Suez Canal University
Faculty of Computers and Informatics
Department of Basic Science

Final Exam -1st Term 2022/2023

Program: software	Course: Discrete mathematics	Course Code: MAT101
Level: First Year	Lecturer: Dr. Asmaa Awad	Date: 16/1/2023
Total Pages: 9	Total Marks: 70	Time Allowed: 2 Hours

Answer the following Questions

Question No.1: Choose the correct answer. **1-point for each one**

1. What are the value of this sum? $\sum_{j=0}^{8}(2^{j+1}-2^{j})$.

A. 511 B.510 C. 512 D.513

2. Find AB if, $A=\begin{bmatrix}1 & -1\\ 0 & 1\\ 2 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & -2 & -1\\ 1 & 0 & 2\end{bmatrix}$.

A. $AB=\begin{bmatrix}2 & -2 & -3\\ 9 & 0 & 2\\ 1 & -2 & 4\end{bmatrix}$

B. $AB=\begin{bmatrix}2 & -2 & -3\\ 1 & 0 & 2\\ 9 & -4 & 4\end{bmatrix}$

C. $AB=\begin{bmatrix}2 & -2 & -3\\ 1 & 0 & 2\\ 1 & -2 & 4\end{bmatrix}$

D. Does don't exist

3. Let p and q be the propositions. Let p : You drive over 65 miles per hour. q : You get a speeding ticket. Write these propositions using p and q and logical connectives. You will get a speeding ticket if you drive over 65 miles per hour.

A. $p \leftrightarrow q$ B. $p \rightarrow q$ C. $q \leftrightarrow p$ D. $q \rightarrow p$

4. Find the truth set of this predicate where the domain is the set of integers, $Q(x): x^2 > x$.

A. The truth set is $\{x \in Z \mid x^2 > x\} = \{\ldots,-1,1,2,3,4,\ldots\}$.

B. The truth set is $\{x \in Z \mid x^2 > x\} = \{\ldots-1,0,1,2,3,4,\ldots\}$.

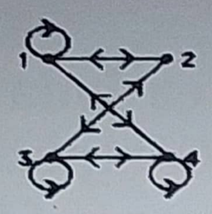
C. The truth set is $\{x \in Z \mid x^2 > x\} = \{\ldots,-2,0,-1,2,3,4,\ldots\}$.

D. The truth set is $\{x \in Z \mid x^2 > x\} = \{\ldots,-2,-1,2,3,4,\ldots\}$.

5. Determine whether each of these statements is true.

A. $\{\emptyset\} \subseteq \{\emptyset\}$ B. $\emptyset \in \{0\}$ C. $\{0\} \subset \emptyset$ D. None of them

6. Find the matrix that represent by the direct graph.



A. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

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7. List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only $a|b$.

A. $R = \{(1,1), (2,2), (3,3), (4,4)\}$

B. $R = \{(1,1), (2,2), (2,4)(3,3), (4,2), (4,4)\}$

C. $R = \{(1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)\}$

D. $R = \{(0,1), (0,2), (0,3), (0,4), (1,1), (1,2), (1,3), (2,1), (2,2), (3,3), (4,2), (4,4)\}$

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8. Provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list $1, 0, 2, 0, 4, 0, 8, 0, 16, 0, \ldots$

A. $2^{\frac{n-1}{2}}$, $n = 1,2,3,4, \ldots$

B. $\frac{1}{2} + \frac{1}{2}(n+1)$, $n = 1,2,3,4, \ldots.$

C. $\frac{n(n+1)}{2}$, $n = 1,2,3,4, \ldots$

D. *None of them*

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9. State the contrapositive of the conditional statement. I come to class whenever there is going to be a quiz.

A. " If I don't come to class, then there won't be a quiz."
B. "If there is not going to be a quiz, then I don't come to class."
C. " It is not the case that, I come to class only if there's going to be a quiz."
D. " It is not the case that, if there is going to be a quiz, then I will come to class."

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10. Evaluate the expression of the bitwise. $(0\,1010 \oplus 1\,1011) \oplus 0\,1000$.

A. 11011 B. 10001 C. 11001 D. 10110

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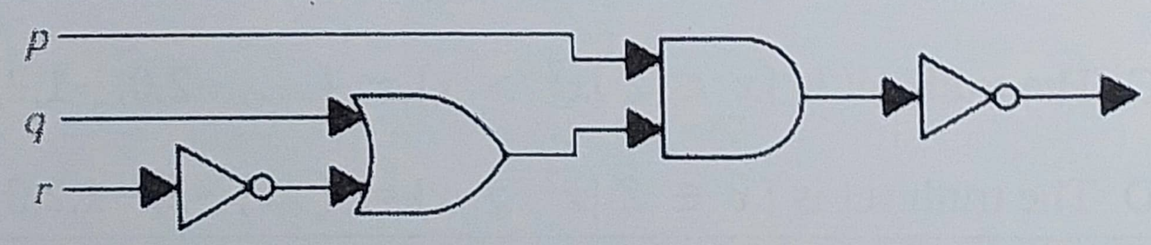
11. Find the output of each of this combinatorial circuit.



A. $\neg(p \wedge (q \vee \neg r))$ B. $\neg(p \vee (q \wedge \neg r))$ C. $(p \wedge \overline{(q \vee \neg r)})$ D. $\neg(p \vee (q \vee \neg r))$

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12. Which one of the following equivalences to the biconditional $p \leftrightarrow q$.

A. $(p \wedge q) \vee (\neg p \wedge \neg q)$ B. $\neg p \leftrightarrow \neg q$ C. a&b D. None of them

13. How many elements does this set have where a and b are distinct elements? $P(\{\emptyset, b, \{a\}, \{\{a\}\}\})$.

A. 18 B. 16 C. 8 D. 6

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14. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i, $A_i = \{-i, i\}$.

A. $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}, \bigcap_{i=1}^{\infty} A_i = \mathbb{Z}$

B. $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}^+, \bigcap_{i=1}^{\infty} A_i = 0$

C. $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}, \bigcap_{i=1}^{\infty} A_i = \Phi$

D. $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}, \bigcap_{i=1}^{\infty} A_i = \mathbb{Z}^-$

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15. Determine whether each of these functions is a bijection from R to R.

A. $f(x) = 2x + 1$ B. $f(x) = x^2 + 1$ C. $f(x) = \frac{x^2+1}{x^2+2}$ D. $f(x) = \sqrt{x^2 + 2}$

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16. Let $S = \{-1, 0, 2, 4, 7\}$. Find $f(S)$ if, $f(x) = \left\lceil \frac{x^2+1}{3} \right\rceil$.

A. $f(S) = \{0,1,6,17\}$ B. $f(S) = \{1,2,,6,17\}$ C. $f(S) = \{0,1,5,16\}$ D. $f(S) = \{0,5,16,17\}$

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17. Let $an = 2^n + 5 \cdot 3^n$ $for\ n = 0, 1, 2, \ldots$. Find a_0, a_1, a_2, a_3 and a_4.

A. $(a_0, a_1, a_2, a_3, a_4) = (6,14,49,134,421)$

B. $(a_0, a_1, a_2, a_3, a_4) = (6,14,94,134,412)$

C. $(a_0, a_1, a_2, a_3, a_4) = (6,17,49,143,421)$

D. $(a_0, a_1, a_2, a_3, a_4) = (6,17,49,134,142)$

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18. Determine which one of these statements is true, if the domain for all variables consists of all integers.

A. $\forall n,\ (n^2 > 0)$ B. $\forall n,\ (n^2 \geq n)$ C. $\exists n,\ (n^2 = 2)$ D. $\exists n,\ (n^2 < 0)$

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19. Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out $\forall x, \neg P(x)$ using disjunctions, conjunctions, and negations.

A. $\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

B. $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

C. $\neg(\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4))$

D. $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

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20. Compute the follow double sum, $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$.

A. 82 B. 87 C. 78 D. 80

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21. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.
$a_n = na_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1.$

A. $(a_0, a_1, a_2, a_3, a_4) = (1,1,7,27,401)$

B. $(a_0, a_1, a_2, a_3, a_4) = (1,1,6,28,204)$

C. $(a_0, a_1, a_2, a_3, a_4) = (1,1,7,72,401)$

D. $(a_0, a_1, a_2, a_3, a_4) = (1,1,6,27,204)$

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22. Let, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Find a formula for A^n, whenever n is a positive integer.

A. $A^n = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}$

B. $A^n = \begin{bmatrix} 1 & n+1 \\ 0 & 1 \end{bmatrix}$

C. $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

D. $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

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23. Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1, or\ 2,\ y = 0\ or\ 1\ and\ z = 0\ or\ 1$. Write out $\exists x \neg Q(x, 0, 1)$ using disjunctions and conjunctions.

A. $\neg(Q\ (0,0,1) \vee Q\ (1,0,1) \vee Q\ (2,0,1))$

B. $\neg(\neg Q\ (0,0,1) \wedge \neg Q\ (1,0,1) \wedge \neg Q\ (2,0,1))$

C. $\neg Q\ (0,0,1) \wedge \neg Q\ (1,0,1) \wedge \neg Q\ (2,0,1)$

D. $\neg Q\ (0,0,1) \vee \neg Q\ (1,0,1) \vee \neg Q\ (2,0,1)$

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24. What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}, B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

A. $\{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$.

B. $\{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,1,0), (1,2,1), (1,2,2)\}$.

C. $\{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,0,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$.

D. $\{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,1,2)\}$.

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25. Let, $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find $A \wedge B$.

A. $A \wedge B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $A \wedge B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $A \wedge B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D. $A \wedge B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is symmetric.

A. $\{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$

B. $\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

C. $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

D. $\{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4)\}$

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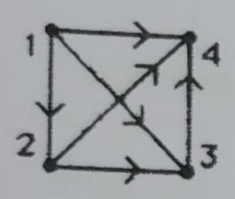
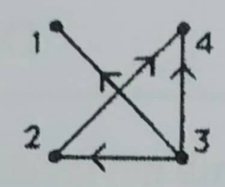
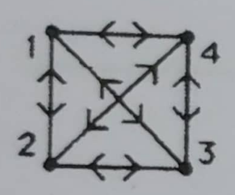
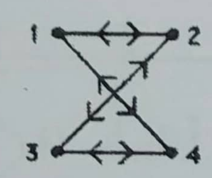
27. Draw the directed graph representing the relation: $\{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$.

A. 

B. 

C. 

D. 

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28. What is the relation between $(p \to q) \to r$, *and* $p \to (q \to r)$.

A. Not Logically Equivalent

B. Logically Equivalent

C. Distributive laws

D. Tautology

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29. Let $R(x, y, z)$ denote the statement $x + y = z$. When values are assigned to the variables $x, y, and\ z$, this statement has a truth value. What are the truth values of the propositions $R(1,2,3)$ *and* $R(0,0,1)$?

A. $R(1,2,3)$ *is true and* $R(0,0,1)$*is true*

B. $R(1,2,3)$ *is true and* $R(0,0,1)$*is false*

C. $R(1,2,3)$ *is false and* $R(0,0,1)$*is flase*

D. $R(1,2,3)$ *is false and* $R(0,0,1)$*is true*

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30. Determine the truth value that (false) of each of these statements if the domain consists of all integers.

A. $\forall n,\ n + 1 > n$

B. $\exists n,\ 2n = 3n$

C. $\forall n,\ 3n \leq 4n$

D. $\exists n,\ n = -n$

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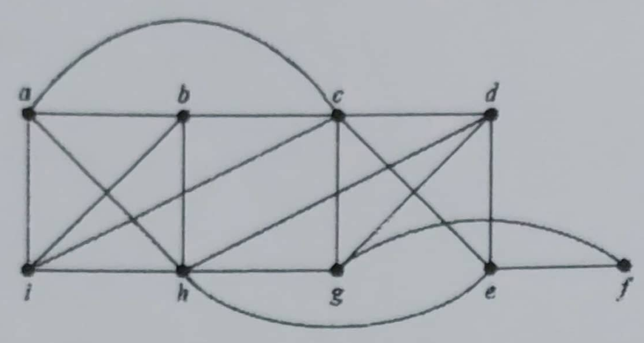
31. Let R_1 be the function from $\{l, m, n\}$ to itself such that $R_1(l) = m,\ R_1(m) = n,\ R_1(n) = l$ and R_2 be the function from $\{l, m, n\}$ *to* $\{-1,0,1\}$ such that $R_2(l) = 1,\ R_2(m) = 0,\ R_2(n) = -1$ .Find $(R_1 0\ R_2)$?

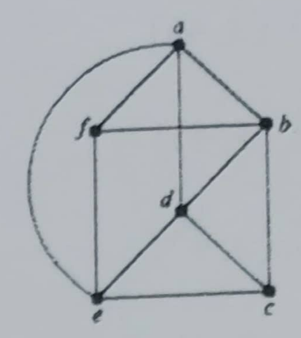
A- $R_1 0\ R_2 = \{0, -1, 1\}$
B- $R_1 0\ R_2 = \{1, 0, -1\}$
C- $R_1 0\ R_2 = \{m, n, l\}$
D- Not defined

32. Determine whether the given graph has an Euler circuit.

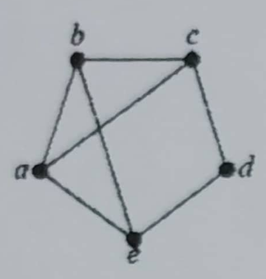
A.



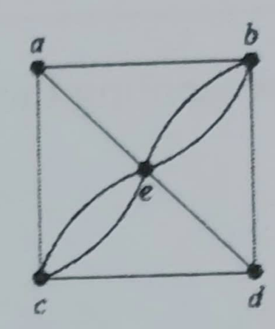
B.



C.



D.



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33. Find these terms a_0, a_1, a_4, a_5 of the sequence $\{a_n\}$, $a_n = 2.(-3)^n + 5^n$.

A. $a_0 = 3, a_1 = -1,\ a_4 = 768,\ a_5 = 1536$
B. $a_0 = 3, a_1 = -1,\ a_4 = 787,\ a_5 = 2639$
C. $a_0 = 3, a_1 = -1,\ a_4 = 687,\ a_5 = 2569$
D. $a_0 = 3, a_1 = -1,\ a_4 = 878,\ a_5 = 3926$

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34. Find the matrix representing the relations SoR, where the matrices representing R and S are

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

A. $M_{SoR} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

B. $M_{SoR} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

C. $M_{SoR} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

D. $M_{SoR} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

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35. Determine whether the relation R on the set of all integers is reflexive and symmetric, where $(x, y) \in R$ if and only if.

A. $x \neq y$

B. $xy \geq 1$

C. $x = y + 1$ *or* $x = y - 1$

D. x and y are both negative or both nonnegative.

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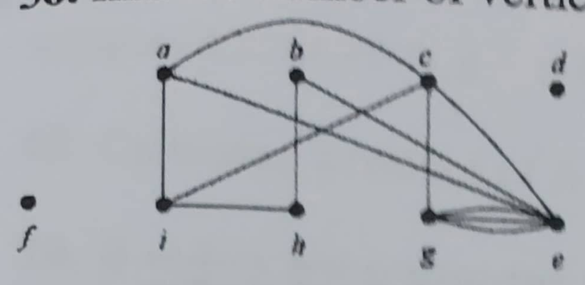
36. find the number of vertices, the number of edges, and the degree of vertex(e) in the given undirected graph.



A. 9 vertices, 11 edge and deg(e) = 6

B. 9 vertices, 12 edge and deg(e) = 5

C. 9 vertices, 12 edge and deg(e) = 6

D. 9 vertices, 11 edge and deg(e) = 5

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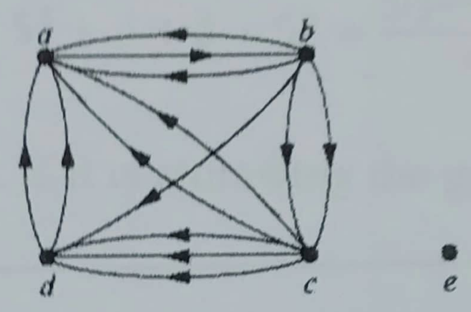
37. Find the in-degree and out-degree of each vertex for the given directed multigraph.



A. $deg^-(a) = 6, deg^+(a) = 1, deg^-(b) = 1, deg^+(b) = 6, deg^-(c) = 2, deg^+(c) = 5,$

$deg^-(d) = 2, deg^+(d) = 4, deg^-(e) = deg^+(e) = 0$

B. $deg^-(a) = 6, deg^+(a) = 1, deg^-(b) = 1, deg^+(b) = 6, deg^-(c) = 2, deg^+(c) = 5,$

$deg^-(d) = 4, deg^+(d) = 1, deg^-(e) = deg^+(e) = 0$

C. $deg^-(a) = 6, deg^+(a) = 1, deg^-(b) = 1, deg^+(b) = 6, deg^-(c) = 5, deg^+(c) = 2,$

$deg^-(d) = 4, deg^+(d) = 2, deg^-(e) = deg^+(e) = 0$

D. $deg^-(a) = 6, deg^+(a) = 1, deg^-(b) = 1, deg^+(b) = 6, deg^-(c) = 2, deg^+(c) = 5,$

$deg^-(d) = 4, deg^+(d) = 2, deg^-(e) = deg^+(e) = 0$

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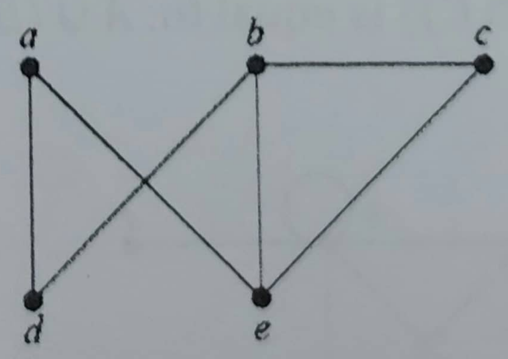
38. Does each of these lists of vertices form a simple path in the following graph?. And which are circuits?



A. a, e, b, c, b, a B. c, b, d, a, e, c C. a, e, d, b, c, a D. e, b, d, a, e, c, b, e

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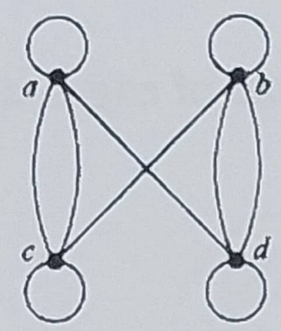
39. Use an adjacency list to represent the given graph.



A.

Vertex	Adjacent vertex
a	a,c,d
b	b,c,d
c	c,a,b
d	d,b,a

B.

Vertex	Adjacent vertex
a	c,d
b	c,d
c	a,b
d	b,a

C.

Initial	Terminal
a	a,c,d
b	b,c,d
c	c,a,b
d	d,b,a

D.

Initial	Terminal
a	c,d
b	c,d
c	a,b
d	b,a

---

40. Determine a sequence if the first 10 terms are 5, 11, 17, 23, 29, 35, 41,47, 53, 59?

A- $6.\left(\frac{1}{5}\right)^n$

B- 5.6^n

C- $5 + 6(n-1)$

D- None of them

Question No.2 : True OR False, if True chose (A), if False chose (B) **1-point for each one**

41. In direct graph the loop at vertex contributes 1 to both $deg^-(v)$ and $deg^+(v)$ of this vertex.

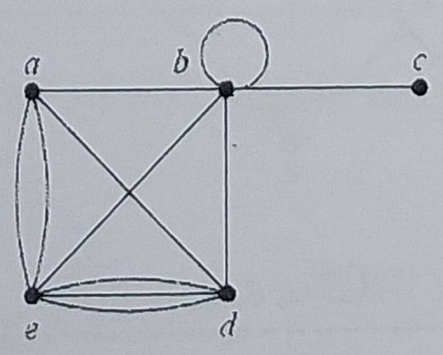
42. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$,then $B \times A = \{(y,a), (y,b), (y,c), (y,d), (a,a), (z,b), (z,c), (z,d)\}$.

43. $(p \wedge q) \rightarrow (p \vee q)$ is Tautology logical statement.

44. Let $A = \{1,2,3,4\}$. Define the relation $R : a \rightarrow b$ if $a \leq b$. So the relation is Transitive and symmetric relation.

45. Let A, B, and C be sets. Show that $\overline{(A \cup (B \cap C))}$ is equal to: $\overline{\bar{A} \cup (B \cap \bar{C})}$

46. $deg(b) = 6$, $N(b) = \{a, b, c, d, e\}$



47. The bitwise XOR of these strings (0110110110 and 1100011101) is 1010101011.

48. Universal Quantifier $\forall x, P(x)$, where $P(x)$ is the statement $x^2 < 10$ and the domain consists of the positive integers not exceeding 3 is false.

49. Cartesian product of A and B denoted by $A \otimes B = \{(a, b), a \in A \wedge b \in B\}$.

50. $R = \{(a, b) | \ a \ divides \ b\}$ is reflexive if $A = \{1,2,3,4\}$.

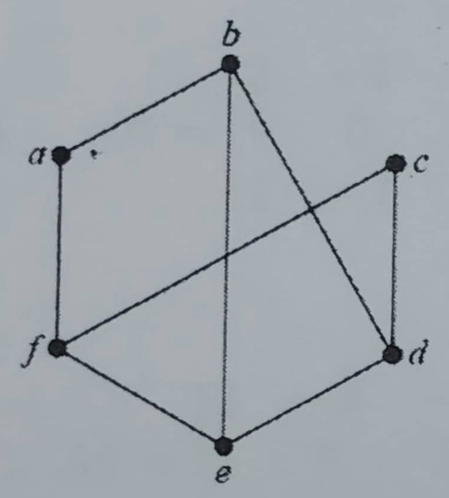
Question No.3: Answer the following questions. **5-point for each one**

I. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.

II. A proof by contraposition, show that if n is an integer and $n^3 + 5$ is odd, then n is even.

III. Use mathematical induction to prove summation formulae. Be sure to identify where you use the inductive hypothesis. Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = \frac{3(5^{n+1} - 1)}{4}$, whenever n is a nonnegative integer.

V. Determine whether the graph is bipartite. If it is redrawing the graph into the new groups.



---Good Luck---