Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 13-1-2020; Time: 3- Hours

Final Examination of Discrete Mathematics BS - 103

الامتحان يقع في ورقة من صفحتين.

First Question (10 Marks)

Choose the appropriate signs "✓" or "x" for the following:

[1] The day of the week will it be after 200 days from Monday is Friday. (.........)

[2] If $R_2 = R \cup R^{-1}$, then R_2 should be symmetric. (.........)

[3] If $x \bmod y = r$, then y divides $x - r$ (.........)

[4] The general term of the sequence: 3, 0.3, 0.03, … is of the form: $\left(\frac{3}{10^n}\right)$ for $n \geq 1$ (.........)

[5] A simple path is a path with no repeated vertices. (.........)

[6] The degree of a vertex in an in-directed graph is the number of edges incident with it. (.........)

[7] ~~The Big O notation is used to give an upper bound of the running time of an algorithm.~~ (.........)

[8] If $f: R \to R^{+}, f(x) = \sin(x^2)$, then f is one to one function. (.........)

[9] If x is rational number, then $\lceil x \rceil = \lfloor x \rfloor + 1$. (.........)

[10] The number of ways to select two persons from a group of 4 persons is 6 (.........)

Second Question (10 Marks)

Choose the correct answer

[11] The infinite series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ is $\{\frac{1}{2^n}, \frac{1}{2^n+1}, \frac{n}{n+1}\}$

[12] ~~Big O of the function $f(n) = 2n^3 + 5n^2 - 2n + 1$, $n \geq 1$~~ equals $\{O(n^3), O(n), O(n^2)\}$

[13] If the relation R satisfies that: $xRy \leftrightarrow yRx$, then R should be {transitive, symmetric}

[14] The value of a that makes the function $f(x) = x^2 + ax$ even is: {0, 1, -1}

[15] If $f(x): \mathcal{R} \to \mathcal{R}^{+}: x \to e^x$ the f is {Onto, Bijective}

[16] $3\lceil 1.5 \rceil + 2\lfloor -1.5 \rfloor = \cdots$ {2, 10, 8}

[17] The range of the function $f(x) = \text{Log}(x)$ is $\{\mathcal{R}, \mathcal{R}^{+}, \mathcal{R}^{-}\}$

[18] $-3 \times h(210) + 12 \bmod 5$ equals {1, -1, 0}

[19] $-44 \bmod 4$ equals {0, 1, 4}

[20] The number of ways in which 2 persons can be selected from a group of 6 persons is: {30, 15, 20}

Third Question (30 Marks)

❶ Given the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ to:

(1) prove by **mathematical induction** that $A^k = \frac{1}{2}\begin{bmatrix} 1+3^k & -1+3^k \\ -1+3^k & 1+3^k \end{bmatrix}$,

(2) **solve** the system of the **recurrence relations** in terms of the initial values x_0 and y_0

$$x_{n+1} = 2x_n + y_n$$
$$y_{n+1} = x_n + 2y_n$$

❷ **Express** the function $f(t)$ by the **unit step function**, where $f(t) = \begin{cases} t^3 + 2t^2 - 1 & 0 \le t \le 1 \\ 2t^2 - t - 1 & 1 < t \le 2 \\ t - 1 & 2 < t \le 3 \\ -1 & t > 3 \end{cases}$.

❸ **Prove** that if n is odd, then $\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$.

$n^2 = n^2+3$

$4n^2 = 4n^2 + 12$

❹ Construct the **tree** of the following expressions: $\left(3 \div \left(2 \div \left(11 \div (9-4)\right)\right)\right) \div \left(2 + \left(3 + (4+7)\right)\right)$

, and then find the **height** of the tree.

Fourth Question (30 Marks)

❶ Use **Taylor series** to **approximate** the function $f(x) = \sin(2x) + e^{5x}$ to just three terms and then use this to approximate the value $f(0.1)$.

❷ Change the lower **index** of the summation $\sum_{k=1}^{k=n} a_{k-1}a_{n-k}$ to start with $k = 3$.

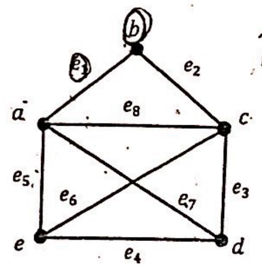
❸ (a) From the opposite figure, complete:

(1) The adjacency matrix is A =

(2) The **Laplacian** matrix is L =

(3) The **incident** matrix is I =

(b) Find the **degree** of each vertex.



❹ Let $A = \{a, b, c, d\}$, and let R be a relation on A such that:

$$M_A = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a,a), (a,d)

(1) Write the relation R as an ordered pairs,

(2) ~~Construct a linked list representation, VERT, TAIL, HEAD and NEXT for the relation R.~~

------------- (إنتهت الأسئلة) -------------

لجنة الممتحنين:

(١) أ.د.م/ ياسر محمد عبد الستار (٢) أ.د.م/ عبد المنعم مجاهد (٣) د. مصطفى عبد العزيز

٢

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2020

Q1

① البت الأصلالاثنين الثلاثاء الأربعاء الخميس الجمعة

6 5 4 3 2 1 0

$\frac{202}{7} = 28$

$202 - 28 \times 7$

$=$

$(2+200) \% 7 = 6$ الجمعة

mod

② ✓

③ ✓

④ ~~✗~~ $\frac{3}{10^n}$ $n \geq 0$

⑤ ~~$\frac{3}{10}$~~ ✓

⑥ ✓

⑦ ملغية

⑧ X

⑨ ✓

⑩ ${}^4C_2 = 6$ (ج)

⑪ $\frac{1}{2^n}$

⑫ X ملغية

⑬ symetric

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14) $f(x) = x^2 + ax$

$f(-x) = x^2 - ax$

$x^2 + ax = x^2 - ax$

$2ax = 0$

$2a = 0$

$\boxed{a = 0}$

(15) Bijective

(16) $3\lceil 1.5 \rceil + 2\lfloor -1.5 \rfloor =$

$3 * 2 + 2 * (-2) = 2$

(17) R^+

(18) $-3 \times h(210) + 12 \bmod 5$

$-3 \times 1 + 2 = (-1)$

$\frac{210}{11} = 19.\cancel{09}$

$210 - 19 * 11 = 1$

(19) $-44 \bmod 4 = 0$ $44 \bmod 4 = 0$

(20) ${}^6C_2 = \frac{6!}{2!\,4!} = 15$

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ Prove $A^K = \frac{1}{2}\begin{bmatrix} 1+3^K & -1+3^K \\ -1+3^K & 1+3^K \end{bmatrix}$ ②

$K=1$ $$A^1 = \frac{1}{2}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

let $K = m$ $$A^m = \frac{1}{2}\begin{bmatrix} 1+3^m & -1+3^m \\ -1+3^m & 1+3^m \end{bmatrix}$$

بالضرب المصفوفة في A

اثبات

$K = m+1$ $$A^{m+1} = \frac{1}{2}\begin{bmatrix} 1+3^m & -1+3^m \\ -1+3^m & 1+3^m \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2+2 * 3^m - 1 + 3^m & 1+3^m-2+2 * 3^m \\ -2+2 * 3^m + 1+3^m & -1+3^m+2+2 * 3^m \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1+3\,3^m & -1+3\,3^m \\ -1+3\,3^m & 1+3\,3^m \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1+3^{m+1} & -1+3^{m+1} \\ -1+3^{m+1} & -1+3^{m+1} \end{bmatrix}$$

(4'3) ②

$$f(t) = \begin{cases} t^3 + 2t^2 - 1 & 0 \le t \le 1 \\ 2t^2 - t - 1 & 1 < t \le 2 \\ t - 1 & 2 < t \le 3 \\ -1 & t > 3 \end{cases}$$

$$(t^3 + 2t^2 - 1)(u(t) - u(t-1)) + (2t^2 - t - 1)(u(t-1) - u(t-$$
$$(t-1)(u(t-2) - u(t-3)) - u(t-3)$$

$$= (t^3 + 2t^2 - 1)\,u(t) + (-t^3 - 2t^2 + 1)\,u(t-1) + (2t^2 - t - 1)\,u(t-1)$$
$$+ (-2t^2 + t + 1)\,u(t-2) + (t-1)\,u(t-2) + (-t+1)\,u(t-3)$$
$$- u(t-3)$$

$$= (t^3 + 2t^2 - 1)\,u(t) + (-t^3 - t)\,u(t-1) + (-2t^2 + 2t)$$
$$u(t-2) + (-t)\,u(t-3)$$

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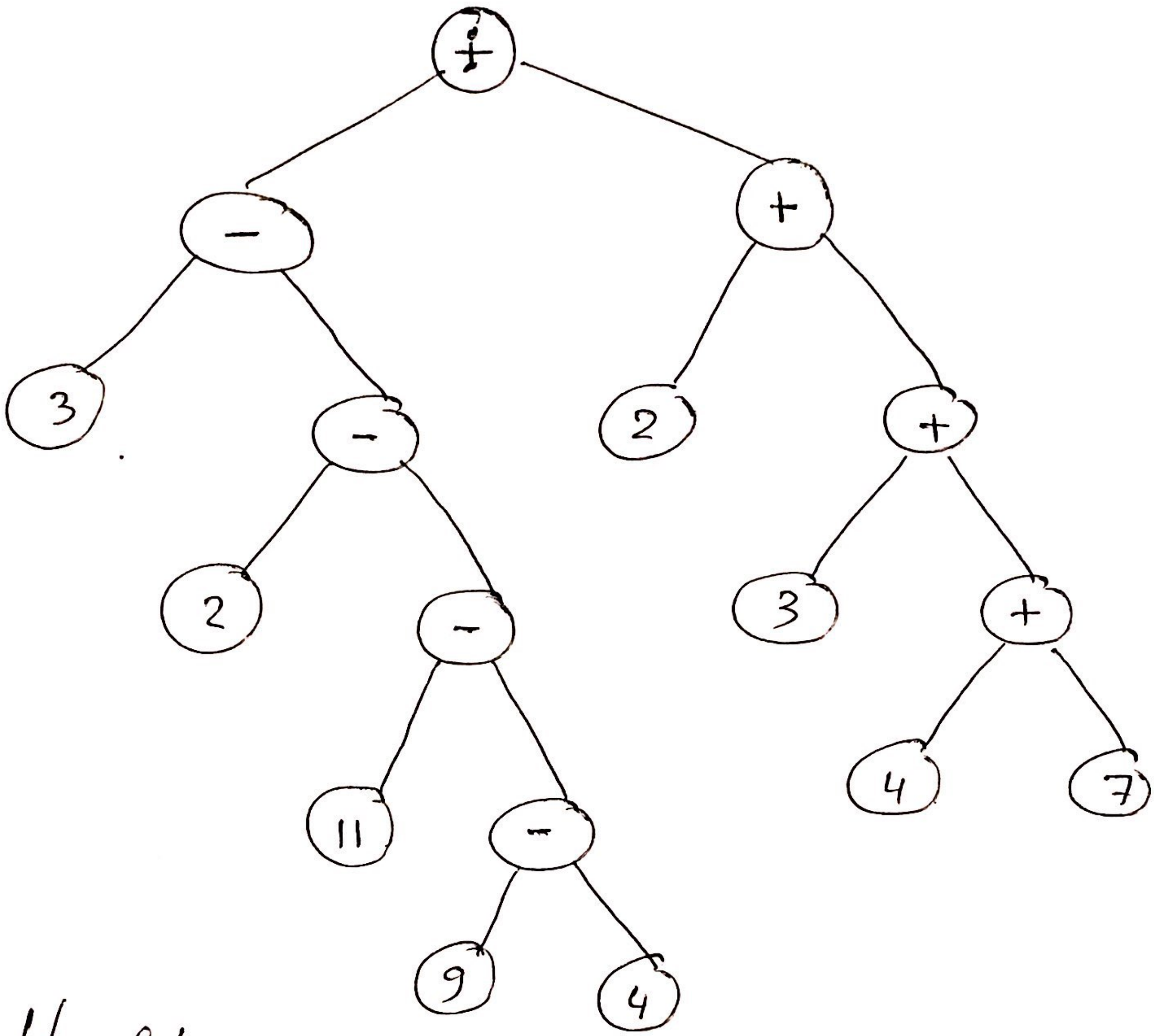

Height = 5

level عدد −1

$f(x) = \sin(2x) + e^{5x}$ ⑤

$f(x) = \sin(2x) + e^{5x}$	$f(0) = 1$
$f'(x) = 2\cos 2x + 5e^{5x}$	$f'(0) = 7$
$f''(x) = -4\sin 2x + 25e^{5x}$	$f''(0) = 25$

$$f(x) = 1 + \frac{7}{1!}x + \frac{25}{2!}x^2 + \cdots$$

$$f(0.1) = 1 + 7 * 0.1 + \frac{25}{2!}(0.1)^2 = 1.7125$$

$$\sum_{K=1}^{K=n} a_{K-1}\, a_{n-K}$$

$K = 1$

$\delta = 3$

$\delta = K + 2 \longrightarrow K = \delta - 2$

$$\sum_{\delta-2=1}^{\delta-2=n} a_{\delta-2-1}\, a_{n-(\delta-2)}$$

$$\sum_{\delta=3}^{\delta=n+2} a_{\delta-3}\, a_{n-\delta+2} \longrightarrow \boxed{\sum_{K=3}^{K=n+2} a_{K-3}\, a_{n-K+2}}$$

③ adjacency

$$
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
a & 0 & 1 & 1 & 1 & 1 \\
b & 1 & 0 & 1 & 0 & 0 \\
c & 1 & 1 & 0 & 1 & 1 \\
d & 1 & 0 & 1 & 0 & 1 \\
e & 1 & 0 & 1 & 1 & 0
\end{array}
$$

Laplacian

$$
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
a & 4 & -1 & -1 & -1 & -1 \\
b & -1 & 2 & -1 & 0 & 0 \\
c & -1 & -1 & 4 & -1 & -1 \\
d & -1 & 0 & -1 & 3 & -1 \\
e & -1 & 0 & -1 & -1 & 3
\end{array}
$$

incident

$$
\begin{array}{c|cccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
\hline
a & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
b & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
c & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
d & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
e & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$

degree $\rightarrow d(a) = 4$

$d(b) = 2$

$d(c) = 4$

$d(d) = 3$

$d(e) = 3$
