AU311, Pattern Recognition Tutorial (Fall 2019) **Homework: 1. Linear Regression**

1. Linear Regression

Lecturer: Xiaolin Huang `xiaolinhuang@sjtu.edu.cn`

Student: XXX `xxx@sjtu.edu.cn`

Problem 1

We know that $Y = Xw$ with $X \in R^{m \times n}$ could be solved if the rank of X is larger than the dimension of w. Now if $m < n$, compressive sensing is still possible to find the real solution, if it is sparse. To verify the recovery capability, you could consider the following situation:

$n = 40, n = 100$

the sparsity K, i.e., the number of non-zero components of $\bar{w}$ (the underlying signal), varies from 1 to 100.

Here, the elements of X follow a Gaussian distribution; the non-zero components of $\bar{w}$ are randomly selected uniformly; and for those non-components, their value follows a Gaussian distribution.

- Suppose the solution of the following problem is $\hat{w}$,

$$\min_{w} \quad \|w\|_1, \quad \text{s.t. } Y = Xw,$$

  then the ratio of *successful recovery* ($\|\hat{w} - \bar{w}\| < 10^{-2}$) could be calculated.

- Furthermore, if there is noise on observations, i.e., $Y = Xw + n$ with n being a Gaussian noise with zero-mean (the signal-to-noise ratio is 20), then we need the following problem, of which the solution is denoted by $\hat{w}$,

$$\min_{w} \quad \|w\|_1 + \frac{\lambda}{2}\|Y - Xw\|_2^2,$$

  and the *recovery accuracy* could be measured by $\|\hat{w} - \bar{w}\|_2 / \|\bar{w}\|_2$

You are required to report i) Matlab code; ii) the ratio of successful recovery for noise-free case; iii) the recovery accuracy for noise-corrupted case (you need to choose a good λ by cross-validation).