Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date:20-11-2021; Time: **1.5- Hours**

Module (A) **Mid-Term Examination 2021-2022** **Module(A)**

Discrete Mathematics (BS-103)

First Question (10- Marks)

- **Choose the correct sign "✓ or ×" for the followings:**

[1]	$P \vee T$ is always true provided that p is true.	(...✓...)
[2]	The vertical asymptote line for the function $f(x) = \frac{1}{x-1}$ is $y = 1$	(...✗...)
[3]	$\overline{\bigcup_{n=1}^{N} A_n} = \{x \mid \forall n, x \in A_n\}$	(...✓...)
[4]	A *proof* is a clear explanation for the truth of a proposition	(...✓...)
[5]	$p \rightarrow q \equiv \bar{p} \rightarrow \bar{q}$	(...✗...)
[6]	$\overline{\bar{p} \vee F} = p$	(...✓...)
[7]	Union between the relation R with the diagonal relation Δ makes R reflexive relation.	(...✓...)
[8]	$\overline{P \wedge T} \equiv T$	(...✗...)
[9]	For the whole numbers greater than two, being odd is necessary to being prime.	(...✓...)
[10]	Let $P(x, y, z): xy < x + z + 1$, then $p(x, x, x)$ is always true $\forall x \in z^+$	(...✗...)
[11]	$p \rightarrow (p \vee q) \equiv T$	(...✓...)
[12]	If p is $4 \geq 2$ and q is "$1 \leq 2$" then $p \oplus q$ is true	(...✗...)
[13]	The path abcd of some relation is of length 4	(...✗...)
[14]	If $R: X \rightarrow X, X = \{a, b, c, d\}, R = \{(a, b), (a, c), (b, a), (b, d), (a, d), (b, c),$ then R is transitive	(...✗...)
[15]	R^{∞} is always transitive relation.	(...✓...)

- **Choose the correct answer for the following statements:**

[1]	$p: \sin\theta = 0.5$ is ....condition for $q: \theta = 30^{\circ}$	(necessary , sufficient)
[2]	$\neg[\exists x, P(x)] \equiv \cdots$.	$\{\forall x, \bar{P}(x);\ \exists x, \bar{P}(x);\ \forall x, P(x)\}$
[3]	The power set that can be formed from the set $A = \{\{1, 3\}, \{5\}, 2, 6\}\}$ equals....	$\{8, 16, 32\}$
[4]	If $R_1 = R \cup R^{-1}$, then R_1 should be .... ..Closure	(reflexive, transitive, symmetric)
[5]	$\forall x, y, z \in z^+$ Let $P(x, y, z): x + y + z$, then $p(x^2, x^2 - 1, x^2 + 1)$ equals	$\{x^3, 3x, 3x^2\}$

Solve behind the page if necessary.

Second Question (10- Marks)

❶ If $R: X \to X, X = \{a, b, c, d\}, R = \{(a,b), (a,c), (b,a), (b,d), (a,d), (b,c)\}$, find:

① X^2, ② R^{-1}, ③ $\bar{R}$, ④ the graph the relation,

⑤ the matrix of the relation relative to the ordering $badc$,

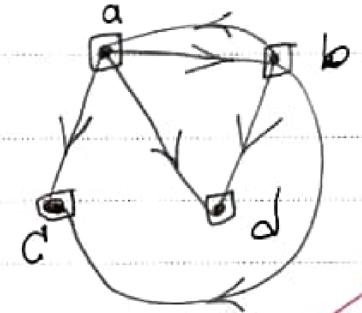
⑥ and finally classify the relation if it is reflexive, symmetric, and transitive or not.

$X^2 = \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d)\}$ 6

$R^{-1} = \{(b,a), (c,a), (a,b), (d,b), (a,d), (c,b)\}$

$\bar{R} = X^2 - R = \{$

$$A_R = \begin{array}{c|cccc} & b & a & d & c \\ \hline b & 0 & 1 & 1 & 1 \\ a & 1 & 0 & 1 & 1 \\ d & 0 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 \end{array}$$



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❷ Prove by contradiction that if $a^2 - 2a + 7$ is even, then a is even. (Solve Behind this paper)

Third Question (10- Marks)

❶ If $S_n = \{2kn \mid k = 2, 3\}$, find $\bigcap_{n=2}^{3} S_n$.

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$\bigcap_{n=2}^{3} S_n = S_2 \cap S_3$

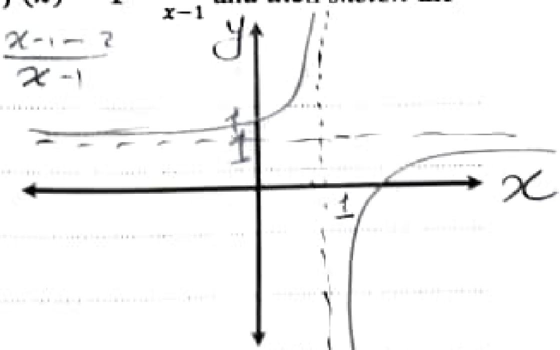
$S_2 = \{4k \mid k = 2,3\} = \{8, 12\}$

$S_3 = \{6k \mid k = 2,3\} = \{12, 18\}$

$\rightarrow \bigcap_{n=2}^{3} S_n = \{12\}$

❷ Find the *horizontal* and the *vertical* asymptote of the function $f(x) = 1 - \frac{2}{x-1}$ and then *sketch* the graph of the function.

$\frac{x-1-2}{x-1}$



$F(x) = \frac{x-3}{x-1}$

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vertical $x - 1 = 0 \rightarrow x = 1$

horizon $y = \lim_{x \to \infty} \frac{x-3}{x-1} = 1$

(انتهت الأسئلة)

Solve behind the page if necessary.

$P: a^2-2a+7$ is even $\rightarrow$ $\overline{P}: a^2-2a+7$ is odd

$q: a$ is even $\rightarrow$ $\overline{q}: a$ is odd

Starting with $\overline{q}=a$ is odd should be tends to

$\overline{P}: a^2-2a+7$ is also odd

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Let a is odd

$a = 2k+1 \quad , \quad k \geq 0$

$$
\begin{aligned}
a^2-2a+7 &= (2k+1)^2-2(2k+1)+7 \\
&= 4k^2+1+\cancel{4k}-\cancel{4k}-2+7 \\
&= 4k^2+6 \\
&= 2(2k^2+3) \quad , \text{ let } m = 2k^2+3 \geq 0 \\
&= 2m \quad \text{which is even.}
\end{aligned}
$$

and this is a contradiction with the first hypothesis.