Networked Life: Homework 2

Guo Ziqi - 1000905
Zhao Juan - 1000918
Zhang Hao - 1000899

1. **Answer:**
First we calculated the mean of all filled entries to be 3.125. Then we identified matrix A and c as below, and calculate b based on exact solution.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, c = \begin{bmatrix} 1.875 \\ 1.875 \\ 0.875 \\ -2.125 \\ -2.125 \\ 0.875 \\ 0.875 \\ -2.125 \\ -1.125 \\ 0.875 \\ -0.125 \\ 0.875 \\ -0.125 \\ -2.125 \\ 1.875 \\ -0.125 \end{bmatrix}, b = \begin{bmatrix} 1.52 \\ -1.21 \\ -0.39 \\ 0.07 \\ 0.07 \\ -0.19 \\ -0.01 \\ -0.37 \\ 0.62 \end{bmatrix}$$

Then we calculate the baseline predictor $\hat{R}$:

$$\hat{R} = \begin{bmatrix} 4.46 & 4.64 & 4.27 & 5 \\ 1.73 & 1.91 & 1.55 & 2.55 \\ 2.55 & 2.73 & 2.36 & 3.36 \\ 3 & 3.18 & 2.82 & 3.82 \\ 3 & 3.18 & 2.82 & 3.82 \end{bmatrix}$$

2. **Answer:**
First we calculated $\tilde{R}$ by calculating the discrepancy between R and $\hat{R}$:

$$\tilde{R} = \begin{bmatrix} 0.55 & - & 0.73 & -1.27 \\ - & -0.91 & -0.55 & 1.45 \\ 1.45 & -1.73 & -0.36 & 0.64 \\ 0 & 0.82 & - & -0.82 \\ -2 & 1.82 & 0.18 & - \end{bmatrix}$$

Then using formula $d_{ij} = \frac{\tilde{r}_i^T \tilde{r}_j}{\|\tilde{r}_i\|_2 \|\tilde{r}_j\|_2}$, we can get all pairs of d_{ij}:

$$d_{AB} = -0.94, d_{AC} = -0.24, d_{AD} = 0.09$$

$$d_{BC} = 0.80, d_{BD} = -0.82, d_{CD} = -0.98$$

Now we can obtain the $\hat{R}$ considering neighbourhood:

$$\hat{R} = \begin{bmatrix} 4.31 & 4.94 & 4.97 & 4.88 \\ 2.56 & 1.23 & 1 & 3.26 \\ 4.00 & 1.65 & 1.24 & 4.35 \\ 2.35 & 3.56 & 3.64 & 3.45 \\ 1.51 & 4.25 & 3.64 & 2.89 \end{bmatrix}$$

3. **Answer:**

(a) Using the formula $b = (A^T A)^{-1} A^T c$, the b vector without regularization is:

$$b = \begin{bmatrix} 1.04 \\ 0.21 \\ 0.54 \end{bmatrix}$$

(b) Taking derivative of the expression with regularization:

$$\frac{d(\|Ab - c\|_2^2\| + \lambda\|b\|_2^2)}{db} = 2b^T(A^T A) - 2c^T A + 2\lambda b^T = 0$$

$$b^T(A^T A) - \lambda b^T = c^T A$$

$$(A^T A - \lambda I)b = c^T A$$

$$b = (A^T A - \lambda I)^{-1} A^T c$$

Varying λ from 0 to 5.0, we solved b using each λ and plotted the two curves against λ. We can observe that as we increase λ, the norm of vector b will be constrained to be smaller, and the error term will be larger.

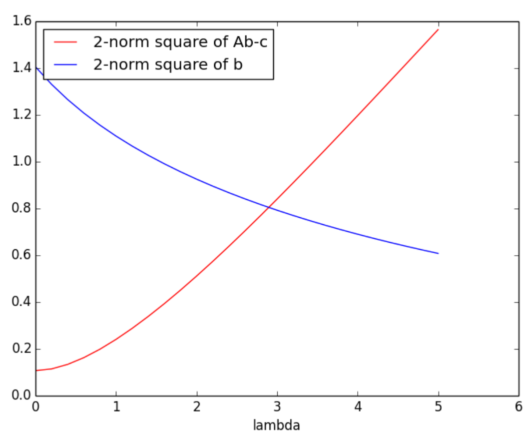

Figure 1: $\|Ab - c\|_2^2$ and $\|b\|_2^2$ against varying λ