Coupled Oscillators

2.1. Consider two objects of mass $m_1 = m_2 = 1kg$ connected by springs $k_1 = 1N/m$, $k_{12} = 1N/m$, $k_2 = 2N/m$, as illustrated in Fig. 1.

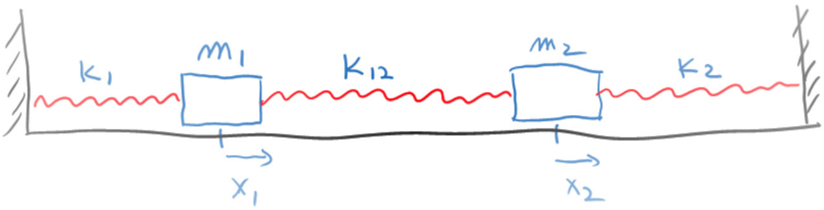

Figure 1: Distinct Coupled Oscillators

(a) Find the normal modes using the matrix method.
(b) Determine the general solution.
(c) Write the solution for the initial conditions $x_1(0) = 1m$, $x_2(0) = 0m$, $v_1(0) = 0m/s$, $v_2(0) = 0m/s$.

2.2. Consider three objects all of equal mass m connected by identical springs with constant k to each other and walls, as illustrated in Fig. 2.

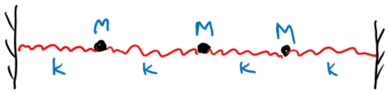

Figure 2: Three Coupled Oscillators

(a) Write the Newton equations for the three coupled masses illustrated in Fig. 2
(b) Find the normal modes using the matrix method.
(c) Determine the general solution.

2.3. Consider three coupled identical pendulums of mass M and length l, coupled by springs of constant K, as illustrated in Fig. 3.

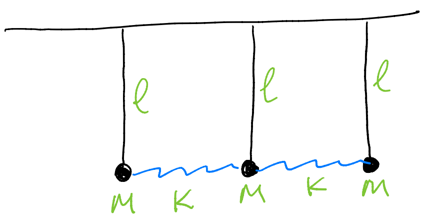

Figure 3: 3 Coupled Pendulums

(a) Write the Newton equations for small oscillation amplitudes.
(b) What is the main difference between the 3 coupled oscillators in Fig. 2 and that of the pendulums in Fig. 3? What is its physical significance?
(c) Imagine an arrangement of three masses whose Newton equations will look formally the same as those of the three pendulums.
(d) Find the normal modes (amplitudes and frequencies).
(e) Write the general solution.

2.4. Consider again two objects of mass m_1 and m_2 connected by springs k_1, K_{12}, k_2, as illustrated in Fig. 1. In the problem 2.1 above we discussed how to solve this system by looking for normal modes via the matrix method. In the lectures we solved a similar (symmetric version of this) problem by finding normal coordinates. Consider a general change in coordinates as

$$\begin{aligned} y_1 &= \alpha\, x_1 + \beta\, x_2, \\ y_2 &= \gamma\, x_1 + \delta\, x_2\,, \end{aligned}$$

and show that this is equivalent to the matrix method.

2.5. Consider an infinite chain of coupled pendulums. Let the pendulums have length l and mass M and be coupled by springs of constant K.

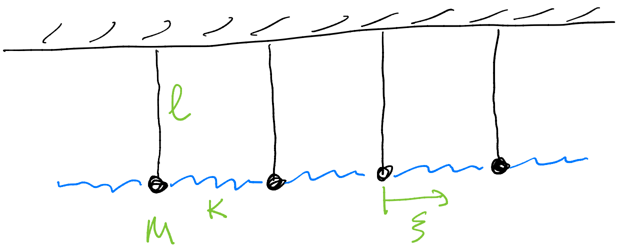

Figure 4: Infinite chain of coupled pendulums

(a) Write the Newton equations for this problem.
(b) Find the normal modes and their dispersion relation (Hint: Use a trial form for the time-dependence of the masses and for the variation of their amplitudes along the chain, as done in the lectures.)
(c) Compare the dispersion relation with that of a chain of masses coupled with springs to one another, as shown in Fig. 5, that was derived in the lectures.

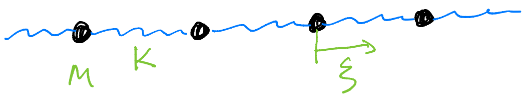

Figure 5: Infinite chain of coupled masses

(d) In the limit of long wavelengths, is the mode dispersive or non-dispersive?

2.6. Consider three objects all of equal mass m connected by identical springs with constant k , as illustrated earlier in Fig. 2. Find the normal modes for this system using the trial function method described for the case of a chain of N masses in the lectures and compare the result with the matrix method result you got in problem 2.2.