الرياضيات الغير منفصلة

Suez Canal University
Faculty of Computer& Informatics
First Semester 2015-2016
Time: Three Hours
Date : 19-1-2016

Final Examination
Discrete Mathematics (BS-103)

Answer all the following four questions.

First Question - (20 Marks)

❶ If $f(x) = 3x^2 - 5x|z| + x,\ x \neq 0$. Determine the value (s) of z which makes $f(x)$ **even** function.

❷ Let f be a function from $X = \{0, 1, 2, 3, 4\}$ to X defined by $f(x) = 3x \bmod 2$, where $x \in X$. Write f(x) as a set of ordered pairs. Check if the function f is **one to one**, **onto**, **both one to one and onto** or **neither one to one nor onto**?

❸ Show that why the relation

$R = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)\}$

is **reflexive**, **symmetric** and **transitive**?

❹ Apply **Taylor series** to expand the function $f(x) = e^x$ with $x_0 = 0$ and then prove that $0 \leq (1 + x)^m \leq e^{mx}$ for positive m and $x \geq -1$

Second Question - (20 Marks)

❶ Let $P(x, y, z)$ be the predicate $xy \geq x + z$, the domain is the positive real numbers. **Write out** the predicate $P(x, x, x - 1)$ and check its **truth value**.

❷ Find the **inverse** of the function $f(x) = x^2 + 8x - 9$ for $x \leq -4$, and then find $f^{-1}(0)$.

❸ Express the function $f(t)$ in terms of **unit step functions**

$$f(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 5 \\ 2t & 5 < t < 8 \\ t^2/7 & t \geq 8 \end{cases}$$

❹ **Choose** the correct answer:

① $\overline{p \wedge T} \equiv \cdots$ $\{T, F, \bar{p}\}$
② $\lceil -4.1 \rceil = \cdots + 1$ $\{\lfloor -1.2 \rfloor, \lceil -5.2 \rceil, \lfloor -3.2 \rfloor\}$
③ The function $f(x) = e^{3x}$, where $f: \mathcal{R} \to \mathcal{R}^+$ is {Onto, One to one, Bijective}
④ The statement $\lceil x \rceil = \lfloor x \rfloor + 1\ \forall\ (x \notin Z)$^ (x is rational) {true, false}
⑤ A tree with 2 −vertices has exactely .... edge {1, 4, 6}
⑥ If $2x + 2\lfloor 2.01 \rfloor - 3\lceil 4.3 \rceil = (15\ mod\ 2)$, then x equals $\{\lceil 4.3 \rceil, \lceil 5.3 \rceil, \lfloor 7.3 \rfloor\}$
⑦ The value of $|-3| - 2\lfloor -17.2 \rfloor$ is $\{\lceil 31 \rceil, \lceil -31.9 \rceil, \lfloor 39.9 \rfloor\}$
⑧ The function $f(x) = \sin(x), f: \mathcal{R} \to [-1, 1]$ is not {onto, one to one}
⑨ The function $f(x) = \mathrm{Ln}(x)$ from $(0, \infty)$ to $\mathcal{R}$ is {injective, surjective, bijective}
⑩ The statement $(A \cap \overline{B}) = \emptyset \iff A \subset B$ is {true, false}

Date: 22-1-2015 First Semester

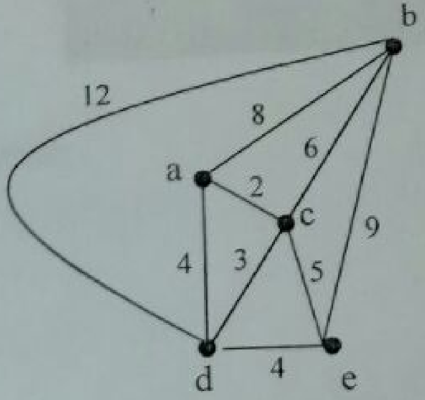
Final Examination
Discrete Mathematics (BS-103)

First Question - (20 Marks)

❶ **Calculate** the value of x, where $2x + 2\lfloor 2.7 \rfloor - 3\lceil 5 \rceil = (15 \; mod \; 2)$

❷ Let f be the function from $X = \{0, 1, 2, 3, 4\}$ to X defined by $f(x) = 2x \; \boldsymbol{mod} \; 2$. **Write** f as a set of ordered pairs. **Check** if the function f is one to one, onto or both?

❸ For the opposite diagraph, complete the following:



① - Is it a directed graph?
② - **What** is the number of vertices and the number of edges? **Find** the relation between them.
③ - Find the **optimal** path.
④ - Define a **simple** cycle of length 4.
⑤ - Write the **adjacency matrix**.

❹ Check the following logical equivalence: $p \vee (p \wedge q) \equiv p$, then verify your answer using **truth table**.

Second Question - (20 Marks)

❶ Use "**indirect proof**" to prove that for all real numbers $d,\ d_1,\ d_2$ and x
"If $d = \min\{d_1, d_2\}$ and $x \le d$ then $x \le d_1$ and $x \le d_2$".

❷ Find the **inverse** of the function $f(x) = x^2$ for $x \ge 0$, and then find $f^{-1}(4)$.

❸ Find $(f \circ g)(0)$ and $(g \circ f)(1)$ for the functions: $f(x) = x^3 + 3$ and $g(x) = x^2$.

❹ **Choose** the correct answer:

① $\overline{p \wedge T} \equiv \cdots$	$\{T, F, \bar{p}\}$
② $\lceil -4 \rceil = \cdots + 1$	$\{\lfloor -4.2 \rfloor, \lceil -5 \rceil, \lfloor -5.2 \rfloor\}$
③ The function $f(x) = e^x$ is	{Onto, One to one, Bijective}
④ If $f(x) = 2\lceil x \rceil$, then 12 mod 5 − 0.5 $f(-0.5)$ equals	$\{1.5,\ 2,\ 0\}$
⑤ A tree with 2 −vertices has exactely .... edge	$\{1,\ 4,\ 6\}$

Third Question[1]- (20 Marks)

❶ Let $A_i = \{i+2,\ i+3,\ i+4\}$. Find $\bigcup_{i=1}^{4} A_i$ and $\bigcap_{i=1}^{4} A_i$.

❷ Solve the recurrence relation $s_n = 2s_{n-1}$ subject to the initial condition $s_0 = 1$.

❸ Prove that $(xy)^n = x^n y^n$ for all real numbers.

❹ Find an explicit formula for the **Fibonacci sequence**:
$f_n - f_{n-1} - f_{n-2} = 0, n \ge 3$ with the initial conditions: $f_1 = 1$, and $f_2 = 2$.

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 4-1-2018; Time: 3- **Hours**

Final Examination of Discrete Mathematics BS - 103

First Question (16 Points)

❶ **Calculate** the value of $(12 \bmod 5) + h(15) - 3\lfloor -4.7 \rfloor + 2\lceil 0.3 \rceil$.

❷ Solve the 1st order **difference** equation: $a_n = a_{n-1}$, with $a_1 = 4$.

❸ **Change** the lower index of the sum $\sum_{k=1}^{n} a_{n-k}$ to start with $k = 0$.

❹ Find the **check digit** of the opposite bar code :

4 710088 41253

Second Question (16 Points)

❶ Find a numerical value of $\sqrt{50}$ approximated to four digits.

❷ **Solve** the recurrence relation $P_n = a + s\,P_{n-1}$ of the economic model where, a and s are parameters depend on the model.

❸ If $f(x) = x^2$, **Find** the $\mathrm{Lim}_{n \to 1} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$.

❹ **Give** three **different** examples of odd functions.

Third Question (20 Points)

❶ **Calculate** the numerical value of the summation: $\sum_{k=1}^{k=800} (2k+1)$.

❷ Department consists of 10 men and 5 women. **How many ways** to conform a

committee consists of 4 persons provided that at least 2 men are selected?

❸ Find Taylor series expansion (just 4 terms) for the function: $f(x) = \cos x$ about $x = 0$.

❹ **Choose** the correct answer:

① $\lceil x \rceil = \lfloor x \rfloor + 1$ for all $x \in \cdots$	$\{z, Q - z, \mathcal{R}\}$
② The function $f(x) = \sin(3x)$ is	{even, odd}
③ If $f(x) = \lceil x + 1 \rceil$, then $10 \bmod 5 - 0.5\, f(-1.5)$ equals	{ 1, 2, 0}
④ A tree with 3 −vertices has exactely .... edges	{2, 4, 8}
⑤ Every relation is a function?	{Yes, No}

الإمتحان يقع فى صفحتين من نفس الورقة.

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 13-1-2019; Time: 3- Hours

Final Examination of
Discrete Mathematics BS - 103

الامتحان يقع فى ورقة من صفحتين.

First Question (20- Marks)

Fill the circle with the appropriate signs "✓ for the circle A" or "× for the circle B"

[1] The day of the week will it be after 100 days from Saturday is **Monday**. (..........)

[2] If $R_2 = R \cup R^{-1}$, then R_2 should be **symmetric**. (..........)

[3] If $x \bmod y = r$, then y **divides** $x - r$ (..........)

[4] $0 \bmod 7 = 1$ (..........)

[5] The functions $f(x) = \lceil x \rceil$ and $f(x) = \lfloor x \rfloor$ map from $\mathcal{R}$ to z is **one to one**. (..........)

[6] The **horizontal** asymptotes of the function $y = 1/x$ is the line $y = 1$. (..........)

[7] The **Big** $\boldsymbol{O}$ notation is used to give an upper bound of the running time of an algorithm. (..........)

[8] If $f: R \to R^+$, $f(x) = \sin(x^2)$, then f is **one to one** function. (..........)

[9] If $f: R^+ \to R$, $f(x) = \mathrm{Log}(x)$, then f is **onto** function. (..........)

[10] The **domain** of the function $f(x) = 2\cos(x)$ is R. (..........)

[11] The **range** of the function $f(x) = 3\sin(2x - 1) - 3$ is $[-3, 3]$. (..........)

[12] The **vertical asymptotes** of the function $y = \frac{1}{x-1}$ is the line $x = 0$. (..........)

[13] If $a, b \in D_f$, $a < b$, and $f(a) < f(b)$, then f is **increasing** function. (..........)

[14] If $f(x) = x$ is the **identity function**, then $f^{-1}(x) = x$. (..........)

[15] The infinite series $\frac{2}{10} + \frac{2}{10^2} + \cdots + \frac{2}{10^n} + ..$ is **divergent**. (..........)

[16] $\lfloor x \rfloor = \lceil x \rceil - 1$ for all $x \in z^+$. (..........)

[17] **Big** $\boldsymbol{O}$ **notation** is used to describe how closely a series approximates a given function. (..........)

[18] The **general term** of the sequence: -1, -3, -5, … is of the form: $(1 - 2n)$. (..........)

[19] If the function $f(x)$ have an **inverse**, then $(f \circ f^{-1})(x)$ should equals $1/x$ (..........)

[20] The **number** of ways for a number of two digits that can be formed from $\{1, 2, 3, 8\}$ without repeating is 16. (..........)

ملحوظة

السؤال الأول والثانى يتم إجابته فى الجزء اليمين فى النموذج الإلكترونى للإجابة بنفس التسلسل من 1-30 فى حين أن السؤال الثالث يتم إجابته فى كراسة الإجابة العادية.

Fourth Question- (20 Marks)

❶ Construct the tree of the following mathematical expression:

$\big(11-(11\times(11\times 11))\big)+\big(11+(11\times 11)\big)$, and then

① Number of **levels**, ② Number of **paths**, ③ **Height** of the tree.

❷ Find the **check digit** of the opposite bar code :

4 710088 41253

❸ Let $A=\{a,b,c,d\}$, and let R be a relation on A such that:

$$M_R=\begin{array}{c} \\ a\\ b\\ c\\ d\end{array}\begin{array}{c}\begin{array}{cccc} a & b & c & d\end{array}\\ \begin{bmatrix}1&1&0&1\\0&1&1&0\\0&1&1&1\\1&1&1&1\end{bmatrix}\end{array}$$

① **Construct** the digraph represents this relation.

② **Construct** a linked list representation, VERT, TAIL, HEAD and NEXT for the relation R.

❹ Given the **second row** of an extended **Prufer code** of a labeled tree on 8 nodes as, 2321020. **Construct** the first row, and **find** the tree represents this code.

The End..
Dr. Y.M. Hamada

Third Question - (20 Marks)

❶ **Change** the lower index of the sum $\sum_{k=1}^{k=n} a_{k-1}\, a_{n-k}$ to start with $k = 3$

❷ **If** $u_n = \frac{1}{2}\, u_{n-1}$, $u_0 = 1$, **prove** that $u_n = \left(\frac{1}{2}\right)^n$.

❸ **Prove** that if n is odd, then $\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$, [Hint: put $n = 2k + 1$]

❹ **If** $f(x) = 1 - x^2$, **prove** that $\text{Lim}_{n\to\infty}\left[\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)\right] = 2/3$

Fourth Question - (20 Marks)

❶ **Construct** the tree of the following mathematical expression:

$\left(3 - \left(2 \times (x \times 5)\right)\right) + \left(x + (x \times y)\right)$, and then find

① Number of **levels**, ② Number of **edges**, ③ **Height** of the tree.

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❷ Show that if the ***check digit*** is correct or not for the 10-ISBN

❸ Let $A = \{a, b, c, d\}$, and let R be a relation on A such that:

$$M_R = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

① **Construct** the digraph represents this relation.

② **Construct** a linked list representation, VERT, TAIL, HEAD and NEXT for the relation R.

The End..
Dr. Y.M. Hamada

Second Question (10- Marks)

Choose the correct answer where the first choice represents A and the second is B and so on...

[21]	The infinite series $\frac{2}{10^0}+\frac{2}{10^1}+\cdots+\frac{2}{10^n}+..$ is	{geometric, arithmetic}
[22]	The general term of the sequence: $-1,-3,-5,-7,\ldots$ is	$\{1-2n, -2n, 1-n\}$
[23]	$-10 \bmod 3 + 10 \bmod 3 = \cdots$	{3,-3, 0}
[24]	The infinite series $3+\frac{3}{4}+\cdots+\frac{3}{4^n}+..$ is	{divergent, convergent}
[25]	If $f(x): \mathcal{R} \to \mathcal{R}^+ : x \to e^x$ the f is	{Onto, Bijective}
[26]	$2\lfloor \overset{-4}{-2.5} \rfloor + 3\lfloor 2.5 \rfloor = \cdots$	{2, -2, 0}
[27]	The range of the function $f(x) = Log(x)$ is	$\{R, R^+, R^-\}$
[28]	If $\overset{y=x^2-3x}{f(x)} = x^2 - 3x,\ x \geq 1.5$, has an inverse, then $f^{-1}(0)$ equals	{3, 0, 4}
[29]	If $f(x) = 2x + a$ have the inverse $f^{-1}(x) = (x+3)/2$, then a equals...	{-3, 3, 2}
[30]	The number of ways in which 2 persons can be selected from a group of 6 persons is:	{30, 15, 20}

$f(g^{-1}) = 2\frac{(x+3)}{2} + a = x$

10/6

Third Question (50- Marks distributed as follows: 8; 7; 7; 7; 7; 7; 7).

❶ Use **Taylor series** to **approximate** the function $f(x) = \sqrt[3]{3x+1}$ to just three terms.

❷ **Prove** that: $\sinh^{-1}x = \text{Ln}\left(x + \sqrt{x^2+1}\right)$.

❸ **Show** that $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_0, a_1, \ldots, a_n \in \mathcal{R}$ is $O(x^n)$.

❹ **Express** the function $f(t)$ by the **unit step function**, where $f(t) = \begin{Bmatrix} t^2+7 & 0 \leq t < 1 \\ t^2 & 1 \leq t < 3 \\ 7 & t \geq 3 \end{Bmatrix}$.

❺ Find the **horizontal** and **vertical** asymptotes of the function and then **sketch** the graph, $f(x) = \frac{3x-9}{x-2}$.

Find the **domain** and the **range** of f.

❻ Use **mathematical induction** to prove that: $1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$, $n = 1, 2, \ldots$

❼ Give three **different** examples of **odd** functions and then give their plots.

(انتهت الأسئلة)

لجنة الممتحنين:

(١) أ.د.م/ ياسر محمد عبد الستار

(٢) أ.د. محمد حلمي مهران

(٣) أ.د.م/ عبد المنعم مجاهد

(٤) د. سامية محمود محمد

ملحوظة

السؤال الأول والثاني يتم إجابته في الجزء اليمين في النموذج الإلكتروني للإجابة بنفس التسلسل من ١-٣٠ في حين أن السؤال الثالث يتم إجابته في كراسة الإجابة المائية

Course:	Discrete Mathematics		Suez Canal University
Time:	Three Hours		Faculty of Computer& Informatics
Date:	June-2014		First Semester 2014-2015

Final Examination -- تخلفات
Discrete Mathematics (BS-103)

First Question - (22 Marks)

❶ Calculate the value of: $x = \lfloor -1.4 \rfloor - 3\lfloor 3.1 \rfloor - 3\lceil -1.7 \rceil \times (17 \; mod \; 3)$.

❷ Let f be the function from $X = \{1,2,3,4\}$ to X defined by $f(x) = 2x \; \textbf{mod} \; 3$. Write f as a set of ordered pairs. Check if the function f is <u>one to one</u> , <u>onto</u> or both?

❸ Expand $(x^2 + 1)^3$.

Second Question - (22 Marks)

❶ Show that $p \to q \equiv \bar{q} \to \bar{p}$

❷ Let $A_i = \{i+2 \,,\; i+3,\; i+4\}$. Find $\bigcup_{i=1}^{4} A_i$ and $\bigcap_{i=1}^{4} A_i$

❸ If $X = \{1,2,3\}$, $Y = \{1,2,3,4\}$

Define the relation R from X to Y by $(x,y) \in R$ where $x < y$

❹ Department consists of 10 men and 5 women. How many ways to conform a committee consists of 4 persons provided that at least 2 men are selected?

Third Question- (16 Marks)

❶ Find the fourth term in the expansion $(2x^3 - 3y^2)^5$.

❷ Prove that $(xy)^n = x^n y^n$ using mathematical induction.

❸ Solve the recurrence relation $s_n = 2s_{n-1}$ subject to the initial condition $s_0 = 1$

Fourth Question- (20 Marks)

❶ Construct the tree of the following mathematical expression:

$$\big(((2 \times 7) + x) \div y\big) \div (3 - 11)$$

❷ Find the inverse of the function $f(x) = x^2$ for $x \geq 0$, and then find $f^{-1}(4)$

❸ Find $(f \circ g)(0)$ and $(g \circ f)(1)$ for the following functions

$f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$

The End..

Dr/Yasser.M.Hamada

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 22-1-2017; Time: 3- Hours

Final Examination
Discrete Mathematics BS - 103

First Question (20 Points)

❶ **Calculate** the values of x that satisfy the equation: $x^2 + \lfloor -4.7 \rfloor x + (13 \bmod 7) = 0$.

❷ **Solve** the 2^d order **difference** equation: $a_n = 7a_{n-1} - 10a_{n-2}$, with $a_0 = 5, a_1 = 16$.

❸ **Change** the lower index of the sum $\sum_{k=1}^{k=n} a_{n-k}$ to start with $k = 3$.

Second Question (20 Points)

❶ **Graph** the function: $f(x) = \begin{cases} 0 & 0 < t < 2 \\ t - 2 & 2 \le t < 4 \\ 1 & t \ge 4 \end{cases}$, and then **express** it using **unit step** function.

❷ **Solve** the **first** order **difference** equation: $a_n = 2a_{n-1}$, with $a_1 = 5$.

❸ If $f(x) = 1 - x^2$, **prove** that $\mathrm{Lim}_{n\to\infty}\left\{\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)\right\} = \frac{2}{3}$.

Third Question (20 Points)

❶ **Compute** the following sum: $\sum_{k=1}^{k=10}(k^2 + 3k + 2)$.

❷ Find the first four terms of **Taylor series** expansion to approximate the function: $f(x) = \sin x$ about $x_0 = 0$.

❸ Use **binomial theorem** to approximate the function $f(x) = \sqrt[3]{x+1}$ for $|x| < 1$.

Fourth Question (20 Points)

❶ Construct the **tree** of the following mathematical **expression**:

$$\big(x + (11 \times (3 \times 4))\big) \div \big(11 + (3 + 4)\big)$$, and then find

① Number of **levels**, ② Number of **paths**, ③ **Height** of the tree.

... function: $f(x) = x^2 + 8x - 9,\ x \le -4$.

Third Question - (20 Marks)

❶ **Change** the lower index of the sum $\sum_{k=1}^{k=n} a_{k-1}\, a_{n-k}$ to start with $k = 3$

❷ If $u_n = \frac{1}{2}\, u_{n-1}$, $u_0 = 1$, **prove** that $u_n = \left(\frac{1}{2}\right)^n$.

❸ **Prove** that if n is odd, then $\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$, [Hint: put $n = 2k + 1$]

❹ If $f(x) = 1 - x^2$, **prove** that $\text{Lim}_{n\to\infty}\left[\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)\right] = 2/3$

Fourth Question - (20 Marks)

❶ **Construct** the tree of the following mathematical expression:

$\big(3 - (2 \times (x \times 5))\big) + (x + (x \times y))$, and then find

① Number of **levels**, ② Number of **edges**, ③ **Height** of the tree.

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❷ Show that if the *check digit* is correct or not for the 10-ISBN

❸ Let $A = \{a, b, c, d\}$, and let R be a relation on A such that:

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

① **Construct** the digraph represents this relation.

② **Construct** a linked list representation, VERT, TAIL, HEAD and NEXT for the relation R.

The End..
Dr. Y.M. Hamada

Fourth Question (28 Points)

❶ **Construct** the **tree** of the following mathematical **expression**:

$$\big((2 \times x) + (3 -- (4 \times x))\big) + (x - (3 \times 11))$$

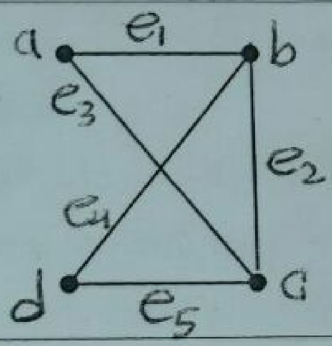
---

❷ For the opposite diagraph:

(1) Find the **adjacency** matrix.

(2) Find the **Laplacian** matrix.

(3) Find the **incident** matrix.



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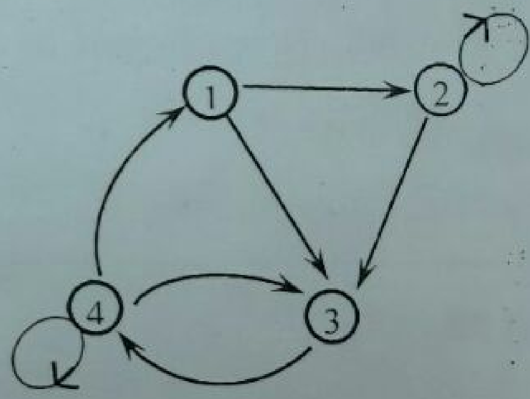
❸ For the opposite graph:

(1) **Write** the ordered pairs for the relation R

(2) **Construct** a linked list representation,

VERT, TAIL, HEAD and NEXT.



The End.

Dr/ Yasser M. Hamada

Final Examination of
Discrete Mathematics BS - 103

First Question (16 Points)

- Calculate the value of $(12 \bmod 5) + h(15) - 3|-4.7| + 2|0.3|$.
- Solve the 1^{st} order difference equation: $a_n = a_{n-1}$, with $a_1 = 4$.
- Change the lower index of the sum $\sum_{k=1}^{n} a_{n-k}$ to start with $k = 0$.
- Find the check digit of the opposite bar code :

4 710088 41255

Second Question (16 Points)

- Find a numerical value of $\sqrt{50}$ approximated to four digits.
- Solve the recurrence relation $P_n = a + s\,P_{n-1}$ of the economic model where, a and s are parameters depend on the model.
- If $f(x) = x^2$. Find the $\text{Lim}_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$.
- Give three different examples of odd functions.

Third Question (20 Points)

- Calculate the numerical value of the summation: $\sum_{k=1}^{k=800}(2k+1)$.
- Department consists of 10 men and 5 women. How many ways to conform a committee consists of 4 persons provided that at least 2 men are selected?
- Find Taylor series expansion (just 4 terms) for the function: $f(x) = \cos x$ about $x = 0$.
- Choose the correct answer:

① $|x| = |x| + 1$ for all $x \in \cdots$	$\{\mathbb{Z}, \mathbb{Q} - \mathbb{Z}, \mathbb{R}\}$
② The function $f(x) = \sin(3x)$ is	{even, odd}
③ If $f(x) = |x + 1|$, then $10 \bmod 5 - 0.5\,f(-1.5)$ equals	{1, 2, 0}
④ A tree with 3 −vertices has exactely .... edges	{2, 4, 8}
⑤ Every relation is a function?	{Yes, No}

الإمتحان يقع في صفحتين من نفس الورقة.

Fourth Question (28 Points)

- **Construct the tree of the following mathematical expression:**

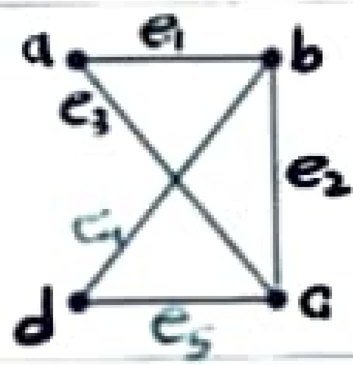
$$\big((2 \times x) + (3 - (4 \times x))\big) + (x - (3 \times 11))$$

- **For the opposite diagraph:**

  (1) Find the adjacency matrix.

  (2) Find the Laplacian matrix.

  (3) Find the incident matrix.

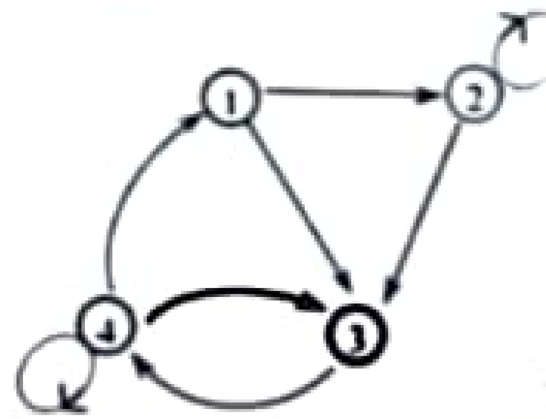


- **For the opposite graph:**

  (1) Write the ordered pairs for the relation *R*

  (2) Construct a linked list representation.

  VERT, TAIL, HEAD and NEXT.



The End.

Dr/ Yasser M. Hamada

Final 2018

First Question :

1- Calculate Value of

$(12 \bmod 5) + h(15) - 3\lfloor -4.7 \rfloor + 2\lceil 2.3 \rceil$

Sol. :-

$15 \bmod 11$

$2 + 4 - 3(-5) + 2(1) = 23$

2- $a_n = a_{n-1}$ $\qquad a_1 = 4$

$a_2 = a_1 = 4$

$a_3 = a_2 = 4$

$a_4 = a_3 = 4$

$\vdots$

$a_n = 4$ #

3- $K = 1$ to $j = 0$ $\quad \therefore K = 1 + j$ at Lower at $K = 1$

$\sum_{k=1}^{k=n} a_{n-k}$ to $K = 0$ $\quad$ at upper: at $K = n$

$\therefore n = j + 1$

$j = n - 1$

$\because$ Sum start from $\underset{\text{Lower}}{k = 1}$ to $\underset{\text{upper}}{K = n}$

$K = 1 \qquad K = n$

$k = 1 + j \qquad K = 1 + j$

Put $K = 1$ $\therefore j = 0$ # $\qquad j = n - 1$ #

$$\sum_{j=0}^{j=n-1} a_{n-(j+1)} = \sum_{j=0}^{n-1} a_{n-j-1} \quad \#$$

(4) $x_{13} = 10 - \left[(x_1 + 3x_2 + x_3 + 3x_4 + \cdots + 3x_{12}) \bmod 10 \right]$

$= 10 - [(4 + 3(7) + 1 + 3(0) + 0 + 3(8) + 8 + 3(4) + 1 + 3(2)$
$+ 5 + 3(3))] \bmod 10$

$= 10 - (91 \bmod 10) = 9$

Second Question :

1- $\sqrt{50} = (49+1)^{1/2} = \left[49\left(1 + \frac{1}{49}\right) \right]^{1/2}$ $\because \left| \frac{b}{a} \right| < 1$

$\therefore \sqrt{50} = \sqrt{49} \left[1 + \frac{\left(\frac{1}{2}\right)}{1!}\left(\frac{1}{49}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{1}{49}\right)^2 \right.$
$\left. + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{1}{49}\right)^3 + \cdots \right] \simeq 7.071$

الحل في الكتاب
Page 184

2- $P_n = a + sP_{n-1}$
$= a + s(a + sP_{n-2}) = a + as + s^2 P_{n-2}$
$= a + as + s^2(a + sP_{n-3}) = a + as + as^2 + s^3 P_{n-3}$
$\vdots$
$a + as + as^2 + \cdots + as^{k-1} + s^k P_{n-k}$

Put $n-k=0$ initial value $\therefore n = k$

$= (a + as + as^2 + \cdots + as^{n-1}) + s^n P_0$

Sum of n terms of geometric series

$S_n = \frac{a(r^n - 1)}{r-1}$, $r = \frac{a_{n+1}}{a_n} = \frac{as^n}{as^{n-1}} = s$ كنا ممكن نوجدها بمجرد النظر

$a = a$

$P_n = \frac{a(s^n - 1)}{s-1} + s^n P_0$ #

(3) if $f(x) = x^2$ Find the $\lim\limits_{n \to 1} \sum\limits_{k=1}^{n} f\left(\frac{k}{n}\right)$

$\because f(x) = x^2$

$\therefore f\left(\frac{k}{n}\right) = \left(\frac{k}{n}\right)^2 = \frac{k^2}{n^2}$

لازم نحلها باستخدام القوانين بولا و (n) عشان نقدر نوصل لها النهاية

$$\lim_{n \to 1} \sum_{k=1}^{n} \frac{k^2}{n^2} = \lim_{n \to 1} \frac{1}{n^2} \sum_{k=1}^{n} k^2$$

$$= \lim_{n \to 1} \left[\frac{1}{n^2} \left(\frac{n(n+1)(2n+1)}{6} \right) \right]$$

Put $n = 1$

$$= \frac{2(3)}{1 \times 6} = 1 \ \#$$

(4) $f(x) = x$ is odd function

$f(x) = x^3$ is odd function

$f(x) = \sin x$ is odd function

$f(x) = x^3 + 5x$ is odd function

$f(x) = \sin x + x^5$ is odd function

Third Question

1- Calculate the numerical value of the summation

$$\sum_{K=1}^{K=800}(2K+1) = 2\sum_{K=1}^{800} K + \sum_{K=1}^{800} 1$$

$$= 2\left[\frac{800(800+1)}{2}\right] + 1(800) = 641600 \ \#$$

شركة ومؤسسة تحتوي على 10 men & 5 women

2- كم عدد الطرق التي يمكن فيها تكوين لجنة من 4 اشخاص

بشرط يكون باللجنة دي فيها على الاقل رجلين

$$C_2^{10}\, C_2^{5} + C_3^{10}\, C_1^{5} + C_4^{10}\, C_0^{5} = 1260$$

احتمال يكون رجلين من 10 رجال و 2 ست من اصل 5

احتمال يكون 3 رجال من اصل 10 وست واحده من 5

احتمال يكونوا الاربع رجال ولا يوجد ست واحده

(3) $f(x) = \cos x$ about $x = 0$

$f(x) = \cos x$	$f(0) = 1$
$f'(x) = -\sin x$	$f'(0) = 0$
$f''(x) = -\cos x$	$f''(0) = -1$
$f'''(x) = \sin x$	$f'''(0) = 0$
$f^{(4)}(x) = \cos x$	$f^{(4)}(0) = 1$
$f^{(5)}(x) = -\sin x$	$f^{(5)}(0) = 0$
$f^{(6)}(x) = -\cos x$	$f^{(6)}(0) = -1$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\,(x)^n}{n!} \;*$$

$$\therefore \cos x = \frac{1}{0!} + \frac{0}{1!} x + \frac{-1}{2!} x^2 + \frac{0}{3!} x^3 + \frac{1}{4!} x^4 + \frac{0}{5!} x^5 + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad \#$$

لأنه طالب 4 حدود ∴ لازم نوجد الحدود تكون غير صفرية
يعني لازم نوجد 4 terms

(4) 1- $\mathbb{Q} - \mathbb{Z}$
2- odd
3- 0
4- 2
5- No

1- كل rational ماعدا الجزء الـ integer فيها
لا لأننا لو فيه كل الاعداد الـ integer ايضاً اعداد ratio

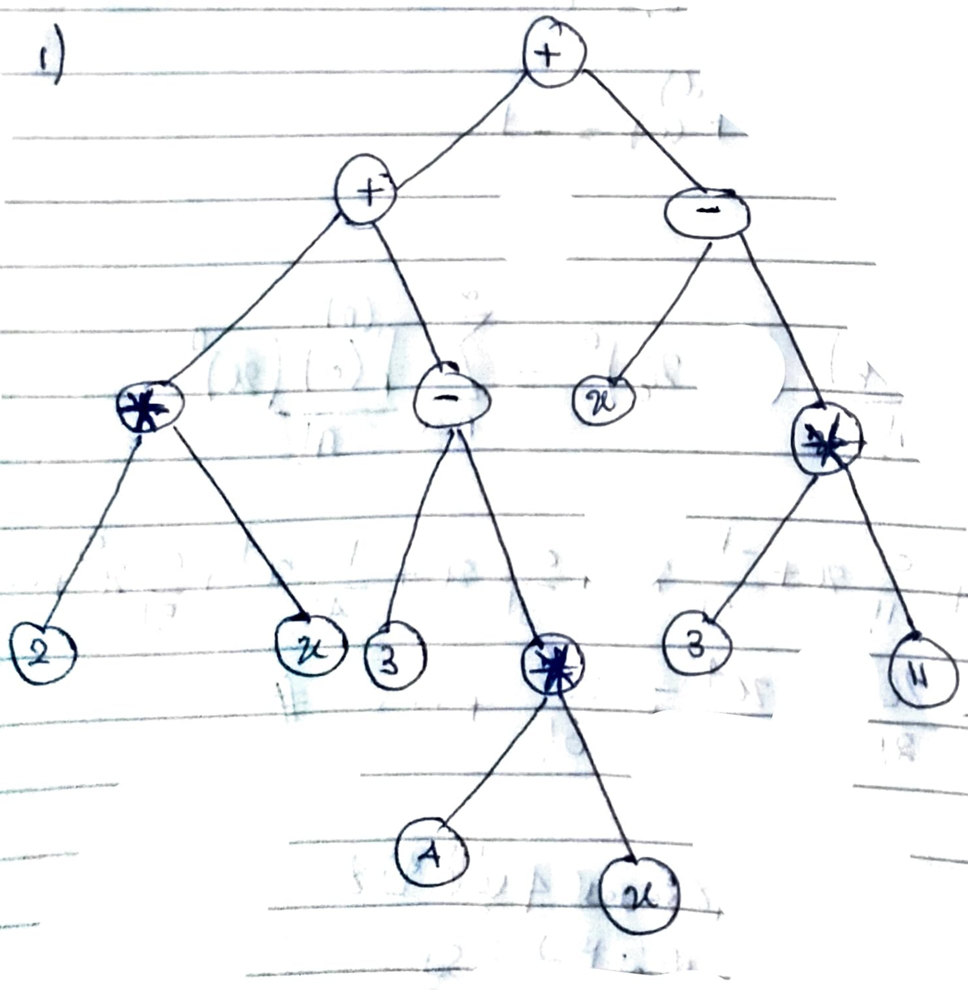
3- $10 \bmod 5 - 0.5 \lfloor(-1.5)\rfloor$

$= -0.5 \lfloor(-1.5)\rfloor = -0.5 \lceil -1.5 + 1 \rceil$
$= -0.5 \lceil -.5 \rceil$
$=$ zero

4- A tree with n vertices has exactly n-1 edges

Fourth Question

1)



(2)

(1) Find incident Matrix

بحط 1 عند كل عنصر والمسار اللي خارج منه

$$
\begin{array}{c|ccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 \\
\hline
a & 1 & 0 & 1 & 0 & 0 \\
b & 1 & 1 & 0 & 1 & 0 \\
c & 0 & 1 & 0 & 1 & 1 \\
d & 0 & 0 & 0 & 1 & 1
\end{array}
$$

(2) بحط 1 بين كل two Vertex بينهم relation

لو مفيش علاقة بين two vertex Zero

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 0 & 1 & 1 & 0 \\
b & 1 & 0 & 1 & 1 \\
c & 1 & 1 & 0 & 1 \\
d & 0 & 1 & 1 & 0
\end{array}
$$

(3) LalPaCiean Matrix

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 2 & -1 & -1 & 0 \\
b & -1 & 3 & -1 & -1 \\
c & -1 & -1 & 3 & -1 \\
d & 0 & -1 & -1 & 2
\end{array}
$$

1- القطر الرئيسي عبارة عن عدد ~~edge~~ edges لكل vertex

2- باقي ال Matrix عبارة عن 0 , 1-

3- لو في علاقة بين two vertex نكتب (1-)

مفيش علاقة نكتب Zero

(3)

(1) $R = \{(2,2), (4,4), (1,2), (1,3), (2,3), (3,4), (4,1), (4,3)\}$. #

ملحوظه حل السؤال الاخير ملغي 😊

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date:28-11-2020; Time: **1.5- Hours**

Module(A) **Mid-Term Examination 2020-2021** **Module(A)**

Discrete Mathematics (BS-103)

First Question (12.5- Marks)

❶ Choose the correct sign "✓ or ×" for the followings:

[1]	$P \vee T$ is always true iff at least one of p or q is true.	(… … …)
[2]	$p \wedge \bar{p}$ is always false	(… … …)
[3]	$\forall x \in \mathcal{R}, p(x): (x^2 - 1) = (x - 1)(x + 1)$	(… … …)
[4]	The domain of a function is contained in the codomain	(… … …)
[5]	$p \to q \equiv \bar{p} \to \bar{q}$	(… … …)
[6]	$\overline{\bar{p} \vee F} = p$	(… … …)
[7]	$\bar{p} \vee q \equiv \overline{p \wedge \bar{q}}$	(… … …)
[8]	$P \wedge T \equiv T$	(… … …)
[9]	$\forall x \in \mathcal{R}, \quad p(x): x^2 > x$	(… … …)
[10]	Let $P(x, y, z): xy < x + z + 1$, then $p(x, x, x)$ is always true $\forall x \in z$	(… … …)
[11]	$\exists x \in \mathcal{R}, p(x): x^2 - 5x + 6 = 0$	(… … …)
[12]	If p is $4 \geq 2$ and q is "$5 \leq 2$" then $p \oplus q$ is true	(… … …)
[13]	$(A \subset B) \wedge (B \subset A) \Leftrightarrow A = B$	(… … …)
[14]	The relation RUR^{-1} refers to reflexive closure.	(… … …)
[15]	$\overline{p \wedge F} = T$	(… … …)

❷ Choose the correct answer for the following statements:

[1]	The identity function, $f: \mathcal{R} \to \mathcal{R}^+, f(x) = x$ is	{one to one, onto, both}
[2]	If $f: \mathcal{R} \to \mathcal{R}^+, f(x) = 2^{x^2}$, then f is not	{one to one, onto, both}
[3]	The range of the function $f(x) = 3\cos(2x - 1)$ is	$\{[-1, 1], [-3, 3], [-2, 2]\}$
[4]	For the exponential function $f: \mathcal{R} \to \mathcal{R}^+, f(x) = a^x, a \in \cdots$	$\{\mathcal{N}, z^+, z^+ - \{1\}\}$
[5]	The domain of the function $f(x) = \mathrm{Log}(x)$ is ….	$\{\mathcal{R}, \mathcal{R}^+, \mathcal{R}^-\}$
[6]	If p: 2 is a positive integer and q: is $\sqrt{2}$ is a rational number, then $p \wedge q$ is true.	{True, False}
[7]	$A - \overline{B} = \cdots$	$\{A \cup B, A \cap B, A - B\}$
[8]	$A \oplus B = (A \cup B) - (A \cap B)$	{True, False}
[9]	If $R_2 = R \cup R^{-1}$, then R_2 should be ….	{reflexive, transitive, symmetric}
[10]	The domain of the function $f(x) = 3\cos(2x - 1)$ is	$\{\mathcal{R}^+, \mathcal{R}\}$

Solve behind the page if necessary.

Second Question (12.5- Marks)

❶ Behind this paper, use the **laws of logic** to show that the statement: $(p \wedge \bar{q}) \wedge (\bar{p} \vee q) \wedge r$ is always false.

❷ Prove by contradiction that if $a^2 - 2a + 7$ is even, then a is even.

Third Question (15- Marks)

❶ Use indirect proof to prove that if x^2 is odd, then x is odd

❷ If $X = \{2, 3, 4\}$ and $Y = \{4, 5, 6, 8\}$,

① define the relation R from X to Y which defined by X divides Y,

② give the matrix of the relation R relative to the ordering $3, 4, 2$ and $\{5, 6, 8, 4\}$,

③ show that if R is reflexive, symmetric or transitive,

④ if ③ is not satisfied use the closure concept to make it reflexive, symmetric, transitive.

(انتهت الأسئلة)

Solve behind the page if necessary.

Mid-Tearm Module A

Question 1 :

1) True

2) True

3) True $(x-1)(x+1) = x^2 + x - x - 1 = x^2 - 1$

4) False

5) False Since $F \to T \equiv T$ but $T \to F \equiv F$ not equivalent

6) True $\overline{\bar{P} \vee F} \equiv \bar{\bar{P}} \wedge \bar{F} \equiv P \wedge T \equiv T$

7) True $\overline{P \wedge \bar{q}} \equiv \bar{P} \vee \bar{\bar{q}} \equiv \bar{P} \vee q$

8) False

9) False $\left(\frac{1}{2}\right)^2 > \frac{1}{2}$ $\left(\frac{1}{4}\right) > \frac{1}{2}$ False

10) False $x^2 - 2x - 1 < 0$

$+ + + + \quad - - - - \quad + + +$

$1-\sqrt{2} \qquad 1+\sqrt{2}$

$x_1 = 1 - \sqrt{2}$, $x_2 = 1 + \sqrt{2}$

$P(x)$ is True at $x \in \,]1-\sqrt{2}, 1+\sqrt{2}[$

$P(x)$ is false at $x \in R - \,]1-\sqrt{2}, 1+\sqrt{2}[$

11) True Since at $x = 2, 3$ $P(x)$ is True

$\therefore$ for some $x \in R$ $P(x) = 0$

12) True $\quad p : 4 \geq 2 \equiv T$

$q : 5 \ngeq 2 \equiv F$

$p \oplus q \equiv T$

13) True

14) False Symmetric closure

15) True

$\overline{p \wedge F} \equiv \bar{p} \vee \bar{F} \equiv \bar{p} \vee T \equiv T$

$\overline{p \wedge F} \equiv \overline{F} \equiv T$

2

1) both

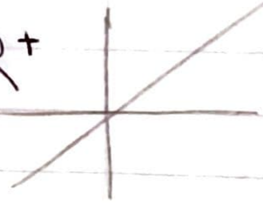
one-to-one

onto since Codomain $= R^{+}$

والدالة متصلة ويمكن كلها صور لعناصر x

في R^{+} كلها

$f(x) = x$



2) one-to-one

$x = \pm 1 \quad f(x) = 2$

$x = \pm 2 \quad f(x) = 2^{4}$, ...

3) $[-3, 3]$

4) $Z^{+} - [1$ ٤]

5) R^{+}

6) False $\quad p : 2$ Positive $\equiv T \quad q : \sqrt{2}$ rational $\equiv F$

$p \wedge q \equiv$ False

7) $A \cap B$

8) True

9) Symmetric

10) R

Question 2:

1)

$$(P \cap \bar{q}) \cap (\bar{p} \vee q) \wedge r \equiv$$

$$\overline{\overline{(P \cap \bar{q})}} \cap (\bar{p} \vee q) \cap r \equiv$$

$$\overline{(\bar{p} \vee q)} \cap (\bar{p} \vee q) \wedge r \equiv$$

$$F \wedge r \equiv F$$

2) $P: a^2 - 2a + 6$ is even

q : a is even

by Contradiction $\bar{q}$: a is odd

$a = 2k + 1$ $k = 0, 1, 2, \ldots$

$$a^2 - 2a + 6 = (2k+1)^2 - 2(2k+1) + 6$$
$$= 4k^2 + 4k + 1 - 2(2k+1) + 6$$
$$= 4k^2 + 4k + 1 - 4k - 2 + 6$$
$$= 4k^2 + 5$$
$$= 4k^2 + 4 + 1$$
$$= 4(k^2 + 1) + 1$$

or. $2(2k^2 + 2) + 1$

$\therefore$ $a^2 - 2a + 6$ is odd

This is Contradiction
Since we suppose $a^2 - 2a + 6$ is even

$\therefore$ a is even

Question 3:

use indirect Proof:

if x^2 is odd Then x is odd

Proof

indirect Proof $\bar{q} \longrightarrow \bar{P}$

let P : x^2 is odd

$\bar{P}$: x^2 is even

q : x is odd

$\bar{q}$: x is even

$\bar{q}$: x is even $\quad x = 2K \quad K = 0, 1, 2, \dots$

$x^2 = (2K)^2 = 4K^2 = 2\underbrace{(2K^2)} \quad K = 0, 1, 2, \dots$

let $2K^2 = M$ integer

$x^2 = 2M$ is even

$\bar{q} \longrightarrow \bar{P} \equiv P \longrightarrow q$

2)

1) $R = [(2,4), (2,6), (2,8),$
$(3,6),$
$(4,4), (4,8)]$

2) $Y \rightarrow$ / $X \downarrow$

$$\begin{array}{c|cccc} & 5 & 6 & 8 & 4 \\ \hline 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \end{array}$$

1) *) not reflexive

since $(2,2), (3,3), (5,5), (6,6), (8,8) \notin R$

$xRx \quad \forall x \in X, Y$

*) not symmetric

since $(2,4), (2,6), (2,8), (3,6), (4,8) \in R$

but $(4,2), (6,2), (8,2), (6,3), (8,4) \notin R$

if $xRy \in R$

then $yRx \in R$

$\forall x, y \in X, Y$

if xRy & yRz then xRz

Transitive $\quad \forall x, y, z \in X, Y$

$$(2,4)\begin{cases}(4,4) \longrightarrow (2,4) \in R \\ (4,8) \longrightarrow (2,8) \in R\end{cases}$$

(2,6) , (2,8) ملهاش زوج يكملها
(3,6)

$(4,4)$ & $(4,8) \longrightarrow (4,8) \in R$

* ∴ Transitive

لازم لو وجدنا $(x,y),(y,z)$ في العلاقة

لازم نلاقي ـــ تالتهم

reflexive
Closure ~~symmetric~~ $\qquad xRx \quad \forall x \in \text{set}$

$$R_1 = R \cup \Delta$$

$$= \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8)\}$$
$$\cup \{(2,2),(3,3),(5,5),(6,6),(8,8)\}$$

$$R_1 = \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8),(2,2),(3,3),(5,5),(6,6),(8,8)\}$$

Closure symmetric

$$R_1 = R \cup R^{-1}$$

$$= \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8)\}$$
$$\cup \{(4,2),(6,2),(8,2),(6,3),(8,4)\}$$

$$R_1 = \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8),(4,2),(6,2),(8,2),(6,3),(8,4)\}$$

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 28-11-2020; Time: **1.5- Hours**

Module(B)

Mid-Term Examination 2020-2021
Discrete Mathematics (BS-103)

Module(B)

First Question (12.5- Marks)

❶ Choose the correct sign "✓ or ✗" for the followings:

[1]	$\neg[\forall x\ P(x)] \equiv \exists x \neg P(x)$	(… … …)
[2]	$p \wedge \bar{p}$ is always true	(… … …)
[3]	$\forall x \in \mathcal{R}, p(x): (x^3 - 1) = (x-1)(x^2+x+1)$	(… … …)
[4]	The domain of a function is contained in the codomain	(… … …)
[5]	If n is an integer then $(\forall\ n)(3n+1 > n)$	(… … …)
[6]	$\overline{p \to q} = p \wedge \bar{q}$	(… … …)
[7]	If $R: X \to Y$ is a relation, then its inverse is $R^{-1}: Y \to X$	(… … …)
[8]	$P \wedge T \equiv T$	(… … …)
[9]	The relation "less than" is reflexive	(… … …)
[10]	Let $P(x,y,z): xy < x + z + 1$, then $p(x, x, x)$ is always false $\forall x \in z$	(… … …)
[11]	$\exists x \in \mathcal{R}, \quad p(x): \dfrac{x^2-1}{2x+1} \geq 0$	(… … …)
[12]	If p is $4 \geq 2$ and q is "$5 \leq 2$" then $p \oplus q$ is false	(… … …)
[13]	The range of the exponential function is $\mathcal{R}$	(… … …)
[14]	The relation $R \cup R^{-1}$ refers to reflexive closure.	(… … …)
[15]	$\overline{p \wedge F} = T$	(… … …)

❷ Choose the correct answer for the following statements:

[1]	The Logarithmic function $f(x) = \text{Log}_a(x), f: \mathcal{R}^+ \to \mathcal{R}$, is	{one to one, onto, both}
[2]	If $f: \mathcal{R} \to \mathcal{R}^+, f(x) = 2^{x^2}$, then f is not	{one to one, onto, both}
[3]	The range of the function $f(x) = 5\sin(2x-1) + 2$ is	$\{[-1,1], [-3,7], [-3,3]\}$
[4]	For the exponential function $f: \mathcal{R} \to \mathcal{R}^+, f(x) = a^x, a \in \cdots$	$\{\mathcal{N}, z^+, z^+ - \{1\}\}$
[5]	The range of the function $f(x) = \text{Log}(x)$ is ….	$\{\mathcal{R}, \mathcal{R}^+, \mathcal{R}^-\}$
[6]	$1 + \cot^2(x) = \cdots$	$\{cosec^2(x),\ sec^2(x)\}$
[7]	The domain of $y = \tan(x)$ is $\mathcal{R} - \{\ldots + k\pi\},\ \ k \in z$	$\{\frac{\pi}{2}, \pi,\ 2\pi\}$
[8]	$A - \bar{B} = \cdots$	$\{A \cup B, A \cap B, B - A\}$
[9]	$A \oplus B = (A \cup B) - (A \cap B)$	{True, False}
[10]	The domain of the function $f(x) = 5\sin(2x-1) + 2$ is	$\{\mathcal{R}^+, \mathcal{R}, [-1,1]\}$

Solve behind the page if necessary.

Second Question (12.5- Marks)

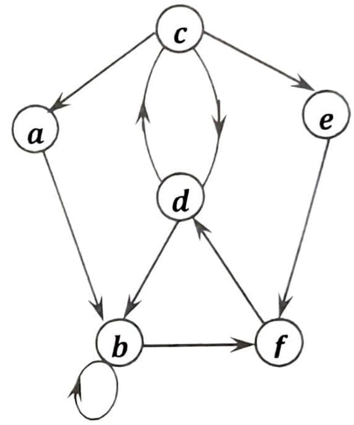
❶ Behind this paper, prove that $\sinh^{-1} x = \text{Ln}\left(x + \sqrt{x^2 + 1}\right)$

❷ Use the laws of logic to simplify: $\overline{p \to (p \wedge q)}$

Third Question (15- Marks)

① If $x = 2$, then $3x - 5 \neq 10$. Prove that this statement is true by contradiction.

② From the opposite diagraph



❶ Write R as an ordered pairs,

❷ List all paths of length 2 starting from vertex c,

❸ Find the symmetric closure of R,

❹ Find the reflexive closure of R,

❺ Find the matrix A for the diagraph (or the relation)

(انتهت الأسئلة)

Solve behind the page if necessary.

Module (B)

Question 1:

(1) True

(2) False $\quad p \wedge \bar{p} \equiv F$

(3) True $\quad (x-1)(x^2+x+1) = x^3 + x^2 + x - x^2 - x - 1$

$= x^3 - 1$

(4) False $\quad$ Range of fun. is Contained in the Codomain

(5) False $\quad n \in \mathbb{Z} = \{\dots, -1, 0, 1, 2, \dots\}$

at $n = -1 \qquad -3 + 1 > -1$

$-2 > -1$ False

(6) True $\quad \overline{p \to q} \equiv \overline{\bar{p} \vee q} \equiv \bar{\bar{p}} \wedge \bar{q} \equiv p \wedge \bar{q}$

(7) True

(8) False $\quad p \wedge T \equiv p$

(9) False

Since less than

$(1,2), (1,3), (1,4), \dots$

$(2,3), (2,4), (2,5), \dots$

$\vdots$

all ordered Pairs $\in R$

but $(1,1), (2,2), (3,3), \dots \notin R$

$\therefore R$ not reflexive

$\boxed{10}$ False

$$P(x,y,z) : xz < x + z + 1$$
$$P(x,x,x) : x^2 < 2x + 1$$
$$x^2 - 2x - 1 < 0 \longrightarrow (*)$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = 1 \pm \sqrt{2}$$

+ + +   - - - - - - - - -   + + + +

-1 $1 - \sqrt{2} \simeq -0.4$ 2 $1 + \sqrt{2} \simeq +2.4$ 3

في الفترة اللي هتخلي المعادلة (*) اقل من Zero يعني
تحقق المتباينة

at $x = (-1)$
$$(-1)^2 - 2(-1) - 1 = 2 > 0$$ مبتحققش

at $x = 2$
$$(2)^2 - 2(2) - 1 = -1 < 0$$ بتحقق

at $x = 3$
$$(3)^2 - 2(3) - 1 = 2 > 0$$ مش بتحقق

$\therefore$ $P(x)$ is True at $x \in \,]1-\sqrt{2}, 1+\sqrt{2}[$
$P(x)$ is False at $x \in R - \,]1-\sqrt{2}, 1+\sqrt{2}[$

الاجابة False لانها مش دايماً False لكل قيم x

11 True

Since $\exists x \in R$

بعض قيم الـ x

اللى بتنتمى للأعداد الحقيقة

$\therefore$ at $x=0$, $\frac{x^2-1}{2x+1} = \frac{-1}{1} = -1 \ngeq 0$ مش هتتحقق

at $x=2$ $\frac{(2)^2-1}{2(2)+1} = \frac{4-1}{4+3} = \frac{3}{7} \geq 0$ متحقق

$\therefore$ بعض القيم لـ x تتحقق مش شرط كل القيم

12) False

$p : 4 \geq 2$ $\quad p \equiv T$

$q : 5 \leq 2$ $\quad q \equiv F$

$T \oplus F \equiv T$ هنا بيطلع True لو الاتنين مختلفين

13) False $\quad D_F = \overset{a}{R} \quad R_F = R^+$

14) False $\quad R \cup R^{-1}$ refers to symmetric closure

15) $\overline{P \wedge F} \equiv \overline{F} \equiv T$

True

Question 2:

1) both

2) one-to-one

3) $[-3, 7]$

4) $Z^+ - [1\ 4]$

5) R

6) $\operatorname{Cosec}^2(x)$

7) $\frac{\pi}{2}$

8) $A \cap B$

9) True

10) R

$$\sin^2 x + \cos^2 x = 1$$

$\div \sin^2 x$

$$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$

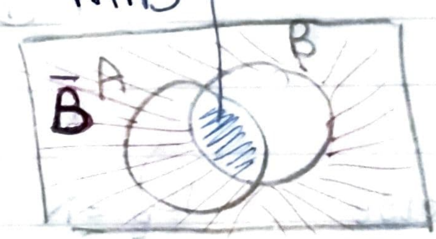
$\div \cos^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

داخل A ولكنه ليس داخل $\bar{B}$

$A \cap B$



$A - \bar{B}$: موجود في A مش موجود في $\bar{B}$

∴ الموجود في A ومش موجود في $\bar{B}$

هو الجزء التقاطع

1. Prove that $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$

let $\sinh^{-1} x = y$

$x = \sinh y$

$$x = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y} \qquad \times e^y$$

$$2xe^y = e^{2y} - 1$$

$$e^{2y} - 2xe^y - 1 = 0 \qquad \text{let } e^y = z$$

$$z^2 - 2xz - 1 = 0 \qquad z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2x) \pm \sqrt{4x^2 - 4}}{2}$$

$$z_{1,2} = \frac{2x \pm \sqrt{4(x^2-1)}}{2} = \frac{2x \pm 2\sqrt{x^2-1}}{2} = \frac{2(x \pm \sqrt{x^2-1})}{2}$$

$$z_{1,2} = x \pm \sqrt{x^2-1}$$

Substitute z by e^y

$$e^y = x \pm \sqrt{x^2-1}$$

$$y = \ln(x \pm \sqrt{x^2-1})$$

$$= \ln(x + \sqrt{x^2-1})$$

Since $\ln$(negative value) undefined

(2) use Laws of Logic to simplify

$$\overline{p \to \overline{(p \cap q)}}$$ use $p \to q \equiv \bar{p} \vee q$

$$\equiv \overline{\bar{p} \vee \overline{(p \cap q)}}$$ using deMorgen's Law

$$\equiv \bar{\bar{p}} \cap \overline{\overline{(p \cap q)}} \equiv p \cap (p \cap q)$$

$$\equiv (p \cap p) \cap q \equiv p \cap q \ \#$$

Question Three:

1) if $x = 2$ Then $3x - 5 \neq 10$

Prove that this statement is Tree by Contraduction

Proof:

let $P: x = 2$

$q: 3x - 5 \neq 10$

$\bar{q}: 3x - 5 = 10$

Start with $\bar{q}$

$3x - 5 = 10 \rightarrow 3x = 15 \rightarrow x = 5$

This Contradiction with $P = 2$

2) 1) $R = \{ (c,a), (c,d), (c,e), (a,b), (d,b), (d,c), (b,f), (e,f), (f,d), (b,b) \}$

2)

$\pi_1 : c, a, b$

$\pi_2 : c, d, c$

$\pi_3 : c, d, b$

$\pi_4 : c, e, f$

$\pi_5 : c, d, c$

3) Symmetric closure of R

$$R_1 = R \cup R^{-1}$$

$$= \{ (c,a), (c,d), (c,e), (a,b), (d,b), (d,c), (b,f), (e,f), (f,d), (b,b) \}$$

$$\cup \{ (a,c), (d,c), (e,c), (b,a), (b,d), (c,d), (f,b), (f,e), (d,f), (b,b) \}$$

$$= \{ (a,c), (c,a), (c,d), (d,c), (c,e), (e,c), (a,b), (b,a), (d,b), (b,d), (d,c), (c,d), (b,f), (f,b), (e,f), (f,e), (f,d), (d,f), (b,b) \}$$

4) reflexive closure of R

$R_1 = R \cup \Delta$

$= \{ (c,a), (c,d), (c,e), (a,b), (d,b),$
$(d,c), (b,f), (e,f), (f,d), (b,b) \}$

$\cup$

$\{ (a,a), (b,b), (c,c), (d,d), (e,e), (f,f) \}$

$R_1 = \{ (c,a), (c,d), (c,e), (a,b), (d,b), (d,c), (b,f), (e,f)$
$(f,d), (b,b), (a,a), (c,c), (d,d), (e,e), (f,f) \}$

$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 1 & 0 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 & 0 & 1 \\ c & 1 & 0 & 0 & 1 & 1 & 0 \\ d & 0 & 1 & 1 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 0 & 1 \\ f & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

الأسم إسلام محمد محمد علي

Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 13-1-2019; Time: 3- Hours

Final Examination of
Discrete Mathematics BS - 103

الامتحان يقع فى ورقة من صفحتين.

First_Question (20- Marks)

Fill the circle with the appropriate signs "✓ for the circle A" or "x for the circle B"

[1] The day of the week will it be after 100 days from Saturday is **Monday**. (...........)

[2] If $R_2 = R \cup R^{-1}$, then R_2 should be **symmetric**. (...........)

[3] If $x \bmod y = r$, then y **divides** $x - r$ (...........)

[4] $0 \bmod 7 = 1$ (...........)

[5] The functions $f(x) = \lceil x \rceil$ and $f(x) = \lfloor x \rfloor$ map from $\mathcal{R}$ to z is **one to one**. (...........)

[6] The **horizontal** asymptotes of the function $y = 1/x$ is the line $y = 1$. (...........)

[7] The **Big O** notation is used to give an upper bound of the running time of an algorithm. (...........)

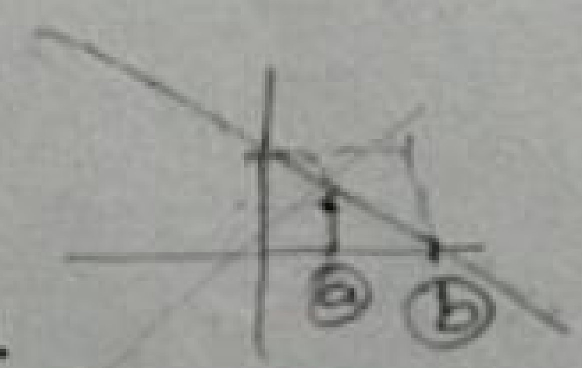
[8] If $f: R \to R^+, f(x) = \sin(x^2)$, then f is **one to one** function. (...........)

[9] If $f: R^+ \to R, f(x) = \mathrm{Log}(x)$, then f is **onto** function. (...........)

[10] The **domain** of the function $f(x) = 2\cos(x)$ is R. (...........)

[11] The **range** of the function $f(x) = 3\sin(2x - 1) - 3$ is $[-3, 3]$. (...........)

[12] The **vertical asymptotes** of the function $y = \frac{1}{x-1}$ is the **line** $x = 0$. (...........)



[13] If $a, b \in D_f$, $a < b$, and $f(a) < f(b)$, then f is **increasing** function. (...........)

[14] If $f(x) = x$ is the **identity function**, then $f^{-1}(x) = x$. (...........)

[15] The infinite series $\frac{2}{10} + \frac{2}{10^2} + \cdots + \frac{2}{10^n} + ..$ is **divergent.** (...........)

[16] $\lfloor x \rfloor = \lceil x \rceil - 1$ for all $x \in z^+$. (...........)

[17] **Big O notation** is used to describe how closely a series approximates a given function. (...........)

$-1 - 2n + 2 = 3 - 2n = -1 + -2(n-1)$

[18] The **general term** of the sequence: -1, -3, -5, … is of the form: $(1 - 2n)$. (...........)

[19] If the function $f(x)$ have an **inverse**, then $(f \circ f^{-1})(x)$ should equals $1/x$ (...........)

[20] The **number of ways** for a number of two digits that can be formed from $\{1, 2, 3, 8\}$ without repeating is 16. (...........)

ملحوظة

السؤال الأول والثانى يتم إجابتة فى الجزء اليمين فى النموذج الإلكترونى للإجابة بنفس التسلسل من 1-30 فى حين أن السؤال الثالث يتم إجابتة فى كراسة الإجابة العادية.

Second Question (10- Marks)

Choose the correct answer where the first choice represents A and the second is B and so on…

[21]	The infinite series $\frac{2}{10^0}+\frac{2}{10^1}+\cdots+\frac{2}{10^n}+..$ is $\frac{1}{10}$	{geometric, arithmetic}
[22]	The general term of the sequence: $-1,-3,-5,-7,\ldots$ is	$\{1-2n, -2n, 1-n\}$
[23]	$-10 \bmod 3 + 10 \bmod 3 = \cdots$	{3,-3, 0}
[24]	The infinite series $3+\frac{3}{4}+\cdots+\frac{3}{4^n}+..$ is	{divergent, convergent}
[25]	If $f(x): \mathcal{R}\to\mathcal{R}^+ : x \to e^x$ the f is	{Onto, Bijective}
[26]	$2\lceil -2.5\rceil + 3\lfloor 2.5\rfloor = \cdots$ $(2\times-2)+3\times 2=$	{2, -2, 0}
[27]	The range of the function $f(x)=Log(x)$ is	{ R, R^+, R^- }
[28]	If $f(x)=x^2-3x,\ x\geq 1.5$, has an inverse, then $f^{-1}(0)$ equals	{3, 0, 4}
[29]	If $f(x)=2x+a$ have the inverse $f^{-1}(x)=(x+3)/2$, then a equals…	{-3, 3, 2}
[30]	The number of ways in which 2 persons can be selected from a group of 6 persons is:	{30, 15, 20}

Third Question (50- Marks distributed as follows: 8; 7; 7; 7; 7; 7; 7).

❶ Use **Taylor series** to **approximate** the function $f(x)=\sqrt[3]{3x+1}$ to just three terms. $x_0=0$

❷ **Prove** that: $\sinh^{-1}x = \text{Ln}\left(x+\sqrt{x^2+1}\right)$.

❸ **Show** that $f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$, where $a_0,a_1,\ldots,a_n\in\mathcal{R}$ is $\boldsymbol{O(x^n)}$.

❹ **Express** the function $f(t)$ by the **unit step function**, where $f(t)=\begin{cases} t^2+7 & 0\leq t<1 \\ t^2 & 1\leq t<3 \\ 7 & t\geq 3 \end{cases}$.

❺ Find the **horizontal** and **vertical** asymptotes of the function and then **sketch** the graph, $f(x)=\frac{3x-9}{x-2}$.

Find the **domain** and the **range** of f.

❻ Use **mathematical induction** to prove that: $1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6},\ n=1,2,\ldots$

❼ **Give** three **different** examples of **odd** functions and then give their plots.

(انتهت الأسئلة)

لجنة الممتحنين:

(١) أ.د.م/ ياسر محمد عبد الستار (٢) أ.د. محمد حلمى مهران

(٣) أ.د.م/ عبد المنعم مجاهد (٤) د. سامية محمود محمد

$\frac{1}{x^3}$ $\sqrt{x}$ $\frac{-1}{x}$

$\cancel{f(-x)=\sqrt{-x}}$ $f(-x)=\frac{1}{-x}=-\frac{1}{x}=-f(x)$