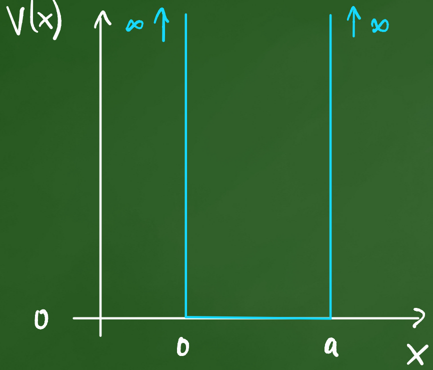
8. Infinite Square Well: Energy Eigenstates

8.1 General properties of energy eigenstates

We now start our study of QM of particles in potential wells

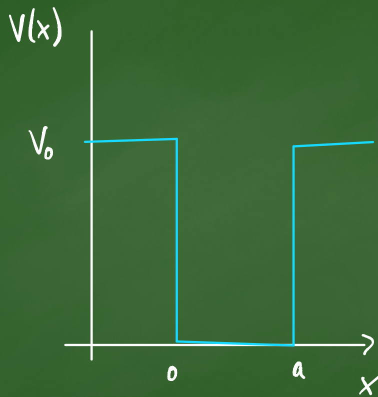
this course: 3 wells



Infinite sq. well

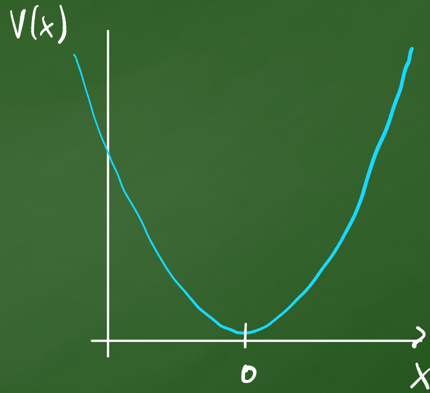
$$V(x) = \begin{cases} 0 & 0 \leqslant x \leqslant a \\ \infty & \text{otherwise} \end{cases}$$

"perfectly rigid walls"



finite sq well

$$V(x) = \begin{cases} 0 & 0 \leqslant x \leqslant a \\ V_0 & \text{otherwise} \end{cases}$$



Harmonic Oscillator

$$V(x) = \tfrac{1}{2} M \omega^2 x^2$$

- math simple
- See: discrete energy levels

- Solve graphically
- See: tunnelling

- good approx. to many forces / wells
- dimensionless co-ords.
- trial & error

Strategy for solving

1. Find energy eigenstates (= stationary states)

   i.e. Solve TISE $\hat{H}u(x) = Eu(x)$

2. Show how to write initial wavefunction (state) $\psi(x,0)$ as a superposition of energy eigenstates.

3. Write down the wavefunction $\psi(x,t)$ at time t.

   (evolution of superposition = superposition of evolutions)

4. look at properties e.g. position, momentum

General properties of energy eigenstates

1. Because E in TISE is energy it is a real number.

   → TISE is a real eqn $\quad Eu(x) = -\frac{\hbar^2}{2M}\frac{d^2u}{dx^2} + V(x)u(x)$

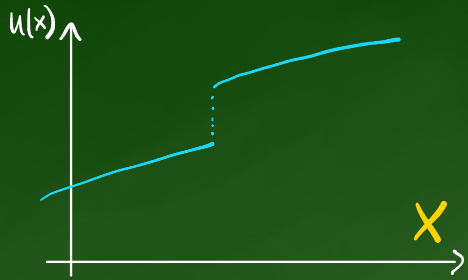
   → we can always find real solutions $\quad$ no complex number necessary → simpler

2. Solutions $u(x)$ cannot have an 'jumps' or 'kinks'

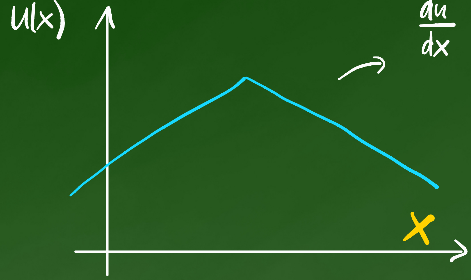
   'jumps': discontinuity $\quad$ 'kinks': discontinuous 1st derivative.

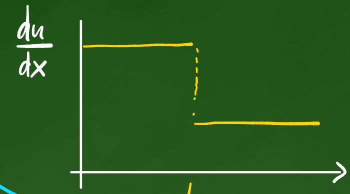
   i.e. $u(x)$ & $\frac{du}{dx}$ are continuous functions

BAD:



u(x) discontinuous





$\frac{du}{dx}$ discontinuous

Exception At points where $V(x) = \infty$ then $\frac{du}{dx}$ is allowed to be discontinuous

u(x) must remain continuous