

## Logic and SET theory

1.

\* Logic :- المنطق (تحديد قيمة العبارة)

\* Proposition (Statement) :- فرضية مدققة

عبارة عن جملة مفيدة وهذه الجملة يمكن تأكيد F أو T.

\* Logical variable :- المتغيرات المنطقية

is variables has value True or false and using the previous symbols p,q,r,s, ...

T or F = P, Q, R, S, ... دالة ما تم برمجها إلى رموز Statement

\* Logical operators : المعمرات

تساعدنا في تبسيط الجمل المنطقية المعقدة

1- negative operator موثر النegation  $\rightarrow$  reflex operator

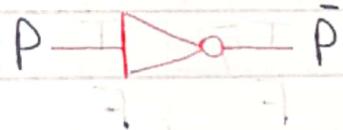
2- conjunction operator موثر الدفع  $\rightarrow$  AND

3- disjunction operator موثر الارتباط  $\rightarrow$  OR

4- Implication operator موثر النتائج  $\rightarrow$

5- bidirectional operator موثر المزدوج  $\rightarrow$

1- negative operator: NOT



P	$\neg P$ or $\sim P$
T	F
F	T

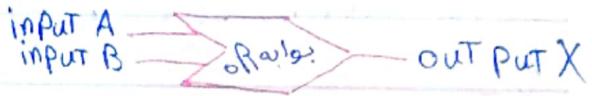
$$P \leftarrow \neg Q \text{ AND } 0 \equiv \neg Q$$

2- Conjunction operator:

عسان تكون T لازم الجملتين مع بعض  
يتكونوا T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F



3- disjunction operator ( $\text{OR}$ )  $\vee$ 

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

تكون T لو واحدة مبنية على

الذقل تكون T

تكون F لو الجملتين F

we get Two kind of OR

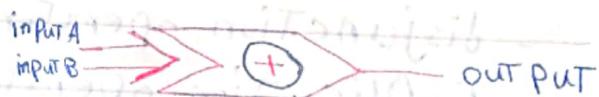
inclusive OR (v)

IF  $p \vee q$  or Both True  
then  $P \vee q$  is True

Exclusive OR (+)

if  $p \oplus q$  (But not Both)  
True then  $P \oplus q$  True

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

4- Implication operator ( $\rightarrow$ ) (P implies q)

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

ذالكت ونبحث (طبعي)

ذالكت وسقط (غير طبعي)

هذا يتحقق ونبحث (طبعي غير)

هذا يتحقق ولنبحث (طبعي)

5 Bidirection :-  $(P \leftrightarrow q)$   
 (bi implication) (iff) (if and only if)

P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

يتكون True لو الحدين متساويان  
 عكس  $\oplus$  يكون True لو الحدين مختلفين  
 $P \leftrightarrow q \equiv (P \rightarrow q) \wedge (q \rightarrow P)$

الأقواس  $\rightarrow$  NOT  $\rightarrow$  AND  $\rightarrow (\neg R, \neg \neg R)$

ترتيب العمليات

**TYPES OF PROPOSITIONS :-**

**Tautology :-** A proposition is said to be a Tautology if its Truth-values always True

**Contradiction :-** A proposition is said to be a contradiction if its Truth-values always False

**Contingency :-** A contingency proposition is neither a Tautology nor a contradiction.

\* **De Morgan's Laws :-**  $P \leftrightarrow q$  or  $P = q$

1<sup>ST</sup> Law:  $\neg(P \wedge q) \Leftrightarrow \neg P \vee \neg q$

2<sup>ST</sup> Law:  $\neg(P \vee q) \Leftrightarrow \neg P \wedge \neg q$

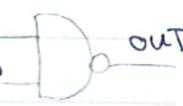
\* **Distributive law:-**

1<sup>ST</sup> Law:  $P \vee (q \wedge r) \Leftrightarrow (P \vee q) \wedge (P \vee r)$

2<sup>ST</sup> Law:  $P \wedge (q \vee r) \Leftrightarrow (P \wedge q) \vee (P \wedge r)$

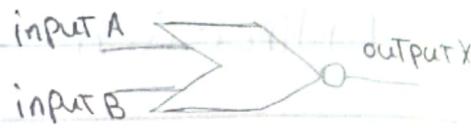
\* **Complement of AND :-**  $\neg(P \wedge q)$  (NAND)

$\neg(P \wedge q) = \text{invert } (P \wedge q) \equiv \neg(P \wedge q)$



\* Complement of OR : ( $\text{NoR}$ )

$$\overline{P \vee Q} \equiv \text{invert}(P \vee Q) \equiv \overline{P \oplus Q}$$



\* Complement of NOR : ( $\text{No}(P \oplus Q)$ )

\* List of Identities

- 1-  $P \equiv P \vee P$  and  $P \equiv P \wedge P$
- 2-  $P \wedge Q \equiv Q \wedge P$  and  $P \vee Q \equiv Q \vee P$
- 3-  $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$  and  $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
- 4-  $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$  and  $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$
- 5-  $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$  and  $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
- 6-  $P \vee T \equiv T$  and  $P \wedge T \equiv P$
- 7-  $P \vee F \equiv P$  and  $P \wedge F \equiv F$
- 8-  $P \vee \overline{P} \equiv T$  and  $P \wedge \overline{P} \equiv F$
- 9-  $\overline{(P)} \equiv P$
- 10-  $P \rightarrow Q \equiv \overline{P} \vee Q$
- 11-  $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$
- 12-  $P \rightarrow Q \equiv \overline{Q} \rightarrow \overline{P}$
- 13-  $(P \wedge Q) \rightarrow R \equiv P \rightarrow (Q \rightarrow R)$

# Exercises 1

Date: \_\_\_\_\_ no: \_\_\_\_\_

1- Assuming that  $p$  and  $r$  are false that  $q$  and  $s$  are true, Find the truth-value of each the following proposition.

$$\textcircled{1} \quad p \rightarrow q \quad F \rightarrow T = T$$

$$\textcircled{2} \quad \bar{p} \rightarrow \bar{q} \quad T \rightarrow F = F$$

$$\textcircled{3} \quad \bar{p} \rightarrow \bar{q} \quad \bar{T} = F$$

$$\textcircled{4} \quad (p \rightarrow q) \rightarrow (q \rightarrow r) = (\bar{p} \rightarrow T) \rightarrow (T \rightarrow F) = T \rightarrow F = F$$

$$\textcircled{5} \quad (p \rightarrow q) \rightarrow r = (F \rightarrow T) \rightarrow F = T \rightarrow F = F$$

$$\textcircled{6} \quad p \rightarrow (q \rightarrow r) = F \rightarrow (T \rightarrow F) = F \rightarrow F = T$$

$$\textcircled{7} \quad (s \rightarrow (p \wedge \bar{r})) \wedge ((p \rightarrow (r \vee q)) \wedge s) = (T \rightarrow (F \wedge T)) \wedge (F \rightarrow (F \wedge T) \wedge T)$$

$$\textcircled{8} \quad (\cancel{p \wedge r \wedge s}) \rightarrow (\cancel{p \vee q}) = (T \rightarrow F) \wedge ((F \rightarrow T) \wedge T)$$

$$= F \wedge (T \wedge T) = F$$

$$\textcircled{9} \quad (p \wedge r \wedge s) \rightarrow (p \vee q) = (F \wedge F \wedge T) \rightarrow (F \vee T)$$

$$= F \rightarrow T = T$$

2- Verify the second De Morgan Law,  $\overline{p \wedge q} \equiv \bar{p} \vee \bar{q}$

$p$	$q$	$\bar{p}$	$\bar{q}$	$\bar{p} \vee \bar{q}$	$p \wedge q$	$\bar{p} \wedge \bar{q}$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

3- Show that  $(p \rightarrow q) \equiv (\bar{p} \vee q)$

$p$	$q$	$p \rightarrow q$	$\bar{p}$	$\bar{p} \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

⑥ For each predicate give (if possible) an integer  $n$  for each which the predicate is True and another integer for which it is False

$$\textcircled{1} \quad (n+1=n) \vee (n=5)$$

at  $n=5$  predicate  $(n+1=n)$  is true

$$(6=5) \vee (5=5) = F \vee T = T$$

at  $n \neq 5$  predicate  $(n=5)$  is false

$$(3+1=5) \vee (3=5) = F \vee F = F$$

$$\textcircled{2} \quad (n > 7) \vee (6 \leq n) < 9$$

at  $n=8, 9, \dots$  predicate is true at  $n=3, 2, 1, \dots$

$$(n > 7) \vee (n < 9)$$

$$(n > 7) \vee (n < 4)$$

$$= F \vee F = F$$

$$= F \vee T = T$$

$$\begin{array}{ccccccccc} & & & & & & & & \\ & T & & T & & T & & T & \\ \hline & F & & F & & F & & F & \end{array}$$

intervall  $n \in [6, 7]$  اى رقم صحيح في الفترة

i.e. at  $n \in \mathbb{Z} - [6, 7]$  صحيح في حالة

$$\textcircled{3} \quad (n > 7) \wedge (n < 4) \text{ is always false}$$

لدين كما طلبنا في السؤال السابق وجدنا ان استحالة نوجد قيمة لـ  $n$

حيث الـ AND statement Two statements True صحيح في نفس الوقت لـ  $n$

مش الحال True غير لو الديفين

(4)  $(n < 7) \vee (n > 4)$  is true always on all  $\mathbb{Z}$   
 $(n < 7) \equiv T$  at  $n = 6, 5, 4, \dots$   
 $(n > 4) \equiv T$  at  $n = 5, 6, 7, \dots$

[9] Let  $P(x, y, z)$  be the predicate  $xy < x+z+1$ .  
the domain is the positive real numbers, write out each of these predicates

①  $P(1, 2, 3)$   $1 \cdot 2 < 1+2+1$

$(1)(2) < 1+3+1 \Leftrightarrow 2 < 4$  (is True)

[10] which of these statements are True?  
 $n$  is an integer, and  $x$  is real number

①  $\forall n (n+3 \geq n)$  always true  $\begin{cases} \oplus & 2+3 \geq 2 \\ \ominus & -5+3 \geq -5 \end{cases}$

②  $\forall x (x+3 \geq x)$  True

③  $\forall n (3n > n)$  False  $\begin{cases} \oplus 3(2) > 2 \equiv T \\ \ominus 3(-2) > 2 \equiv F \end{cases}$

لأن مثلاً كل قيمة  $n$  تتحقق

④  $\forall x (3x > x)$  False

⑤  $\forall n (3n+1 > n)$  False  $\begin{cases} \oplus 3(2)+1 > 2 = T \\ \ominus 3(-2)+1 > 2 = F \end{cases}$

⑥  $\forall x (3x+1 > x)$  False

⑦  $\forall x (\text{if } x > 1, \text{ then } x+1 > 1) \equiv \text{True}$

if  $p$  then  $q$   $p \rightarrow q$

$p \equiv (x > 1) \equiv F$  لأن كل رقم  $x$  ليس أكبر من 1

$q \equiv (x+1 > 1) \equiv F$   $F \rightarrow F \equiv T$

⑧  $\forall x (x^2 - 1 > 0) \equiv \text{False}$

STATEMENT  $\exists x$  such that  $x^2 - 1 > 0$  is False  
 $x \in \mathbb{R}$

at  $x = 0 \quad (0)^2 - 1 > 0$  False

$x = 1 \quad 1 - 1 > 0$  False

$x = 2 \quad (2)^2 - 1 > 0$  True

10. For every positive integer  $n$ , if  $n$  is even then  $n^2 + n + 19$  is prime  
False

$2^2 + 2 + 19 = 25$  is not prime