Mathematical Physics
Formula Sheet

Useful Identities

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B) \quad \cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x = \sqrt{\pi}$$

Kronecker Delta

$$\delta_{ij} = \begin{cases} 1 \text{ for } i = j \\ 0 \text{ otherwise} \end{cases} \qquad \sum_i a_i \delta_{ij} = a_j$$

Probabilities

$$P(A \text{ and } B) = P(A)\cdot P(B|A) = P(B)\cdot P(A|B), \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

A, B independent: $P(B|A) = P(B)$, $P(A|B) = P(A)$
A, B mutually exclusive: $P(A \text{ and } B) = 0$

Probability Distributions

Important distributions

name	formula	mean	std-dev (σ)
Gaussian	$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ
Poisson	$P(r) = \frac{\lambda^r}{r!} e^{-\lambda}$	λ	$\sqrt{\lambda}$
Binomial	$P(r) = \left(\begin{array}{c} N \\ r \end{array}\right) p^r (1-p)^{N-r}$	Np	$\sqrt{Np(1-p)}$

Key properties

Discrete probability distribution: $\sum\limits_{\text{all x}} P(x) = 1$; Continuous: $\int p(x)\,\mathrm{d}x = 1$, $p(y) = p(x)\left|\frac{\mathrm{d}x}{\mathrm{d}y}\right|$

Mean/expectation value, variance, standard deviation

	mean/expectation value	Variance V (note: $\sigma = \sqrt{V}$)
Data	$\overline{x} = \frac{1}{N}\sum\limits_i^N x_i \quad , \quad \overline{f(x)} = \frac{1}{N}\sum\limits_i^N f(x_i)$	$V = \frac{1}{N}\sum\limits_i^N (x_i - \overline{x})^2 = \overline{x^2} - \overline{x}^2$
Discrete Prob	$\langle x \rangle = \sum\limits_{\text{all x}} xP(x)$, $\langle f(x) \rangle = \sum\limits_{\text{all x}} f(x)P(x)$	$V = \sum\limits_{\text{all x}} (x - \langle x \rangle)^2 P(x) = \langle x^2 \rangle - \langle x \rangle^2$
Contin. Prob	$\langle x \rangle = \int x p(x)\,\mathrm{d}x$, $\langle f(x) \rangle = \int f(x)p(x)\,\mathrm{d}x$	$V = \int (x - \langle x \rangle)^2 p(x)\,\mathrm{d}x = \langle x^2 \rangle - \langle x \rangle^2$

CLT

Set of N randomly distributed, independent variables $\{x_i\}$, $Y = \sum_{i=1}^{N} x_i$, $Z = \frac{1}{N}\sum_{i=1}^{N} x_i$. Then:

$$\overline{Y} = \sum \overline{x_i}, \qquad V(Y) = \sum V(x_i); \qquad \overline{Z} = \frac{1}{N}\sum \overline{x_i}, \qquad V(Z) = \frac{1}{N^2}\sum V(x_i)$$

and for large N, both Y and Z are Gaussian distributed.

Coordinate Systems & Basis Vectors

$$\begin{aligned} x &= \rho\cos\phi = r\sin\theta\cos\phi \\ y &= \rho\sin\phi = r\sin\theta\sin\phi \\ z &= z \qquad\; = r\cos\theta \end{aligned} \qquad \begin{aligned} \rho^2 &= x^2 + y^2 \\ r^2 &= x^2 + y^2 + z^2 \end{aligned} \qquad \begin{aligned} \tan\phi &= y/x \\ \cos\theta &= z/r \end{aligned}$$

$$\begin{aligned} \hat{e}_\rho &= \cos\phi\ \hat{\imath} + \sin\phi\ \hat{\jmath} \\ \hat{e}_\phi &= -\sin\phi\ \hat{\imath} + \cos\phi\ \hat{\jmath} \\ \hat{e}_z &= \hat{k} \end{aligned} \qquad \begin{aligned} \hat{e}_r &= \sin\theta\cos\phi\ \hat{\imath} + \sin\theta\sin\phi\ \hat{\jmath} + \cos\theta\ \hat{k} \\ \hat{e}_\theta &= \cos\theta\cos\phi\ \hat{\imath} + \cos\theta\sin\phi\ \hat{\jmath} - \sin\theta\ \hat{k} \\ \hat{e}_\phi &= -\sin\phi\ \hat{\imath} + \cos\phi\ \hat{\jmath} \end{aligned}$$

div$\vec{F}$	$\vec{\nabla}T$	d$\vec{r}$
$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\frac{\partial T}{\partial x}\hat{\imath} + \frac{\partial T}{\partial y}\hat{\jmath} + \frac{\partial T}{\partial z}\hat{k}$	$\mathrm{d}x\hat{\imath} + \mathrm{d}y\hat{\jmath} + \mathrm{d}z\hat{k}$
$\frac{1}{\rho}\frac{\partial(\rho F_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$	$\frac{\partial T}{\partial\rho}\hat{e}_\rho + \frac{1}{\rho}\frac{\partial T}{\partial\phi}\hat{e}_\phi + \frac{\partial T}{\partial z}\hat{k}$	$\mathrm{d}\rho\hat{e}_\rho + \rho\mathrm{d}\phi\hat{e}_\phi + \mathrm{d}z\hat{k}$
$\frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$	$\frac{\partial T}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial T}{\partial\theta}\hat{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi}\hat{e}_\phi$	$\mathrm{d}r\hat{e}_r + r\,\mathrm{d}\theta\hat{e}_\theta + r\sin\theta\,\mathrm{d}\phi\,\hat{e}_\phi$

Area and Volume Elements

$\mathrm{d}\vec{S} = \frac{\partial\vec{S}(u,v)}{\partial u} \times \frac{\partial\vec{S}(u,v)}{\partial v}\ \mathrm{d}u\ \mathrm{d}v$. For $x-y$ plane: $\mathrm{d}\vec{S}_{\mathrm{xy}} = \mathrm{d}x\ \mathrm{d}y\ \hat{k} = r\,dr\,d\phi\,\hat{k}$

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \left|\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{pmatrix}\right| \mathrm{d}u\,\mathrm{d}v\,\mathrm{d}w = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Geometric Objects

Name	dim	constraint	example parametrisation
line	1	$\frac{x-a_x}{v_x} = \frac{y-a_y}{v_y} = \frac{z-a_z}{v_z}$	$\vec{a} + \vec{v}\,t$
circle in x, y plane	1	$x^2 + y^2 = R^2, z = 0$	$\begin{pmatrix} R\cos\phi \\ R\sin\phi \end{pmatrix}, \begin{pmatrix} \pm\sqrt{R^2 - y^2} \\ y \end{pmatrix}$
ellipse in x, y plane	1	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$	$\begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$
flat $\perp \hat{n}$, through $\vec{a}$	2	$\vec{r} \cdot \hat{n} = \vec{a} \cdot \hat{n}$	as below; find $v_{1,2}$ through $\vec{v}_i = \vec{X} \times \hat{n}$ (e.g. $\vec{X} = \hat{\imath}, \hat{\jmath}, \hat{k}$)
flat, through $\vec{a}$, $\parallel \vec{v}_1, \vec{v}_2$	2	as above with $\hat{n} \propto \vec{v}_1 \times \vec{v}_2$	$\vec{a} + \vec{v}_1 \cdot t_1 + \vec{v}_2 \cdot t_2$
flat through $\vec{a}, \vec{b}, \vec{c}$	2	as above with $\vec{v}_1 = \vec{b} - \vec{a}, \vec{v}_2 = \vec{c} - \vec{a}$	
disk in x, y plane	2	$x^2 + y^2 \leq R^2, z = 0$	$\begin{pmatrix} \rho\cos\phi \\ \rho\sin\phi \end{pmatrix}, \begin{pmatrix} \pm\sqrt{\rho^2 - y^2} \\ y \end{pmatrix}, \rho \leq R$
sphere	2	$x^2 + y^2 + z^2 = R^2$	$\begin{pmatrix} R\sin\theta\cos\phi \\ R\sin\theta\sin\phi \\ R\cos\theta \end{pmatrix}, \begin{pmatrix} \sqrt{R^2 - z^2}\cos\phi \\ \sqrt{R^2 - z^2}\sin\phi \\ z \end{pmatrix}$
ellipsoid	2	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	$\begin{pmatrix} a\sin\theta\cos\phi \\ b\sin\theta\sin\phi \\ c\cos\theta \end{pmatrix}$
paraboloid	2	$z = \alpha(x^2 + y^2)$	$\begin{pmatrix} x \\ y \\ \alpha(x^2 + y^2) \end{pmatrix}, \begin{pmatrix} r\cos\phi \\ r\sin\phi \\ \alpha r^2 \end{pmatrix}$
cylindrical surface w/o 'lids'	2	$x^2 + y^2 = R^2$	$\begin{pmatrix} R\cos\phi \\ R\sin\phi \\ z \end{pmatrix}$
cone	2	$x^2 + y^2 = (az)^2$	$\begin{pmatrix} az\cos\phi \\ az\sin\phi \\ z \end{pmatrix}$
ball	3	$x^2 + y^2 + z^2 \leq R^2$	$\begin{pmatrix} r\sin\theta\cos\phi \\ r\sin\theta\sin\phi \\ r\cos\theta \end{pmatrix}, r < R$

Note that these are special cases (e.g. circles and spheres *centered at zero*,) that you might need to adapt before applying them. E.g. if you want to move an object by $\vec{c}$, for the constraint, change $x \longrightarrow x - c_x, y \longrightarrow y - c_y, z \longrightarrow z - c_z$, and for the parametrisation, add $\vec{c}$.

Integral theorems

Gradient	Stokes'	Divergence (Gauss)
$\int\limits_{\gamma(\vec{a}\to\vec{b})} -\vec{\nabla}(\phi) \cdot \mathrm{d}\vec{r} = \phi(\vec{a}) - \phi(\vec{b})$	$\iint\limits_S \vec{\nabla} \times \vec{F} \cdot \mathrm{d}\vec{S} = \oint\limits_{\partial S} \vec{F} \cdot \mathrm{d}\vec{r}$	$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\ \mathrm{d}V = \oiint\limits_S \vec{F} \cdot \mathrm{d}\vec{S}$

Vector identities and differentiation rules

$$\begin{aligned}
\vec{A}\times\left(\vec{B}\times\vec{C}\right) &= \vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)\\
\vec{\nabla}(\phi\psi) &= \phi\vec{\nabla}\psi+\psi\vec{\nabla}\phi\\
\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) &= \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right)+\left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\vec{B}\times\left(\vec{\nabla}\times\vec{A}\right)+\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}\\
\vec{\nabla}\cdot\phi\vec{A} &= \phi\vec{\nabla}\cdot\vec{A}+\vec{A}\cdot\vec{\nabla}\phi\\
\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) &= \vec{B}\cdot\vec{\nabla}\times\vec{A}-\vec{A}\cdot\vec{\nabla}\times\vec{B}\\
\vec{\nabla}\times\left(\phi\vec{A}\right) &= \phi\vec{\nabla}\times\vec{A}-\vec{A}\times\vec{\nabla}\phi\\
\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) &= \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}+\vec{A}\left(\vec{\nabla}\cdot\vec{B}\right)-\left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}-\vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)\\
\overline{\left(\vec{\nabla}\cdot\vec{A}\right)\vec{B}} &= \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)\\
\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) &= \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)-\left(\vec{\nabla}\cdot\vec{\nabla}\right)\vec{A}\\
\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) &= 0\\
\vec{\nabla}\times\vec{\nabla}\phi &= \vec{0}
\end{aligned}$$

The overline indicates the extent of the operation of the $\vec{\nabla}$ operator in a case where this would otherwise be ambiguous.

Scalar Product and Orthogonality for Functions

The scalar product of $f(x)$ and $g(x)$ over the interval $[a,b]$ with respect to the weight function $w(x)>0$ is

$$\langle f,g\rangle \equiv \int_a^b f^*(x)\,g(x)\,w(x)\;\mathrm{d}x$$

and f and g are orthogonal if $\langle f,g\rangle = 0$

Delta Functions

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\;\mathrm{d}x = f(x_0)$$

$$\int_{-\infty}^{\infty} f(x)\,\delta\big(g(x)\big)\;\mathrm{d}x = \sum_i \frac{f(x_i)}{|g'(x_i)|}$$

where x_i are all the points where $g(x)=0$.

Fourier Series

For a periodic function $f(x)=f(x+L)$:

$$f(x) \equiv \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\frac{2n\pi x}{L}+\sum_{n=1}^{\infty} b_n\sin\frac{2n\pi x}{L}$$

$$\begin{aligned}
a_n &= \frac{2}{L}\int_{x_0}^{x_0+L}\cos\frac{2n\pi x}{L}f(x)\,\mathrm{d}x\\
b_n &= \frac{2}{L}\int_{x_0}^{x_0+L}\sin\frac{2n\pi x}{L}f(x)\,\mathrm{d}x
\end{aligned}$$

Orthogonality relations (non-zero m and n):

$$\begin{aligned}
\frac{2}{L}\int_{x_0}^{x_0+L}\cos\frac{2n\pi x}{L}\cos\frac{2m\pi x}{L}\,\mathrm{d}x &= \delta_{nm}\\
\frac{2}{L}\int_{x_0}^{x_0+L}\sin\frac{2n\pi x}{L}\sin\frac{2m\pi x}{L}\,\mathrm{d}x &= \delta_{nm}\\
\frac{2}{L}\int_{x_0}^{x_0+L}\sin\frac{2n\pi x}{L}\cos\frac{2m\pi x}{L}\,\mathrm{d}x &= 0
\end{aligned}$$

Parseval's Theorem:

$$\frac{|a_0|^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}(|a_n|^2+|b_n|^2)=\frac{1}{L}\int_{x_0}^{x_0+L}|f(x)|^2\,\mathrm{d}x$$

Complex form of Fourier Series

For a periodic function $f(x)=f(x+L)$

$$f(x)=\sum_{n=-\infty}^{\infty}c_n e^{ik_n x}$$

where $k_n = 2\pi n/L$ and

$$c_n=\frac{1}{L}\int_{x_0}^{x_0+L}f(x)e^{-ik_n x}\,\mathrm{d}x$$

Parseval's theorem:

$$\sum_{n=-\infty}^{\infty}|c_n|^2=\frac{1}{L}\int_{x_0}^{x_0+L}|f(x)|^2\,\mathrm{d}x$$

Fourier Transforms

For a function $f(x)$ with $\int_{-\infty}^{\infty}|f(x)|^2\,\mathrm{d}x$ finite

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{f}(k)e^{ikx}\,\mathrm{d}k \qquad \text{and} \qquad \tilde{f}(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-ikx}\,\mathrm{d}x$$

Parseval's theorem:

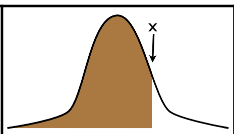
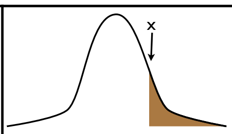
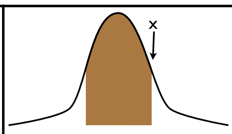
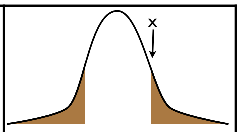
$$\int_{-\infty}^{\infty}|f(x)|^2\,\mathrm{d}x=\int_{-\infty}^{\infty}|\tilde{f}(k)|^2\,\mathrm{d}k$$

If $h(x)$ is the convolution defined by

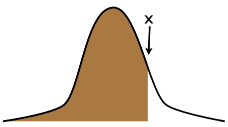
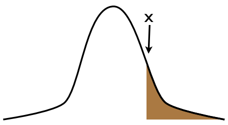
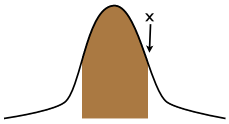
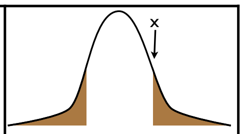
$$h(x)=\int_{-\infty}^{\infty}f(x')g(x-x')\,\mathrm{d}x'$$

$$\tilde{h}(k)=\sqrt{2\pi}\,\tilde{f}(k)\tilde{g}(k)$$

Gaussian Integration Table (normal)

				
$s = \dfrac{x-\mu}{\sigma}$	$\int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$1 - \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$\int_{-s}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$1 - \int_{-s}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$
0	0.500	5.00E-01	0.000	1.00E+00
0.1	0.540	4.60E-01	0.080	9.20E-01
0.2	0.579	4.21E-01	0.159	8.41E-01
0.3	0.618	3.82E-01	0.236	7.64E-01
0.4	0.655	3.45E-01	0.311	6.89E-01
0.5	0.691	3.09E-01	0.383	6.17E-01
0.6	0.726	2.74E-01	0.451	5.49E-01
0.7	0.758	2.42E-01	0.516	4.84E-01
0.8	0.788	2.12E-01	0.576	4.24E-01
0.9	0.816	1.84E-01	0.632	3.68E-01
1	0.841	1.59E-01	0.683	3.17E-01
1.1	0.864	1.36E-01	0.729	2.71E-01
1.2	0.885	1.15E-01	0.770	2.30E-01
1.3	0.903	9.68E-02	0.806	1.94E-01
1.4	0.919	8.08E-02	0.838	1.62E-01
1.5	0.933	6.68E-02	0.866	1.34E-01
1.6	0.945	5.48E-02	0.890	1.10E-01
1.7	0.955	4.46E-02	0.911	8.91E-02
1.8	0.964	3.59E-02	0.928	7.19E-02
1.9	0.971	2.87E-02	0.943	5.74E-02
2	0.977	2.28E-02	0.954	4.55E-02
2.1	0.982	1.79E-02	0.964	3.57E-02
2.2	0.986	1.39E-02	0.972	2.78E-02
2.3	0.9893	1.07E-02	0.9786	2.14E-02
2.4	0.9918	8.20E-03	0.9836	1.64E-02
2.5	0.9938	6.21E-03	0.9876	1.24E-02
2.6	0.9953	4.66E-03	0.9907	9.32E-03
2.7	0.9965	3.47E-03	0.9931	6.93E-03
2.8	0.9974	2.56E-03	0.9949	5.11E-03
2.9	0.9981	1.87E-03	0.9963	3.73E-03
3	0.99865	1.35E-03	0.99730	2.70E-03
3.1	0.99903	9.68E-04	0.99806	1.94E-03
3.2	0.99931	6.87E-04	0.99863	1.37E-03
3.3	0.99952	4.83E-04	0.99903	9.67E-04
3.4	0.99966	3.37E-04	0.99933	6.74E-04
3.5	0.99977	2.33E-04	0.99953	4.65E-04
3.6	0.999841	1.59E-04	0.999682	3.18E-04
3.7	0.999892	1.08E-04	0.999784	2.16E-04
3.8	0.999928	7.23E-05	0.999855	1.45E-04
3.9	0.999952	4.81E-05	0.999904	9.62E-05
4	0.999968	3.17E-05	0.999937	6.33E-05

Gaussian Integration Table (small significances)

				
$s = \frac{x-\mu}{\sigma}$	$\int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$1 - \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$\int_{-s}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$	$1 - \int_{-s}^{s} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}s^2} ds$
0	0.5000	5.000E-01	0.000	1.0E+00
0.01	0.5040	4.960E-01	0.008	9.9E-01
0.02	0.5080	4.920E-01	0.016	9.8E-01
0.03	0.5120	4.880E-01	0.024	9.8E-01
0.04	0.5160	4.840E-01	0.032	9.7E-01
0.05	0.5199	4.801E-01	0.040	9.6E-01
0.06	0.5239	4.761E-01	0.048	9.5E-01
0.07	0.5279	4.721E-01	0.056	9.4E-01
0.08	0.5319	4.681E-01	0.064	9.4E-01
0.09	0.5359	4.641E-01	0.072	9.3E-01
0.1	0.5398	4.602E-01	0.080	9.2E-01
0.11	0.5438	4.562E-01	0.088	9.1E-01
0.12	0.5478	4.522E-01	0.096	9.0E-01
0.13	0.5517	4.483E-01	0.103	9.0E-01
0.14	0.5557	4.443E-01	0.111	8.9E-01
0.15	0.5596	4.404E-01	0.119	8.8E-01
0.16	0.5636	4.364E-01	0.127	8.7E-01
0.17	0.5675	4.325E-01	0.135	8.7E-01
0.18	0.5714	4.286E-01	0.143	8.6E-01
0.19	0.5753	4.247E-01	0.151	8.5E-01
0.2	0.5793	4.207E-01	0.159	8.4E-01
0.21	0.5832	4.168E-01	0.166	8.3E-01
0.22	0.5871	4.129E-01	0.174	8.3E-01
0.23	0.5910	4.090E-01	0.182	8.2E-01
0.24	0.5948	4.052E-01	0.190	8.1E-01
0.25	0.5987	4.013E-01	0.197	8.0E-01
0.26	0.6026	3.974E-01	0.205	7.9E-01
0.27	0.6064	3.936E-01	0.213	7.9E-01
0.28	0.6103	3.897E-01	0.221	7.8E-01
0.29	0.6141	3.859E-01	0.228	7.7E-01
0.3	0.6179	3.821E-01	0.236	7.6E-01
0.31	0.6217	3.783E-01	0.243	7.6E-01
0.32	0.6255	3.745E-01	0.251	7.5E-01
0.33	0.6293	3.707E-01	0.259	7.4E-01
0.34	0.6331	3.669E-01	0.266	7.3E-01
0.35	0.6368	3.632E-01	0.274	7.3E-01
0.36	0.6406	3.594E-01	0.281	7.2E-01
0.37	0.6443	3.557E-01	0.289	7.1E-01
0.38	0.6480	3.520E-01	0.296	7.0E-01
0.39	0.6517	3.483E-01	0.303	7.0E-01
0.4	0.6554	3.446E-01	0.311	6.9E-01