

**Module (B)****Mid-Term Examination 2021-2020
Discrete Mathematics (BS-103)****Module(B)****First Question (10- Marks)****① Choose the correct sign “✓ or ✗” for the followings:**

- [1] $\neg[\forall x, P(x)] \equiv \exists x, \neg P(x)$ (✓)
- [2] If p is true and q is false, then $p \wedge \bar{q}$ is always true (✓)
- [3] $\bigcap_{n=1}^N A_n = \{x | \forall n, x \in A_n\}$ (✓)
- [4] The function is onto, when every value in the range has at least one value in the domain that maps to it. (✓)
- [5] If n is rational, then $(\forall n)(3n + 2 < n)$ (✗)
- [6] $\overline{p \rightarrow q} = p \wedge \bar{q}$ (✓)
- [7] If $R: X \rightarrow Y$ is a relation, then its inverse is $R^{-1}: Y \rightarrow X$ (✓)
- [8] $\bar{P} \wedge T \equiv P$ (✗)
- [9] The relation "greater than or equal" is reflexive (✓)
- [10] Union between the relation R with its inverse makes R symmetric. (✓)
- [11] R^∞ is always transitive relation. (✓)
- [12] If p is $5 \geq 2$ and q is " $3 \leq 2$ " then $p \oplus q$ is false (✗)
- [13] The vertical asymptote line for the function $f(x) = \frac{1}{x+1}$ is $y = -1$ (✗)
- [14] The relation RUR^{-1} refers to reflexive closure. (✗)
- [15] For the whole numbers greater than two, being odd is necessary to being prime. (✓)

② Choose the correct answer for the following statements:

- [1] A number being divisible by 2 is for its being even. (necessary, sufficient)
- [2] $\neg[\forall x, \bar{P}(x)] \equiv \dots$ $\{\exists x, \bar{P}(x), \exists x, P(x); \forall x, P(x)\}$
- [3] $A - \bar{B} = \dots$ $\{A \cup B, A \cap B, B - A\}$
- [4] $A \oplus B = (A \cup B) - (A \cap B)$ (True, False)
- [5] $\forall x, y, z \in \mathbb{Z}^+$ Let $P(x, y, z): x + y = z$, then $P(x, x - 1, x + 1)$ equals $\{x^3, 3x, x\}$

Second Question (10- Marks)

- Use indirect proof to prove that if x^2 is odd, then x is odd. (solve behind this paper) 4

- If $R: X \rightarrow X, X = \{1, 2, 3, 4\}, R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 4), (4, 2), (4, 3)\}$, find:

① X^2 , ② R^{-1} , ③ \bar{R} , ④ the graph the relation, 6

⑤ the matrix of the relation relative to the ordering 1432.

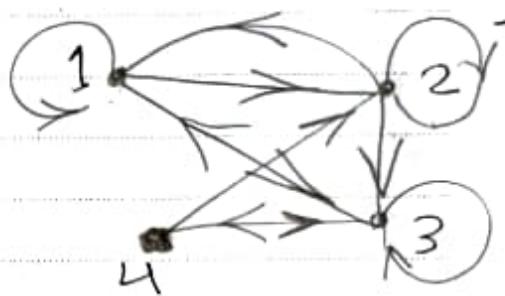
⑥ and finally classify the relation if it is reflexive, symmetric, and transitive or not.

① $X^2 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

② $R^{-1} = \{(1,1), (2,2), (3,3), (2,1), (3,1), (1,2), (3,2), (1,3), (2,4), (3,4)\}$

③ $\bar{R} = X^2 - R = \{ \dots \}$

④ $A_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$



Third Question (10- Marks)

- If $S_n = \{n^2 k^2 \mid k = 1, 2\}$, find $\bigcup_{n=2}^3 S_n$. 5

$$\bigcup_{n=2}^3 S_n = S_2 \cup S_3$$

$$S_2 = \{4k^2 \mid k=1, 2\} = \{4, 16\}$$

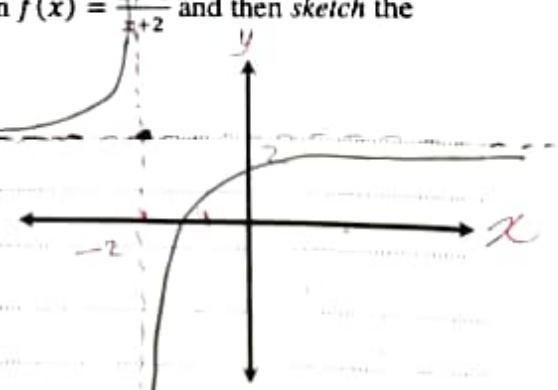
$$S_3 = \{9k^2 \mid k=1, 2\} = \{9, 36\}$$

$$\bigcup_{n=2}^3 S_n = S_2 \cup S_3 = \{4, 16, 9, 36\}$$

- Find the horizontal and the vertical asymptote of the function $f(x) = \frac{2x-3}{x+2}$ and then sketch the graph of the function. 5

vertical asymptote

$$x + 2 = 0 \rightarrow x = -2$$



horizontal asymptote

$$y = \lim_{x \rightarrow \infty} \frac{2x-3}{x+2} = 2$$

(النهاية الأصلية)

- IF x^2 is odd, then x is odd

$$P: x^2 \text{ is odd} \rightarrow \bar{P}: x^2 \text{ is even}$$
$$q: x \text{ is odd} \rightarrow \bar{q}: x \text{ is even.}$$

let x is even $\rightarrow x = 2k$, $k \geq 0$

$$\Rightarrow x^2 = 4k^2 = 2(2k^2) = 2m, m \geq 0$$

then x^2 is even, i.e., $\bar{q} \rightarrow \bar{P} \equiv P \rightarrow q$