Lecture 8

Chapter 5

Graphs and Trees

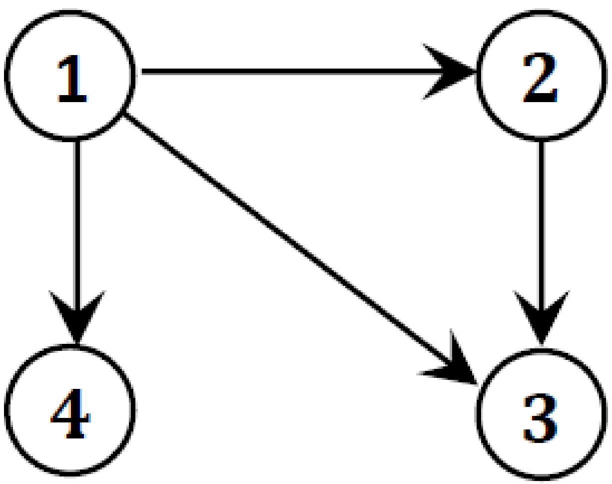
Graphs

The **graph G** is a pattern consists of

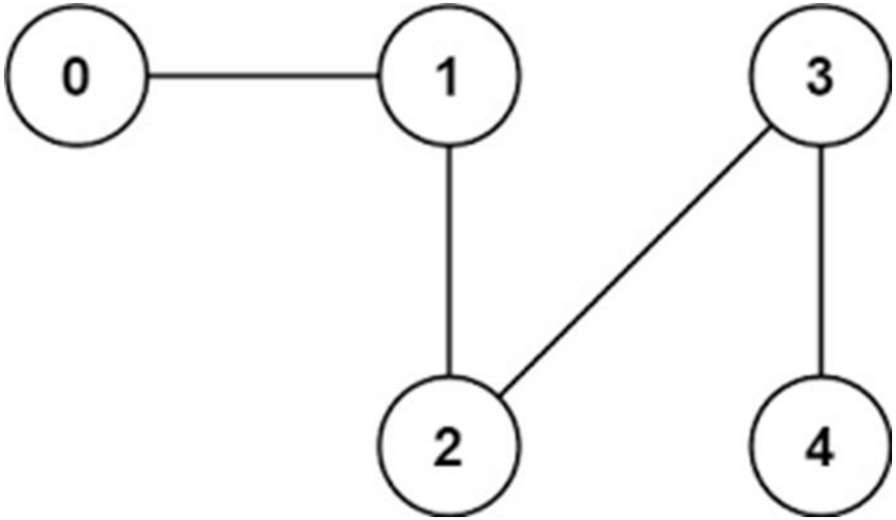
finite set of **vertices (*V*, set of vertices)**

with

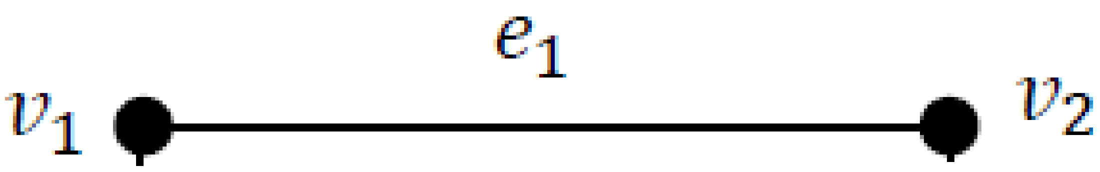
finite set of lines called **edges** (***E*, set of edges**).



Path: is a tour start from v_1 to v_n.



Edge: the edge $\boldsymbol{e_1}$ associated with the ordered pair (v_1, v_2)

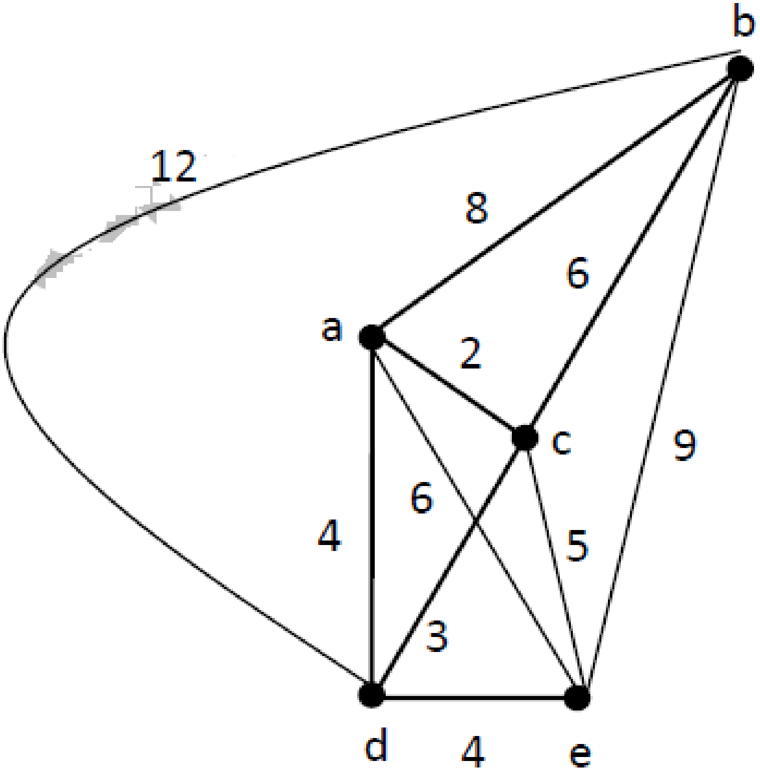


Loop: an edge where its endpoints are at the **same vertex**

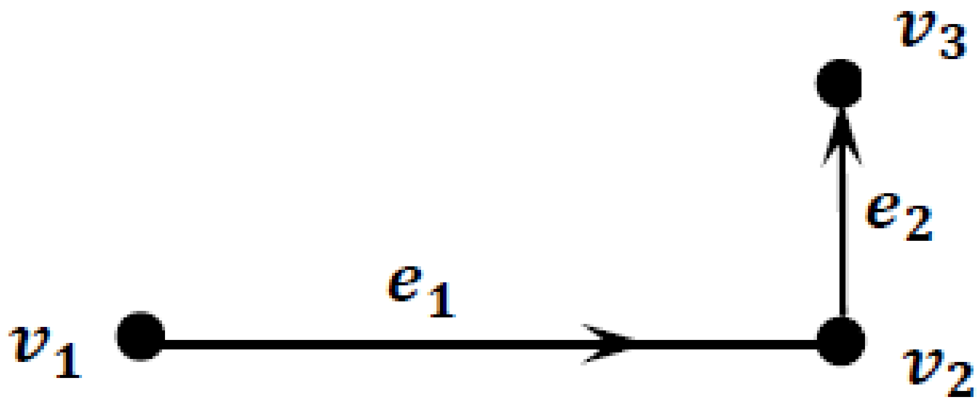
Weighted Graph: is a graph with numbers on edges

this number may be

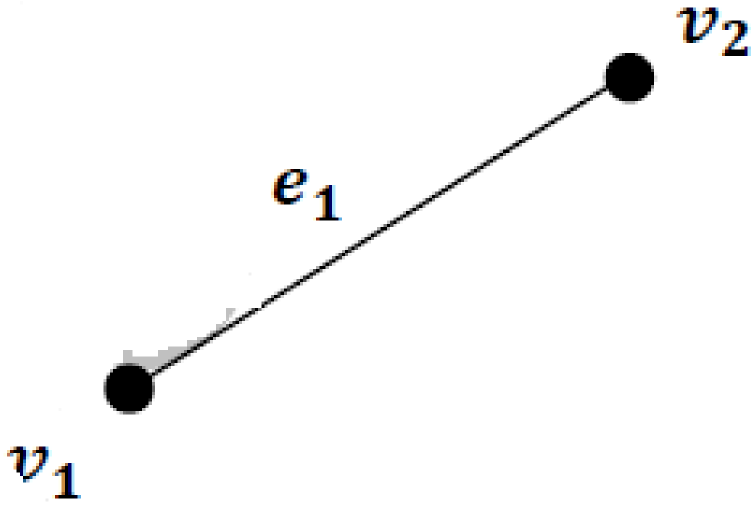
length, or **time**.



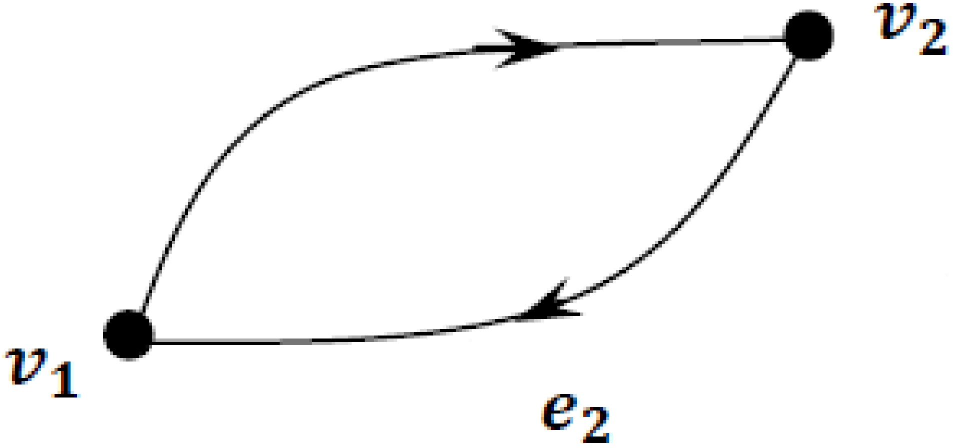
Directed graph: is the graph whose edges have a direction



undirected edge



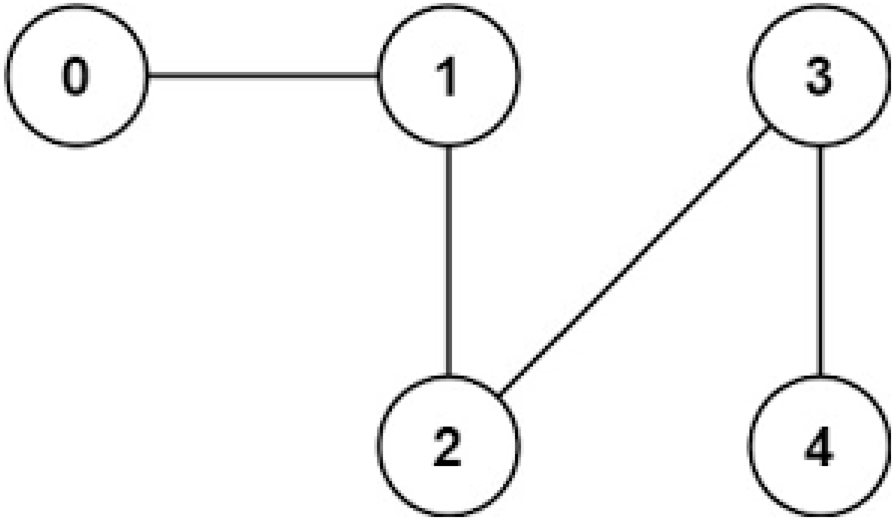
Parallel edge



Length of the path

is the **number of edges** of the path

length = 4

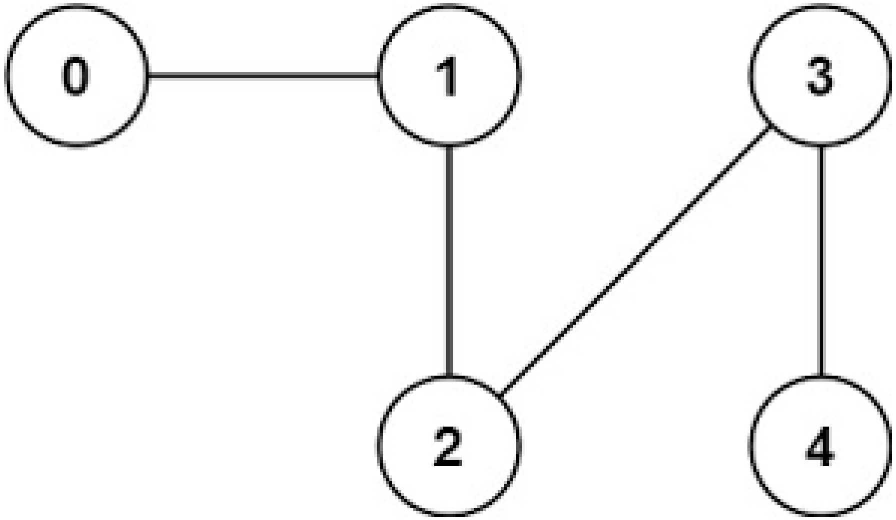


A **path** from v_0 to v_n

length = $\boldsymbol{n}$

Edges= $\boldsymbol{n}$

vertices = $\boldsymbol{n}$+1



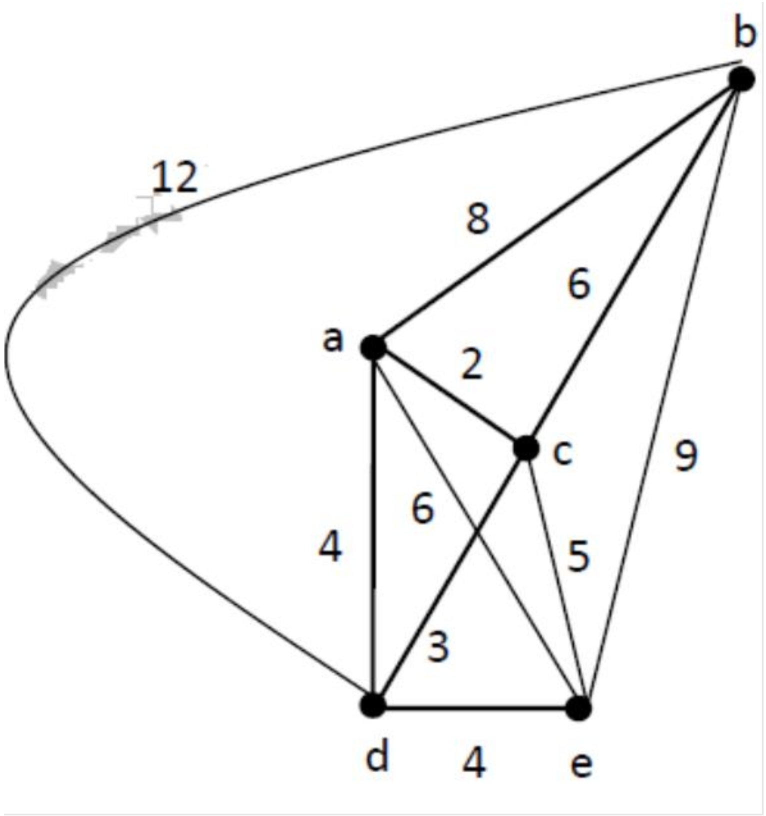
length = 4

Edges= 4

vertices = 5

degree of the vertex $\boldsymbol{v}$

it is the number of edges connected with it



$d(a)=4$

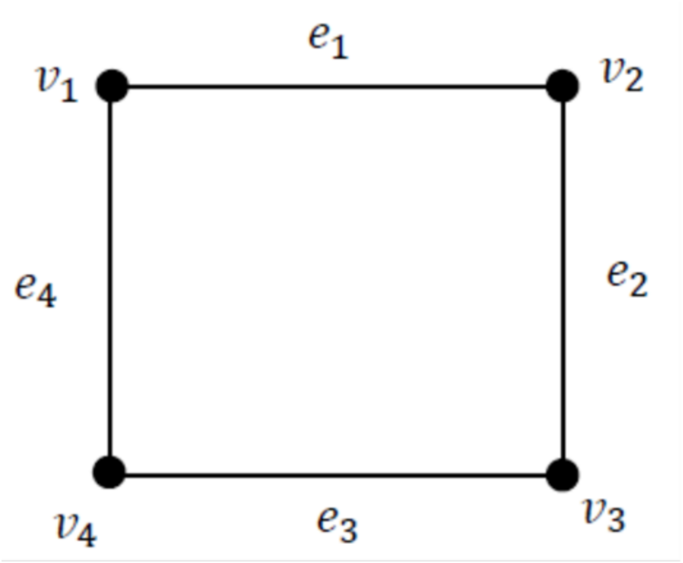
$d(c)=4$

$d(d)=4$

Let G=(V,E) be a graph.

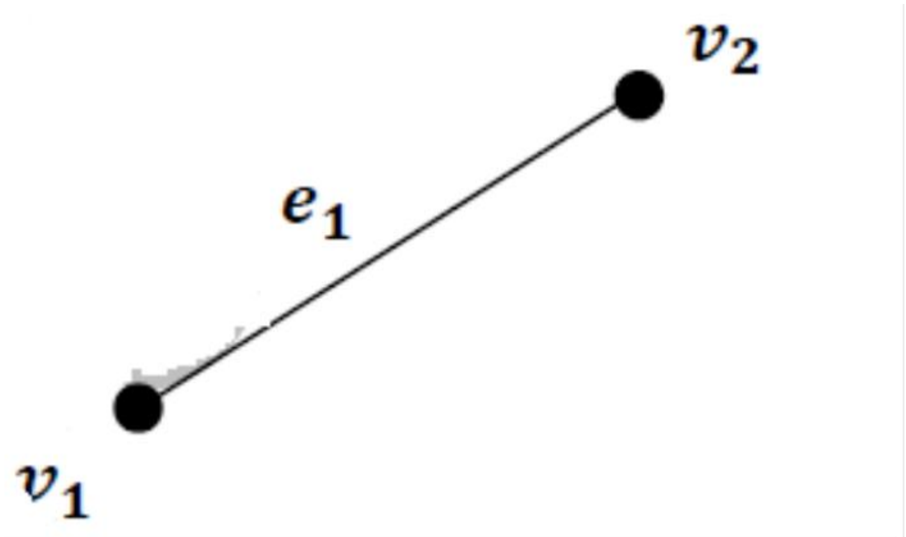
Then the sum of the **degrees of all vertices** is equal to twice the **number of edges**

$$\sum_{v \in V} \deg(V) = 2|E|$$

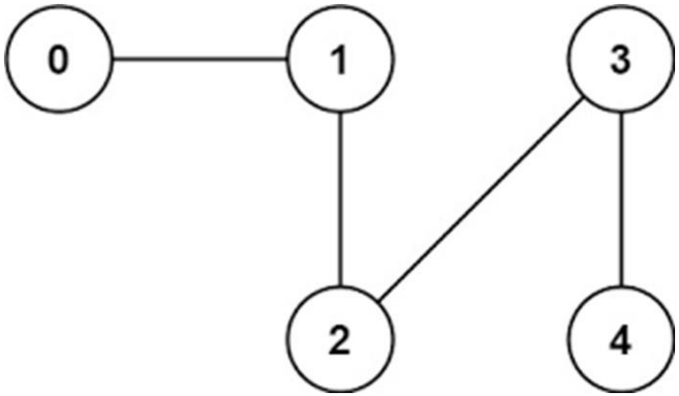


Adjacent vertices

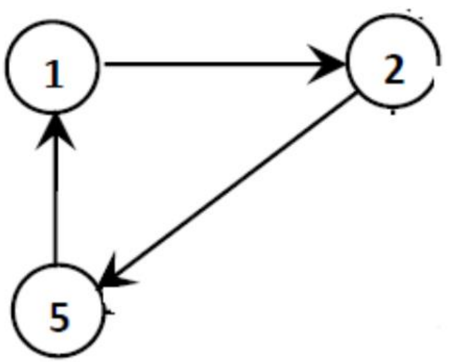
The two vertices v and w are said to be **adjacent** if there is an **edge** joining them.



- **A simple path** from v to w is a path from v to w with **no repeated vertices**



- A path that begins and ends at the same vertex is called a **cycle**



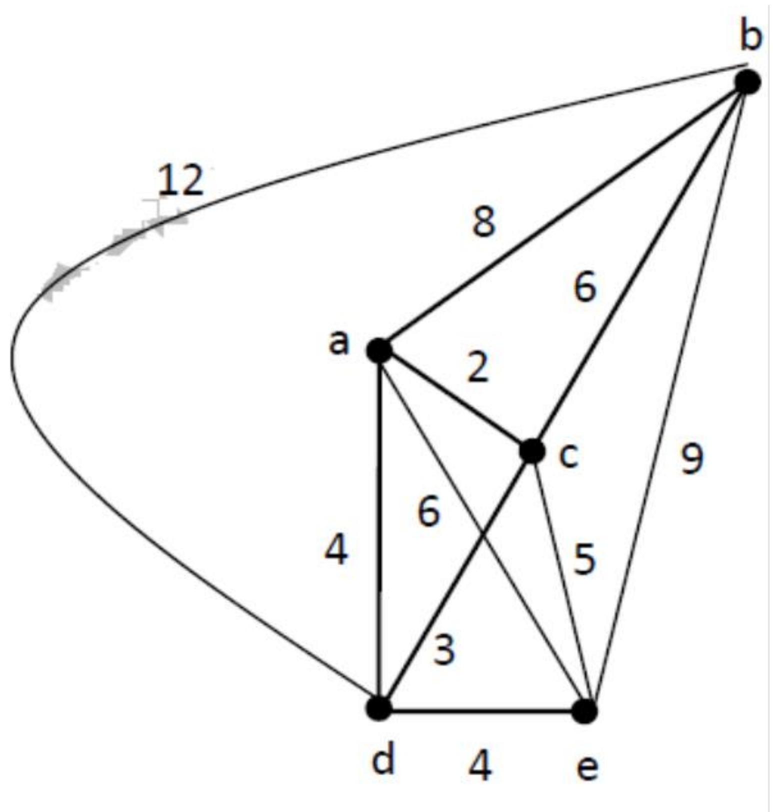
Path	Simple Path?	Cycle?	Simple Cycle?
(6, 5, **2**, 4, 3, **2**, 1)	No	No	No
(6, 5, 2, 4)	Yes	No	No
(**2**, 6, 5, **2**, 4, 3, **2**)	No	Yes	No
(**5**, 6, 2, **5**)	No	Yes	Yes
(7)	Yes	No	No

Optimal Path

A path of **minimum length** that visits every vertex exactly **one time**

mple

Paths	Length
a, b, c, d, e	21
a, b, d, c, e	28
a, c, b, d, e	24
a, c, d, b, e	26
a, d, b, c, e	27
a, d, c, b, e	22

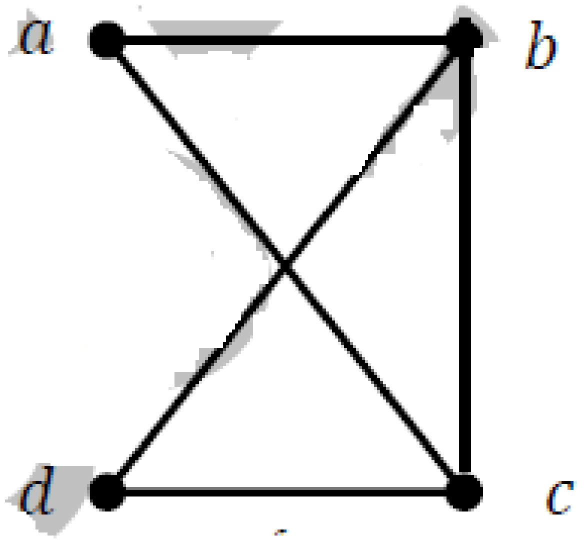


Thus, the **optimal path** is ***abcde*** which equals **21**

Matrix Representation

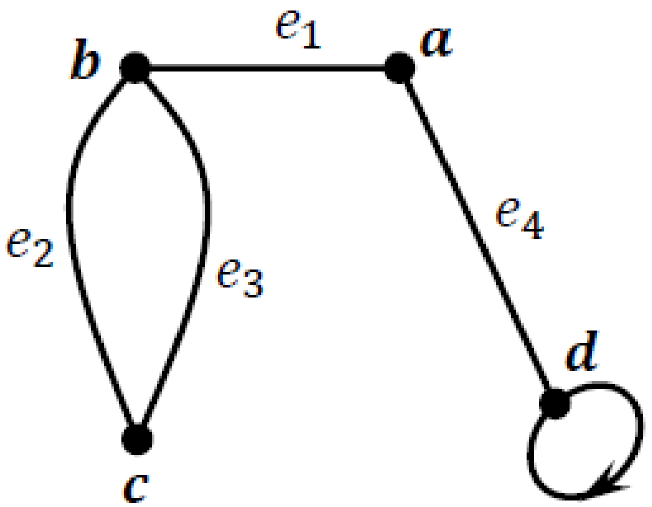
1- Adjacency Matrix

$$A = \begin{array}{c} \\ \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{array} \begin{array}{c} \begin{array}{cccc} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} & \boldsymbol{d} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



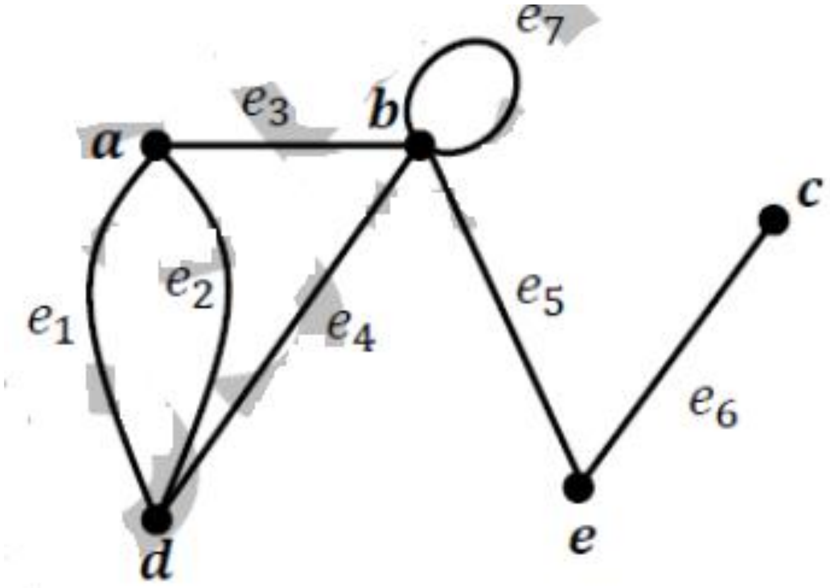
2- Incident Matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



Incident Matrix

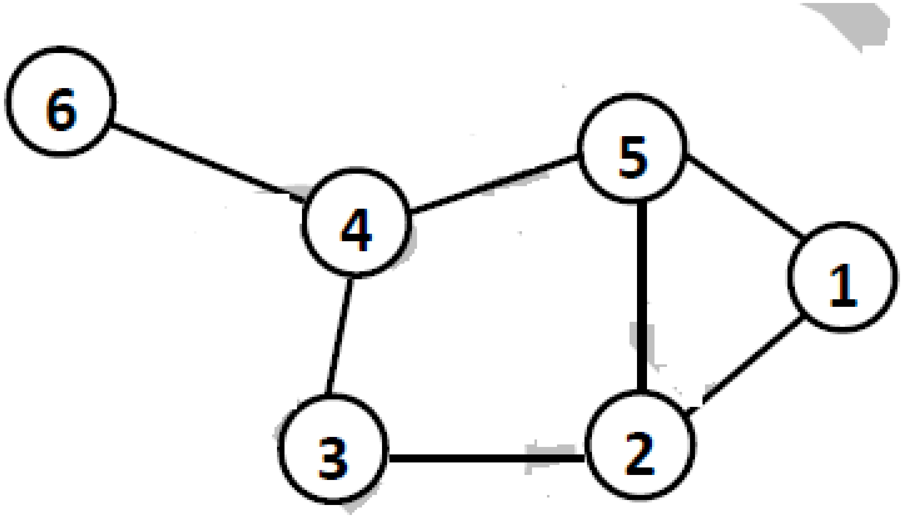
$$\begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline a & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ c & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ d & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$



3- Laplacian Matrix

$$L = \left[\ell_{ij}\right]$$

$$\ell_{ij} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

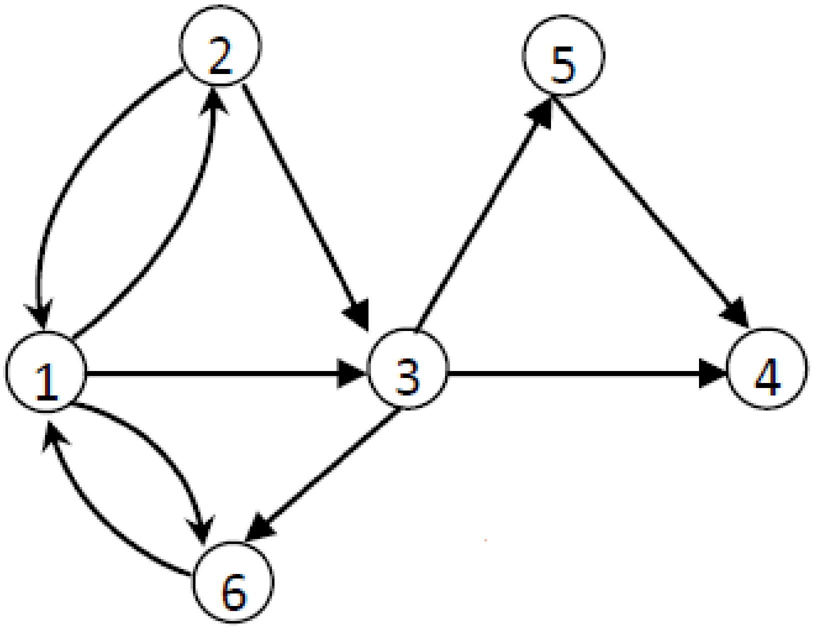


$$\begin{array}{c c}
 & \begin{array}{cccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \end{array} \\
\begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \end{array} &
\begin{bmatrix}
2 & -1 & 0 & 0 & -1 & 0 \\
-1 & 3 & -1 & 0 & -1 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 3 & -1 & -1 \\
-1 & -1 & 0 & -1 & 3 & 0 \\
0 & 0 & 0 & -1 & 0 & 1
\end{bmatrix}
\end{array}$$

Methods of Storing Data

Linked List representation

Construct a linked list representation, **VERT**, **TAIL**, **HEAD** and **NEXT** for the relation *R*



الخارج من ال Vertex

Vertex 1

(1,2), (1,3), (1,6)

Vertex 2

(2,1), (2,3)

Vertex 3

(3,4), (3,5), (3,6)

Vertex 4

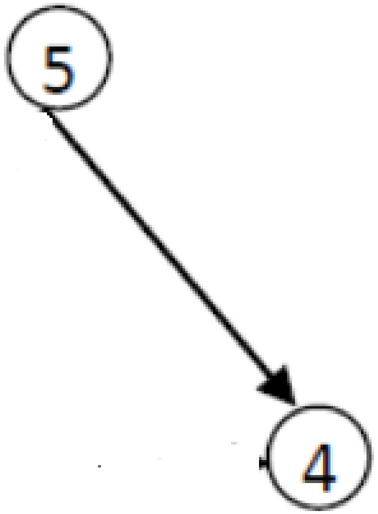
(,)

Vertex 5

(5,4)

Vertex 6

(6,1)



VERT	TAIL	HEAD	NEXT
1	1	2	2
4	1	3	3
6	1	6	0
0	2	1	5
9	2	3	0
10	3	4	7
	3	5	8
	3	6	0
	5	4	0
	6	1	0

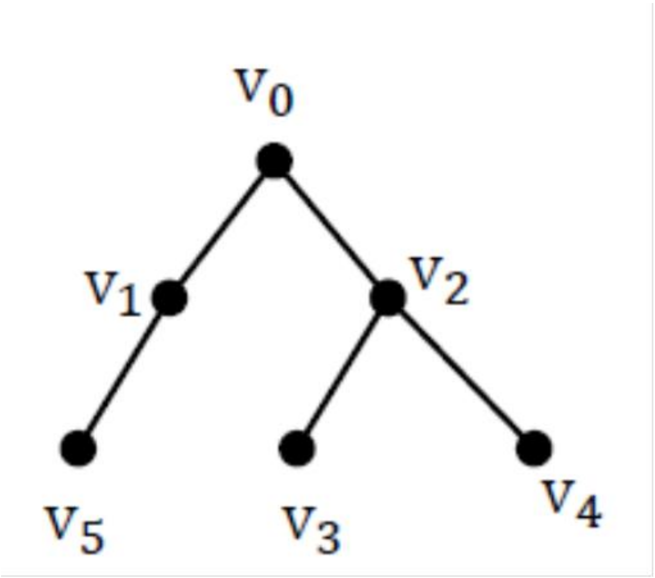
VERT	TAIL	HEAD	NEXT
10	**1**	**2**	**0**
2	2	3	3
4	2	1	0
0	3	5	6
5	5	4	0
8	3	4	7
	3	6	0
	6	1	0
	1	**6**	**1**
	1	**3**	**9**

VERT	TAIL	HEAD	NEXT
9	1	2	0
3	2	3	0
6	2	1	2
0	3	5	7
5	5	4	0
8	3	4	4
	3	6	0
	6	1	0
	1	6	10
	1	3	1

Trees

The Tree

An undirected diagraph is called a tree if each **pair of distinct vertices** has **exactly one path**

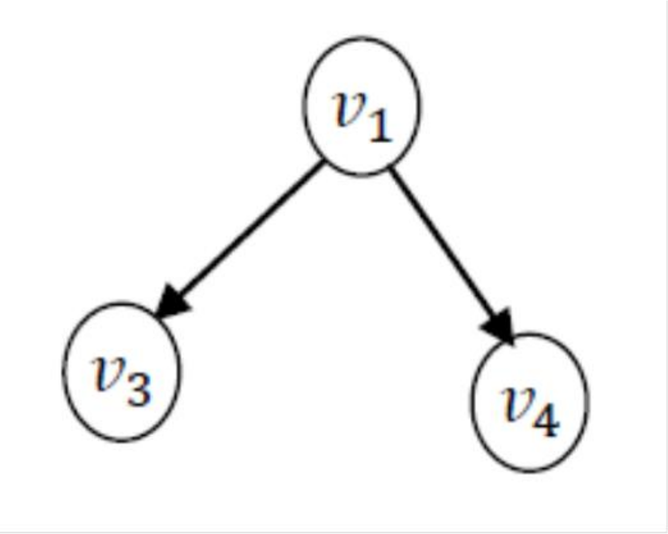


a tree has **no parallel edges**

and **no loops**

Binary tree

Each vertex has at **most two children**

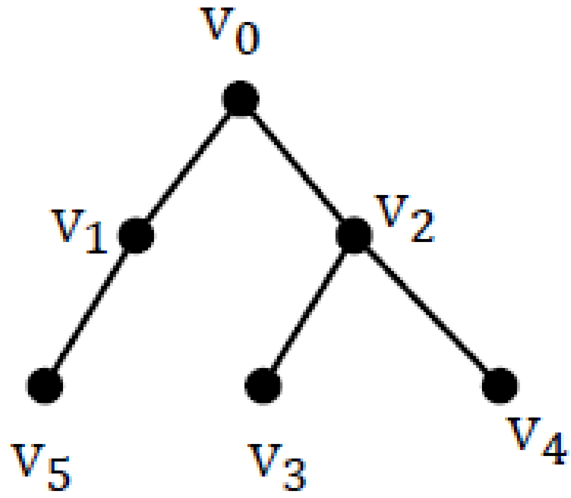


Theorem

A tree with $\boldsymbol{n}$ vertices has exactly $\boldsymbol{n-1}$ edges.

Vertices = 6

Edges = 5

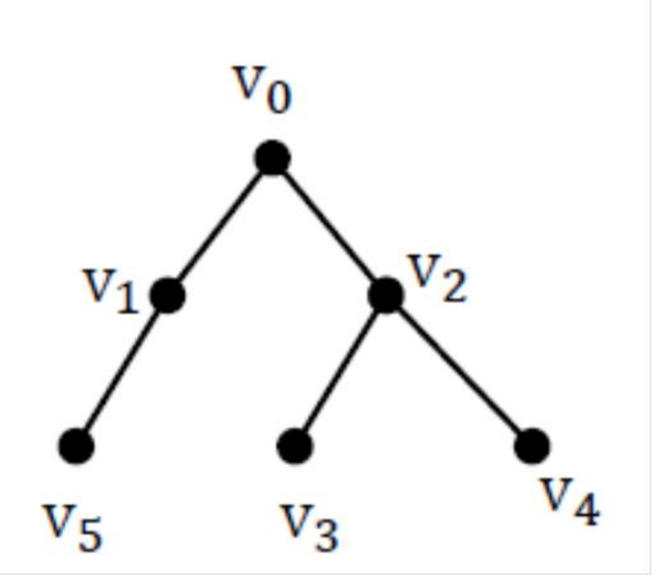


labeled tree

vertices are labeled with numbers $(0,1,2,...,n-1)$ as names

A rooted tree

A tree with a specified root (**labeled**)



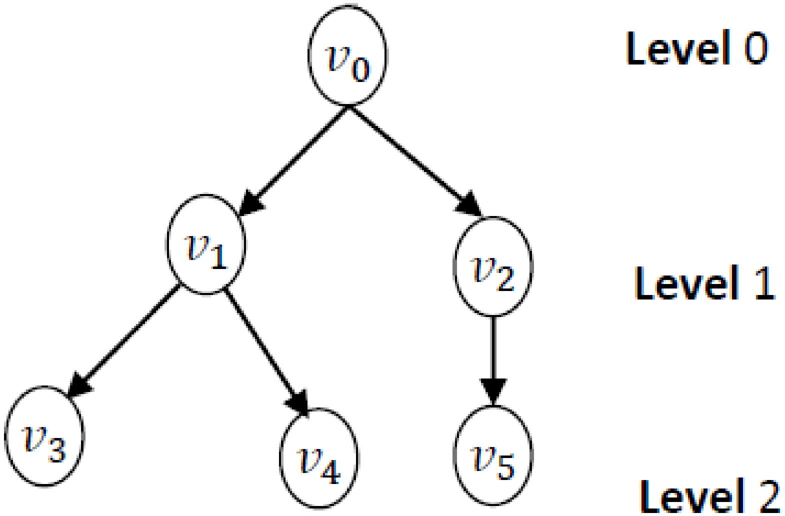
Since each vertex is specified with **a label**

Levels of the tree

Level-0 has the vertex v_0

Level-1 has the vertex v_1, v_2

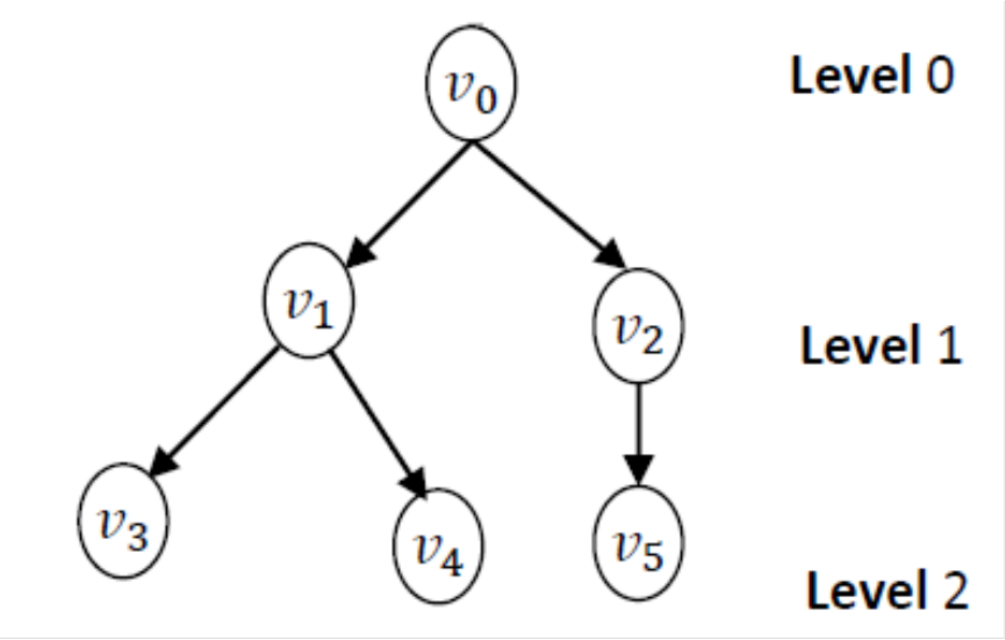
Level-2 has the vertex v_3, v_4, v_5



height of the tree

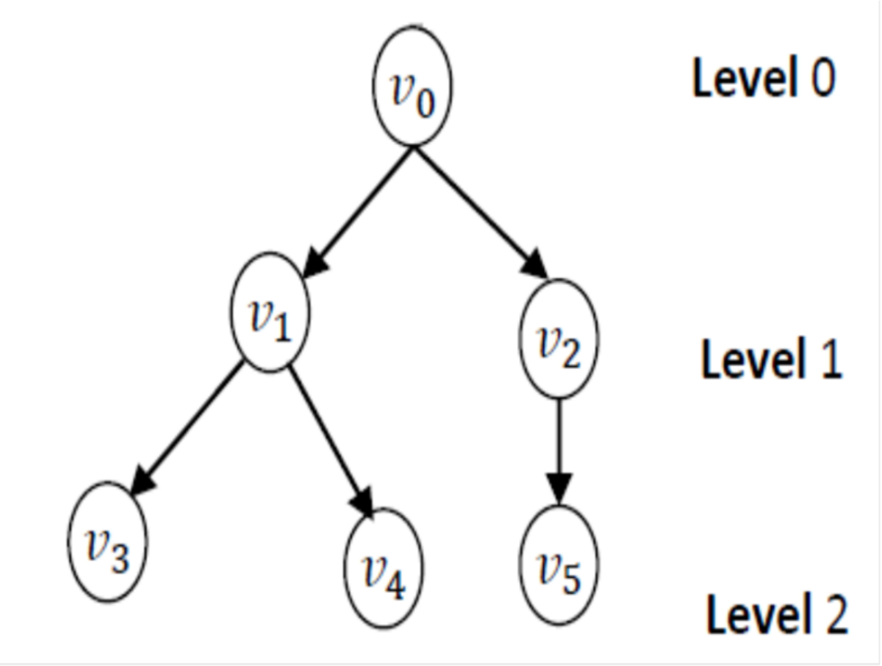
The largest level number of the tree

Height = 2



leaves of the tree

v_3, v_4, v_5 are called leaves of the tree

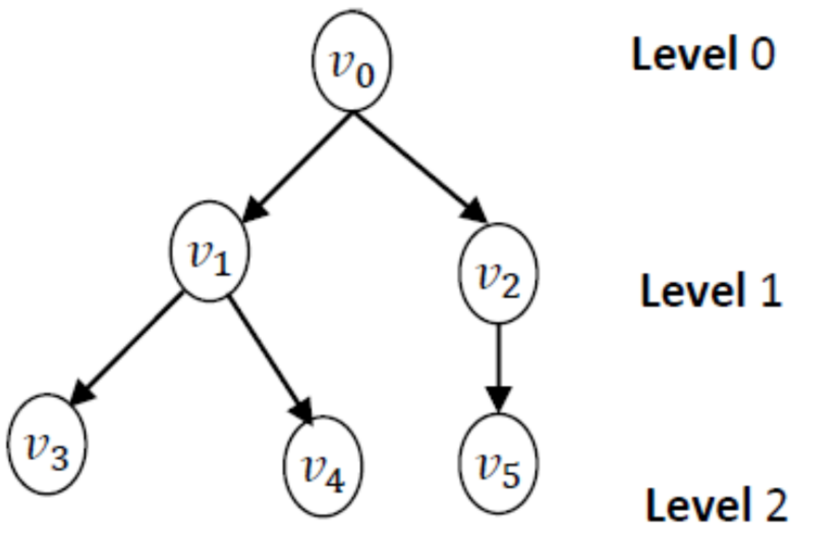


The parent

v_0 is the parent of v_1, v_2

v_1 is the parent of v_3, v_4

v_2 is the parent of v_5

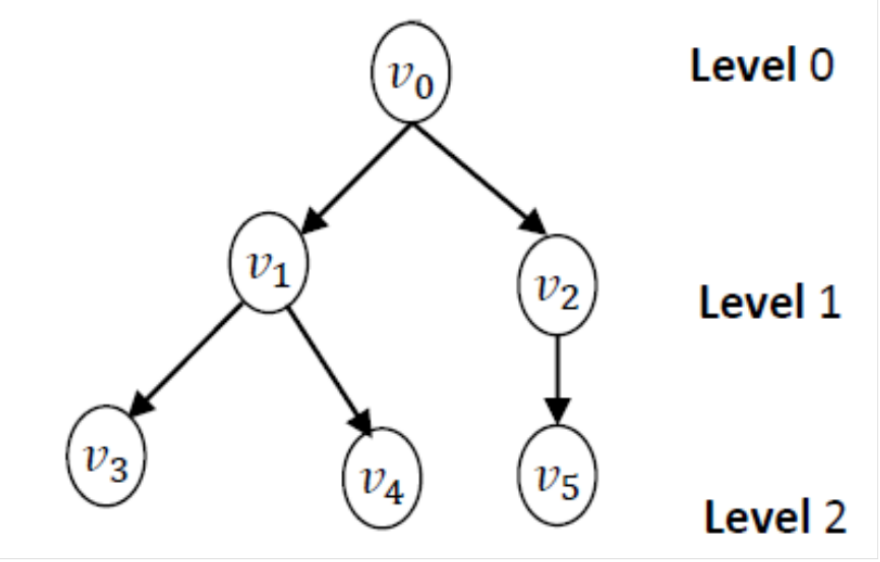


The children

the children of v_0 is $v_1, \ v_2$

the children of v_1 is $v_3, \ v_4$

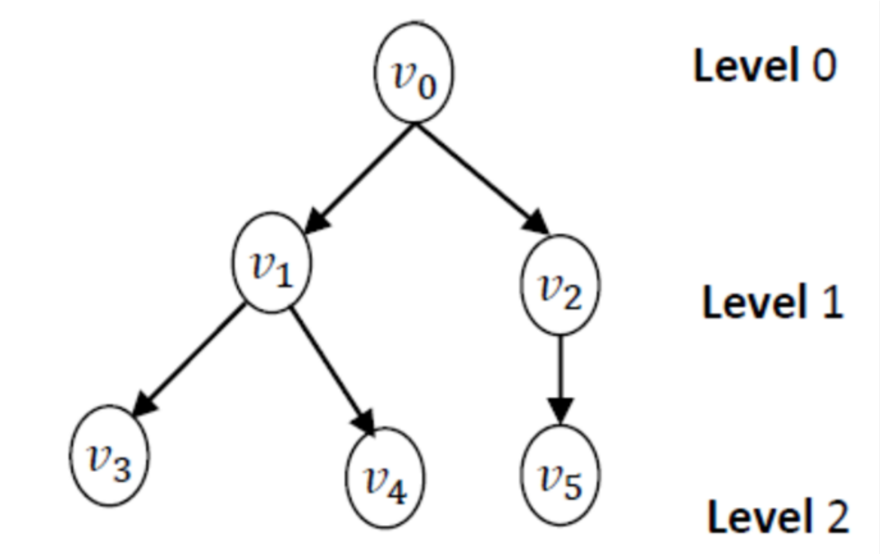
the children of v_2 is v_5



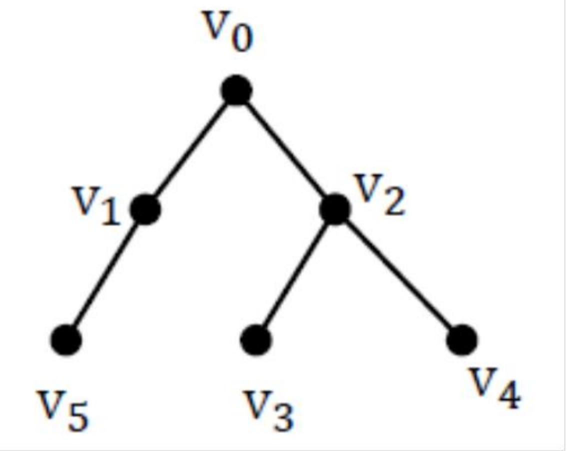
Offspring of a level

are the vertices of this level

- ❑ the offspring of level $-$ 0 are v_0
- ❑the offspring of level $-$ 1 are $v_1,\ v_2$
- ❑the offspring of level $-$ 2 are $v_3,\ v_4, v_5$



Example



For the following tree:

1- Is it a rooted tree?

2- Is it binary?

3- How many vertices (nodes) are there? How many paths? What is the relation between these two numbers?

4- Find the level of each vertex and the height of the tree.

5- Find the parent of v3 and find the children of v2

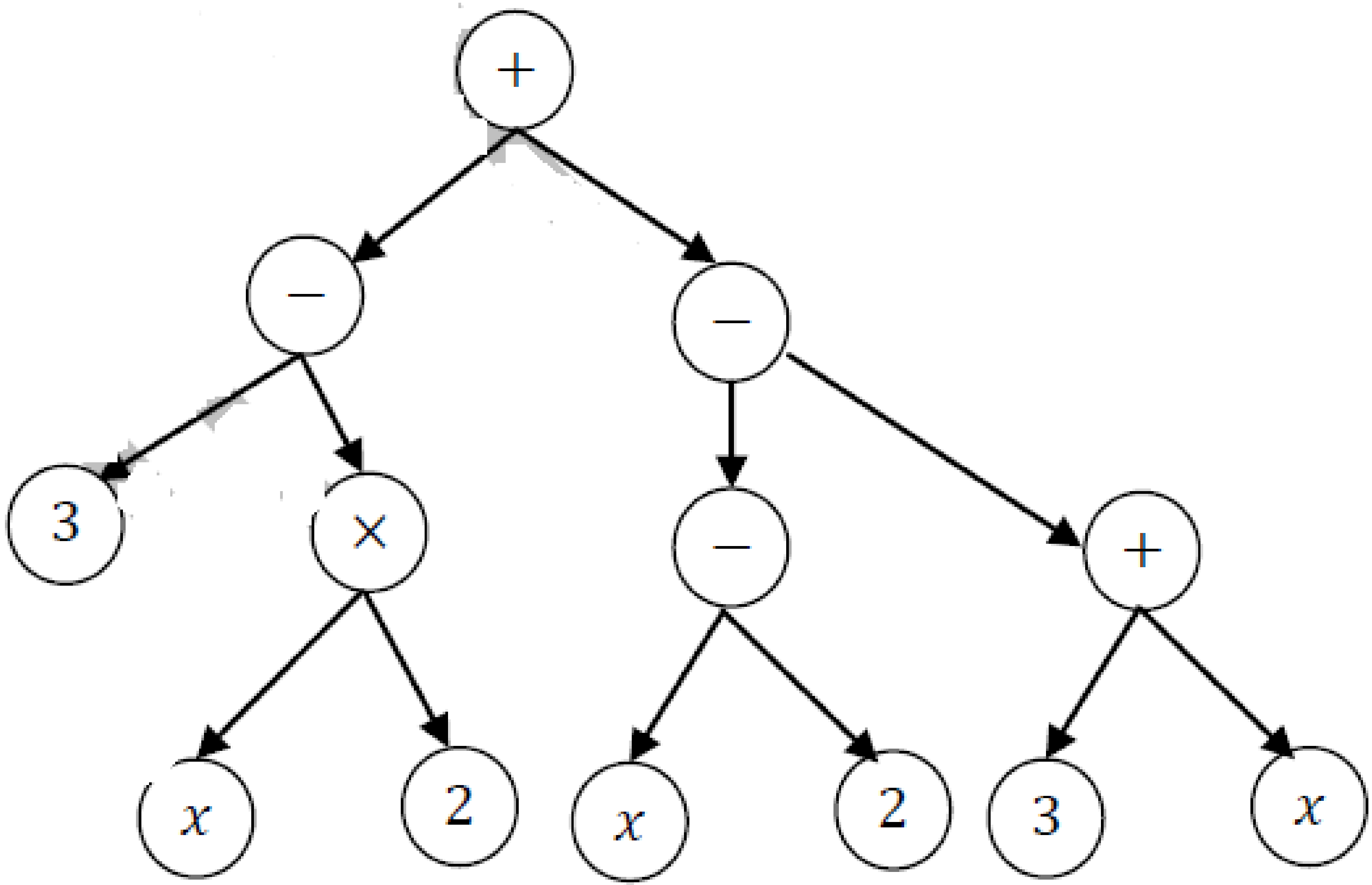
6- Find the offspring of level 1 and level 2.

7- Find the leaves of the tree

Example

Construct a **binary tree** of the algebraic expression

$$(3-(2\times x))+((x-2)-(3+x))$$



+
−
−
3
×
−
+
x
2
x
2
3
x

Exercises

$$(x+(y-(x+y)))\times((3\div(2\times7))\times4)$$

$$(11-(11\times(11\times11)))+(11+(11\times11))$$

$$(3-(2-(11-(9-4))))\div(2+(3+(4+7)))$$