



Final Examination of
Discrete Mathematics BS - 103

الامتحان يقع في ورقة من صفحتين.

First Question (20- Marks)

Fill the circle with the appropriate signs "✓ for the circle A" or "✗ for the circle B"

- [1] The day of the week will it be after 100 days from Saturday is **Monday**. (.....)
- [2] If $R_2 = R \cup R^{-1}$, then R_2 should be **symmetric**. (.....)
- [3] If $x \bmod y = r$, then y **divides** $x - r$ $\frac{10}{3} \bmod 3 = 1$ (.....)
- [4] $0 \bmod 7 = 1$ (.....)
- [5] The functions $f(x) = [x]$ and $f(x) = |x|$ map from \mathbb{R} to \mathbb{Z} is **one to one**. (.....)
- [6] The **horizontal asymptotes** of the function $y = 1/x$ is the line $y = 1$. $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ (.....)
- [7] The **Big O** notation is used to give an upper bound of the running time of an algorithm. (.....)
- [8] If $f: \mathbb{R} \rightarrow \mathbb{R}^+$, $f(x) = \sin(x^2)$, then f is **one to one** function. (.....)
- [9] If $f: \mathbb{R}^+ \rightarrow \mathbb{R}$, $f(x) = \log(x)$, then f is **onto** function. (.....)
- [10] The **domain** of the function $f(x) = 2 \cos(x)$ is \mathbb{R} . (.....)
- [11] The **range** of the function $f(x) = 3 \sin(2x - 1) - 3$ is $[-3, 3]$. (.....)
- [12] The **vertical asymptotes** of the function $y = \frac{1}{x-1}$ is the **line** $x = 0$. (.....)
- [13] If $a, b \in D_f$, $a < b$, and $f(a) < f(b)$, then f is **increasing** function. (.....)
- [14] If $f(x) = x$ is the **identity function**, then $f^{-1}(x) = x$. (.....)
- [15] The infinite series $\frac{2}{10} + \frac{2}{10^2} + \dots + \frac{2}{10^n} + \dots$ is **divergent**. (.....)
- [16] $[x] = [x] - 1$ for all $x \in \mathbb{Z}^+$. (.....)
- [17] **Big O notation** is used to describe how closely a series approximates a given function. (.....)
- [18] The **general term** of the sequence: $-1, -3, -5, \dots$ is of the form: $(1 - 2n)$. (.....)
- [19] If the function $f(x)$ have an **inverse**, then $(f \circ f^{-1})(x)$ should equals $1/x$ (.....)
- [20] The **number of ways** for a number of two digits that can be formed from $\{1, 2, 3, 8\}$ without repeating is 16. (.....)

ملحوظة

السؤال الأول والثاني يتم إجابتة في الجزء اليمين في النموذج الإلكتروني للإجابة بنفس التسلسل من 1-30 في حين أن السؤال الثالث يتم إجابتة في كراسة الإجابة العادي.

Second Question (10- Marks)

Choose the correct answer where the first choice represents A and the second is B and so on...

- [21] The infinite series $\frac{2}{10^0} + \frac{2}{10^1} + \dots + \frac{2}{10^n} + \dots$ is {geometric, arithmetic}
- [22] The general term of the sequence: $-1, -3, -5, -7, \dots$ is { $1 - 2n, -2n, 1 - n$ }
- [23] $-10 \bmod 3 + 10 \bmod 3 = \dots$ {3, -3, 0}
- [24] The infinite series $3 + \frac{3}{4} + \dots + \frac{3}{4^n} + \dots$ is {divergent, convergent}
- [25] If $f(x): \mathcal{R} \rightarrow \mathcal{R}^+$: $x \rightarrow e^x$ the f is {Onto, Bijective}
- [26] $2[-2.5] + 3[2.5] = \dots$ {2, -2, 0}
- [27] The range of the function $f(x) = \log(x)$ is {R, R⁺, R⁻}
- [28] If $f(x) = x^2 - 3x$, $x \geq 1.5$, has an inverse, then $f^{-1}(0)$ equals {3, 0, 4}
- [29] If $f(x) = 2x + a$ have the inverse $f^{-1}(x) = (x + 3)/2$, then a equals... {-3, 3, 2}
- [30] The number of ways in which 2 persons can be selected from a group of 6 persons is: {30, 15, 20}

Third Question (50- Marks distributed as follows: 8; 7; 7; 7; 7; 7; 7).

- ❶ Use Taylor series to approximate the function $f(x) = \sqrt[3]{3x+1}$ to just three terms.
- ❷ Prove that: $\sinh^{-1}x = \ln(x + \sqrt{x^2 + 1})$.
- ❸ Show that $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where $a_0, a_1, \dots, a_n \in \mathcal{R}$ is $\mathcal{O}(x^n)$.
- ❹ Express the function $f(t)$ by the unit step function, where $f(t) = \begin{cases} t^2 + 7 & 0 \leq t < 1 \\ t^2 & 1 \leq t < 3 \\ 7 & t \geq 3 \end{cases}$.
- ❺ Find the horizontal and vertical asymptotes of the function and then sketch the graph, $f(x) = \frac{3x-9}{x-2}$.
Find the domain and the range of f .
- ❻ Use mathematical induction to prove that: $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$, $n = 1, 2, \dots$
- ❼ Give three different examples of odd functions and then give their plots.

(انتهت الأسئلة)

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$$\frac{1}{x^3} \quad \sqrt{x} \quad \frac{-1}{x}$$

$$f(-x) = \sqrt{-x} \quad f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$$