Networked Life: Homework 6

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1. **Answer:**

   Based on how the graph is connected, we can tell that node 6 is the destination. Therefore, the steps of the Bellman-Ford Algorithm are tabulated below:

t	d6	d5	d4	d3	d2	d1
t=0	0	∞	∞	∞	∞	∞
t=1	0	5	2	9	∞	8
t=2	0	5	2	9	3	7
t=3	0	5	2	9	3	7

   Table 1: Steps of the Bellman-Ford Algorithm

   From $t = 2$ to $t = 3$, the shortest distances were not updated any more. Therefore we obtained the shortest path from all points to node 6.

2. **Answer:**

   (1) $N_d = C/1 = C$.

   (2) $P(X > N_d) < \gamma$ is equivalent with $P(X \leq N_d) \geq 1-\gamma$. So we just have to choose N_s such that:

   $$P(X \leq N_d) = \sum_{i=1}^{N_d} \binom{N_s}{i} p^i (1-p)^{N_s - i} \geq 1 - \gamma$$

   Plugging in $N_d = C$ and $\gamma = 0.01$ and $p = 0.1$, we increment N_s from C to find the last number that still maintains $P(X \leq C) \geq 0.99$. This number is the maximum number of users in the system. Following this method, we obtained: $N_s = 50$ when $C = 10$; $N_s = 122$ when $C = 20$; $N_s = 200$ when $C = 30$. There appears to be a linear relationship between N_s and the value of C. $\sum_{i=N_d+1}^{N_s} P(X = i)$ when $N_d = 10$ is 0.00935.

   (3) Let's take the case when $C = 10$ as an example. $N_s = 50$ in this case.

   In System A, an N_s equal to 122 can be achieved. In System B, the total N_s is simply 2 times the original $N_s = 50$, which is 100. Comparing these two values, System A can fit the largest amount of users. This is because when the resources are pooled together, the system is more flexible and tolerant. In System B, there might be a case where one link is crowded but the one link is empty. When resource pooling is used, such uneven distribution of demand can be eliminated, resulting in a more robust system.

3. **Answer:**

We can re-express $1/E[n, \rho]$ using formula (1):

$$\frac{1}{E[n,\rho]} = \frac{e^{\rho} \int_{\rho}^{\infty} \frac{x^n e^{-x}}{n!} dx}{\rho^n / n!}$$

Cancelling $n!$ and plugging in t, we get:

$$\frac{1}{E[tn,t\rho]} = \frac{e^{t\rho} \int_{t\rho}^{\infty} x^{tn} e^{-x} dx}{(t\rho)^{tn}} = \frac{\int_{t\rho}^{\infty} x^{tn} e^{t\rho - x} dx}{(t\rho)^{tn}}$$

Re-indexing the range of the summation, we get:

$$\frac{1}{E[tn,t\rho]} = \frac{\int_{0}^{\infty} (x + t\rho)^{tn} et\rho - x - t\rho dx}{(t\rho)^{tn}} = \int_{0}^{\infty} (\frac{x + t\rho}{t\rho})^{tn} e^{-x}$$

Only the middle term $(\frac{x+t\rho}{t\rho})^{tn}$ contains t. So we only need to compare which term, $(x + t\rho)^{tn}$ or $(t\rho)^{tn}$ grows faster as t gets large. It is obvious that given the same index, the nominator will increase faster than the denominator does. Therefore, $1/E[tn, t\rho]$ is increasing with t. As a result, it has been proven that the original function $E[tn, t\rho]$ is strictly decreasing in t.