Networked Life: Homework 3

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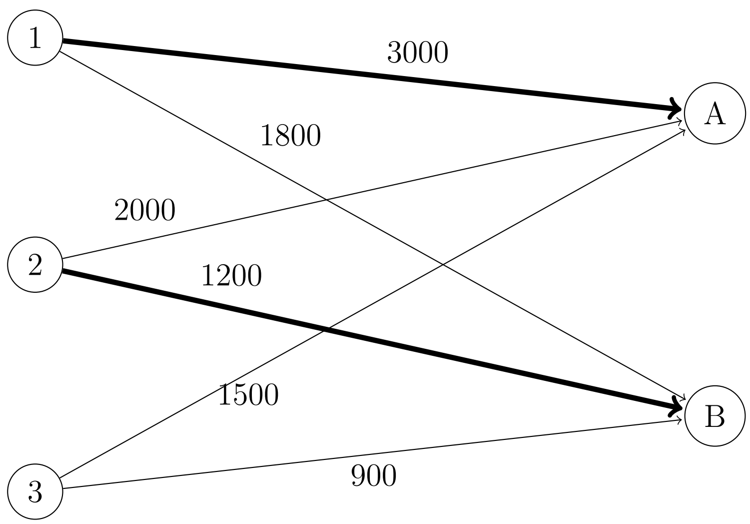
1. **Answer:**

   (a) Time the average revenues per click together with the clickthrough rates, we can get the following bipartite graph with the edges indicating values per hour and maximum matching in bold lines:



   (b) Because $\$6 > \$4 > \$3$, so ad space A will be assigend to advertiser 1, and ad space will be assigned to advertiser 2.

   The price charged to 1 would be \$4 per click, or $\$4 \times 500 = \$2000$ per hour. The price charged to 2 would be \$3 per click, or $\$3 \times 300 = \$900$ per hour.

   The payoff for 1 is $\$6 - \$4 = \$2$ per click, or $\$2 \times 500 = \$1000$ per hour. The payoff for 2 is $\$4 - \$3 = \$1$ per click, or $\$1 \times 300 = \$300$ per hour.

2. **Answer:**

Table 1: Bidding history of all three bidders over reach day of the auction

Bidder	Day 1	Day 2	Day 3	Day 4	Day 5
1	(1) 7	(3) 9.5	(5) 10→11		(8) 27.45
2		(2) 9.25		(6) 13.65	
3			(4) 9.75→11.25	(7) 13.9	
Ask price	7.25	9.75	11.5	14.15	27.7

According to the bidding progression, bidder 1 wins the bid and pays \$27.45.

3. **Answer:**

   (a) If $b_1 > b_2$, Alice wins the slot with clickthrough rate of 500 per hour. Her payoff is $\$(r - b_2)$ per click, or equivalently, $\$500(r - b_2)$ per hour.

   If $b_1 < b_2$, Alice wins the slot with clickthrough rate of 300 per hour. Alice's payoff is then $\$r$ per click, or equivalently, $\$300r$ per hour.

   (b) Alice's dominant strategy should yield maximum payoff under all conditions.

   When $r < b_2$, it doesn't make a difference whether Alice bids truthfully or underbids.

   When $r > b_2$, Alice needs to decide whether she should bid truthfully, or underbid so that $b_1 < b_2$. We can compare the payoffs of these two scenarios. Set $500(r - b_2) = 300r$. So if Alice estimates that $r > 2.5b_2$, Alice should bit truthfully to maximize her chance getting the top slot. On the other hand, if $r < 2.5b_2$, then she should just underbid to lose the bid intentionally, which results in her getting the second slot.

4. **Answer:**

   (a) Bidder 2 in this case would like to secure 1 seat. However, he won't spend more than \$10 on one seat. So Bidder 1 could always outbid Bidder 2. Therefore, the strategy of Bidder 1 is to make sure not to invest too much on one seat prematurely. After each bid made by Bidder 2, Bidder 1 just needs to make a bid that is one minimum increment higher. In this way, Bidder 1 can make sure his left budget is always more than that of Bidder 2.

   That said, Bidder 1, as long as he bids strategically, will win the bid. Assuming the minimum increment is \$m, the price charged to Bidder 1 will be $\$(12 + 2m)$. His payoff is $\$(3 - 2m)$.

   (b) In this case, Bidder 1 just needs to always bid on the package consisting of both seats. Regardless of whether Bidder 2 bids on individual seats or package, Bidder 1 just needs to top Bidder 2's total bid by minimum increment.

   Therefore, Bidder 1 can still win the bid if package bidding is used. The price charged is $\$(12 + m)$. His payoff is $\$(3 - m)$.

5. **Answer:**

   The price charged to the winners Bidder 1 and 4 are represented by p_1 and p_4:

$$M* = (1, 1), (4, 2)$$

$$V* = 10 + 9 = 19$$

$$p_1 = (7 + 8) - 9 = 6$$

$$p_4 = (10 + 8) - 10 = 8$$

6. **Answer:**

   Vector $\pi[k]$ will exhibit periodic behavior as k becomes large. In particular, $\pi[0] = [1/2, 1/2, 0, 0]^T, \pi[1] = [1/2, 0, 0, 1/2]^T, \pi[2] = [0, 0, 1/2, 1/2]^T, \pi[3] = [0, 1/2, 1/2, 0]^T$, and then the pattern repeats itself.

   Therefore, there is no solution for steady-state probabilities π^* such that $\pi^{*T} = \pi^{*T} H$.

7. **Answer:**

   When $\theta = 0.1$, $\pi_1 = 0.211, \pi_2 = 0.205, \pi_3 = 0.200, \pi_4 = 0.193, \pi_5 = 0.190$;
   When $\theta = 0.3$, $\pi_1 = 0.236, \pi_2 = 0.221, \pi_3 = 0.196, \pi_4 = 0.177, \pi_5 = 0.170$;
   When $\theta = 0.5$, $\pi_1 = 0.270, \pi_2 = 0.249, \pi_3 = 0.183, \pi_4 = 0.153, \pi_5 = 0.145$;
   When $\theta = 0.85$, $\pi_1 = 0.385, \pi_2 = 0.369, \pi_3 = 0.102, \pi_4 = 0.073, \pi_5 = 0.071$;

   As θ increases, the difference of probabilities grows larger. It is more and more obvious that π_1 and π_2 have larger significance. This trend is because θ indicates how much we value the original distribution of transition probabilities. So as θ becomes larger, the probabilities at equilibrium are mostly determined by the structural connectivity of the graphs.

8. **Answer:**

   (a) $[\pi_A^*, \pi_B^*]^T = [0.827, 0.173]^T$

   (b) $[\pi_1^*, \pi_2^*]^T = [0.5, 0.5]^T$
   $[\pi_3^*, \pi_4^*, \pi_5^*]^T = [0.416, 0.168, 0.416]^T$

   (c) The approximate π^* calculated by the given equation is $[0.413, 0.413, 0.072, 0.029, 0.072]^T$.

   The time complexity of this approximate method is less than that of the directly computing π^*. No matter if we use inversion or matrices or iterative method, expensive matrix operations like inversion or multiplication is involved. As the matrix dimension n gets larger, computational load would increase exponentially. Splitting the whole matrix into smaller matrices would make computaiton less costly. In this case, updating each iteration would cost around $5^2 = 25$ multiplications for the original matrix. But once we use the approximation, only $3^2 + 2^2 + 2$=17 multiplications are needed.