



Mid-Term Exam (10/30) + Academic Assignments (20/30)

Mid-Term Exam of Discrete Math for Summer  
Course (BSD-103)

Module-A

Module-A

### **First Question (15 Marks)**

- Use mathematical induction to  $x^{2n} - y^{2n}$  is divisible by  $(x + y)$  for  $n \in \{0, 1, 2, \dots\}$

---

  - For the relation  $R$  on  $X = \{1, 2, 3, 4\}$  defined by the rule  $(x, y) \in R, x + 1 \leq y$ 
    - ① Find the relation  $R$
    - ② Give the graphical representation
    - ③ Give the matrix representation of  $R$
    - ④ Identify that if  $R$  is reflexive, symmetric or transitive

استخدم حلقة الورقة عند ينطاك الأمر

### **Second Question (15 Marks)**

- Prove that if  $x^2$  is even, then  $x$  is even

• Let  $S_n = \{kn^2 \mid k = 1, 2, 3\}$ . find  $\bigcup_{n=1}^{\infty} S_n$

- Simplify the following statement:  $(p \wedge q) \vee (p \wedge \bar{q}) \vee (\bar{p} \wedge \bar{q})$

## first Question

①

$$\text{Step 1: } S_n = x^{2n} - y^{2n}$$

at  $n=1$   $S_1 = x^2 - y^2 = (x+y)(x-y)$   
is divisible by  $(x+y)$

$$\begin{aligned} \text{at } n=2 & \quad S_2 = x^4 - y^4 \\ &= (x^2 + y^2)(x^2 - y^2) \\ &= (x^2 + y^2)(x - y)(x + y) \\ &\text{is divisible by } (x+y) \end{aligned}$$

$\therefore S_n = (x^{2n} - y^{2n})$  is divisible by  $(x+y)$

step 2: at  $n+1$

$$\begin{aligned} S_{n+1} &= x^{2n+2} - y^{2n+2} \\ &= x^2 x^{2n} - y^2 y^{2n} \\ &= x^2 x^{2n} - x^2 y^{2n} + x^2 y^{2n} - y^2 y^{2n} \\ &= \underbrace{x^2 (x^{2n} - y^{2n})}_{\text{from step 1}} + \underbrace{y^{2n} (x^2 - y^2)}_{\text{from step 1}} \end{aligned}$$

By the first step  $S_n = x^{2n} - y^{2n}$  is  
divisible by  $(x+y)$

By the second step  $S_{n+1} = x^2(x^{2n} - y^{2n}) + y^{2n}(x^2 - y^2)$   
is also divisible by  $(x+y)$

Date: \_\_\_\_\_

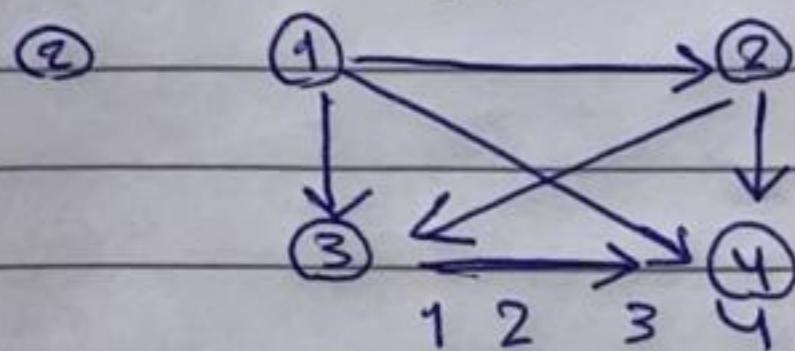
Subject: \_\_\_\_\_

$$\textcircled{2} \quad X = \{1, 2, 3, 4\}$$

R defined by The rule  $(x, y) \in R$   
 $x + 1 \leq y$

$$\textcircled{1} \quad R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$$

~~Reflexive~~



\textcircled{3}

$$M_R = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

\textcircled{4} R is not reflexive, not symmetric & Transitive

Date: \_\_\_\_\_

Subject: \_\_\_\_\_

## Second Question

(1)

P:  $x^2$  is even number

q:  $x$  is also even number

if  $\neg q$ :  $x$  is odd number

$$x = 2k + 1$$

$$x^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Then  $x^2$  is odd number

This mean That if  $x^2$  is even, Then  $x$  is even

$$\textcircled{2} \quad S_1 = \{k \mid k=1, 2, 3\} = \{1, 2, 3\}$$

$$S_2 = \{4k \mid k=1, 2, 3\} = \{4, 8, 12\}$$

$$S_3 = \{8k \mid k=1, 2, 3\} = \{8, 16, 24\}$$

$$\bigcup_{n=1}^3 S_n = S_1 \cup S_2 \cup S_3 = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

*↓ Example ↓*

③ simplify  $(P \wedge q) \vee (P \wedge \neg q) \vee (\neg P \wedge \neg q)$

$$(P \wedge q) \vee (P \wedge \neg q) \vee (\neg P \wedge \neg q)$$

$$\equiv P \wedge (q \vee \neg q) \vee (\neg P \wedge \neg q)$$

$$\equiv (P \wedge T) \vee (\neg P \wedge \neg q)$$

$$\equiv P \vee (\neg P \wedge \neg q)$$

$$\equiv (P \wedge \neg P) \vee (P \wedge \neg q)$$

$$\equiv T \vee (P \wedge \neg q)$$

$$\equiv P \vee \neg q \quad \#$$