#### ORIGINAL ARTICLE

# Crowded and Expensive: Density Shift as a Measure of Demand in Large US Apartment Markets

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# **Abstract**

We develop a density-based demand index for rental housing and show how it segments and forecasts market behavior. Drawing on annual populationperunit changes, we derive supply and demand curves for the 100 largest U.S. metropolitan areas (2010-2023) and compute each markets implied clearing rent and inventory. By comparing these benchmarks with the actual rents and units, we classify each market-year as over- or undersupplied and over- or under-priced. These four regimes strongly predict twelve-month rent growth: undersupplied/over-priced markets see the fastest real-rent gains, while oversupplied/under-priced markets lag. Our methodology offers a clear, actionable measure of consumer housing demand, illuminates household space-vs-price trade-offs, and quantifies the ripple effects of new supply.

#### KEYWORDS

density, supply and demand, multifamily rent-growth, apartment markets

# 1 INTRODUCTION

Housing shortages and affordability concerns have risen to the forefront of policy debates in major urban markets. In the United States, housing production has consistently lagged population growth for decades, contributing to an estimated national shortfall of 4.4 million housing units (Betancourt, 2022). Nearly half of U.S. renter households now spend over 30% of their income on housing (Bureau, 2022), and from 2000 to 2024, the consumer price index (CPI) for shelter exceeded the CPI for all other items by 30% (of St. Louis, 2024). At the same time, select high-growth regions have recently experienced rent declines due to a glut of new supply (Mott, 2024). This juxtaposition of chronic national undersupply with localized oversupply underscores a deeper issue: the lack of a reliable metric to measure consumer housing demand.

Traditional indicators of demand in multifamily real estate, such as occupancy rates and net absorption, are informative but fundamentally supply-constrained. Occupancy rates are naturally bounded at 100%, and absorption cannot exceed the rate of new deliveries (Mueller & Laposa, 1999). These constraints obscure excess demand: when all available and affordable units are occupied, latent demand becomes invisible to market participants and researchers alike (Gabriel & Nothaft, 2001; Sirmans, Sirmans, & Benjamin, 1991; Pyhrr, Cooper, & Wofford, 1999). This problem inhibits clear attribution of rent increases to supply versus demand dynamics (Pennington, 2021; Molloy, 2022). Moreover, ongoing academic debate persists around whether rising rents in constrained markets result more from supply-side limitations (Saiz, 2010) or from heightened demand for desirable locations (Davidoff, 2015). Without a transparent, consistent demand-side metric, attempts to assess equilibrium conditions remain incomplete.

In this paper, we introduce a novel empirical measure of rental housing demand: the *rental density index* (RDI), defined as the number of people per existing rental unit in a given metropolitan area. Unlike occupancy or absorption, RDI is not inherently bounded and therefore can reflect intensifying demand even in fully occupied markets. The conceptual foundation is straightforward: as rents rise, renters economize on space—whether by delaying household formation, taking on roommates, or crowding—thus increasing the ratio of people per rental unit. By observing shifts in RDI over time, we capture underlying demand pressures that traditional metrics obscure.

Abbreviations: MSA, Metropolitan Statistical Area; RDI, relative density index;

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Present trends support this reframing of demand, as density, rental prices, and rental preferences have all shifted meaningfully. For most of its history, the U.S. was less densely populated than its high-income peers, but in 1992 this changed. In that year, the US grew to 28 people per square kilometer, outpacing the High Income countries whose average was 23 people per square kilometer (Bank, 2025). The most recent figures report the US with a density of 36 and the High Income nations with an average density of 27. Over this same period, rental prices have outpaced both wages and non-shelter inflation (Feiveson et al., 2024; of St. Louis, 2024), with younger cohorts increasingly preferring rental housing (Mae, 2023).

Prior literature in urban economics and real estate has studied density extensively, often linking increased geographic density to higher wages, rents, and productivity due to agglomeration effects (Titman, Wang, & Yildirmaz, 2024; Liu, Pan, & Li, 2018). However, these studies generally define density in terms of spatial density: people per square mile or apartments per square mile. Our focus differs: we define density as the quotient of population over occupied rental units, which provides a clearer window into the number of people actively competing for housing. Unlike geographic density, this measure is directly responsive to shifts in demand, especially during periods of supply constraint or shocks.

Our framework builds on the standard assumption that, all else equal, households derive higher utility from larger living spaces (Muth, 1969; Molloy, 2022). In high-rent markets, however, the extra price per square foot may eventually exceed the marginal benefit of additional space. To economize, renters may respond by sharing units (adding roommates), downsizing to smaller apartments, or relocating to less expensive regions. Conversely, when real rents fall or new supply comes online, the marginal cost of extra space might drop below its marginal utility, enticing households to spread out, move to larger units, or reduce unitshare. The aggregate effect of these choices combined with the new supply additions or subtractions determines the direction the Rental Density Index change. Because the RDI directly captures these spaceconsumption margins, yearoveryear changes in RDI proxy for shifts in aggregate housing demand at prevailing prices.



**FIGURE 1** Relationship between annual change in RentalDensityIndex (horizontal) and annual real rent growth (vertical) across the 100 largest MSAs ină2024. Slopeă=ă0.41(p<0.01).

Focusing on a panel of the 100 largest metropolitan statistical areas (MSA), their population, rental unit count, and change in rent, we estimate implied demand of each MSA at each of 10 years from 2013 to 2023. We plot this against rent to derive a demand curve. We then take the observed supply growth of each MSA and plot it against rent to derive a supply curve. The intersection of those two curves is the derived point of supply and rent equilibrium. Comparing a market's actual rent to the market's derived rent suggests over or underpricing. Comparing a market's actual supply growth to the market's derived supply growth provides evidence of over or undersupply. We show that by segmenting markets as over or undersupplied and over or underpriced each year, we can effectively forecast the market's next-year rent growth.

This empirical strategy reveals consistent rent growth differences across market classifications. We find that markets classified as both overpriced and undersupplied experience significantly higher next-year rent growth than those that are underpriced and oversupplied. Specifically, markets in the former category averaged next-year relative real rent growth approximately 1.25% higher than those in the latter category. These findings suggest that density-based classifications have predictive power and reflect latent demand more accurately than traditional indicators alone.

Our paper makes two principal contributions. First, we introduce the Rental Density Index (RDI), a novel, supplyunbounded metric of housing demand that can be computed at scale from readily available population and unitstock data. Second, we demonstrate RDIs empirical value: it not only uncovers market disequilibrium but also delivers outofsample forecasts of twelvemonth rent growth with statistically significant accuracy.

These findings carry immediate applications for all housingmarket stakeholders. For policymakers, RDI functions as an early-warning gauge of emerging shortages, enabling targeted zoning reforms, calibrated subsidy programs, or expedited permitting in precisely those submarkets under the greatest pressureand subsequently measuring the efficacy of those measures. Institutional investors and residential developers can embed RDI trends into feasibility and risk models, avoiding the twin hazards of overbuilding in cooling metros or underbuilding in tightening ones. Finally, RDI sharpens the housing affordability debate by identifying the locales where rent increases are most likely to outstrip income growth, thereby guiding tenantprotection policies, rentassistance programs, and other affordability interventions to the communities that need them most.

In the next section we discuss research on other measures of demand before presenting details on our proposed variable. The Data and Descriptive Statistics section describes and analyzes the density data we used, while the Emperical Analysis section presents evidence and illustrates statistical tests performed to evaluate the validity of the classifications. The final sections discuss the results of our tests before concluding with implications and further research suggestions.

# 2 BACKGROUND AND LITERATURE REVIEW

The multifamily housing market has long relied on a narrow set of demand indicators, notably occupancy rates and net absorption. These indicators, while intuitively appealing and widely used by practitioners, are fundamentally constrained by the available stock of housing units. Occupancy rates cannot exceed 100%, and net absorption can never exceed new supply (Mueller & Laposa, 1999; Gabriel & Nothaft, 2001). Consequently, they fail to capture periods of latent demand where tenants cohabitate. (Sirmans et al., 1991; Pyhrr et al., 1999). Occupancy in particular has become an especially measure of demand since the advent of algorithmic pricing systems. So-called revenue management systems adjust rent pricing with the goal of keeping occupancy very near 96%, (CalderWang & Kim, 2024) the result of which is a variable with little signal.

This measurement limitation is consequential. Numerous empirical studies find that real rent growth often occurs in periods of high occupancy, yet they typically ascribe this to supply constraints rather than to excess demand (Goodman, 1992; Wheaton, 1991). While such studies validate the predictive value of these metrics, their theoretical bounds limit their explanatory reach, particularly when trying to assess equilibrium or derive true demand elasticities (Pennington, 2021; Molloy, 2022).

Alternative approaches to measuring demand have included econometric estimates of demand elasticity (Green, Malpezzi, & Mayo, 2002), consumer preference surveys (Malpezzi & Green, 1996), and utility-based choice models (Rosenthal, 1997). However, these methods either lack the spatial and temporal granularity needed for policy or investment use, or they are not publicly available in standardized formats.

The broader urban economics literature has focused on density as a related but distinct construct. Seminal work by (Glaeser et al., 2001) and (Duranton & Puga, 2004) positions geographic density—typically measured as people or housing units per square mile—as a proxy for agglomeration benefits. These studies find that higher density correlates with increased productivity, innovation, and wages. However, they stop short of using density as a direct measure of housing demand.

More recent research has explored density in the context of housing affordability. (Ahlfeldt & Pietrostefani, 2019) and (Albouy et al., 2015) argue that densification can both alleviate and exacerbate affordability issues, depending on its implementation. For instance, densification may increase supply and reduce rents in the long term but may also create localized price pressures or quality-of-life tradeoffs that drive demand elsewhere.

Our work contributes to this literature by redefining density as *people per rental unit*, rather than per geographic area. We call this the Rental Density Index (RDI). This formulation allows demand to exceed supply in a measurable way: if population grows faster than units, density rises. If renters prefer space and are observed to cohabitate only at higher prices, then shifts in RDI reveal the slope of the underlying demand curve (Muth, 1969; Molloy, 2022).

Unlike geographic density, which may be influenced by zoning and land use policy, RDI responds directly to demographic pressures and consumer decisions. Its changes over time— $\Delta$ RDI—can be interpreted as demand shocks, analogous to shifts in labor force participation or consumption behavior in macroeconomic models. By linking  $\Delta$ RDI to rent outcomes, we recover a market-clearing framework that identifies over- and under-supplied markets and projects likely rent growth trajectories.

This approach also aligns with emerging calls in the real estate literature to develop forward-looking, demand-side indicators that complement the traditional focus on supply elasticity (Glaeser & Gyourko, 2019). In contrast to existing studies, our method uses widely available data—population, rental unit inventory, and rent—to produce a scalable, repeatable measure of housing demand at the metro level.

In sum, our contribution lies in reinterpreting a well-studied spatial metric—density—through the lens of consumer housing decisions. By focusing on the population-to-unit ratio rather than geographic dispersion, we provide a new empirical tool to better understand and forecast real estate market dynamics.

# 3 DATA AND DESCRIPTIVE STATISTICS

Our data are sourced primarily from Costar and supplemented with inflation measures from the U.S. Bureau of Labor Statistics. We restrict our analysis to the 100 largest U.S. metropolitan statistical areas (MSAs) by multifamily inventory as of 2001, ensuring robust sample representation and consistency over time.

To normalize pricing data across time, we convert all nominal rent figures into real terms using the Consumer Price Index for All Urban Consumers (CPI-U), all-items, provided by the BLS. Although MSA-level inflation indices excluding rent would be ideal, such data are unavailable at a consistent and granular level for our study period. Therefore, real effective rent per square foot and rent growth metrics are adjusted using national CPI.

We convert the rent growth figures to nominal as well, by subtracting the annual growth in CPI. Finally, we standardize the figures by subtracting the year's median rent growth. We do this to remove the impact of national macroeconomic shocks (i.e., COVID-19). This also serves to stationarize the data and remove the autoregressive tendencies of timeseries data.

Our novel demand variable, the Rental Density Index (RDI), is calculated by dividing population by rental inventory at the MSA level. We compute its year-over-year percentage change— $\Delta$ RDI—to reflect demand dynamics more accurately. This measure avoids the bounded structure of traditional demand indicators like occupancy and absorption, which cap at 100%.

Other variables include supply growth, computed as the ratio of delivered units to the previous year's inventory, and lagged variables prior-year rent growth.

Importantly, no records were discarded during preprocessing. Our dataset retains all 100 MSAs over all years between 2000 and 2024. Some records are extreme, such as post-Katrina New Orleans, but we preserve fidelity to real-world dynamics by keeping them.

TABLE 1	Ke	v variables	used in	the em	pirical	analy	sis

Variable	Description		
CPI	Consumer Price Index for All Urban Consumers: All Items in U.S. City Average		
real_rentpsf	Costar's Rent Per Square Foot divided by indexed CPI		
real_rent_growth	Costar's Effective Rent Growth 12M less Annual percent change in CPI		
relative_real_rent_growth	real_rent_growth less that year's median real_rent_growth		
inventory	Existing multifamily rental stock		
delivered	Units delivered in the current year		
pop	Total MSA population (Costar/Moodys)		
RDI	Population divided by rental inventory		
delta_RDI	Yearoveryear percentage change in RDI		
supply_growth	Delivered units as a % of prior year's inventory		

# 3.1 Variable Construction

The key outcome variable in this study is the *Rental Density Index* (RDI), calculated as the total population divided by the number of rental housing units in a given MSA-year:

$$RDI_{it} = \frac{Population_{it}}{Rental\ Units_{it}}.$$

Within the 100 MSAs in the study, this metric ranges from 6.8 to 64.8. It reflects the number of people per rental unit.

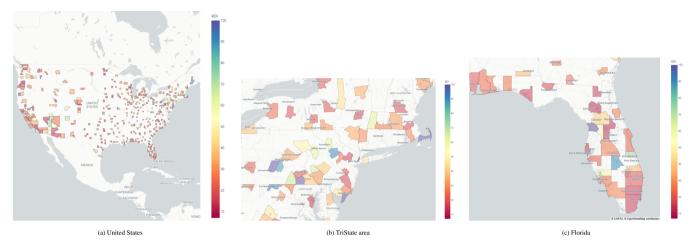


FIGURE 2 RentalDensityIndex (RDI) choropleths at three spatial scales.

While the RDI itself is useful for identifying how tight a market is at a given point in time, its absolute level is shaped by long-run demographic and structural trends such as rentership rates and changes in household formation. Accordingly, we focus on the *year-over-year change in RDI*:

$$\Delta RDI_{it} = RDI_{it} - RDI_{it-1}$$
.

The RDI and can move sharply in times of natural disasters, as shown by the figures in table 2, but so can real rent growth.

TABLE 2 Extreme observations in RDI and real rent growth

Category	Year	MSA	Population	Inventory	RDI	Real rent growth
Highest RDI	2001	Long Island-NY	2,785,876	43,235	64.435	-0.0079
Lowest RDI	2024	Fargo-ND	264,314	38,634	6.841	-0.0128
Highest real rent growth	2006	NewăOrleans-LA	1,055,155	57,719	18.281	0.2809
Lowest real rent growth	2005	NewăOrleans-LA	1,300,912	57,679	22.554	-0.1915

The change in RDI ( $\Delta$ RDI) offers several advantages. First, it mitigates issues of nonstationarity in the level RDI, enabling cross-market comparison. Second, it reduces the impact of varying rentership percentages across cities. Most importantly,  $\Delta$ RDI captures demand pressure on housing stock: when it rises, it indicates more people are consolidating into fewer rental units, signaling tightening demand. When it falls, it implies renters are spreading out and absorbing more space, suggesting slack.

Theoretically, under the assumption that renters prefer more space to less, the change in RDI reveals when prices are sufficiently high to induce cohabitation and when price relief allows individuals to live separately. Thus,  $\Delta$ RDI, especially when combined with rent growth, helps identify periods of excess demand or slack and allows us to trace out implied demand curves. As we show later, this approach allows us to locate the intersection between demand and supply curves using observable outcomes.

# 3.2 | Summary Statistics

Table 3 provides descriptive statistics for the key variables across the full panel. On average, the RDI across all MSAs and years is approximately 2.31 persons per rental unit, with a standard deviation of 0.35. The average real rent per square foot is \$1.22, and the average annual growth in rental supply is 1.9%. The missing records exist only in the beginning or ending years as the prior period and next-period values do not exist.

TABLE 3 Summary Statistics (2000–2024)

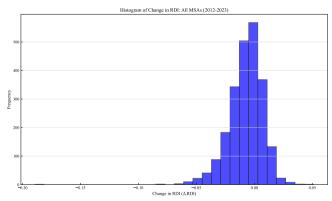
Variable	Mean	Std. Dev.	Min	25thăPct.	Median	75thăPct.	Max	Missing
Population	2,012,976	2,169,231	175,653	758,060	1,224,731	2,422,333	14,849,020	0
Inventory	133,435	190,354	20,783	35,735	66,261	153,774	1,572,425	0
Supply growth	0.0169	0.0152	0.0173	0.0057	0.0059	0.0239	0.1061	100
RDI	18.259	6.852	6.841	13.737	17.067	20.967	64.795	0
RDI growth	0.0078	0.0151	0.1895	0.0163	0.0059	0.0022	0.0512	100
Real rent (\$/sqft)	0.9489	0.3551	0.5584	0.7164	0.8373	1.0436	2.9110	0
Real rent growth	0.0035	0.0300	0.1915	0.0215	0.0046	0.0114	0.2809	100

# 3.3 Coverage and Representativeness

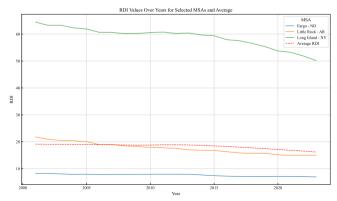
The sample includes approximately 1,300 MSA-year observations, covering a mix of large coastal cities, fast-growing Sun Belt metros, and slower-growing Midwestern regions. Together, these MSAs account for over 16 million of the 22 million U.S. apartment units, providing a representative snapshot of national rental dynamics.

# 3.4 | Preliminary Observations

Figure 3 traces the national Rental Density Index (RDI) from 2000 to 2024. Contrary to conventional wisdom about an acute housing shortage, we observe a steady decline in people per unit (from roughly 16 people-per-unit in 2000 to 13 people-per-unit in 2024). A companion histogram of RDI is stationary but skewed slightly negative, confirming that most years see more units added per new person than vice versa.







(b) A line graph showing the RDI values (population divided by apartment units) for the MSAs that were the min, max, and median in year 2001

 $FIGURE\ 3\quad \hbox{Histogram of Change in RDI, and Line Graph of RDI}$ 

This pattern poses a puzzle: if supply growth has outstripped population growth, why is everyone still worried about shortages? Occupancy simply tells you *that* units are full–it cannot distinguish whether they are full by choice (everyone prefers roommates) or by necessity (everyone must share).

By contrast, year-over-year changes in RDI isolate the relative pace of new households versus new units. When rents rise faster than new construction can absorb, RDI spikes, revealing latent demand pressure. When deliveries overwhelm demand, RDI falls, signaling surplus. In the next section we exploit that property, regressing RDI on rent and supply growth to recover the underlying demand curve.

# 4 | EMPERICAL FRAMEWORK

Our objective is to evaluate  $\Delta$ RDI as a robust measure of housing demand by deriving supply and demand curves for each metropolitan area and year. For each of the 100 largest MSAs, and for each year from 2010 through 2022, we construct demand and supply curves using observed data on real rent per square foot and changes in RDI and supply.

# 4.1 Constructing the Demand and Supply Curves

We estimate reduced-form linear models to approximate demand and supply responses to price, using ten years of historical data for each MSA:

$$\Delta RDI_{it} = \alpha_i + \beta_i p_{it} + \epsilon_{it}$$
 (Demand)  
SupplyGrowth<sub>it</sub> =  $\gamma_i + \delta_i p_{it} + \nu_{it}$  (Supply)

The coefficient  $\beta_i$  reflects how changes in rent induce renters to economize on space by cohabitating, while  $\delta_i$  captures developers' responsiveness to rent signals via new supply. We estimate these relationships using ordinary least squares (OLS) for each market-year observation, pooling data from the prior ten years.

The intersection of these two curves yields the market's implied equilibrium rent  $p^*$  and equilibrium inventory growth  $q^*$ . We compare these values to observed market outcomes (p,q) in that year. If  $p^* > p$ , we infer that the market is underpriced; if  $p^* < p$ , it is overpriced. Likewise, if  $q^* > q$ , the market is deemed undersupplied; otherwise, oversupplied. The following pseudocode outlines the segmentation logic applied to each market-year observation:

By way of example, we can look at a supply shock as experienced by Phoenix in 2022 (figure 4). The supply line is constructed by regressing the change in supply and the real rent per-square-foot. The demand line is constructed by regressing the change in RDI and the real rent per-square-foot. Both use the data from 2012-2021. Their intersection (red circle) gives the implied 2021 marketclearing rent (\$0.76/sf) and inventory growth (0.74%). But the observed values are in 2022 (blue circle) were both greater greater than derived, so this market would be classified as overpriced and oversupplied, characteristics of a bubble. It is more overpriced than it is oversupplied, and so it returns to equilibrium through a price decline in the years that follow.

These states correspond to economic intuition. Overpriced and undersupplied markets—what we refer to as *landlord-favorable* markets—are expected to exhibit stronger rent growth in the following year as prices can adjust faster than supply can be added. Conversely, oversupplied and underpriced markets—*renter-favorable* markets—should experience weaker or negative rent growth for the same reason: prices can adjust faster than more people can arrive. The final two situations are partial disequilibria and their next year depends on which factor is more out of line with observed values. Overpriced and oversupplied markets are similar to late-cycle bubbles, while underpriced undersupplied markets are are similar to early-cycle growth.

TABLE 4 Hypothesized Next-Year Rent Growth by Market Classification

		Pricing					
		Overpriced	Underpriced				
Supply _	Oversupplied	Neutral	Renter-Favorable				
	Undersupplied	Landlord-Favorable	Neutral or				

#### Algorithm 1 Segment markets into pricesupply regimes

```
for y \leftarrow 2010 to 2022 do
   for each market m in the top 100 MSAs do
     estimate demand curve: \Delta RDI \leftarrow f(rent_{psf})
     estimate supply curve: inventory growth \leftarrow g(\text{rent}_{psf})
      solve for equilibrium (q^*, p^*)
     obtain actual values (q,p)
     if q^* > q then
         if p^* > p then
            assign (m, y) to UndersuppliedUnderpriced
            assign (m, y) to UndersuppliedOverpriced
         end if
     else
         if p^* > p then
            assign (m, y) to OversuppliedUnderpriced
         else
            assign (m, y) to OversuppliedOverpriced
         end if
      end if
   end for
end for
```

We frame this as a partial equilibrium exercise, in which observed price and quantity may deviate from the implied marketclearing levels due to frictions such as construction lags, policy constraints, or imperfect price discovery (Wheaton, 1991; Glaeser & Gyourko, 2019).

This segmentation yields market classifications that form the basis for our predictive rent growth analysis in the following section.

# 5 | RESULTS

This section presents empirical findings from our market classification framework. Using the derived supply and demand curves, we compute equilibrium prices  $(p^*)$  and quantities  $(q^*)$  for each of the 100 largest MSAs from 2010 to 2022. By comparing  $(p^*, q^*)$  to the observed market outcomes (p, q), we segment markets into four states based on whether they are overpriced and/or oversupplied.

# 5.1 | Event Study: Rent Growth Before and After Regime Entry

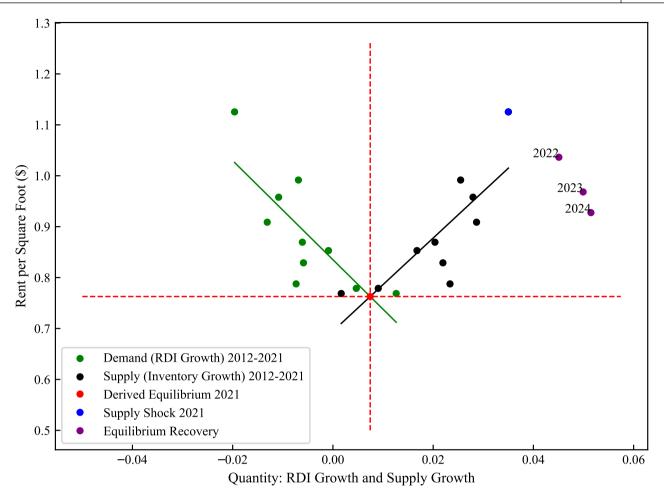
To complement our classification-based forecasting approach, we implement an event study that traces rent dynamics around the time a market enters the *Overpriced & Oversupplied* regime. This regimecharacterized by both above-equilibrium rents and excessive new supplyis hypothesized to precede periods of slower rent growth or correction.

We define the event year t = 0 as the first year an MSA is classified as Overpriced & Oversupplied. We construct an event time index ranging from t = -2 (two years before the regime entry) to t = +2 (two years after entry). For each event-time year, we compute the average real rent growth across all MSAs that experienced the regime switch.

This structure ensures that the rent growth observed at t+1 and t+2 is entirely out-of-sample with respect to the classification year t=0. To contextualize the results, we compare these patterns to MSAs that never entered the Overpriced & Oversupplied regime during the study period, as well as to those that experienced alternative classification transitions.

Figure ?? displays average real rent growth by event year for each group.

Density Shift as a Measure of Demand



**FIGURE 4** Recovering equilibrium for a high growth market. Black circles: supply growth vs real rent 20122022, Phoenix MSA. Green circles: RentalDensityIndex growth (RDI) vs real rent, same years.

The results show that real rent growth slows meaningfully following entry into the Overpriced & Oversupplied regime. While pre-entry years may exhibit moderate rent growth, the post-entry period is characterized by significantly lower average rent changes. This pattern supports the hypothesis that this classification regime is forward-looking and useful in anticipating market cooling or correction.

# 5.2 | Group Differences in Rent Growth

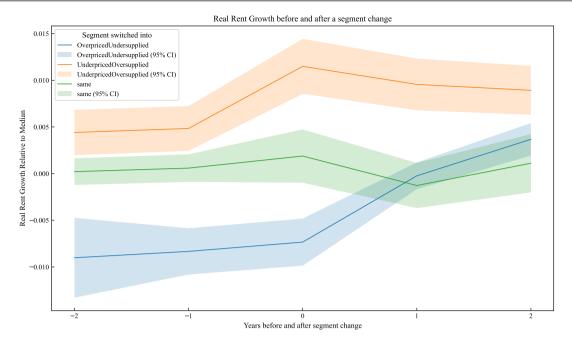
We assess whether these classifications are associated with meaningful differences in future rent performance. Table ?? reports average next-year real rent growth and relative rent growth (defined as deviation from the within-year median) across the four groups formed by the boolean indicators 'overpriced' and 'oversupplied'.

To aid interpretation, Table ?? maps the binary groups to intuitive market types.

These results support our core hypothesis: landlord-favorable markets exhibit the strongest subsequent rent growth, while renter-favorable markets experience the weakest. Balanced and bubble-like markets show intermediate outcomes.

# 5.3 | Statistical Tests of Group Differences

We conduct a two-way ANOVA on **relative rent growth** using 'overpriced' and 'oversupplied' as independent variables.



**FIGURE 4** Event study of average real rent growth before and after entry into the Overpriced & Oversupplied regime (t = 0).

**TABLE** 5 Mean Next-Year Real-Rent Growth and Standard Errors

Group	Mean (bps)	SE (bps)	$n_{\rm obs}$
Balanced	25	11	173
Landlord-favourable	94***	30	63
Price-Bubble	14	15	684
Renter-favourable	-23*	12	380

Note: Significance indicators: \*\*\* p < 0.01, \*\* p < 0.05, \* p < 0.10.

TABLE 6 Two-Way ANOVA

Source	Sum Sq	df	F	PR(>F)
C(overpriced) C(oversupplied) Residual	0.005178 0.006205 0.545538	1.0 1.0 1297.0	12.311490*** 14.751522*** —	0.000466 0.000129

Note: \*\*\* p < 0.01, \*\* p < 0.05, \* p < 0.10. Robust standard errors are used.

These patterns do not appear in the raw rent growth values, reinforcing that our framework is best applied in relative termsbenchmarking each MSA against its peers in a given year.

# 5.4 | Visual and Quantitative Robustness

Figure ?? shows the distribution of next-year rent growth across market types, highlighting the predictive spread between groups. The spread in median relative rent growth between landlord-favorable and renter-favorable markets is 117 basis points. This exceeds the cross-sectional standard deviation of rent growth in most years, reinforcing the economic magnitude of these classification effects.

TABLE 7 Tukey HSD Post-Hoc Test

Group1	Group2	Mean Diff	p-adj	Lower	Upper	Reject Null
Balanced	Landlord-favourable	0.0069	0.1012	-0.0009	0.0147	False
Balanced	Price-Bubble	-0.0012	0.9082	-0.0057	0.0033	False
Balanced	Renter-favourable	-0.0049	0.0468	-0.0097	-0.0000	True
Landlord-favourable	Price-Bubble	-0.0081	0.0150	-0.0150	-0.0011	True
Landlord-favourable	Renter-favourable	-0.0118	0.0001	-0.0190	-0.0046	True
Price-Bubble	Renter-favourable	-0.0037	0.0243	-0.0071	-0.0003	True

Note: The Tukey HSD Post-Hoc Test compares mean differences between groups at a family-wise error rate (FWER) of 0.05.

TABLE 7 Mean Rent Growth by Market State. All rent figures are real

Group (overpriced, oversupplied)	Relative Rent Growth (bps)	Next-Year Rent Growth (bps)
(False, False)	25	78
(False, True)	-23	64
(True, False)	94	113
(True, True)	14	55

TABLE 7 Market Classification by Pricing and Supply

Overpriced	Oversupplied	Label
False	False	Balanced Market
False	True	Renter-Favorable
True	False	Landlord-Favorable
True	True	Price Bubble

TABLE 7 Two-Way ANOVA Results

Source	Sum Sq	df	F	p-value
Overpriced	0.0052	1	12.38	0.0004
Oversupplied	0.0063	1	14.97	0.0001
Residual	0.5454	1297	_	-

TABLE 7 Tukey HSD Pairwise Comparisons

Group 1	Group 2	Mean Diff	p-adj	Lower	Upper	Reject
False-False	False-True	0.0067	0.1204	-0.0011	0.0144	No
False-False	True-False	-0.0050	0.0408	-0.0099	-0.0001	Yes
False-False	True-True	-0.0013	0.8912	-0.0058	0.0033	No
False-True	True-False	-0.0117	0.0002	-0.0188	-0.0046	Yes
False-True	True-True	-0.0079	0.0167	-0.0148	-0.0010	Yes
True-False	True-True	0.0038	0.0213	0.0004	0.0071	Yes

# 5.5 Distributional Evolution and Market Cycles

While the groupings are not evenly distributed across years, this reflects underlying market behavior rather than methodological bias. In early years such as 2011, few MSAs exhibit characteristics of overpriced and oversupplied markets, consistent with post-crisis housing conservatism. By contrast, between 2020 and 2022, the vast majority of MSAs fall into the Price Bubble category, despite negative or flat rent growth. This asymmetry suggests the framework captures real cyclical shifts, where high observed rents and heavy supply growth outpace renter willingness or ability to absorb space. The sharp contrast between high pricing and poor performance in these years further reinforces the frameworks value in signaling impending cooling phases or market corrections.

# 5.6 | Predictive Power and Caveats

We estimated a linear model using 2010–2022 data to forecast 2023 rent growth. The model included an interaction term between implied demand, supply growth, and contemporaneous rent growth. Despite this models simplicityrelying only on a single interaction variableit achieved an out-of-sample  $R^2$  of 0.31 when predicting next-year rent growth.

To evaluate economic significance, we compared the top 20 MSAs by predicted growth to the bottom 20. The top group saw next-year rent growth that exceeded the bottom group by 246 basis points. This suggests the implied demand signal can be used to rank markets by expected future performance, offering both interpretability and predictive power.

While this forecasting result is encouraging, we acknowledge a potential limitation: market pricing and supply choices may themselves reflect expectations about future rent growth. This opens the door to endogeneity. Although our model uses trailing 10-year trends to construct classifications, future work should explore this causal directionality more rigorously.

Together, these findings validate the informational value of market misalignment relative to derived equilibrium, offering a parsimonious and predictive classification of multifamily rent dynamics.

# 6 | DISCUSSION

sec6

Our findings reinforce the utility of rental density—measured as population per occupied rental unit—as a robust proxy for demand in multifamily housing markets. Traditional demand-side variables such as occupancy and absorption are bounded above by 100%, rendering them structurally incapable of expressing excess demand. By contrast, our proposed measure, the change in Rental Density Index ( $\Delta$ RDI), accommodates marginal and nonlinear shifts in tenant behavior, particularly around unit sharing or household formation.

The empirical evidence supports this theoretical proposition. When supply and demand curves were derived from historical  $\Delta$ RDI and supply growth data, we observed that the relative position of actual market conditions to the implied equilibrium—quantified as overpriced/underpriced and oversupplied/undersupplied—was significantly associated with next-year rent growth. Specifically, markets that were both overpriced and undersupplied experienced statistically higher rent growth than other combinations. Conversely, renter-favorable markets (underpriced and oversupplied) exhibited lower growth or even declines.

This segmentation has both explanatory and predictive power. Our simplified ANOVA model revealed statistically significant differences in next-year rent growth across these market states. Furthermore, we employed a univariate linear model using the interaction of implied demand, supply growth, and current rent growth to forecast future rent changes. The model yielded an  $R^2$  of 0.31 for 2023, indicating a moderate yet meaningful explanatory fit. The top 20 forecasted markets outperformed the bottom 20 by [PLACEHOLDER] percentage points in next-year rent growth.

While the number of MSAs in each state was not evenly distributed across years, this asymmetry reflects genuine market dynamics rather than bias or misclassification. For example, in periods of expansion, we naturally observe more overpriced or undersupplied markets, consistent with cyclical pressures on housing. Nonetheless, even in years with few markets in a given segment, those groupings consistently displayed expected behavior in rent trajectories.

These findings reinforce the value of  $\Delta RDI$  as a continuous, forward-looking measure of housing demand, especially when contrasted with backward-looking or constrained alternatives. They also suggest that equilibrium misalignment—in both price and quantity—can be quantified in a way that is both actionable for investors and meaningful for policymakers.

# 7 | CONCLUSION

sec7

This paper contributes to the literature on housing market dynamics by introducing the change in Rental Density Index ( $\Delta$ RDI) as a scalable, interpretable measure of demand. Unlike occupancy or absorption,  $\Delta$ RDI captures variation in consumer preference through household formation behavior—a critical but often overlooked channel of adjustment in rental markets.

By modeling supply and demand curves separately and estimating their intersection, we were able to derive implied market-clearing quantities and prices for each metropolitan area in our dataset. These derived values allowed for the segmentation of MSAs into four quadrants based on price and supply misalignment. Across the 100 largest U.S. markets from 2010 to 2023,

these quadrants were statistically associated with next-year rent growth in ways consistent with theory: undersupplied and overpriced markets performed best; oversupplied and underpriced markets performed worst.

This demand proxy also showed moderate predictive power when used in a linear forecasting framework, suggesting that it may serve as a leading indicator of rent pressure—particularly when combined with supply metrics. The ability to anticipate future rent dynamics is valuable to developers, institutional landlords, and policymakers seeking to manage affordability and stability in rental housing.

Future work should explore refinement of  $\Delta RDI$  to account for compositional changes in renter populations and household structures. Additional improvements might include the integration of MSA-specific CPI indices or disaggregated demand inputs, such as migration, income distributions, or age cohorts. Nonetheless, the present analysis establishes  $\Delta RDI$  as a conceptually valid and empirically useful measure of multifamily housing demand, with applications in forecasting, pricing, and supply planning.

#### **AUTHOR CONTRIBUTIONS**

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#### ACKNOWLEDGMENTS

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#### FINANCIAL DISCLOSURE

None reported.

# CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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#### SUPPORTING INFORMATION

Additional supporting information may be found in the online version of the article at the publishers website.

How to cite this article: Larriva M.. Density Shift as a Measure of Demand Real Estate Econ. 2021;00(00):1–18.

#### **APPENDIX**

#### A PROGRAM CODES APPEAR IN APPENDIX

Using the package listings you can add non-formatted text as you would do with \begin{verbatim} but its main aim is to include the source code of any programming language within your document.

Use \begin{lstlisting}...\end{lstlisting} for program codes without mathematics.

The listings package supports all the most common languages and it is highly customizable. If you just want to write code within your document, the package provides the lstlisting environment; the output will be in Computer Modern typewriter font. Refer to the below example:

#### Listing 1 Descriptive caption text

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{ do nothing }
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#### C EXAMPLE OF ANOTHER APPENDIX SECTION

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# **AUTHOR BIOGRAPHY**

