7.3 *A citation network and matrix multiplication* **

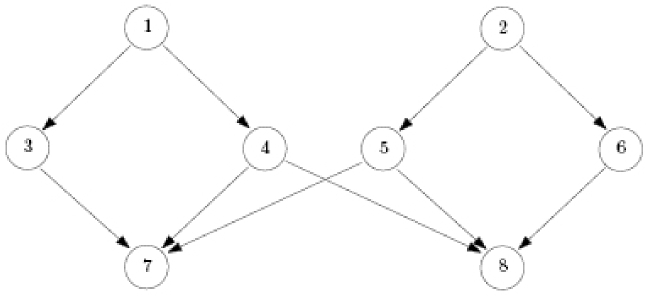

Figure 7.16 A directed graph to represent a citation network's topology.

Consider a set of eight papers with their citation relationships represented by the graph in Figure 7.16. Each paper is a node, and a directed edge from node i to node j means paper i cites paper j.

(a) Write down the adjacency matrix **A** (which we will talk much more about in the next chapter), where the (i, j) entry is 1 if node i points to node j, and 0 otherwise.

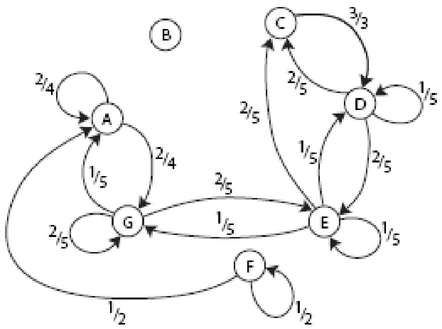

Figure 7.17 A network showing the transition diagram corresponding to the chorus of *Oh Susanna*. Each node is a diatonic pitch. Each link is a possible transition, with the link weight representing the probability of transition.

(b) Compute the matrix **C** defined as

$$\mathbf{C} = \mathbf{A}^T \mathbf{A},$$

and compare the values C_{78} and C_{75}. In general, what is the physical interpretation of the entries C_{ij}?

(c) Now compute

$$\mathbf{A}^2 = \mathbf{AA},$$
$$\mathbf{A}^3 = \mathbf{A}^2\mathbf{A}.$$

Is there anything special about $\mathbf{A}^3$? In general, what do the entries in $\mathbf{A}^m$ (where $m = 1, 2, \ldots$) represent?

8.1 *Computing centrality and betweenness* *

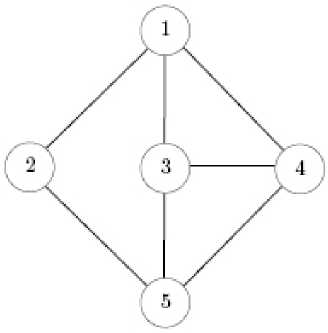

Figure 8.19 A simple graph for computing centrality measures.

(a) Compute the degree, closeness, and eigenvector centrality of each node in the graph in Figure 8.19.

(b) Compute the node betweenness centrality of nodes 2 and 3.

(c) Compute the link betweenness centrality of the links (3, 4) and (2,5).

8.2 *Contagion* *

Consider the contagion model in the graph in Figure 8.20 with p = 0:3.

(a) Run the contagion model with node 1 initialized at state-1 and the other nodes initialized at state-0.

(b) Run the contagion model with node 3 initialized at state-1 and the other nodes initialized at state-0.

(c) Contrast the results from (a) and (b) and explain in terms of the cluster densities of the sets of initially state-0 nodes.

8.3 *SIRS infection model***

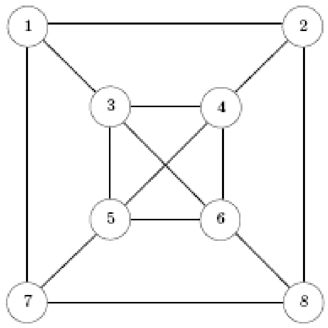

Figure 8.20 A simple graph for studying contagion.

We consider an extension to the SIR model that allows nodes in state R to go to state S. This model, known as the SIRS model, accounts for the possibility that a person loses the acquired immunity over time.

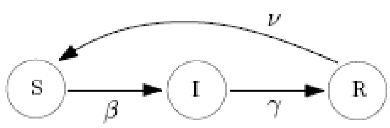

Figure 8.21 The state-transition diagram for the SIRS infection model.

Consider the state diagram in Figure 8.21. We can write out the set of differential equations as

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) + \nu R(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t) - \nu R(t).$$

Modify the Matlab code **www.network20q.com/hw/simulateSIR.m** for the numerical solution of the SIR model. Solve for $t = 1, 2, ..., 200$ (set the tspan vector in code accordingly) with the following parameters and initial conditions: $\beta = 1$, $\gamma = 1/3$, $\upsilon = 1/50$, $I(0) = 0.1$, $S(0) = 0.9$, and $R(0) = 0$. Describe and explain your observations.

8.4 *Information centrality* **

Consider a weighted, undirected, and connected graph with N nodes, where the weight for link (i, j) is x_{ij}. First construct a matrix **A** where the diagonal entries $A_{ii} = 1 + \Sigma_j x_{ij}$, $A_{ij} = 1 - x_{ij}$ if nodes i and j are adjacent, and $A_{ij} = 1$ otherwise.

Now compute the inverse: $\mathbf{C} = \mathbf{A}^{-1}$. The following quantity is called the **information centrality** of node i:

$$C_I(i) = \frac{1}{C_{ii} + (T - 2R)/N},$$

where $T = \Sigma_i C_{ii}$ is the trace of matrix **C** and $R = \Sigma_j C_{ij}$ is (any) row sum of matrix **C**.

Can you think of why this metric is called information centrality?