c^2)	Flavour	Charge	Mass (GeV/c^2)
e^-	-1	5.11$\times 10^{-4}$	u	$+2/3$	< 0.01
ν_e	0	~ 0	d	$-1/3$	< 0.01
μ^-	-1	0.106	c	$+2/3$	≈ 1.1
ν_μ	0	~ 0	s	$-1/3$	≈ 0.3
τ^-	-1	1.784	t	$+2/3$	≈ 172
ν_τ	0	~ 0	b	$-1/3$	≈ 5.4

Mesons / **Baryons**

Meson	Quarks	Mass (GeV/c^2)	Baryon	Quarks	Mass (GeV/c^2)
π^+	$u\bar{d}$	0.14	p	uud	0.938
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0.13	n	udd	0.940
K^+	$u\bar{s}$	0.49	Λ^0	uds	1.116
K^0	$d\bar{s}$	0.50	Σ^+	uus	1.189
D^+	$c\bar{d}$	1.87	Σ^0	uds	1.193
D^0	$c\bar{u}$	1.86	Σ^-	dds	1.197
B^+	$u\bar{b}$	5.28	Δ^{++}	uuu	1.232
B^0	$d\bar{b}$	5.28	Λ_c^+	udc	2.285
J/ψ	$c\bar{c}$	3.10	Δ^-	ddd	1.232
Υ(1s)	$b\bar{b}$	9.46	Ξ_b^0	usb	5.788

Constants

$c = 2.998 \times 10^8$ m s^{-1}

$\theta_c \approx 13°$

$\alpha = \frac{e^2}{4\pi\hbar c\epsilon_0} \approx 1/137$ (at low energies)

$\alpha_s \approx 1/8$ (at M_Z)

$G_F \approx 1.166 \times 10^{-5}$ GeV^{-2}

1 barn $= 10^{-28}$ m^2

$\hbar c = 197$ MeV fm

N_A = 6.0221$\times 10^{23}$ mol^{-1}

Units

1 Bq	=	1 disintegration s^{-1}
1 Curie (Ci)	=	3.7×10^{10} Bq
1 u	=	931.494 MeV/c^2
1 amu	$\approx$	1.66×10^{-27} kg

Atomic masses

Proton	1.007276 u
Neutron	1.008665 u
$^1_1$H	1.007825 u
$^4_2$He	4.002603 u

Magic numbers

2, 8, 20, 28, 50, 82, 126

Radioactive decay

$$N(t) = N_0 e^{-\lambda t}$$

Uncertainty principle

$$\Delta E \Delta t > \frac{\hbar}{2} \qquad \Delta p \Delta x > \frac{\hbar}{2}$$

Rutherford scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Ze^2}{16\pi\varepsilon_0 K_\alpha}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Semi-empirical Mass Formula

$$\begin{aligned} M(A,Z) &= Zm_p + (A-Z)m_n + Zm_e \\ &\quad -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} \\ &\quad +a_A \frac{(A-2Z)^2}{A} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \frac{a_P}{A^{3/4}} \end{aligned}$$

Mass constants

a_V	=	15.56 MeV/c^2
a_S	=	17.23 MeV/c^2
a_C	=	0.70 MeV/c^2
a_A	=	23.29 MeV/c^2
a_P	=	12.0 MeV/c^2

Answer:

7. *The particle energies are much much larger than their rest mass, so we can ignore those safely and $|p_p| \approx E_p$ and $|p_e| \approx E_e$. The collisions are head on, so if the proton moves in the $+z$ direction, then the electron moves in the $-z$ direction.*

$$\sqrt{s} = \sqrt{E_{tot}^2 - p_{tot}^2} = \sqrt{(E_p + E_e)^2 - (E_p - E_e)^2} = \sqrt{4E_pE_e} = 300\ GeV$$

10. *The total energy $E_{tot} = E_A + m_B$. The total momentum $p_{tot} = p_A = \sqrt{E_A^2 - m_A^2}$. The invariant mass of C is*

$$m_c = \sqrt{E_{tot}^2 - p_{tot}^2} = \sqrt{E_A^2 + m_B^2 + 2m_BE_A - E_A^2 + m_A^2} = \sqrt{2E_Am_B + m_B^2 + m_A^2}$$

35. *Use s conservation.*

$$\begin{aligned} s_{after} &= (m_{\Delta^{++}} + m_\pi)^2 \\ s_{before} &= (E_\pi + m_p)^2 - \vec{p}_\pi^{\,2} = E_\pi^2 + m_p^2 + 2E_\pi m_p - \vec{p}_\pi^{\,2} = m_\pi^2 + m_p^2 + 2E_\pi m_p \\ E_\pi &= \frac{(m_{\Delta^{++}} + m_\pi)^2 - m_\pi^2 - m_p^2}{2m_p} = 524\ MeV \end{aligned}$$

36. *First energy and momentum conservation.*

$$E_{\Sigma^+} = E_{\pi^+} + E_n \qquad \vec{p}_{\Sigma^+} = \vec{p}_{\pi^+} + \vec{p}_n \tag{5}$$

$$\begin{aligned} \vec{p}_n^{\,2} &= (\vec{p}_{\Sigma^+} - \vec{p}_{\pi^+})^2 \\ E_n^2 - m_n^2 &= E_{\Sigma^+}^2 - m_{\Sigma^+}^2 + E_{\pi^+}^2 - m_{\pi^+}^2 - 2\vec{p}_{\Sigma^+} \cdot \vec{p}_{\pi^+} \\ E_n^2 - m_n^2 &= 2E_{\pi^+}^2 + E_n^2 + 2E_{\pi^+}E_n - m_{\Sigma^+}^2 - m_{\pi^+}^2 \\ -m_n^2 &= 2E_{\pi^+}(E_{\pi^+} + E_n) - m_{\Sigma^+}^2 - m_{\pi^+}^2 \\ -m_n^2 &= 2E_{\pi^+}E_{\Sigma^+} - m_{\Sigma^+}^2 - m_{\pi^+}^2 \\ E_{\pi^+} &= \frac{m_{\Sigma^+}^2 + m_{\pi^+}^2 - m_n^2}{2E_{\Sigma^+}} = 183.2\ MeV \end{aligned}$$

55. (a) *The energy is the sum of the electron and positron energy, which is $9.0 + 3.1 = 12.1$ GeV. The momentum in the z direction is trivially 9.0-3.1=5.9 GeV in the direction of the electron. The momentum components in the x and y direction are 0. Hence $p_\Upsilon = (0, 0, 5.9)$.*

    (b) *The only particles that give signals in the muon chambers are the muons. Hence*

$$E_{J/\psi} = 3.776 \qquad p_{J/\psi} = p_1 + p_6 = \begin{pmatrix} -0.874 \\ 0.72 \\ 1.834 \end{pmatrix} \tag{6}$$

$$m = \sqrt{3.776^2 - 0.874^2 - 0.72^2 - 1.834^2} = 3.1\ GeV/c^2 \tag{7}$$

(c) *You already know that two particles, 1 and 6, form the J/ψ. Since the J/ψ and the K^+ and π^- form the B^0, the two tracks of the K^+ and the π^- do not form a complete B^0.*
Hence two of the particles, the K^- and π^+, form the the $\overline{B}^0$. Hence there are 6 combinations left: (2,3), (2,4), (2,5), (3,4), (3,5) and (4,5). One can proceed by calculating the invariant masses for each combination until a combination with the invariant mass of a $\overline{B}^0$ is found.
Here there is a quicker way. In order to obtain an invariant mass of a $\overline{B}^0$, the sum of their energies needs to be higher than the mass of the $\overline{B}^0$. This only allows the combination of 2 and 4.
Proof:

$$E_{\overline{B}^0} = 6.044 \qquad p_{\overline{B}^0} = \begin{pmatrix} 0.283 \\ -0.217 \\ 2.9393 \end{pmatrix} \Rightarrow m_{\overline{B}^0} = 5.269\ GeV/c^2 \tag{8}$$

(d) *There are two possibilities. Either sum the energies and momenta for particle 1, 3, 5 and 6 or use that $E_\Upsilon = E_{B^0} + E_{\overline{B}^0}$ and $p_\Upsilon = p_{B^0} + p_{\overline{B}^0}$. Both yield*

$$E_{B^0} = 6.055 \qquad p_{B^0} = \begin{pmatrix} -0.283 \\ 0.217 \\ 2.96 \end{pmatrix} \tag{9}$$

(e) *Again calculate invariant mass when combining the B^0 and the $\overline{B}^0$. This will yield and energy of 12.1 GeV and a momentum vector of $p = (0, 0, 5.9)$ and thus an invariant mass of 10.6 GeV/c^2.*