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# INTRODUCTION

* *Description of motivation and research problem (2 paragraphs, min. half a page in total)*

In recent years, the observation of the formation of Earth-like planets has gained particular interest of the scientific society. Current focus of modern astronomy is studying the physical properties and chemical processes which can lead to prebiotic conditions in Earth-like planets, which can open the path to understanding the genesis of our own planet.

Earth-like planets can be originated in circumstellar disks and protoplanetary regions. These regions in space are obscured by clouds of gas and dust such that they are the best observable in far-infrared frequencies. This is, however, not possible to carry out from the ground due to the limitations caused by Earth’s atmosphere.

Thus, a space-based interferometric telescope mission called Infrared Astronomy Satellite Swarm Interferometry (IRASSI) was designed. This mission is a joint project between Menlo Systems GmbH, TU Braunschweig Institut für Flugführung, Max-Planck-Institut für Astronomie, and Universität der Bundeswehr München Institut für Raumfahrttechnik and Institut für Navigation. The projuect is funded by German Aerospace Centre (DLR). UniBw ISTA is conducting a feasibility study for that mission. The aim of the mission is to further develop the understanding of star and thus planet formation, by simultaneously implementing new technologies with regard to observation instruments, ranging detection systems and formation flying. According to the plan, the constellation of five satellites, whose inter-satellite distance will be tracked to an unprecedented precision, will be following Halo orbit around second Lagrange point of Sun-Earth-Moon system.

This mission is of great importance for the future of exo-planets discovery and studies, therefore thorough studies have to be carried out before the mission enters the launch phase. One of the most important among these studies is whether the satellite positions are determined with sufficient accuracy to meet the scientific and operational goals of the mission. The first step for this would be the development of a high precision simulation environment for the satellite motion in the vicinity of SEM L2.

**Main objective of the thesis**

Main objective of this Master Thesis project is to create the aforementioned simulation environment in order to propagate the sufficiently precise orbit around SEM L2 point. The simulation will not include the launch and early orbit phase (LEOP). Only the satellite motion during HALO orbit phase will be taken into account and studied. The simulation environment will be used to test the orbit determination performance of one IRASSI mission satellite.

Many astrodynamical tools exist, which are using simplified models such as Circular-Restricted Three Body problem or consider only Sun, Earth and Moon influences. One of the goals of this work is to find out whether those simplifications affect the orbit determination precision and to what extent by comparison. This includes determination of the relevant forces in the vicinity of L2 environment and including them in the precise model.

The next objective is to compare certain well-known numerical integrators for orbit propagation and show which one could be the best choice to propagate the HALO-type orbit around L2.

## Background

* *Advantages of L2 orbits and why it is popular for science/exoplanet missions (1 paragraph)*
* *L2 missions (1 small paragraph for each mission)*
* *Irassi mission description (similar to previous in format, but longer – half a page), (Irassi mission specific information: Buinhas, L., Ferrer-Gil, E., & Forstner, R. (2016, March). IRASSI: InfraRed astronomy satellite swarm interferometry—Mission concept and description. In Aerospace Conference, 2016 IEEE (pp. 1-20). IEEE.*

For scientific missions in space L2 point of the Sun-Earth-Moon system is often utilized. This point allows spacecraft to remain relatively stationary with respect to the Sun and Earth/Moon. This results in a low energy expenditure required for station keeping.

The satellite in an orbit around L2 is much closer to the Earth than if it resided at L4 or L5, which results in reducing the time to command a spacecraft. Light will take only 5 seconds to reach the spacecraft, whereas it will take 9 minutes to reach L4/L5. This, in turn, facilitates the ability to do real time commands, which occasionally might be useful.

Since the spacecraft resides at a distance of 1.5 millions kilometers from Earth it is not troubled by any atmospheric absorption. In addition, the spacecraft avoids any problems caused by thermal infrared radiation from the Earth interfering with observations.

The L2 orbit also prevents the occurrence of temperature changes due to the spacecraft moving in and out of eclipse in an Earth orbit, which are a particular problem for infrared instruments requiring extreme thermal stability. Satellites will be shielded against primary radiation sources which protects the onboard equipment.

Furthermore, L2 orbit offers uninterrupted eclipse-free observations, provided the sufficient size of the orbit. Scientific observatories are pointed away from the Sun, Earth and Moon, therefore from L2 the entire celestial sphere can be observed during the course of one year.

In general, major advantage of L2 orbit would be continuous visibility of the satellite from Earth.

The outline of the IRASSI mission concept study was built on precursor missions and mission concepts such as:

**GAIA**

GAIA’s goal is to chart a three-dimensional map of our Galaxy, the Milky Way, in the process revealing the composition, formation and evolution of the Galaxy. Gaia is placed in an orbit around the Sun, at the second Lagrange point L2. The mission utilizes the Lissajou-type orbit. The orbit period is about 180 days and the size of the orbit is typically 340 000 × 90 000 km. An operational lifetime of 5 years is planned. The spacecraft must perform small manoeuvres every month.

**HERSCHEL**

The Herschel Space Observatory is the largest infrared space observatory launched to date. The observatory resides at the Sun-Earth system Lagrange point (L2) and consists of two spacecrafts. Nominal mission lifetime is three years. The spacecraft is at a Lissajous orbit around L2 with an average amplitude of about 700 000 km and an orbital period of about 178 days. Herschel will use its propulsion system to perform orbit maintenance manoeuvres roughly once each month. **2-4 m/s!**

**PLANCK**

**The Planck mission was devised to collect and characterise radiation from the Cosmic Microwave Background (CMB) using sensitive radio receivers operating at extremely low temperatures.**

Planck spacecraft resides in a Lissajous orbit with an average amplitude of about 400 000 km around the L2 point at a distance of around 1.5 million km from Earth in the anti-Sun direction. The station-keeping maneuvers are performed every 23 days

Also worth mentioning – well-known James Webb Space Telescope will be placed at SEM L2 as well, in HALO orbit.

* *Irassi mission description (similar to previous in format, but longer – half a page), (Irassi mission specific information: Buinhas, L., Ferrer-Gil, E., & Forstner, R. (2016, March). IRASSI: InfraRed astronomy satellite swarm interferometry—Mission concept and description. In Aerospace Conference, 2016 IEEE (pp. 1-20). IEEE.*

**IRASSI MISSION**

As was already mentioned, the telescopes would be operating in far-infrared radiation spectrum in order to look through the clouds of gas and dust present in protoplanetary regions. Specifically range of 1 to 6 THz, spanning wavelengths from 300 down to 50 mkm will be utilized.

Observations in such range requires sophisticated instrumentation. To be able to resolve the processes involved in planets formation, angular resolutions of less than 0.1 arcsec must be provided. The angular resolution has an inverse relation with the diameter of the collecting dish, which means that in order to achieve the required resolution, the dish must be prohibitively large, which, in turn, is not feasible for space-based telescopes. This obstacle can be overcome by employing interferometry. Interferometric systems employ arrays of telescopes to be able to extract information from the radiation source by superimposing electromagnetic wavefronts, which are phase-shifted, in order to measure their interference.

The high angular resolution is achieved by changing the separation between the satellites, also known as baselines. IRASSI mission will use five satellites in free-flying formation. The baselines are foreseen to vary from 7 to 850 meters and each spacecraft ought to be able to measure the baseline relative to the other four spacecrafts. Achieving accurate baseline measurements is a very challenging task and this problem is currently under investigation by Menlo Systems.

It is of great importance to the telescopes that observations are carried out unobstructively and in stable conditions. Which has lead to the consideration of putting the space observatory in an orbit around Lagrangian (or libration) point.

Libration points are analytical solutions of the Circular-Restricted Three-body Problem (CR3BP), which describes the dynamics of a spacecraft under the gravitational influence of two primary massive bodies revolving around their center of mass in circular orbits, without the spacecraft influencing these primary bodies. CR3BP is an approximation of the real world as no other influencing forces, such as radiation pressure and influences of planets, are taken into account. Libration points are special positions, in which the gravitational forces of the two primaries balance out the centripetal force of the spacecraft, therefore in these positions the spacecraft can maintain stationary position with respect to the primary bodies. Also, quasi-periodic orbits exist around these points. There are five such points, and IRASSI mission will be placed around the second point of Sun/Earth-Moon system, which is 1.5 million kilometers away from Earth in the anti-Sun direction. Earth and Moon in CR3BP are treated as a single massive body by considering their barycenter.

A certain type of orbit, called Halo, is possible around libration point. Such orbit is quasi-periodic, allows large amplitudes which may allow eclipse-free observations, which makes Halo orbits ideal to carry out observations in the far-infrared spectrum, satisfying scientific requirement of the IRASSI mission. The orbit will be further discussed in more details in later sections.

## Motivation

* Detailed description of motivation with references (1 page)

Motivation for thesis or for IRASSI? Not sure what to put here

# THEORY OF HALO ORBITS

## Circular Restricted Three Body Problem (CR3BP)

* History of the problem, who studied it (1 small paragraph)
* Assumptions (1 small paragraph)
* Rotating reference frame description with figure (1 small paragraph)(figure)
* Equations of motion in non-dimensional form (equations with assumptions)(approx. 1 page)
* How lagrange points are derived from the equation (optional)(half a page)

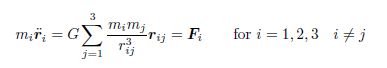
In order to establish the basis for further explanation of Lagrange points and possible orbits around them, first we have to take a look at the Circular Restricted Three-body problem of orbital mechanics in more details.

In general, Three-body problem doesn’t have analytical solutions unless some restrictions are imposed. Those restrictions were found by Lagrange and published in his “Essai sur le Probleme des Trois Corps”. These restrictions force three bodies to remain in an equilateral triangle or collinear formation. More information about those formations can be found in [2]. With respect to Lagrange points, the CR3BP is of particular interest.

In the CR3BP we assume that both primary bodies are very massive objects compared to the third mass. Thus, the Keplerian motion of the first two masses is determined through their respective inverse-square gravitational attraction.

Furthermore, primary bodies are assumed to revolve in circular orbits about their center of mass. This is considered to be a good approximation for celestial couples like Earth-Moon, Sun-Earth and others. [2]

The general equation of motion for the restricted three-body problem in a closed form is shown below: [2]

(1.1)

where G is the universal gravitational constant, Fi is the net resultant force acting in each mass mi, r ij is the relative position vector defined as rij = rj – ri

Graphical representation of CR3BP is shown on the figure below:

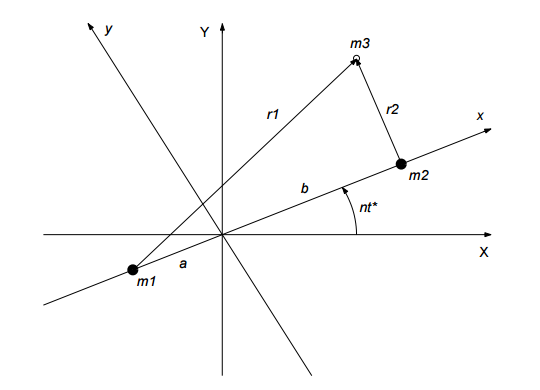


Figure 1.1 Geometry of the CR3BP in sidereal (XY) and synodic (xy) planes [3]

Worth mentioning, that C3RBP are non-linear systems from dynamics perspective.

**Equations of motion in non-dimensional form**

Lagrangian points, i.e. points of equilibrium exist only in the rotating reference frame. Therefore non-dimensionless equations of motion of mass m near the circularly orbiting m1 and m2 should be used.

Since the primaries revolve in a circular orbits under their mutual gravitational attractions, the gravitational and centrifugal forces must balance, therefore:

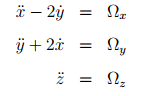
(1.2)

where the distances a, b and l are shown in the figure 1.1, k is the Gaussian gravitational constant and n is the angular velocity of the two bodies around their center of mass.

The constants a,b,l,n, m1 and m2 can be related to one parameter:

 (1.3)

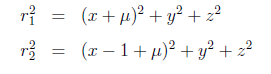
Taking mu = m2 and 1-mu = mu1, the Lagrangian dimensionless equations are:

(1.4)

where subscripts stand for partial derivatives of 1.5 for respective components

(1.5)

and



In equation 1.4 the sum of masses of the primary bodies, the distance between them and their angular velocity are normalized to one.

**USE SCHLAUB EXPLANATION!**

Now the center of mass condition can be written as:

Should I include the information of Jacobian, zero-velocities and other info on what libration points mean mathematically in more details?

*DO I need to delve deep into stability of L-points, zero-velocity-regions and Jacobi integral?*

### Periodic orbits around L points

* Periodic and quasi-periodic orbits, orbit types (distinguish between L2 and other libration points), short description of orbits other than halo (half a page)
* Halo orbits, characteristics, advantages over other libration orbits (half a page)

Should I actually describe the process of finding the HALO orbits here or it is unnecessary?

IRASSI paper p4-5 great info about the orbit!

## Lagrange points

* Description of lagrange points (1 paragraph)
* Figure of lagrange points in the sun-earth-moon system (figure)
* Lagrange point stability (1 paragraph)

## N-Body Problem

* Description (1 paragraph)
* General Equations of Motion (barycentric)
* Difference from CRTBP, why it is more realistic (1 parapraph)

# METHODOLOGY OF PRECISE ORBIT SIMULATION

* Description of the methodology (min. 1 page, max. 3 pages)

## Force Model

* Introductory small paragraph

### Space Environment around SEM L2

* Description of the space environment around L2 and overview of characteristics (min. 1 page, max. 3 pages)
* One big table of the effects with their magnitudes (table)

### Gravitational perturbation sources (approx. 1 page)

* Description (approx. 1 page)
* Table with the effects of all solar system bodies (table)
* Earth centric equations of motion (newtonian dynamics) (approx. 1 page)

### Solar Radiation Pressure

* Description (half a page)
* Equations with the assumptions (max. 2 pages)

### Spacecraft Maneuvers

* Description (half a page)
* Theory (not the implementation) on how to calculate the manuvers (max. 2 pages)

*HERE WOULD BE THE IMPORTANT PART ABOUT HALO COMPUTATION AND APPLYNG MANEUVERS -> Working on it now*

## Numerical Integration

* Describe generic numerical integration process (theory) (1 page)
* Literature search on numerical integrators used in astrodynamics applications (one big paragraph)

### Numerical integrators

* Introductory paragraph to outline which integrators will be described and if they have types e.g. fixed step, extrapolation etc. Montenbruck’s book. (min. half a page)

#### Integrator 1

* Describe the integrator

#### Integrator 2

* Describe the integrator

So on..

# SIMULATION AND TESTS

## Simulation

### Orbit Propagation

* Description of orbit propagation (half a page)

#### Force model

* Describe which model has been used, refer to the equation in theory section and state which values, bodies, libraries, etc. has been used. (approx. 2 pages)
* Describe everything necessary other than theory here for simulation.

#### Numerical Integration

* Describe which integrators has been used, refer to the theory section (approx. 1 page)
* Describe everything you used other than theory here.

### Maneuver Calculation

* Describe methodology of calculating maneuver with a flow chart (min. half a page)(figure: flow chart)
* Describe differential corrector, and the function used (approx.. 1 page)
* Describe event handling, stopping conditions, time frames (approx.. 1 page)

## Test Cases and Results

* Repeat motivation in the introductory paragraph and outline the tests (1 small paragraph)

### Force Model Simplification

#### Test description

* Describe the test, state what we are changing and why, state what remains the same, list the test cases in a table (approx. 1 page) (table)

Should I include numerical computation of halo orbits for different amplitudes?

How do we know which amplitude do we have/need? Given by UniBw? Or has to be calculated?

#### Force Model Simplification Test Results

* Plots of orbits created for each case
* Plots of differences in the coordinates, velocities and calculated maneuvers
* Apply the maneuvers created from simplified case to the full model and show if it is successful or unsuccessful staying in the orbit

#### Comparison of Integrators Test description

* Describe the test, state what we are changing and why, state what remains the same, list the test cases in a table (approx. 1 page) (table)

#### Comparison of Integrators Test Results

* Plots of orbits created for each case
* Plots of differences in the coordinates, velocities and calculated maneuvers
* Apply the maneuvers created from the worst case to the best case and show if it is successful or unsuccessful staying in the orbit

# CONCLUSION

* To be written when chapter 5 is finished.

# REFERENCES

* Make it an automatic table