Homework 4

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1 Exercise 1 (Exercise 7.3 in textbook)

1.1 Solution(a)

$$A = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1.2 Solution(b)

$$C = \begin{pmatrix} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 \end{pmatrix}$$

$$C_{78} = 2$$

$$C_{75} = 0$$

The physical interpretation is the number of common nodes pointing to both i and j. For node 7 and node 8, there are 2 common nodes pointing to them. For node 7 and node 5, there is zero common node.

1.3 Solution (c)

$$A^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} A^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

For A^3, every entry is 0. For A^m, where m = 1,2,..., each entry A^m_{ij} meansmatrix the number of shorted path with length m from node i to node j. For example, from node 1 to node 7, there are 2 shortest paths with length 2. Hence $A^2_{17} = 2$. However, since for each pair of node the shortest path is smaller or equal to 2, the entry for A^3 is zero.

2 Exercise 2(Exercise 8.1 in textbook)

2.1 Solution(a)

$Degree = \begin{pmatrix} 3 & 2 & 3 & 3 & 3 \end{pmatrix}$
$Closeness = \begin{pmatrix} 0.8 & 0.2 & 0.8 & 0.8 & 0.8 \end{pmatrix}$
$Eigenvectorcentrality = \begin{pmatrix} 0.45579856 & 0.31921209 & 0.49122245 & 0.49122245 & 0.45579856 \end{pmatrix}$ To compute the eignenvector centrality, firstly we wrote down the adjacency matrix A:

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

. After that we calculated A's eigenvalues:
$eigenvalues = \big(2.85577251e+00 \quad 2.17740968e+00 \quad 7.39629427e-17 \quad 3.21637174e-01 \quad 1.00000000e$
Since 2.85577251e+00 is the largest eigenvalue, its corresponding eigenvector $\big(0.45579856 \quad 0.31921209 \quad 0.491$
is the eigenvector centrality.

2.2 Solution(b)

Compute the node betweenness between node 2 and node3.
Node 2: 1+1/3+3=13/3
Node 3: 1+1/3+1+2=13/3

2.3 Solution(c)

Compute the link betweenness(3,4) and (2,5).
(3,4)=1
(2,5)=1/3+1/2+1/2+1=7/3

3 Exercise 3(Exercise 8.2 in textbook)

3.1 Solution(a)

Run the contagion model with node 1 initialized at state-1 and the other nodes initialized to state-0. The result is :[0 1 -1 -1 -1 -1 1 2].
Here -1 means healthy, 0 means node 1 is infected initially, 1 means both node 2 and 7 are infected after the first round, 2 means node 8 is infected in the second round.

3.2 Solution(b)

Run the contagion model with node 3 initialized at state-1 and the other nodes initialized to state-0. The result is :[1 2 0 3 3 4 2 3].
Here 0 means node 3 is infected initially, 1 means both node 1 is infected after the first round, 2 means both node 2 and node 7 are infected after the second round.3 means both node 4,5,8 are infected after the third round. 4 means node 6 is infected after the fourth round.

3.3 Solution(c)

For section(b), when node 1 is initialized to state-1, node 3,4,5,6 forms a cluster of density 0.75, which is higher than p=0.3. This prevents complete flipping. However, in section(c), when node 3 is initialized to state-1, there is no cluster of density higher than 0.7. Therefore, complete flipping happened for this case.

4 Exercise 4(Exercise 8.3 in textbook)

4.1 Solution

After 200 iterations, S(t), I(t), R(t) converge to $\begin{pmatrix}0.3334 & 0.0364 & 0.6301\end{pmatrix}$.

5 Exercise 5(Exercise 8.4 in textbook)

One number is 10 more than another. The sum of twice the smaller plus three times the larger,is 55. What are the two numbers?

5.1 Solution