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# INTRODUCTION

* *Description of motivation and research problem (2 paragraphs, min. half a page in total)*

In recent years, the observation of the formation of Earth-like planets has gained particular interest of the scientific society. Current focus of modern astronomy is studying the physical properties and chemical processes which can lead to prebiotic conditions in Earth-like planets, which can open the path to understanding the genesis of our own planet.

Earth-like planets can be originated in circumstellar disks and protoplanetary regions. These regions in space are obscured by clouds of gas and dust such that they are the best observable in far-infrared frequencies. This is, however, not possible to carry out from the ground due to the limitations caused by Earth’s atmosphere.

Thus, a space-based interferometric telescope mission called Infrared Astronomy Satellite Swarm Interferometry (IRASSI) was designed. This mission is a joint project between Menlo Systems GmbH, TU Braunschweig Institut für Flugführung, Max-Planck-Institut für Astronomie, and Universität der Bundeswehr München Institut für Raumfahrttechnik and Institut für Navigation. The projuect is funded by German Aerospace Centre (DLR). UniBw ISTA is conducting a feasibility study for that mission. The aim of the mission is to further develop the understanding of star and thus planet formation, by simultaneously implementing new technologies with regard to observation instruments, ranging detection systems and formation flying. According to the plan, the constellation of five satellites, whose inter-satellite distance will be tracked to an unprecedented precision, will be following Halo orbit around second Lagrange point of Sun-Earth-Moon system.

This mission is of great importance for the future of exo-planets discovery and studies, therefore thorough studies have to be carried out before the mission enters the launch phase. One of the most important among these studies is whether the satellite positions are determined with sufficient accuracy to meet the scientific and operational goals of the mission. The first step for this would be the development of a high precision simulation environment for the satellite motion in the vicinity of SEM L2.

**Main objective of the thesis**

Main objective of this Master Thesis project is to create the aforementioned simulation environment in order to propagate the sufficiently precise orbit around SEM L2 point. The simulation will not include the launch and early orbit phase (LEOP). Only the satellite motion during HALO orbit phase will be taken into account and studied. The simulation environment will be used to test the orbit determination performance of one IRASSI mission satellite.

Many astrodynamical tools exist, which are using simplified models such as Circular-Restricted Three Body problem or consider only Sun, Earth and Moon influences. One of the goals of this work is to find out whether those simplifications affect the orbit determination precision and to what extent by comparison. This includes determination of the relevant forces in the vicinity of L2 environment and including them in the precise model.

The next objective is to compare certain well-known numerical integrators for orbit propagation and show which one could be the best choice to propagate the HALO-type orbit around L2.

## Background

* *Advantages of L2 orbits and why it is popular for science/exoplanet missions (1 paragraph)*
* *L2 missions (1 small paragraph for each mission)*
* *Irassi mission description (similar to previous in format, but longer – half a page), (Irassi mission specific information: Buinhas, L., Ferrer-Gil, E., & Forstner, R. (2016, March). IRASSI: InfraRed astronomy satellite swarm interferometry—Mission concept and description. In Aerospace Conference, 2016 IEEE (pp. 1-20). IEEE.*

For scientific missions in space L2 point of the Sun-Earth-Moon system is often utilized. This point allows spacecraft to remain relatively stationary with respect to the Sun and Earth/Moon. This results in a low energy expenditure required for station keeping.

The satellite in an orbit around L2 is much closer to the Earth than if it resided at L4 or L5, which results in reducing the time to command a spacecraft. Light will take only 5 seconds to reach the spacecraft, whereas it will take 9 minutes to reach L4/L5. This, in turn, facilitates the ability to do real time commands, which occasionally might be useful.

Since the spacecraft resides at a distance of 1.5 millions kilometers from Earth it is not troubled by any atmospheric absorption. In addition, the spacecraft avoids any problems caused by thermal infrared radiation from the Earth interfering with observations.

The L2 orbit also prevents the occurrence of temperature changes due to the spacecraft moving in and out of eclipse in an Earth orbit, which are a particular problem for infrared instruments requiring extreme thermal stability. Satellites will be shielded against primary radiation sources which protects the onboard equipment.

Furthermore, L2 orbit offers uninterrupted eclipse-free observations, provided the sufficient size of the orbit. Scientific observatories are pointed away from the Sun, Earth and Moon, therefore from L2 the entire celestial sphere can be observed during the course of one year.

In general, major advantage of L2 orbit would be continuous visibility of the satellite from Earth.

The outline of the IRASSI mission concept study was built on precursor missions and mission concepts such as:

**GAIA**

GAIA’s goal is to chart a three-dimensional map of our Galaxy, the Milky Way, in the process revealing the composition, formation and evolution of the Galaxy. Gaia is placed in an orbit around the Sun, at the second Lagrange point L2. The mission utilizes the Lissajou-type orbit. The orbit period is about 180 days and the size of the orbit is typically 340 000 × 90 000 km. An operational lifetime of 5 years is planned. The spacecraft must perform small manoeuvres every month.

**HERSCHEL**

The Herschel Space Observatory is the largest infrared space observatory launched to date. The observatory resides at the Sun-Earth system Lagrange point (L2) and consists of two spacecrafts. Nominal mission lifetime is three years. The spacecraft is at a Lissajous orbit around L2 with an average amplitude of about 700 000 km and an orbital period of about 178 days. Herschel will use its propulsion system to perform orbit maintenance manoeuvres roughly once each month. **2-4 m/s!**

**PLANCK**

**The Planck mission was devised to collect and characterise radiation from the Cosmic Microwave Background (CMB) using sensitive radio receivers operating at extremely low temperatures.**

Planck spacecraft resides in a Lissajous orbit with an average amplitude of about 400 000 km around the L2 point at a distance of around 1.5 million km from Earth in the anti-Sun direction. The station-keeping maneuvers are performed every 23 days

Also worth mentioning – well-known James Webb Space Telescope will be placed at SEM L2 as well, in HALO orbit.

* *Irassi mission description (similar to previous in format, but longer – half a page), (Irassi mission specific information: Buinhas, L., Ferrer-Gil, E., & Forstner, R. (2016, March). IRASSI: InfraRed astronomy satellite swarm interferometry—Mission concept and description. In Aerospace Conference, 2016 IEEE (pp. 1-20). IEEE.*

**IRASSI MISSION**

As was already mentioned, the telescopes would be operating in far-infrared radiation spectrum in order to look through the clouds of gas and dust present in protoplanetary regions. Specifically range of 1 to 6 THz, spanning wavelengths from 300 down to 50 mkm will be utilized.

Observations in such range requires sophisticated instrumentation. To be able to resolve the processes involved in planets formation, angular resolutions of less than 0.1 arcsec must be provided. The angular resolution has an inverse relation with the diameter of the collecting dish, which means that in order to achieve the required resolution, the dish must be prohibitively large, which, in turn, is not feasible for space-based telescopes. This obstacle can be overcome by employing interferometry. Interferometric systems employ arrays of telescopes to be able to extract information from the radiation source by superimposing electromagnetic wavefronts, which are phase-shifted, in order to measure their interference.

The high angular resolution is achieved by changing the separation between the satellites, also known as baselines. IRASSI mission will use five satellites in free-flying formation. The baselines are foreseen to vary from 7 to 850 meters and each spacecraft ought to be able to measure the baseline relative to the other four spacecrafts. Achieving accurate baseline measurements is a very challenging task and this problem is currently under investigation by Menlo Systems.

It is of great importance to the telescopes that observations are carried out unobstructively and in stable conditions. Which has lead to the consideration of putting the space observatory in an orbit around Lagrangian (or libration) point.

Libration points are analytical solutions of the Circular-Restricted Three-body Problem (CR3BP), which describes the dynamics of a spacecraft under the gravitational influence of two primary massive bodies revolving around their center of mass in circular orbits, without the spacecraft influencing these primary bodies. CR3BP is an approximation of the real world as no other influencing forces, such as radiation pressure and influences of planets, are taken into account. Libration points are special positions, in which the gravitational forces of the two primaries balance out the centripetal force of the spacecraft, therefore in these positions the spacecraft can maintain stationary position with respect to the primary bodies. Also, quasi-periodic orbits exist around these points. There are five such points, and IRASSI mission will be placed around the second point of Sun/Earth-Moon system, which is 1.5 million kilometers away from Earth in the anti-Sun direction. Earth and Moon in CR3BP are treated as a single massive body by considering their barycenter.

A certain type of orbit, called Halo, is possible around libration point. Such orbit is quasi-periodic, allows large amplitudes which may allow eclipse-free observations, which makes Halo orbits ideal to carry out observations in the far-infrared spectrum, satisfying scientific requirement of the IRASSI mission. The orbit will be further discussed in more details in later sections.

## Motivation

* Detailed description of motivation with references (1 page)

Motivation for thesis or for IRASSI? Not sure what to put here

# THEORY OF HALO ORBITS

## Circular Restricted Three Body Problem (CR3BP)

* History of the problem, who studied it (1 small paragraph)
* Assumptions (1 small paragraph)
* Rotating reference frame description with figure (1 small paragraph)(figure)
* Equations of motion in non-dimensional form (equations with assumptions)(approx. 1 page)
* How lagrange points are derived from the equation (optional)(half a page)

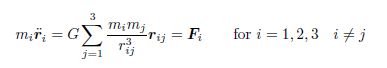
In order to establish the basis for further explanation of Lagrange points and possible orbits around them, first we have to take a look at the Circular Restricted Three-body problem of orbital mechanics in more details.

In general, Three-body problem doesn’t have analytical solutions unless some restrictions are imposed. Those restrictions were found by Lagrange and published in his “Essai sur le Probleme des Trois Corps”. These restrictions force three bodies to remain in an equilateral triangle or collinear formation. More information about those formations can be found in [2]. With respect to Lagrange points, the CR3BP is of particular interest.

In the CR3BP we assume that both primary bodies are very massive objects compared to the third mass. Thus, the Keplerian motion of the first two masses is determined through their respective inverse-square gravitational attraction.

Furthermore, primary bodies are assumed to revolve in circular orbits about their center of mass. This is considered to be a good approximation for celestial couples like Earth-Moon, Sun-Earth and others. [2]

The general equation of motion for the restricted three-body problem in a closed form is shown below: [2]

(1.1)

where G is the universal gravitational constant, Fi is the net resultant force acting in each mass mi, r ij is the relative position vector defined as rij = rj – ri

Graphical representation of CR3BP is shown on the figure below:

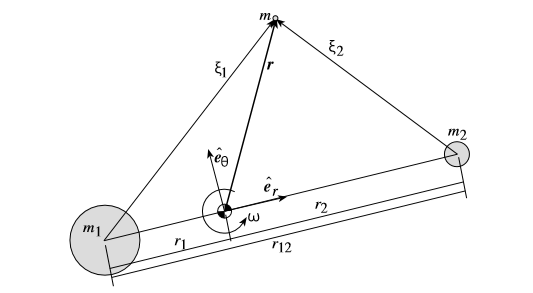


Figure 1.1 Geometry of the CR3BP [3]

Worth mentioning, that C3RBP are non-linear systems from dynamics perspective.

**Equations of motion in non-dimensional form**

Lagrangian points, i.e. points of equilibrium exist only in the rotating reference frame. Therefore non-dimensionless equations of motion of mass m near the circularly orbiting m1 and m2 should be used. Below, the general principle of deriving the non-dimensional equations of motion is shown. (The full detailed derivation can be found in [1] or [2] with different level of details provided)

First, we need to express the inertial position vector r (figure 1.1) with components taking in a rotating reference frame F : (er,eteta,e3). The origin is at the system center of mass.

The angular velocity w is given by:

(1.2)

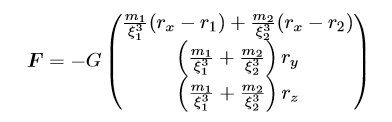
The position vector r in F is expressed as:

(1.3)

Acceleration vector of r is expressed as:

(1.4)

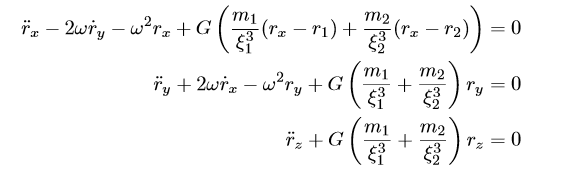
The gravitational force acting on m in F frame components is expressed as:

(1.5)

Where relative distances of mass m to primary bodies are given as:

(1.6)

Thus, the equations of motion of mass m in a rotating reference frame F can be expressed as three coupled differential equations:

(1.7)

These equations of motion can be rewritten in a convenient non-dimensional form. The non-dimensional time variable tau is defined as:

(1.8)

Therefore the non-dimensional time derivative with respect to this time is:

(1.9)

Thus, non-dimensional time derivative is related to previous xdot as:

(1.10)

Scalar distances are non-dimensionalized by dividing them by r12=m1-m2:

(1.11)

Mass quantities are non-dimensionalized by introducing the mass parameter mu defined as:

(1.12)

The center of mass condition was defined as:

m r = -m r1 – m2r2 (1.13)

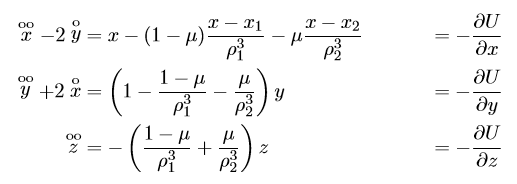
With the new non-dimensional quantities, the center of mass condition is rewritten as:

 (1.14)

Knowing that the distance between primary bodies is normalized to one and given center of mass condition, the non-dimensional coordinates of primary masses m1 and m2 are now expressed in terms of the mass parameter mu:

(1.15)

Now, the equations of motion in Eq. 1.7 are expressed in non-dimensional form as:

(1.16 a b c )

Where non-dimensional relative distances are defined as:

(1.17)

And the corresponding non-dimensional potential function is:

(1.18)

## Lagrange points

* Description of lagrange points (1 paragraph)
* Figure of lagrange points in the sun-earth-moon system (figure)
* Lagrange point stability (1 paragraph)

Lagrange (or libration) points are the natural equilibrium solutions of the restricted three body problem. Setting the relative velocities and accelerations in non-dimensional equations of motion (Eq. 1.16) to zero, the conditions that are satisfied by the stationary points of CR3BP can be found. These stationary points are exactly the Lagrange points, mentioned before.

All stationary points have z=0 and therefore lie in m1-m2 plane. Eq.1.16b is equal zero either if y=0, which corresponds to the collinear solution, or if ro1 = ro2 which corresponds to equilateral solutions.

Lagrange discovered five distinct formations which are invariant when viewed from the rotating reference frame. So, five possible locations for small mass m, for which the net gravitational force of two primary bodies on this smaller mass m is balanced by the centrifugal force of the rotating primaries themselves. That in turn makes those locations appear stationary as seen from rotating frame.

In other words, these are the five locations around a planet’s orbit where the gravitational forces and the orbital motion of the spacecraft and massive bodies interact to create a stable locations from which, for instance, one can make observations. Lagrange points of the 3-body system Sun/Earth-Moon are shown on the figure below:

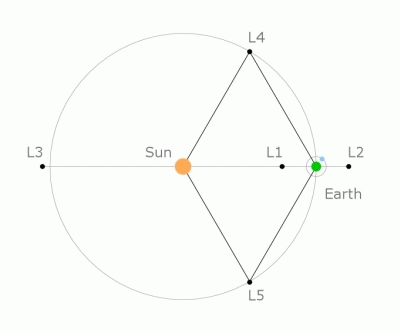


Figure 1.2 Stationary lagrange points [esa]

As seen from the Sun, the L4 and L5 points lie at 60 degrees ahead of and behind Earth, close to its orbit.

**Lagrange point stability**

Unlike the other Lagrange points, L4 and L5 are resistant to gravitational perturbations. Because of this stability, objects such as dust and asteroids tend to accumulate in these regions.

A spacecraft at L1, L2, or L3 is ‘meta-stable’. A little perturbation forces it to move away, so a spacecraft must use frequent rocket firings to stay in orbits around the Lagrangian point.

### Periodic orbits around L points

* Periodic and quasi-periodic orbits, orbit types (distinguish between L2 and other libration points), short description of orbits other than halo (half a page)
* Halo orbits, characteristics, advantages over other libration orbits (half a page)

Stationary points of the circular-restricted 3 body problem are of practical interest. Much effort has been put into finding periodic stationary orbits of RC3BP. Such orbits form closed three-dimensional curves and remain fixed as seen from the rotating frame.

Thousands such orbits were discovered but not all of them are of practical value. Those orbits can be grouped into certain families, thus helping to explain the more general classes of orbits for this 3body problem. [Shaub 2].

In Hamiltonian systems the presence of a periodic orbit involves the characterization of the whole family as isolated periodic orbits do not exist in such systems. Hence, in order to identify a single orbit belonging to the family, a certain parameter must be introduced. For halo orbit, which will be discussed further, this parameter is the out-of-plane amplitudes Az. (for Lyapunov orbit this is in-plane Ax). [3]

Before going deeper into these orbits types, linearized non-dimensional equations of motion (Eq. 1.16) should be expressed in a different form:

****

(1.19)

Thus we obtain the two frequency parameters in xy in-plane (primary bodies plane) and one frequency in the perpendicular plane.

*In [3] it is shown that* linearized dynamics (Eq. 1.19) around collinear libration points, in particular L1 and L2, with only two of these frequencies (in xy plane) allows obtaining some infinitesimnal or **Lyapunov orbits** in a small neighborhood around those points, given an appropriate initial condition. Moreover these planar orbits can be numerically continued until the desired finite size is reached. Different Lyapunov orbits around L1 of the Sun-Earth system are shown on the figure below:

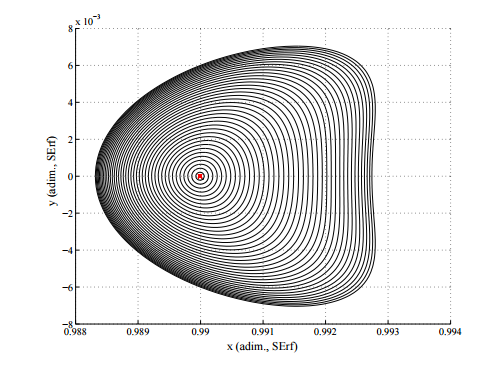


figure 1.3 Finite-size Lyapunov orbits of different amplitudes [3]

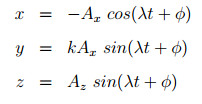
As can be seen, these frequencies actually vary and therefore the orbit changes its shape with growing amplitude. This happens due to the actual non-linearities of the dynamical system.

Such orbits do not allow the out-of-plane motion and therefore not suitable for space application. [3 Mil pages 14-15 answer why]

*Other that [3] see also L2env paper for additional details:*

With frequencies in both planes (if unequal) we would obtain the quasi-periodic closed form path called Lissajous Trajectory, since frequencies λ and v are different. Lissajous orbits can be used in space missions but do not allow big excursions in the out-of-plane direction, which sometimes is requirement in order to meet the mission scientific or operational requirements, which is the case for IRASSI mission.

However, very interesting and applicable for space missions large periodic Halo-orbit types can be obtained, which allow out-of-plane motion. This is possible, if the amplitudes of in-plane and out-of-plane motions are of sufficient magnitude, so that the non-linear contributions to the system produce eigenfrequencies that are equal, in other words, Eq.1.19 becomes: *[3] p 23*

(1.20)

### Halo orbit

So, one of the advantages of halo orbit over other types of orbits around libration points, is that it allows for the big excursions in the out-of-plane direction. This can be help serving the scientific or operational mission-specific goals.

Therefore, choosing an appropriate Az is important since through its value the mission-specific constraints can be formulated. For instance (applies to IRASSI mission) space telescope about the Sun-Earth L2 requires a minimum Az in order to avoid the eclipses and maintain uninterrupted communication link.

In the CR3BP model, halo orbits are both periodic and time independent. Note, however, that if we take into account all the effects of the full solar system, which is among others, the purpose of this work, halo orbits are in fact quasi-periodic and time dependent.

Any orbit around L2 is dynamically unstable in reality, i.e. over time a spacecraft in its orbit will diverge and escape to the outer space. This is due to the effect of the solar radiation pressure, gravitational attraction of other planets and movement of the Moon.

Orbits’ name speaks for itself, it is a ring or “halo” about the libration point.

In order to describe this orbit, in case of IRASSI around L2 point of Sun/Earth-Moon system, the L2-centered reference frame must be defined. The following properties apply: [irassi]

* X-axis is aligned with the Sun-Earth line in the ecliptic plane. Its positive direction points towards outer space
* Z-axis is aligned with the North and South ecliptic poles of the celestial sphere. Positive direction towards the North-Pole
* Y-axis completes the right-handed reference frame

Below is the representation of L2-centered reference frame:

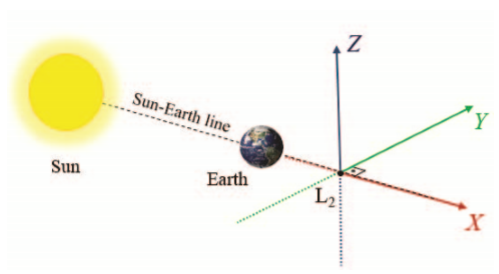


Figure 1.4 L2-centered frame

In general quasi-Halo orbits would be characterized by the following properties: [irassi]

* Quasi-periodic. Frequency in z-axis is nearly the same as in X-Y axes frequencies.
* Large amplitudes in the Y-component
* Spacecraft on such an orbit will not enter the antumbra region, therefore free-of-eclipse observations are possible
* Quasi-free delta-V transfer maneuvers

Therefore, quasi-Halo orbit is chosen for the IRASSI mission as these properties make such orbits ideal for carrying out far-infrared observations and satisfying scientific requirements for the mission.

The spacecraft should not enter the antumbra shadowed region of the Earth at any point during the Nominal Operations phase in order to ensure the constant solar power supply. During the mission analysis study [irassi] it was concluded that the minimum Az to avoid drifting into eclipse within a period of 5 years is 270 000 km. Adequate value of Az is also important in terms of constant direct visibility from the ground stations. As such, an excessive amplitude Az on the negative direction may impair the communications between the satellites and the ground station situated in the Northern Hemisphere.

Maximum amplitude Ay was set to 850 000 km but can be altered in future.

The launch periods were selected in accordance with the possibility to satisfy aforementioned conditions and no maximum amplitudes (Ay) lower than 600 000 km were obtained for the selected launch periods.

Illustration of the quasi-Halo orbit around L2 including transfer from the Earth is shown on the figure below:

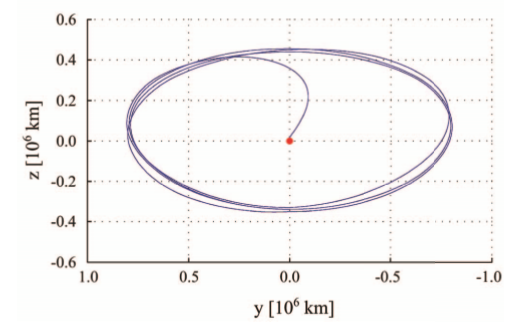


Figure 1.5 Quasi-Halo orbit around L2 SEM [irassi]

As was already mentioned Halo orbits tend to be unstable, therefore station-keeping maneuvers are required to maintain its stability and periodicity. More detailed information about maneuvers calculation and implementation is in the following chapters.

Also worth mentioning, that Halo orbits are difficult to obtain because the problem is highly non-linear and small changes in the initial conditions break the periodicity of the orbits. However, there are some methods allowing for the generation of halo orbits with desired amplitudes. Detailed description of the process goes beyond the scope of this work, but relevant information can be found for instance in Zazzera [3] or in full details in Richardson [7].

## N-Body Problem

* Description (1 paragraph)
* General Equations of Motion (barycentric)
* Difference from CRTBP, why it is more realistic (1 parapraph)

# METHODOLOGY OF PRECISE ORBIT SIMULATION

* Description of the methodology (min. 1 page, max. 3 pages)

In Newtonian physics the motion of the satellite under the influence of the force is described by the differential equation



where r an v are the position and velocity of the satellite in a non-rotating geocentric coordinate system and m denotes the satellite’s mass.

In order to perform precise orbit determination one has to include as many acceleration sources as possible within a certain threshold and decide which forces can be neglected in order to achieve desired precision.

Most significant perturbations arise from gravitational attraction of the celestial bodies, especially Sun, Earth and Moon when we are dealing with the Earth-orbiting satellites. Other dominant planetary contributions stem from Venus and Jupiter.

Satellites with altitudes of several hundred kilometers above the Earth’s surface are subject to a drag force, caused by Earth’s atmosphere. Since the atmospheric density decreases exponentially with increasing height, drag effect mainly affect the low-Earth satellites and is the strongest during the perigee of an orbit. While, the acceleration due to gravitational forces is independent of the satellite’s mass and area, that is not the case for atmospheric drag as well as for other surface forces. Among these the solar radiation pressure is the most significant. The radiation pressure arises when photons transfer the impulse to the satellite and doesn’t vary with altitude. Its main effect is in slight changes in eccentricity and the longitude of perigee. [7]

For near-Earth orbiting satellites quite significant accelerations arise from the fact that Earth is not a perfect sphere, but has the form of an oblate spheroid with an equatorial diameter that exceeds the polar diameter by 20 km. This perturbation is about three orders of magnitudes smaller that the central gravitational attraction. Due to its angular momentum the orbit behaves like a gyroscope and reacts with a precessional motion of the orbital plane, and a shift of the line of nodes by several degrees per day. [7] The effect of various perturbations as a function of geocentric satellite distance is illustrated on a figure below:

## Force Model

* Introductory small paragraph

### Space Environment around SEM L2

* Description of the space environment around L2 and overview of characteristics (min. 1 page, max. 3 pages)
* One big table of the effects with their magnitudes (table)

### Gravitational perturbation sources (approx. 1 page)

* Description (approx. 1 page)
* Table with the effects of all solar system bodies (table)
* Earth centric equations of motion (newtonian dynamics) (approx. 1 page)

### Solar Radiation Pressure

* Description (half a page)
* Equations with the assumptions (max. 2 pages)

### Spacecraft Maneuvers

* Description (half a page)
* Theory (not the implementation) on how to calculate the manuvers (max. 2 pages)

*HERE WOULD BE THE IMPORTANT PART ABOUT HALO COMPUTATION AND APPLYNG MANEUVERS -> Working on it now*

## Numerical Integration

* Describe generic numerical integration process (theory) (1 page)
* Literature search on numerical integrators used in astrodynamics applications (one big paragraph)

### Numerical integrators

* Introductory paragraph to outline which integrators will be described and if they have types e.g. fixed step, extrapolation etc. Montenbruck’s book. (min. half a page)

#### Integrator 1

* Describe the integrator

#### Integrator 2

* Describe the integrator

So on..

# SIMULATION AND TESTS

## Simulation

### Orbit Propagation

* Description of orbit propagation (half a page)

#### Force model

* Describe which model has been used, refer to the equation in theory section and state which values, bodies, libraries, etc. has been used. (approx. 2 pages)
* Describe everything necessary other than theory here for simulation.

#### Numerical Integration

* Describe which integrators has been used, refer to the theory section (approx. 1 page)
* Describe everything you used other than theory here.

### Maneuver Calculation

* Describe methodology of calculating maneuver with a flow chart (min. half a page)(figure: flow chart)
* Describe differential corrector, and the function used (approx.. 1 page)
* Describe event handling, stopping conditions, time frames (approx.. 1 page)

## Test Cases and Results

* Repeat motivation in the introductory paragraph and outline the tests (1 small paragraph)

### Force Model Simplification

#### Test description

* Describe the test, state what we are changing and why, state what remains the same, list the test cases in a table (approx. 1 page) (table)

Should I include numerical computation of halo orbits for different amplitudes?

How do we know which amplitude do we have/need? Given by UniBw? Or has to be calculated?

#### Force Model Simplification Test Results

* Plots of orbits created for each case
* Plots of differences in the coordinates, velocities and calculated maneuvers
* Apply the maneuvers created from simplified case to the full model and show if it is successful or unsuccessful staying in the orbit

#### Comparison of Integrators Test description

* Describe the test, state what we are changing and why, state what remains the same, list the test cases in a table (approx. 1 page) (table)

#### Comparison of Integrators Test Results

* Plots of orbits created for each case
* Plots of differences in the coordinates, velocities and calculated maneuvers
* Apply the maneuvers created from the worst case to the best case and show if it is successful or unsuccessful staying in the orbit

# CONCLUSION

* To be written when chapter 5 is finished.

# REFERENCES

* Make it an automatic table