Exercise

1) let $R_1 = \{(1,1), (1,2), (3,4), (4,2)\}$ and

$R_2 = \{(1,1), (2,1), (3,1), (4,4), (2,2)\}$

Find $R_2 \circ R_1$ and $R_1 \circ R_2$

$R_1 \circ R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)$
$(4,2), (2,4)\}$

لأن: اللى بينتهى بيه R_2 بيبدأ بيه R_1 والناتج عبارة عن زوج مرتب يحتوى
على بداية R_2 ونهاية R_1

$R_2 \circ R_1 = \{(1,1), (1,2), (3,4), (4,1), (4,2)\}$

لأن: اللى بينتهى بيه R_1 بيبدأ بيه R_2 والناتج ~~عبارة~~ عبارة عن
مجموعة تحتوى على ازواج مرتبة الازواج عبارة عن
بداية R_1 ونهاية R_2

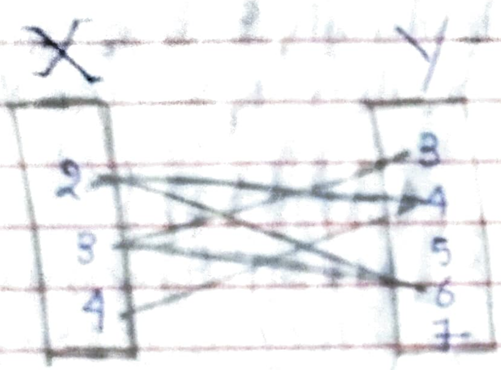
2 Let $X = \{2, 3, 4\}$ & $Y = \{3, 4, 5, 6, 7\}$

① define $(x, y) \in R$ if x divides y with zero remainder

العلاقة R عبارة عن ازواج مرتبة الاول عنصر فيها يقسم تماما العنصر الثاني بدون باقي

$R = \{(2,4), (2,6), (3,3), (3,6), (4,4)\}$

② find the domain and the range



Domain $= \{2, 3, 4\} = X$

CoDomain $= \{3, 4, 5, 6, 7\} = Y$

Range $= \{3, 4, 6\}$

③ check if R is equivalent relation or not

not equal

عشان تكون equivalent لازم تتحقق

$(2,2), (3,3), (4,4), (6,6)$

1) reflexive

2) Symmetric

3) Transverse

3) Determine whether each relation defined on the set $X = \{1,2,3,4,5,6\}$ $\forall x,y \in X$ is reflexive, symmetric and/or transitive

① $(x,y) \in R$ if $x = y^2$

$R = \{(1,1),(4,2)\}$

not reflexive, not symmetric and not Transitive

② $(x,y) \in R$ if $x > y$

$R = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)$
$(5,1),(5,2),(5,3),(5,4),(6,1),(6,2)$
$(6,3),(6,4),(6,5)\}$

not reflexive, Not symmetric, Transitive

③ $(x,y) \in R$ if $x \geq y$

$R = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3)$
$(4,1),(4,2),(4,3),(4,4),(5,1)$
$(5,2),(5,3),(5,4),(5,5),(6,1)$
$(6,2),(6,3),(6,4),(6,5),(6,6)\}$

reflexive : (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

not symmetric since (3,1) موجودة لكن → (1,3) مش موجودة

Transitive : xRy & $yRz \Rightarrow xRz$

(2,1) & (1,1) → (2,1)
(3,2) & (2,1) → (3,1)
(3,3) & (3,2) → (3,2)

(4,1) & (1,1) → (4,1)
(4,2) & (2,2) = (4,2)
(6,4) & (5,4) = (6,4)

(6,2) & (2,1) = (6
(4,3) & (3,2) = (4,

Date: Subject:

④ $(x,y) \in R$ if $x = y$

$R = \{(3,3),(4,4),(5,5),(6,6)\}$ ~~(1,1),(2,2)~~

$\therefore$ reflexive : since $x R x$

not $\therefore$ Transtive : since $x R y$ & $y R z \rightarrow x R z$

"مفيش عناصر مختلفة اصلا"

$\therefore$ $x R x$ & $x R x \rightarrow x R x$ وهكذا باقي العناصر

symmetric : if $x R y \rightarrow y R x$

⑤ $(x,y) \in R$ if x divides y

$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$
$(1,2),(1,3),(1,4),(1,5),(1,6)$
$(2,4),(2,6),(3,6),\}$

$\therefore$ reflexive : Since $x R x$

$\therefore$ Transtive : Since $x R y$ & $y R z \rightarrow x R z$

$(1,2)$ & $(2,4) \rightarrow (1,4)$ موجودة
$(1,2)$ & $(2,6) \rightarrow (1,6)$ موجودة
$(1,2)$ & $(2,4) \rightarrow (1,4)$ "
$(1,2)$ & $(2,2) \rightarrow (1,2)$ "
$(1,3)$ & $(3,3) \rightarrow (1,3)$
$(1,3)$ & $(3,6) \rightarrow (1,6)$
$(1,4)$ & $(4,4) \rightarrow (1,4)$
$(1,5)$ & $(5,5) \rightarrow (1,5)$
$(1,6)$ & $(6,6) \rightarrow (1,6)$

Transitive

not symmetric

Since $x R y \xrightarrow{not} y R x$

4] Determine each of the following relation defined on the set of positive integers is reflexive, symmetric and/or Transitive

① $R_1 = \{(x,y) \in R_1 \text{ and } x = y^2\}$

$R_1 = \{(1,1), (4,2), (9,3), (16,4), (25,5), \dots\}$ not ref.

not sym., not Trans.

$\downarrow$ $(16,4)(4,2) \in R$ but $(16,2) \notin R$

② $R_2 = \{(x,y) \in R_2 \text{ and } x > y\}$

$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), \dots\}$

not ref., not sym., Trans.

③ $R_3 = \{(x,y) \in R_3 \text{ and } x \geq y\}$

$R_3 = \{(1,1), (2,1), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4) \dots\}$

reflexive and Transtive مثلا، $(4,2) \& (2,1) \to (4,1)$

not symmetric مثلا $(3,2)$ $(2,3)$ مش موجود في العلاقة

④ $R_4 = \{(x,y) \in R_4 \text{ and } x = y\}$

ref $R_4 = \{(1,1), (2,2), (3,3), \dots\}$ → [illegible]

reflexive and symmetric & not Transitive

وهنا مش بيتنقل

[illegible] Trans.

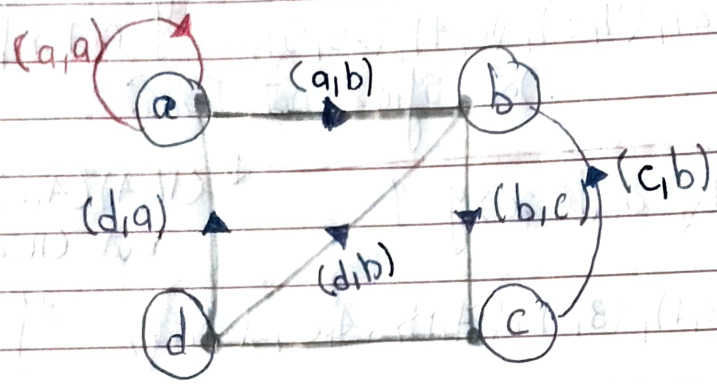
5] Draw the directed graph that represents the relation

$\{(a,a), (a,b), (b,c), (c,b), (d,a), (d,b)\}$

Solution

اول حاجة لازم احدد ال Vertex

a , b , c , d



Transitive

1- لازم الاقي $(x,y)(y,z)$

2- لو لقيت ∴ لازم الاقي التالت بتعهم (x,z)

3- لو ملقتش اصلا" $(x,y)(y,z)$

4- ادور على الزوج اللي بعده اللي يحقق العلاقة

5- لو ملقتش فالعنصر في كلها not Tran.

6- طب لو لقيت لكن (x,z) مش متحقق ∴ not Trans.

7

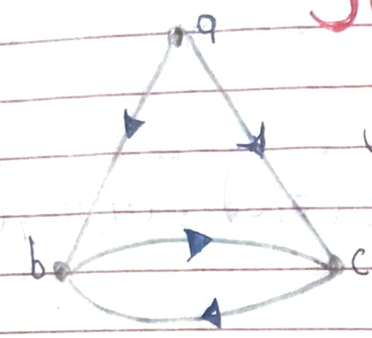
$(1,1), (2,2), (3,3)$ not Trans.

Date.

8 For the following diagraph list the ordered pairs in the relation represented by the directed graphs. then check out each relations if it is reflexive, symmetric or Transitive ?

Solution

(1)



1) Not reflexive :

لا مفيش علاقة xRx لكل الرؤوس

2) not symmetric :

لا مش كل العناصر xRy & yRx

الموصلة التي تحقق

[(a,b)(b,c),(a,c)(c,b)] $bRc \xrightarrow{\text{موجود}} cRb$

علي loop لانه فيه ناقص
b,c

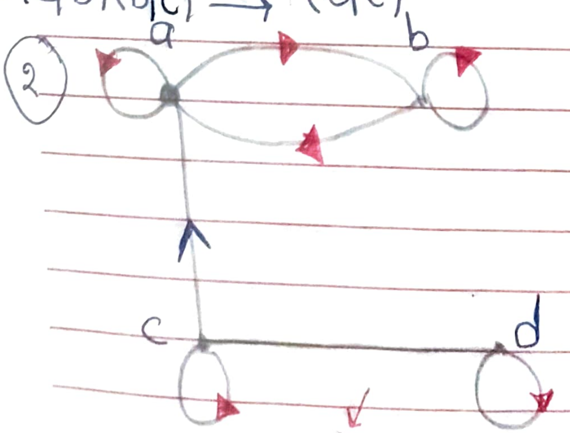
3) Not Transitive :

xRy & $yRz \Rightarrow xRz$ لامفيش هنا مو تتحقق

*(b,c)(c,b) → (b,b)

*(c,b)(b,c) → (c,c)

(2)



1) reflexive

2) not symmetric

3) Not Transtive

R = [(a,a),(b,b),(c,c),(d,d), (a,b),(b,a),(c,a), (c,d),(d,c)]

Date: Subject:

9) Show that why The relation

$R = \{(1,1), (1,3), (1,5), (2,2), (2,4)$
$(3,1), (3,3), (3,5), (4,2), (4,4)$
$(5,1), (5,3), (5,5)\}$

is reflexive, symmetric and Transitive ?

Solution:

1) reflexive : (1,1), (2,2), (3,3), (4,4), (5,5)

2) symmetric :

$(1,1) \rightarrow (1,1)$ $(1,3) \rightarrow (3,1)$
$(2,2) \rightarrow (2,2)$ $(1,5) \rightarrow (5,1)$
$(3,3) \rightarrow (3,3)$ $(2,4) \rightarrow (4,2)$
$(4,4) \rightarrow (4,4)$ $(5,3) \rightarrow (3,5)$
$(5,5) \rightarrow (5,5)$

Transitive: xRy & $yRz \rightarrow xRz$

$(1,3),(3,1) \rightarrow (1,1)$	$(1,3),(3,3) \rightarrow (1,3)$
$(1,5),(5,3) \rightarrow (1,3)$	$(1,3),(3,5) \rightarrow (1,5)$
$(1,5),(5,1) \rightarrow (1,1)$	$(1,5),(5,5) \rightarrow (1,5)$
$(2,4),(4,2) \rightarrow (2,2)$	$(3,5),(5,1) \rightarrow (3,1)$
$(2,4),(4,4) \rightarrow (2,4)$	$(3,5),(5,3) \rightarrow (3,3)$
$(3,1),(1,1) \rightarrow (3,1)$	$(3,5),(5,5) \rightarrow (3,5)$
$(3,1),(1,3) \rightarrow (3,3)$	$(4,2),(2,4) \rightarrow (4,2)$
$(3,1),(1,5) \rightarrow (3,5)$	$(4,4),(4,4) \rightarrow (4,4)$
$(3,3),(3,3) \rightarrow (3,3)$	$(2,2),(2,2) \rightarrow (2,2)$
$(1,1),(1,1) \rightarrow (1,1)$	$(5,1),(1,1) \rightarrow (5,1)$
$(5,1),(1,3) \rightarrow (5,3)$	$(5,3),(3,1) \rightarrow (5,1)$
$(5,1),(1,5) \rightarrow (5,5)$	$(5,5),(5,5) \rightarrow (5,5)$
$(5,3),(3,3) \rightarrow (5,3)$	

10 For the relation R on the set $\{1, 2, 3, 4, 5\}$ defined by the rule $(x, y) \in R$ if $x + y \leq 6$ Find the relation R

Solution

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5)$
$(2,1), (2,3), (2,4), (2,2)$
$(3,1), (3,2), (3,3), (4,1)$
$(4,2), (5,1)\}$

12 if R, S are relation R from A to B

Find $\bar{R}$, $\bar{S}$, $\bar{R} \cap S$ and S^{-1}

Solution

① if $A = B = \{1,2,3\}$

$R = \{(1,1),(1,2),(2,3),(3,1)\}$ & $S = \{(2,1),(3,1),(3,2),(3,3)\}$

الحل

عشان اوجد المكمله لازم اوجد ال $A \times B$

R جزئيه من $A \times B$

$A \times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

اللي موجود في $A \times B$ ومش موجود في R

$\bar{R} = \{(1,3),(2,2),(3,2),(3,3)\}$

اللي موجود في $A \times B$ ومش موجود في S

$\bar{S} = \{(1,1),(1,2),(1,3),(2,2),(2,3)\}$

تقاطع

$\bar{R} \cap S = \{(1,3),(2,2)\}$

$S \cap R = \{(3,1)\}$

$S^{-1} = \{(1,2),(1,3),(2,3),(3,3)\}$

(2) if $A = \{a, b, c\}$, $B = \{1, 2, 3\}$

$R = \{(a,1), (b,1), (b,2), (c,3)\}$

$S = \{(a,1), (a,2), (b,1), (b,2)\}$

Solution

$$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$$

$\bar{R} = \{(a,2), (a,3), (b,2), (b,3), (c,1)\}$

$\bar{S} = \{(a,3), (b,3), (c,1), (c,2), (c,3)\}$

$R \cap S = \{(a,1), (b,1)\}$

$S^{-1} = \{(1,a), (2,a), (1,b), (2,b)\}$

15 For the Following Matrices

① write the relation R, given by the Matrix as a set of ordered Pairs

$R_1 = \{(a,a), (a,c), (c,c), (d,a), (d,b), (d,c), (d,d)\}$

$R_2 = \{(w,w), (w,y), (y,w), (y,y), (z,z)\}$

$R_3 = \{(1,1), (1,3), (2,2), (2,3), (2,4)\}$

② Find inverse of These Matrices

inve: بنعمل دوران المصفوفة

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array} \quad \begin{array}{c c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = M[R_1^{-1}]$$

$$\begin{array}{c c} & \begin{matrix} w & x & y & z \end{matrix} \\ \begin{matrix} w \\ x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = M[R_2^{-1}]$$

$$\begin{array}{c c} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array} = M[R_3^{-1}]$$

Date: Subject:

(3) identify which of them are reflexive or symmetric

reflexive xRx & symmetric xRy & yRx

Tran. xRy & $yRz \rightarrow xRz$

(1) not reflexive

not symmetric

مطلوبش علاقة
↓

not Transitive (a,c) & $(c,d) \longrightarrow (a,d)$

(2) not reflexive $(x,x), (y,y) \notin R_2$

symmetric since $(w,w) = (w,w)$ $(y,y) = (y,y)$

$(w,y) = (y,w)$ $(z,z) = (z,z)$

~~Not~~ Transitive (w,w) & $(w,w) \rightarrow (w,w) = 1$

(w,y) & $(y,y) \rightarrow (w,y) = 1$

محتق (1) اتنين في العلاقة (y,w) & $(w,y) \rightarrow (y,y) = 1$

(y,y) & $(y,w) \rightarrow (y,y) = 1$

(w,w) & $(w,y) \rightarrow (w,w) = 1$

(3) reflexive ، لأنها متحقق شرط الـ لكل عناصر الـ domain

not symmetric $(1,2) \overset{\text{أو لو}}{\leftrightarrow} (2,1)$

not Transitive

$(2,4)$ & $(4,2)$ $(2,3)$ & $(3,\boxed{2})$

↓ مش متحققة ↓

مش متحققة

مفيش عنصر بيشغله فالعب

] Let R be the relation on

{ a, b, c, d } defined by the Matrix

Solution

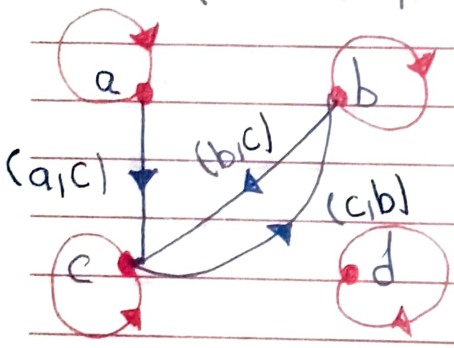
① Draw the graphical representation of R

R = { (a,a), (a,c), (b,b), (b,c), (c,b), (c,c), (d,d) }

وجبنا العلاقة R من الـ Matrix

T تمثل انه يوجد علاقة تسمى (T او 1)

F " " لايوجد " " (F او zero)



سؤال كان .Trans، sym.، ref

② State & give resons, whether R is reflexive symmetric or Transitive

* R is reflexive ~~symmetric~~ since (a,a), (b,b), (c,c), (d,d) ∈ R

* R is not symmetric since (a,c) but not (c,a), ...

* R is not Transitive since (a,c) & (c,b) but not Find (a,b) ∈ R

$\boxed{27}$ if $X = \{2, 3, 4\}$

$Y = \{4, 5, 6, 8\}$

① Define the relation R_1 from X to Y by

$(x, y) \in R$

where X divide Y

Sol:

نوجد R_1 بحيث انه X قاسم لل Y بدون باقي ~ (يقبل القسمة)

$R_1 = \{(2,4), (2,6), (2,8), (3,6), (4,8)\}$

② Define the relation R_2 from X to Y by

$(x, y) \in R$

where $x + y \leq 7$

Sol: $R_2 = \{(2,4), (2,5), (3,4)\}$

③ Show that if these relations are reflexive, symmetric or Transitive

Sol:

R_1 is not reflexive, not symmetric, not Transitive $(2,4)$ & $(4,8) \xrightarrow{\text{لا تؤدي لـ}} (2,8)$

$(2,4) \xrightarrow{\text{لا يؤدي الى}} (4,2)$

not Trans. $(2,4)(4,8) \in R_1$
but $(2,8) \notin R$

R_2 is not symmetric
not reflexive
not Transitive $(2,4)$ & $(2,5)$ not tends to $(4,5)$

Date: Subject:

30 Let R be the relation whose Matrix is

① Find the reflexive closure of R

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

② Find the symmetric closure of R

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

لازم يكون في
تماثل
حول القطر
الرئيسي

31 let $A = \{1,2,3,4\}$ & $B = \{1,2,3\}$

Given the Matrices M_R & M_S of the relations R&S

From A to B Compute:

Solution

(1) $M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

مكمله نحول كل

$1 \longrightarrow 0$

$0 \longrightarrow 1$

$M_{R \cap S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$1 \cap 1 = 1$

otherwise $= 0$

$M_{R \cup S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$0 \vee 0 = 0$

otherwise $= 1$

$M_{R^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

صف عمود

والعمود صف

2

$$M_{\bar{R}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{R \cap S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R \cup S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R^{-1}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Date

34 Let R be a relation whose digraph as follows:

(1) write R as an ordred

Sol: $R = \{(a,b), (b,b), (b,f), (c,e), (c,d), (d,c), (f,d), (d,b), (c,a), (e,f)\}$

مطلوب كل المسارات التي طولها = 2

(2) List all Paths of length 2, starting from vertex C

Sol: π_1 : c, a, b

π_2 : c, d, b → تاملاحظة نكمل المسار لو كان طالب الطول اكبر من 2

π_3 : c, e, f

π_4 : c, d, c

(3) Find a cycle starting at vertex C

Sol: c, d, c

Date: Subject:

(4) Find all Paths of Length 1 and also of length 3

Sol: of Length 1 : are all elements of $R = R$ اللي جبناها كلها في شكل زوج مرتب

of length 3 :

c, d, b, b , c, a, b, b

تبدأ بـ c : c, a, b, f , c, d, b, f , c, e, f, d

تبدأ بـ a : a, b, f, d , a, b, b, f

تبدأ بـ b : b, b, f, d , b, f, d, c , b, f, d, b

تبدأ بـ d : d, b, b, f , d, b, f, d , d, c, e, f
d, c, a, b , d, c, d, b

تبدأ بـ e : e, f, d, b , e, f, c, a , e, f, c, e
e, f, d, c

تبدأ بـ f : f, d, b, f , f, d, c, e , f, d, c, e
f, d, c, a

(5) Reflixive Closure $= R \cup \{(a,a), (c,c), (d,d), (e,e), (f,f)\}$

(6) symmetric Closure =

$R \cup \{(f,e), (f,b), (b,d), (d,f), (b,a), (a,c), (e,c)\}$

$\boxed{13}$ let $A=\{a,b,c,d\}$, $B=\{1,2,3\}$
and R is a relation from A to B
and $R=\{(a,2),(b,1),(c,2),(d,1)\}$ is R a fun.? is R^{-1} a fun.? Explain your answer

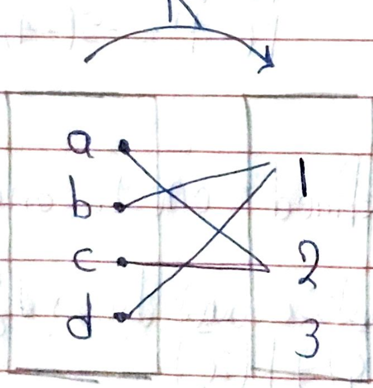
Solution

R is a function since no element in the domain is related with two different elements in the range
and the range $\{1,2\}$ الصور تتكرر عادي لكن
عناصر الـ Domain مينفعش تتكرر

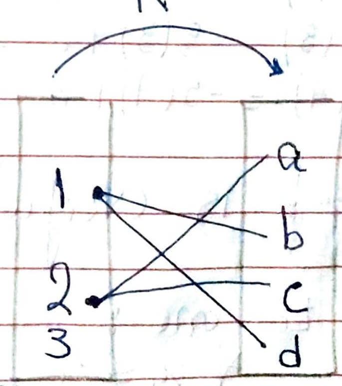
$R^{-1}=\{(2,a),(1,b),(2,c),(1,d)\}$

R^{-1} is not a function, since the element 2 in the domain related with two elements in the range

R



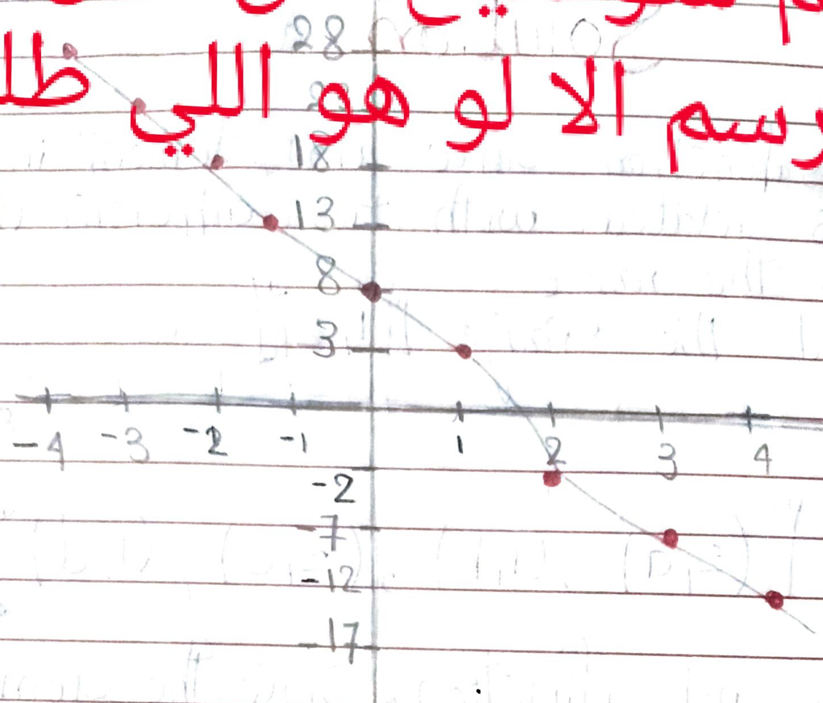
R^{-1}



14 let $A = B = \mathbb{R}$ let $f: A \to B$ be the function defined by $f(x) = -5x + 8$

Show that f is one to one and onto

الرسم للتوضيح لكن مش بنحل بالرسم الا لو هو اللي طلب



$f(0) = -5(0) + 8 = 8$
$f(1) = -5(1) + 8 = 3$
$f(2) = -5(2) + 8 = -2$
$f(3) = -5(3) + 8 = -7$
$f(4) = -5(4) + 8 = -12$

$f(-1) = -5(-1) + 8 = 13$
$f(-2) = -5(-2) + 8 = 18$
$f(-3) = -5(-3) + 8 = 23$
$f(-4) = -5(-4) + 8 = 28$

$\therefore f$ is everywhere defined one to one and onto

one to one : مفيش صورتين لـ y بتقابل قيمتين مختلفين في x

$\forall x_1 \neq x_2 \longleftrightarrow f(x_1) \neq f(x_2)$

onto : all element of y are Image

$$f(x) = -5x + 8$$

to be one-to-one

let $f(x_1) = f(x_2)$

$$-5x_1 + 8 = -5x_2 + 8$$

$$-5x_1 = -5x_2$$

$$\therefore x_1 = x_2$$

$\therefore f(x)$ is one-to-one

اكلك البرهان :-

عشان اعرف إذا كانت
1-to-1 ولا لا ؟
بفرض ان
$f(x_1) = f(x_2)$
ونمشي خطوات جبرية
عادية عشان نوصل
لـ
$x_1 = x_2$
لو طلعت صحيحة تبقى
one-to-one
لو طلعت مثلاً
$x_1 = \pm x_2$
تبقى
not one-to-one

---

one-to-one

$$x_1 \neq x_2 \longrightarrow y_1 \neq y_2$$

or $y_1 = y_2 \longrightarrow x_1 = x_2$

$$p \longrightarrow q$$

$$\bar{q} \longrightarrow \bar{p}$$

---

another Sol.

$$x_1 \neq x_2$$

$$-5x_1 \neq -5x_2$$

$$-5x_1 + 8 \neq -5x_2 + 8$$

$$\therefore f(x_1) \neq f(x_2)$$

لازم اجيب المدى (Range)
لو طلع بيساوي الـ Codomain
اللي هو B في المسألة = R
يبقى onto
طلعوا مش متساويين
$\therefore$ not onto
وعشان اجيب المدى
لازم اجيب المجال

Domain of $f(x)$ is $\mathbb{R}$

$A = \mathbb{R}$ معطى في المسألة

ولو مش معطى فإن كل

الدوال الخطية مجالها $\mathbb{R}$

$$-\infty < x < \infty$$

بنكتبه في صورة فترة

ونمشي خطوات جبرية عشان

نوصل الـ x لـ $f(x)$

$$\infty > -5x > -\infty$$

$$\infty > -5x + 8 > -\infty$$

$$\infty > f(x) > -\infty$$

$\therefore$ Range $=]-\infty, \infty[$

and Codomain of $f(x) = \mathbb{R}$ معطى

$\therefore$ Range of $f(x)$ = Codomain of $f(x)$

$\therefore f(x)$ is onto

(17) let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$ determine whether the relation R from A to B is a function and if it is so given its range

① $R_1 = \{(a,3), (b,2), (c,1)\}$

② $R_2 = \{(a,1), (b,1), (c,1), (d,1)\}$

Solution

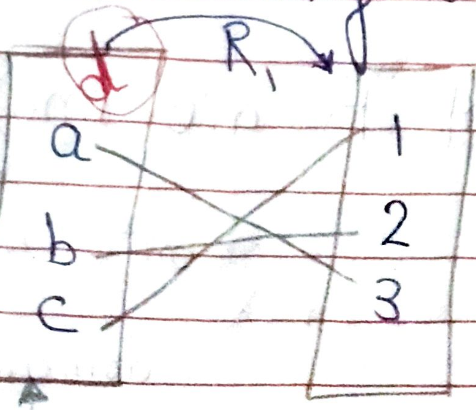
R_1 is not a function, since d element in the domain is not related with element in the range

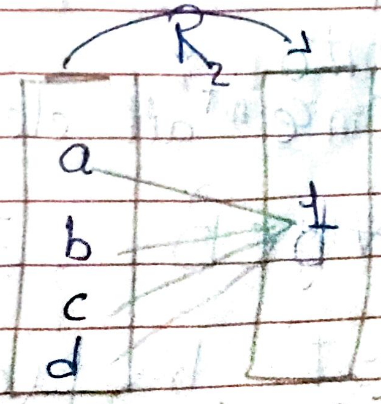
and the range is $\{1, 2, 3\}$

$\exists\, d \in A \quad \nexists\, (d, y) \in R$

R_2 is a function, since no element in the domain is related with two different element in the rang

and the rang is $\{1\}$





∵ مفيش عنصر ليه صورتين

onto not

not one-to-one

∴ fun. one-to-one & onto

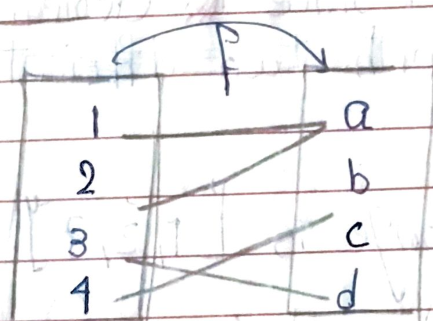
(21) let $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$

and let $f = \{(1,a), (2,a), (3,d), (4,c)\}$

IS f is one to one or onto?

Solution

1) f is a function since no element in the domain is related with two different elements in the range



2) not one to one: Since one to one $\forall\, x_1 \neq x_2 \longleftrightarrow f(x_1) \neq f(x_2)$

$\boxed{x_1 = 1} \neq \boxed{x_2 = 2}$ $\boxed{f(x_1) = a} = \boxed{f(x_2)}$

since $(1,a), (2,a) \in f$

3) not onto: Since not all elements of B are images

$\forall\, b \in B, \exists\, a \in A$

s.t $b = f(a)$

onto لما تكون
لازم
domain = Range

Since Range $= \{a, c, d\} \neq$ Codomain $= \{a, b, c, d\}$

33 let $A = \{1,3,4\}$, $B = \{0,1,2\}$

$C = \{3,4\}$ and $D = \{1,2,3,4\}$

Consider the following three functions from A to B , A to D and B to C respectively على التوالي

① $f_1 = \{(1,0), (3,2), (4,1)\}$

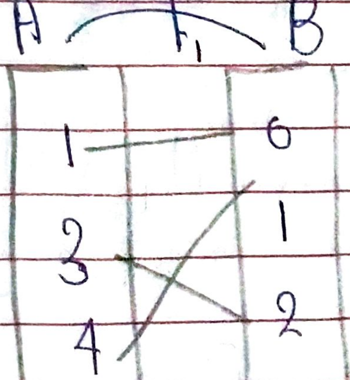
② $f_2 = \{(1,2), (3,1), (4,4)\}$

③ $f_3 = \{(0,4), (1,4), (2,3)\}$

1) Determine whether each function is one-to-one whether each functions is onto and whether each function is every where defined

Solution

F_1 : from A to B , ~~from A to D~~ ~~and B to C~~



① From A to B

one-to-one & onto

*) one-to-one

since $\forall\ x_1 \neq x_2 \longleftrightarrow f(x_1) \neq f(x_2)$

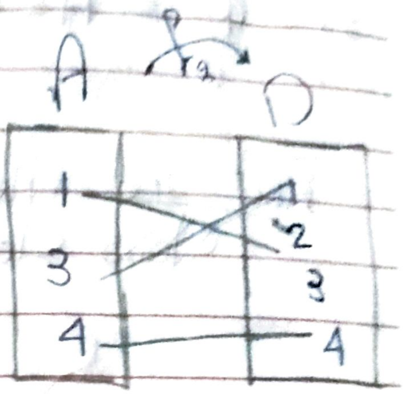
*) onto

since all element of B are images

Date: Subject:

② f_2 : from A to D

one-to-one , not onto



(*) one-to-one : since

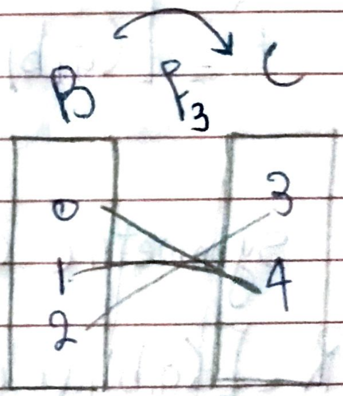
$\forall x_1 \neq x_2 \longleftrightarrow f(x_1) \neq f(x_2)$

(*) not onto : since
not all element of D are Images

---

③ f_3 : from B to C

not one-to-one but onto



*) not one-to-one since

$\forall\ x_1 = 1 \neq x_2 = 0$
$f(x_1) = f(x_2)$

*) onto since all element of C are images

Date:
Subject:

37) if $A = \{a, b, c, d\}$ and $R = \{(a,b), (b,c), (a,c), (c,d)\}$

Find the reflexive, symmetric and Transitive ~~Class~~ Closure of relation R

Solution

- Reflexive Closure $\boxed{R_1 = R \cup \Delta}$

$R_1 = \{(a,b), (b,c), (a,c), (c,d)\} \cup \{(a,a), (b,b), (c,c), (d,d)\}$

$R_1 = \{(a,b), (b,c), (a,c), (c,d), (a,a), (b,b), (c,c), (d,d)\}$

- Symmetric Closure $\boxed{R_1 = R \cup R^{-1}}$

$R_1 = \{(a,b), (b,c), (a,c), (c,d)\} \cup \{(b,a), (c,b), (c,a), (d,c)\}$

$R_1 = \{(a,b), (b,a), (b,c), (c,b), (a,c), (c,a), (c,d), (d,c)\}$

- Transitive Closure $\boxed{R_1 = R^{\infty}}$ Paths of any length

$R_1 = \{(a,b), (b,c), (a,c), (c,d), (b,d), (a,c), (a,d)\}$

$R = \{(a,b), (b,c), (a,c), (c,d)\}$

$(a,b),(b,c) \to (a,c)$ $\quad$ $(a,c),(c,d) \to (a,d)$ $\quad$ $(b,c),(c,d) \to \cancel{(b,d)}$

لازم اتأكد ان الأزواج المرتبة اللي انا ضفتها ايضاً بتحقق Trans. دي لو ضفنا كل الأزواج للـ R عشان تكون Trans.

لازم اشوف الأزواج دي لو لقينا زوج بيبدأ بنهاية الاول زي التاني لازم اضيف الزوج التالت اللي بداية من الزوج الاول ونهاية من التاني.

28) Let $f_1(x) = \dfrac{x}{x+1}$ and $f_2(x) = \dfrac{x}{1-x}$

① Find the domain of each function

domain $f_1 = R - \{-1\}$

domain $f_2 = R - \{1\}$

② Find $(f_1+f_2)(x)$, and then find its domain

$$(f_1+f_2)(x) = f_1(x) + f_2(x)$$

$$= \frac{x}{x+1} + \frac{x}{1-x} = \frac{x(1-x) + x(x+1)}{(x+1)(1-x)}$$

$$= \frac{x - \cancel{x^2} + \cancel{x^2} + x}{\cancel{x} - x^2 + 1 - \cancel{x}} = \frac{2x}{1-x^2}$$

$1 - x^2 = 0 \qquad x^2 = 1 \qquad x = \pm 1$

domain $= R - \{\pm 1\}$

or

domain (f_1+f_2) = domain $f_1 \cap$ domain f_2

بيتقاطعوا فى R فرق ± 1

④ without use of diagraph, Find the range of each fun.

① $f_1 = \frac{x}{x+1} = \frac{x+1-1}{x+1} = \frac{(x+1)}{(x+1)} - \frac{1}{x+1} = 1 - \frac{1}{x+1}$

نطرح ونضيف الواحد عشان أعزل x

Domain $R - \{-1\}$ $]-\infty,-1[$ & $]-1,\infty[$

$-\infty < x < -1$

$-\infty < x+1 < 0$ $\times(-1)$

$\infty > -(x+1) > 0$

$\frac{1}{\infty} < \frac{1}{-(x+1)} < \frac{1}{0}$

$0 < +\frac{1}{-(x+1)} < \infty$

$1 < 1 - \frac{1}{x+1} < \infty$

$-1 < x < \infty$

$0 < x+1 < \infty$

$\infty > \frac{1}{x+1} > 0$

$-\infty < -\frac{1}{x+1} < 0$

$-\infty < 1 - \frac{1}{x+1} < 1$

$R_{f_1} = R - \{1\}$

41) let $f(x) = x^2 - 1$ and $g(x) = \frac{1}{x}$

① Evaluate $f(-1)$, $f(0)$, $f(5a)$, $f(a)$, $f(a+b)$

$f(-1) = (-1)^2 - 1 = 0$

$f(0) = (0)^2 - 1 = -1$

$f(5a) = (5a)^2 - 1 = 25a^2 - 1$

$f(a) = (a)^2 - 1 = a^2 - 1$

$f(a+b) = (a+b)^2 - 1 = a^2 + 2ab + b^2 - 1$

② is $f(a+b) = f(a) + f(b)$

$L.H.S = f(a+b) = (a+b)^2 - 1$

$= a^2 + 2ab + b^2 - 1$

$R.H.S = f(a) + f(b)$

$= (a^2 - 1) + (b^2 - 1) = a^2 + b^2 - 2$

$\therefore f(a+b) \neq f(a) + f(b)$ لا تعتبر متطابقة

Since $f(x) = x^2 - 1$ ليس نفس الصورة

not ~~one-to-one~~

$x_1 \neq x_2$ $f(x_1) = x_1^2 - 1$. $f(x_2) = x_2^2 - 1$ $x_1^2 \cancel{-1} = x_2^2 \cancel{-1}$

$x_1 = \pm x_2$

Date:

Subject:

③ Find the function $h(x) = f^2 + 2f + 3$

$f^2(x) = f \cdot f = (x^2-1)(x^2-1) = x^4 - x^2 - x^2 + 1$

$= x^4 - 2x^2 + 1$

$\therefore h(x) = (x^4 - 2x^2 + 1) + 2(x^2 - 1) + 3$

$= x^4 - \cancel{2x^2} + 1 + \cancel{2x^2} - 2 + 3$

$= x^4 + 2$

Date:

(40) R be the relation on the set Z defined by the rule xRy if $x-y$ divisible by 4 (that is $x-y=4n$) for some integer n)

Show that R is an equivalence relation

Sol.

$n = 0, 1, 2, \ldots$

$R = \{(x,y) \mid x-y = 4n\}$

$R = \{(1,1), (2,2), (3,3), \ldots (4,0), (5,1)$
$(6,2), (7,3), (8,4), \ldots\}$
$(8,0), (10,2), \ldots$ دومجرد للتخيل

1- $\because (x,x) \in R$
since $x-x = 4(0) \in R$
$\therefore$ Ref.

بحيث

2- let $(x,y) \in R$ s.t $x-y = 4n$
$-(y-x) = 4n$
$y-x = 4(-n) = 4n$
$\therefore (y,x) \in R$
$\therefore R$ sym.

3- $(x,y), (y,z) \in R$
$x-y = 4n$ & $y-z = 4m$
$x = 4n+y$ $-z = 4m-y$

$x-z = 4n+\cancel{y}+4m-\cancel{y} = 4(\underbrace{n+m}_{n}) = 4n$
$\therefore (x,z) \in R$ is Tran.