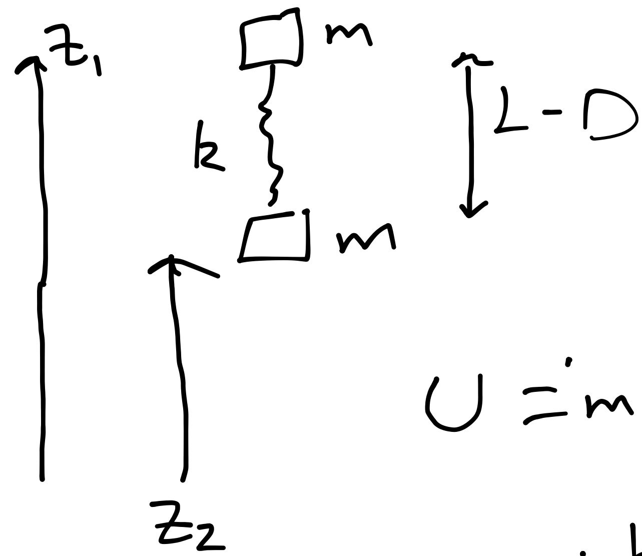
4.4



$$U = mgz_1 + mgz_2 + \frac{k}{2}(z_1 - z_2 - L)^2$$

$$m\ddot{z}_1 = -\frac{\partial U}{\partial z_1} = -mg - k(z_1 - z_2 - L)$$

$$m\ddot{z}_2 = -\frac{\partial U}{\partial z_2} = -mg - k(z_2 - z_1 + L)$$

$$\ddot{z}_1 = -\left(mg - \frac{k}{m}L\right) + \frac{k}{m}(z_1 - z_2)$$

$$\ddot{z}_2 = -\left(mg + \frac{k}{m}L\right) + \frac{k}{m}(z_2 - z_1)$$

Define c.o.m $Z_c = \dfrac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$

$= \dfrac{z_1}{2} + \dfrac{z_2}{2}$

and relative $z_r = z_1 - z_2$

Then $\ddot{Z}_c = - g$

$$\boxed{\text{c.f. problem } 3.8}$$

$\ddot{z}_r = - \dfrac{2k}{m}\left(z_r - L\right)$

$Z_c = Z_c^0 - g t^2/2 \qquad (\dot{z}^0_c = 0)$

$z_r = L + A\cos\omega t + B\sin\omega t$

$\omega = \sqrt{\dfrac{2k}{m}}$

at $t=0$ $z_r = L - D$ $\dot{z}_r = 0$

$\Rightarrow A = -D$, $B = 0$

$z_r = L - D\cos\omega t$

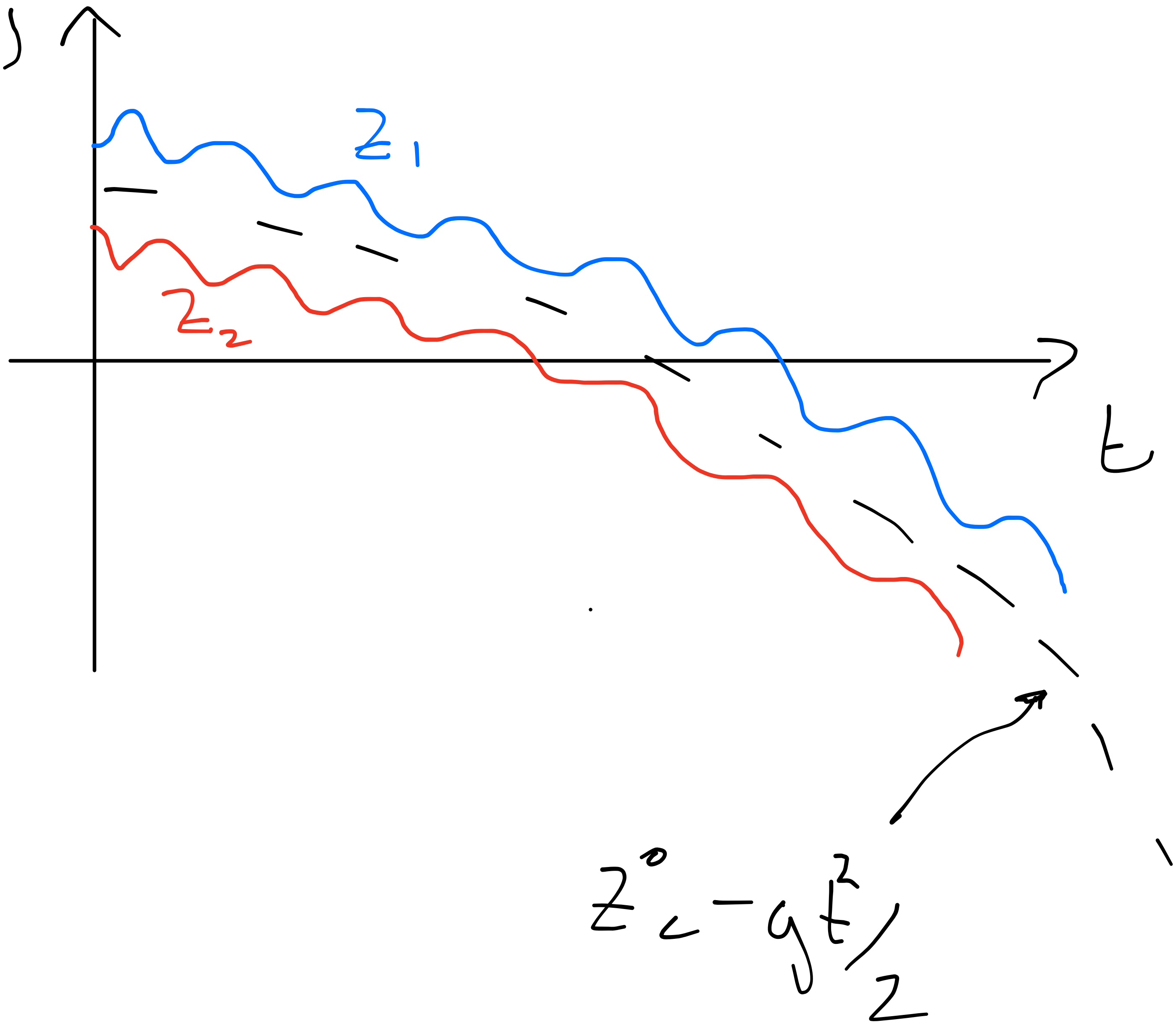
$\Rightarrow$

$$z_1 = z_c + \frac{z_r}{2}$$

$$z_1 = z_c^0 - \frac{gt^2}{2} + \frac{L}{2} - \frac{D}{2}\cos\omega t$$

$$z_2 = z_c^0 - \frac{gt^2}{2} - \frac{L}{2} + \frac{D}{2}\cos\omega t$$

b)



c) $\dot{z}_1 = -gt + \frac{D\omega}{2}\sin\omega t$

for t small

$$z_1 \simeq -\left(g + \frac{D\omega^2}{2}\right) t$$

if $\frac{D\omega^2}{2} > g$ it goes up at $t=0$

lower mass can also go up!

d) mass 1.

$$v_1 = -gt + \omega \frac{D}{2} \sin \omega t$$

we want peak of third upward movement at $\omega t = 4\pi + \pi/2$
$= 9\pi/2$

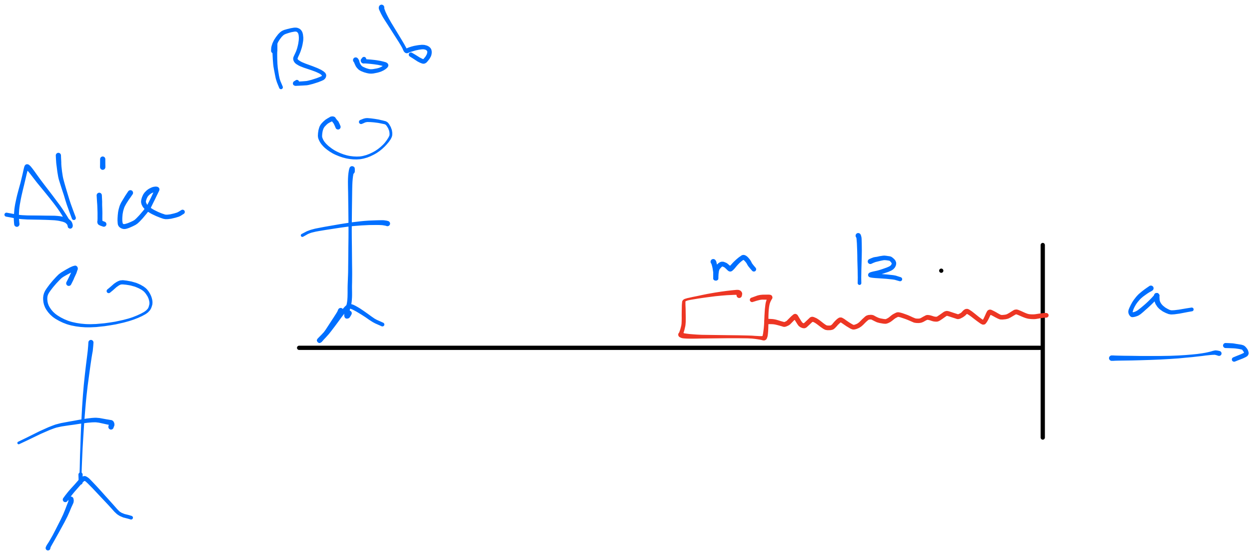
at this time

$$V_1 = -\frac{g}{\omega}\frac{9\pi}{2} + \omega\frac{D}{2}$$

we want $v_1 = 0$

$$\Rightarrow D > \frac{9g\pi}{\omega^2}$$

4.5



$$m\frac{d^2 x_{pB}}{dt^2} = -k\,x_{pB} - \underbrace{ma}_{\text{fictitious force}}$$

or

$$\frac{d^2}{dt^2}\left(x_{pB} + \frac{ma}{k}\right) = -\frac{k}{m}\left(x_{pB} + \frac{ma}{k}\right)$$

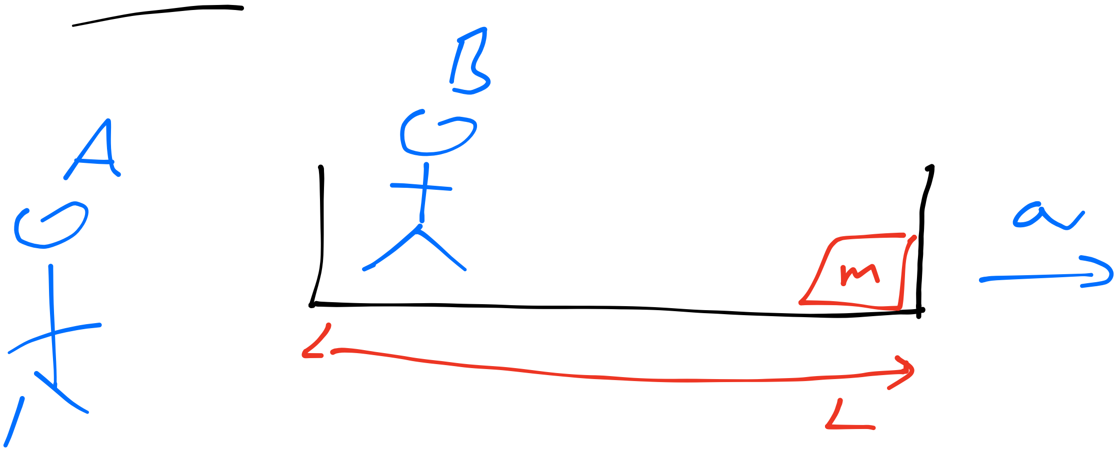
has zero time derivative

$$x_{pB} + \frac{ma}{k} = A\cos(\omega t + \phi) \qquad \omega^2 = \frac{k}{m}$$

$$x_{pB} = A\cos(\omega t + \phi) - \frac{ma}{k}$$

$$x_{pA} = x_{pB} + at^2/2 .$$

4.6



For Bob a fictious force

$$F = -ma$$

What happens?

for $0 < T < \sqrt{2L/a}$ mass slides back:

$$x_B = L - at^2/2$$

$\ulcorner$ at $t = T = \sqrt{\frac{2L}{a}}$ Elastic collision

from Bob's perspective

particle then moves forward with velocity

aT

for Alice it moves forward with

velocity $2aT$ $\lrcorner$

$T < t < 3T$

$$x_B(t) = aT(t-T) - \frac{a(t-T)^2}{2}$$

(suvat) ↗

at $t = T$ $x_B = 0$

$= 2T$ $x_B = L$ ← max

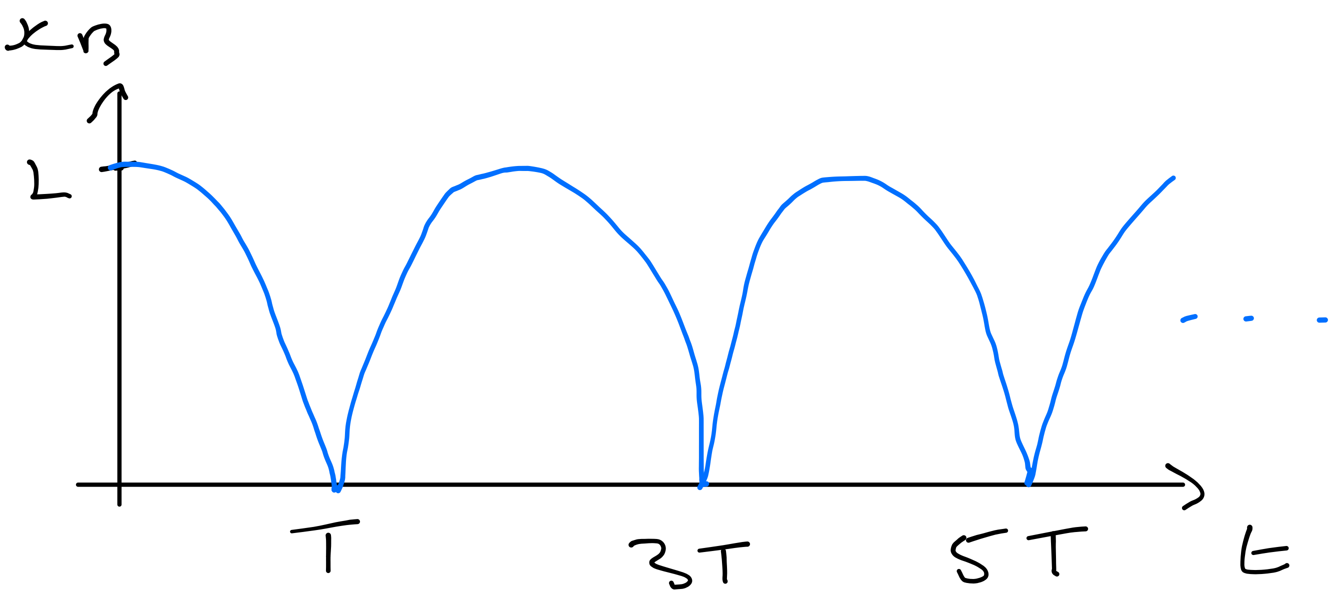
$t = 3T \quad x_B = 0$

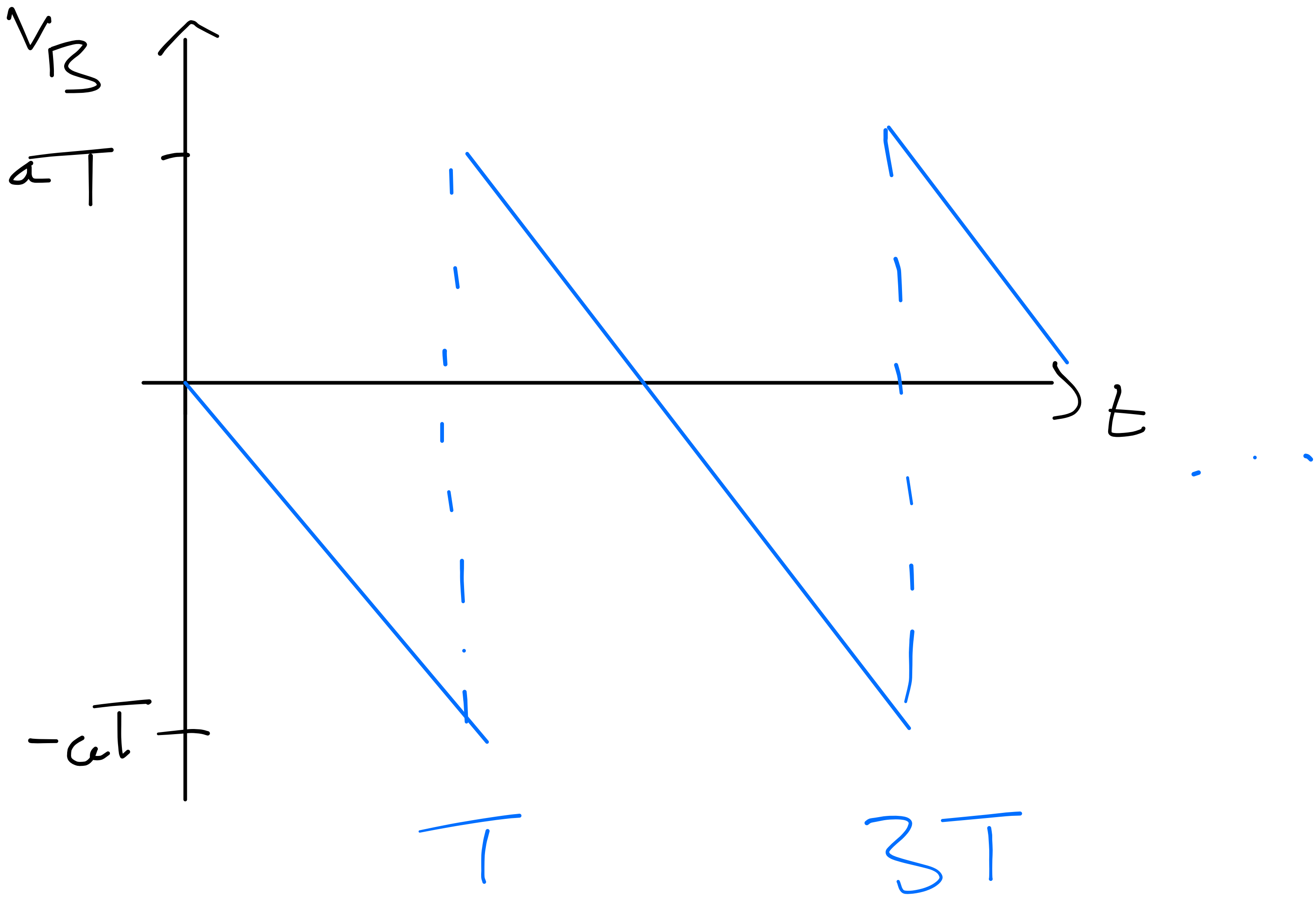
at $t = 3T$ $v_B = \dot{x}_B = -aT$

$\hookrightarrow$ elastic collision

so $x_B = aT(t-3T) - \frac{a}{2}(t-3T)^2$

The pattern continues...





Now Alice

$$x_A = x_B + at^2/2$$

but Alice never seen

forces, except at collisions

$\Rightarrow$ x_A is linear

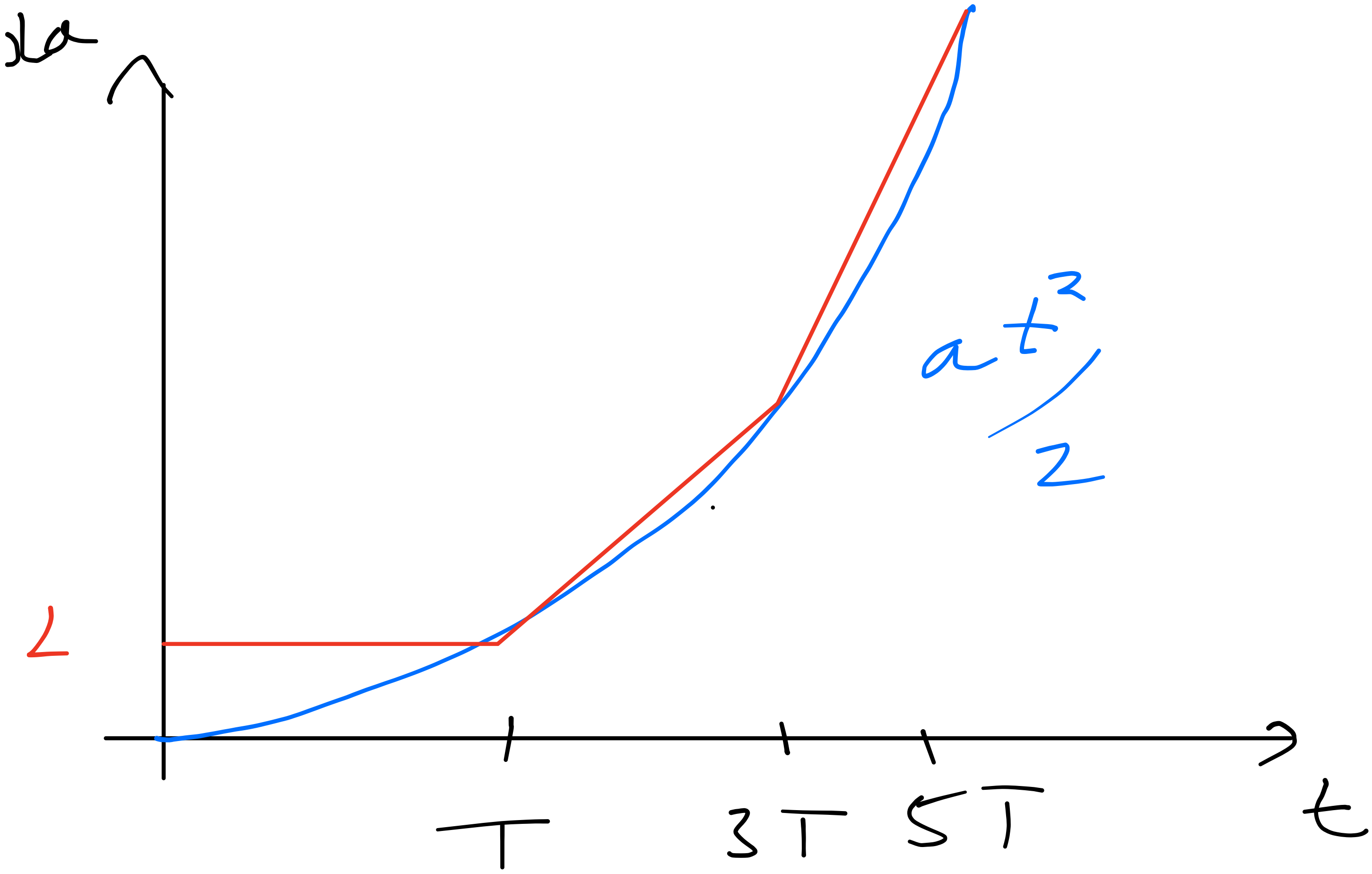
can we see this?

for $2n+1 < \frac{t}{T} < 2n+3$

$$x_B = aT\left(t-(2n+1)T\right) - \frac{a}{2}\left(t-(2n+1)T\right)^2$$

$$\Rightarrow x_A = \frac{aT}{2}\Big[-(3+4n(2+n))T + 4(1+n)t\Big]$$

linear



__4-7__ No solu$^{\underline{n}}$ given