



**Final Examination of  
Discrete Mathematics BS - 103**

الإمتحان يقع فى ورقة من صفحتين.

**First Question (20- Marks)**

Fill the circle with the appropriate signs "✓ for the circle A" or "× for the circle B"

- [1] The day of the week will it be after 100 days from Saturday is **Monday**. (.....)
- [2] If  $R_2 = R \cup R^{-1}$ , then  $R_2$  should be **symmetric**. (.....)
- [3] If  $x \bmod y = r$ , then  $y$  **divides**  $x - r$ .  $10 \bmod 3 = 1$  (.....)
- [4]  $0 \bmod 7 = 1$  (.....)
- [5] The functions  $f(x) = [x]$  and  $f(x) = \lfloor x \rfloor$  map from  $\mathbb{R}$  to  $\mathbb{Z}$  is **one to one**. (.....)
- [6] The **horizontal** asymptotes of the function  $y = 1/x$  is the line  $y = 1$ .  $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$  (.....)
- [7] The **Big O** notation is used to give an upper bound of the running time of an algorithm. (.....)
- [8] If  $f: \mathbb{R} \rightarrow \mathbb{R}^+$ ,  $f(x) = \sin(x^2)$ , then  $f$  is **one to one** function. (.....)
- [9] If  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ ,  $f(x) = \text{Log}(x)$ , then  $f$  is **onto** function. (.....)
- [10] The **domain** of the function  $f(x) = 2 \cos(x)$  is  $\mathbb{R}$ . (.....)
- [11] The **range** of the function  $f(x) = 3 \sin(2x - 1) - 3$  is  $[-3, 3]$ . (.....)
- [12] The **vertical asymptotes** of the function  $y = \frac{1}{x-1}$  is the line  $x = 0$ . (.....)
- [13] If  $a, b \in D_f$ ,  $a < b$ , and  $f(a) < f(b)$ , then  $f$  is **increasing** function. (.....)
- [14] If  $f(x) = x$  is the **identity function**, then  $f^{-1}(x) = x$ . (.....)
- [15] The infinite series  $\frac{2}{10} + \frac{2}{10^2} + \dots + \frac{2}{10^n} + \dots$  is **divergent**. (.....)
- [16]  $[x] = [x] - 1$  for all  $x \in \mathbb{Z}^+$ . (.....)
- [17] **Big O** notation is used to describe how closely a series approximates a given function.  $-1 - 2n + 2 = -1 + -2(n-1)$  (.....)
- [18] The **general term** of the sequence: -1, -3, -5, ... is of the form:  $(1 - 2n)$ . (.....)
- [19] If the function  $f(x)$  have an **inverse**, then  $(f \circ f^{-1})(x)$  should equals  $1/x$ . (.....)
- [20] The **number of ways** for a number of two digits that can be formed from  $\{1, 2, 3, 8\}$  without repeating is 16. (.....)

**ملحوظة**

السؤال الأول والثاني يتم إجابتة فى الجزء اليمين فى النموذج الإلكتروني للإجابة بنفس التسلسل من 1-30 فى حين أن السؤال الثالث يتم إجابتة فى كراسة الإجابة العادية.



## Second Question (10- Marks)

Choose the correct answer where the first choice represents A and the second is B and so on...

- [21] The infinite series  $\frac{2}{10^0} + \frac{2}{10^1} + \dots + \frac{2}{10^n} + \dots$  is {geometric, arithmetic}
- [22] The general term of the sequence:  $-1, -3, -5, -7, \dots$  is { $1 - 2n, -2n, 1 - n$ }
- [23]  $-10 \bmod 3 + 10 \bmod 3 = \dots$  {3, -3, 0}
- [24] The infinite series  $3 + \frac{3}{4} + \dots + \frac{3}{4^n} + \dots$  is {divergent, convergent}
- [25] If  $f(x): \mathcal{R} \rightarrow \mathcal{R}^+: x \rightarrow e^x$  the  $f$  is {Onto, Bijective}
- [26]  $2[-2.5] + 3[2.5] = \dots$  {2, -2, 0}
- [27] The range of the function  $f(x) = \text{Log}(x)$  is { $\mathcal{R}, \mathcal{R}^+, \mathcal{R}^-$ }
- [28] If  $f(x) = x^2 - 3x, x \geq 1.5$ , has an inverse, then  $f^{-1}(0)$  equals {3, 0, 4}
- [29] If  $f(x) = 2x + a$  have the inverse  $f^{-1}(x) = (x + 3)/2$ , then  $a$  equals... {-3, 3, 2}
- [30] The number of ways in which 2 persons can be selected from a group of 6 persons is: {30, 15, 20}

## Third Question (50- Marks distributed as follows: 8; 7; 7; 7; 7; 7; 7).

- ① Use Taylor series to approximate the function  $f(x) = \sqrt[3]{3x+1}$  to just three terms.  $\infty = 0$
- ② Prove that:  $\sinh^{-1}x = \text{Ln}(x + \sqrt{x^2 + 1})$ .
- ③ Show that  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ , where  $a_0, a_1, \dots, a_n \in \mathcal{R}$  is  $\mathcal{O}(x^n)$ .
- ④ Express the function  $f(t)$  by the unit step function, where  $f(t) = \begin{cases} t^2 + 7 & 0 \leq t < 1 \\ t^2 & 1 \leq t < 3 \\ 7 & t \geq 3 \end{cases}$ .
- ⑤ Find the horizontal and vertical asymptotes of the function and then sketch the graph,  $f(x) = \frac{3x-9}{x-2}$ .  
Find the domain and the range of  $f$ .
- ⑥ Use mathematical induction to prove that:  $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, n = 1, 2, \dots$
- ⑦ Give three different examples of odd functions and then give their plots.

(انتهت الأسئلة)

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$$\frac{1}{x^3} \quad \sqrt{x} \quad \frac{-1}{x}$$

$$f(-x) = \sqrt{-x} \quad f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$$