Θέμα 1ο

[redacted]

(α) $y' = (1-y)(x+1) \Rightarrow$

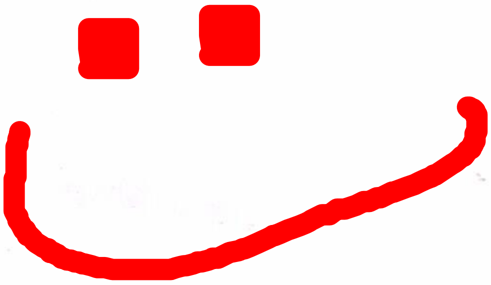
$\int \frac{dy}{1-y} = \int x+1\, dx \Rightarrow -\ln|1-y| = \frac{x^2}{x} + c \xrightarrow{y(0)=2}$

$-\ln|1-2| = c \Rightarrow \boxed{c=0}$

Οπότε έχουμε:

$-\ln|1-y| = \frac{x^2}{2} + x \Rightarrow \ln|1-y| = -\frac{x^2}{2} - x \Rightarrow$

$1-y = e^{-\frac{x^2}{2}-x} \Rightarrow y = 1-e^{-\frac{1}{2}x^2-x}$.



[redacted]

(b) $xy' = y - \sqrt{x^2 - y^2}$

Θέτω $y = vx$, $y' = v'x + v$

$x(v'x+v) = vx - \sqrt{x^2 - v^2x^2} = vx - x\sqrt{1-v^2}$

$v'x + v = v - \sqrt{1-v^2} \Rightarrow v'x = -\sqrt{1-v^2} \Rightarrow$

$\int \frac{dv}{\sqrt{1-v^2}} = -\int \frac{dx}{x} \Rightarrow$ τοξημ$v = -\ln x + c \Rightarrow$

$v =$ ημ$(-\ln x + c) \Rightarrow$

$y = vx = x$ ημ$(-\ln x + c)$.