Networked Life: Homework Q9, Q11

Due date: Thursday, March 30th, during class

Exercise 1. *Computation of C and L (Exercise 9.1)*

Exercise 2. *Kleinberg model (Exercise 9.4)*

Exercise 3. *Congestion prices*

Consider a movie theater where there is a negative externality from the people that share the room (i.e., because everybody eats pop-corn and this makes noise that adds-up). Suppose that the net benefit of a person that decides to watch a movie and the hall has n spectators (including himself) is the value of watching the movie A minus the cost of the noise that is cn where each pop-corn eater generates cost $c < A$ and cost is additive.

1. Find the number of spectators at the equilibrium. To do that, think of each person joining one by one. Initially each person has positive benefit from joining, and eventually when n_e people join, the last of them (as well as any of them) obtains zero net benefit, and if one more person joins, then she would get negative net benefit (like all of them).

2. Find the optimal number of spectators that the social planer would like to have in the hall to watch the movie. Compare these two numbers. Do we have a tragedy of the commons situation? Explain.

3. Find the price that if the social planner charges each person to watch the movie, then the new equilibrium is the socially optimal one. This is an example of a congestion price!

4. Assume that the money the social planner collects is redistributed to the participants in some form (say as a rebate on future movies). What is the net benefit of each participant by the use of the congestion pricing scheme?

Exercise 4. *Usage prices*

Consider 2 users with utility functions for bandwidth $a \log x_1$ and $b \log x_2$ respectively, where $a < b$.

Case 1: Network with finite capacity C

1. Calculate the price for which demand = supply. Calculate the bandwidth allocation to each user.

Case 2: Network with infinite capacity and constant marginal capacity cost c :

2. Calculate the price that maximizes Social Welfare. Calculate the bandwidth allocation to each user if the network operator uses the above optimal price.

3. Assume that the number n of user increases and they have identical utility functions (say $a \log x$). Compute the corresponding prices in Cases 1 and 2 as a function of n. Explain what you observe.

We return to the initial case of the two users, Case 2.

4. Assume that the network has some fixed operating cost B to recover, otherwise it will go bankrupt. How do you suggest that this cost should be recovered from the users? Assume that the network operator does not know the utility functions of the users and also it cannot discriminate (by law). What are the potential problems that can arise?

   Assume now that the network has fixed capacity C, two users

5. Assume that network operator wants to maximize his profit by adding a fixed fee to the charging scheme (besides the usage price). Assume also that he knows the utility functions of the two users but he must use the same charging scheme for both of them (no discrimination).

   For large values of C, what fixed fee will he use? Which of the two users benefits? (Hint: the network operator must only charge his users using equal terms, he is not forced to serve all of them necessarily. Hence your answer must depend on the relative values of a, b. There are two cases: i) both players participate and he charges as a fixed fee enough to make the weak player become indifferent to participate, ii) the fixed fee is large enough and only the strong player participates. Compare the profits of the ISP in these 2 cases)