Research Centre

Process and Product-Engineering (PPE)

Evolution Strategies for Constrained Optimization

Constrained Optimization

The research field of Constrained Optimization is concerned with searching for the optimal parameters of a complex system or a target function with respect to a restricted parameter range.

The mathematical description of a general constrained optimization problem reads

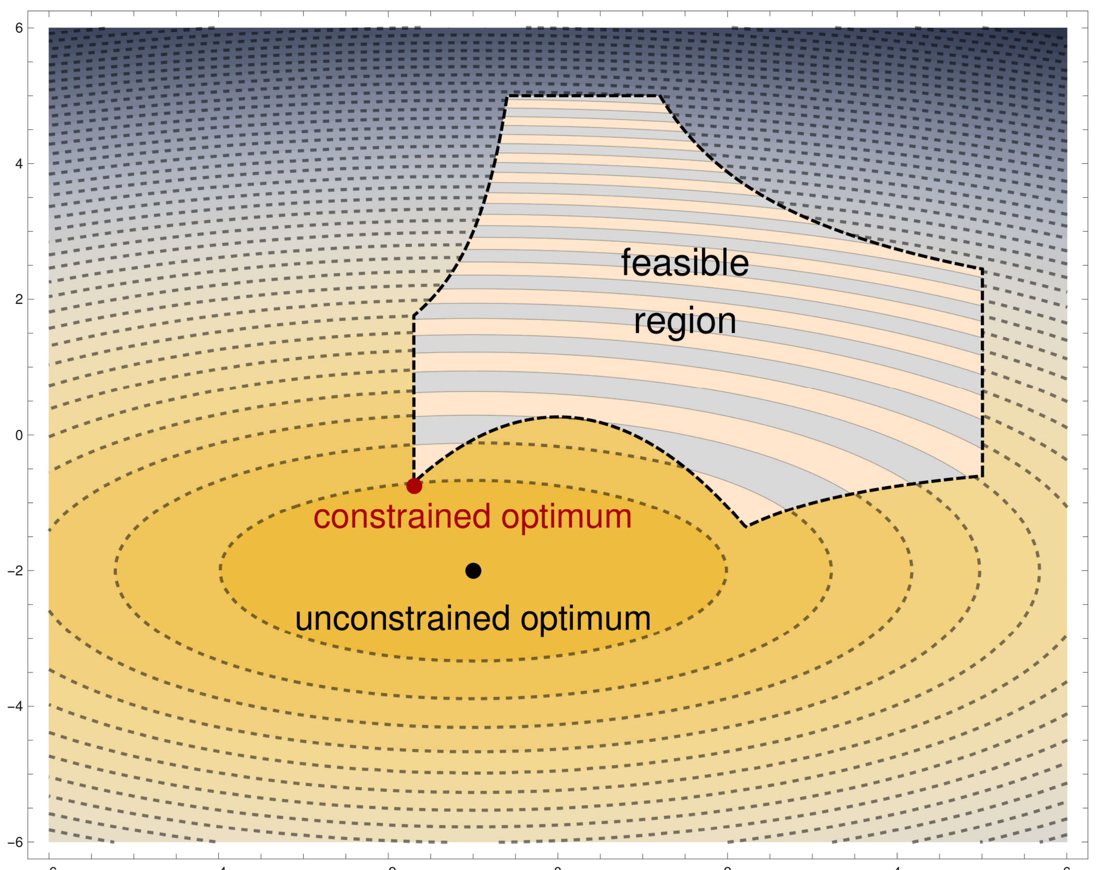
$$\begin{aligned} \min\ & F(\mathbf{y}) \\ \text{s.t.}\ & g(\mathbf{y}) \leq 0 \\ & h(\mathbf{y}) = 0 \\ & \mathbf{y} \in \mathbb{R}^N. \end{aligned}$$

The objective function $F(\mathbf{y})$ usually represents a quantity that either is to be minimized (cost or energy functions) or maximized (profit or benefit functions) over a feasible set M of admissible search space parameter vectors. The feasible set M is determined by the problem-specific constraint functions $g(\mathbf{y})$, and $h(\mathbf{y})$. The constraints formulate necessary conditions on the parameter vector y and thereby limit the feasible parameter range, e.g. with respect to resource availability, physical limitations, or structural dependencies.

Optimization problems under constraints can be found in all scientific disciplines that deal with unknown parameters. Within the Research Centre for Process and Product Engineering (PPE) applications, among others, involve

- Financial Mathematics (Portfolio Optimization),
- Operations Research (Logistics),
- Statistics, and
- Engineering (Design Automation).

Frequently, the derivation of analytical solutions to these optimization problems is limited or, in fact, rarely possible. In such situations, efficient and effective numerical methods are required to provide reasonable solutions.



2D-contour lines of a real-valued, quadratic objective function with non-linearly constrained parameter space

Evolution Strategies

Evolution strategies (ESs), especially versions of the so-called covariance matrix adaptation ES (CMA-ES), are arguably the currently best-performing general purpose direct search methods for unconstrained optimization. These strategies mimic the principles of Darwinian Evolution to generate onward improvement in a population of candidate solutions.

However, up until now, the success of these direct search strategies is rather restricted to the unconstrained case. That is, the incorporation of equality $h(\mathbf{y})$ and inequality constraints $g(\mathbf{y})$ in the design of ESs is still in an infant state when compared to other classes of Evolutionary Algorithms such as Differential Evolution or Genetic Algorithms.

Project Goals

It is the goal of this project to foster the development of ESs for constrained optimization on a theoretically-grounded basis. This is accomplished by connecting the analysis of direct search strategies with theoretically motivated algorithm design, and the evaluation of the strategies developed.

Based on the knowledge gained through this research, a deeper understanding of the working principles of direct search strategies in constrained search spaces is expected. This will not only lead to better performing ESs, but also to general design principles for Evolutionary Algorithms.

Related Publications

i. P. Spettel, H. Beyer and M. Hellwig, "A Covariance Matrix Self-Adaptation Evolution Strategy for Optimization under Linear Constraints", in IEEE Transactions on Evolutionary Computation, IEEE 2018.

ii. Michael Hellwig, Hans-Georg Beyer, Benchmarking evolutionary algorithms for single objective real-valued constrained optimization – A critical review, Swarm and Evolutionary Computation, Elsevier 2018.

iii. Patrick Spettel, Hans-Georg Beyer, Analysis of the (1,λ)-σ-Self-Adaptation Evolution Strategy with repair by projection applied to a conically constrained problem, Theoretical Computer Science, Elsevier 2018.

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