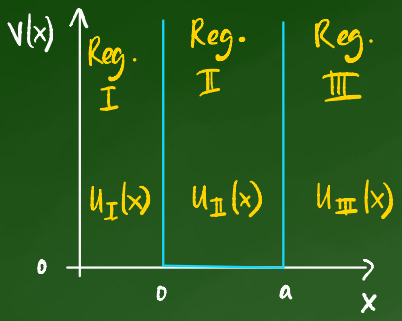
8.3 Energy eigenstates of Infinite Sq. well



$$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

- No forces inside well
- Solve TISE in the different regions where the potential is different.
- 'Stitch' solutions together at boundaries (i.e walls) $x=0,\ x=a$

  → use continuity conditions

Region I & III

$V(x) = \infty$ → only solution is $u_I(x) = 0$, $u_{III}(x) = 0$

TISE: $E\,u(x) = -\frac{\hbar^2}{2M}\frac{d^2u}{dx^2} + \infty \cdot u(x)$

physically: want E finite to make sense.

physically: particle inside well.

[stronger justification in next section]

Region II

$V(x) = 0$

TISE: $E u_{II}(x) = -\frac{\hbar^2}{2M}\frac{d^2 u_{II}}{dx^2}$ $\times -\frac{2M}{\hbar^2}$

$-\frac{2ME}{\hbar^2} u_{II}(x) = \frac{d^2 u_{II}}{dx^2}$ $\quad k^2 = \frac{2ME}{\hbar^2}$ just like for free particle

$-k^2 u_{II}(x) = \frac{d^2 u_{II}}{dx^2}$ SHO differential eq^n

$\rightarrow$ write down solution: $u_{II}(x) = A\cos kx + B\sin kx$

(Notice: Real solutions)

A, B real constants.

$\rightarrow$ use continuity of $u(x)$ at $x=0$ & $x=a$ to find A & B.

N. B. at $x=0$ & $x=a$ $V(x)=\infty$ → in exceptional case

i.e. $\frac{du}{dx}$ is allowed to be discont.

$x=0$ $\quad u_I(0)=0$

$u_{II}(0) = A\underbrace{\cos(0)}_{1} + B\underbrace{\sin(0)}_{0} = A$

Continuity: $u_I(0) = u_{II}(0)$

$$0 = A \quad \rightarrow \quad u_{II}(x) = B\sin kx$$

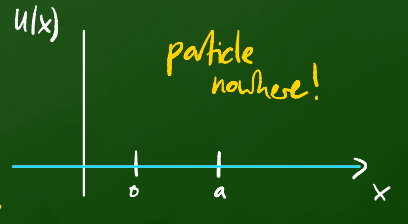
$x=a$ $\quad u_{II}(a) = B\sin ka$

$u_{III}(a) = 0$

Continuity: $u_{II}(a) = u_{III}(a)$

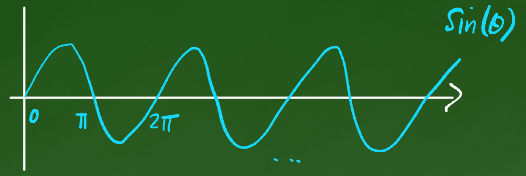
$$0 = B\sin ka$$

two possibilities: (1) $B=0$ → $u_{II}(x)=0$ ✗ Bad



$\rightarrow \quad B \neq 0$

(ii) $\sin ka = 0$



$\sin(\theta) = 0$ at $\theta = n\pi$

n integer

$\rightarrow \quad ka = n\pi$

$$k = \frac{n\pi}{a}$$

$n \neq 0$

$n = 1, 2, 3, \ldots$

positive integers.

Recall: $E = \frac{\hbar^2 k^2}{2M}$

Sub n: $u_{II}(x) = B \sin\left(\frac{n\pi x}{a}\right)$

$\rightarrow$

$$u(x) = \begin{cases} B \sin\left(\frac{n\pi x}{a}\right) & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

Reg II

Regs I & III