Basic stuff

These are just to remind you, but presumed already known.

$$\sum_{k=0}^{\infty} k \cdot p(k) \equiv \langle k \rangle, \tag{1}$$

$$\sum_{k=0}^{\infty} C \cdot f(k) = C \cdot \sum_{k=0}^{\infty} f(k), \tag{2}$$

$$e^{\ln x} = x, \tag{3}$$

$$\ln e^x = x, \tag{4}$$

$$\frac{\log_a x}{\log_a b} = \log_b x, \tag{5}$$

$$\int k^{-\gamma} = \frac{k^{1-\gamma}}{1-\gamma} \tag{6}$$

$$\int k^{-1} = \ln k \tag{7}$$

Approximations

These are new relations that you should just learn by heart.

$$\log 1 + x \approx x \qquad \text{iff } |x| << 1, \tag{8}$$

$$(1 + \frac{k}{n})^n \approx e^k \qquad \text{iff } n >> 1, \tag{9}$$

$$\frac{N!}{(N-k)!} \approx N^k \qquad \text{iff } k << N, \tag{10}$$

$$\frac{k}{N} \approx 0 \qquad \text{for } k << N, \tag{11}$$

$$\sum_{i=0}^{\infty} a_i = \text{const.} \quad \text{if } a_{i+1}/a_i = r < 1. \tag{12}$$

- Note: the notation $|x| << 1$ means: x is very close to zero. $n >> 1$ means: n is very large.
- To see the intuition that Eq. 11 is true, consider $N\,(N-1)$, i.e., $k = 2$. The relative error of the approximation N^2 equals $\frac{N^2 - N(N-1)}{N^2} = \frac{1}{N}$. As $N \to \infty$, this error goes to zero.