Arbitrary Initial Wavefunction

Previously : $\psi(x,0) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ c(k)\, e^{ikx}$ Specified in terms of $c(k)$

Q: How to specify $\psi(x,0)$ & find $c(k)$ i.e. reverse problem.

Method: multiply both sides of above eq^n by $e^{-ik'x}$ & integrate over x

$$\rightarrow \int_{-\infty}^{\infty} \psi(x,0)\, e^{-ik'x}\, dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ c(k)\, e^{ikx}\, e^{-ik'x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ c(k) \underbrace{\int_{-\infty}^{\infty} dx\ e^{i(k-k')x}}_{2\pi\,\delta(k-k')}$$

$$= \sqrt{2\pi}\int_{-\infty}^{\infty} dk\ c(k)\,\delta(k-k')$$

$$= \sqrt{2\pi}\, c(k')$$

Re-arrange: $$\boxed{c(k') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x,0)\, e^{-ik'x}\, dx}$$

c.f $$\boxed{\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k)\, e^{ikx}\, dk}$$

Fourier transform pairs!

Finally: use this to write an expression for $\psi(x,t)$ in terms of $\psi(x,0)$

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, c(k)\, e^{-i\hbar k^2 t/2M}\, e^{ikx}$$

$$\boxed{\psi(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, \underbrace{\int_{-\infty}^{\infty} dy\, \psi(y,0)\, e^{-iky}}_{\sqrt{2\pi}\, c(k)}\, e^{-i\hbar k^2 t/2M}\, e^{ikx}}$$

formidible expression.

* Difficult to evaluate

* Conceptually: everything is here!

c.f.

$$x(t) = x_0 + \frac{p_0 t}{M}$$

$$p(t) = p_0$$