

# Mathematical induction

## الاستقراء الرياضي

عشاد نشئت ای Statement بطریقه الاستقراء الرياضي

بیفرض  $S_n \rightarrow$  Statement

1) Prove that  $S_n$  at  $n=1$  is True

2) Suppose that statement  $S_n$  is True

3) we need to Prove that  $S_{n+1}$  is True

ex: 1

Prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof: -

$$\text{let } S_n = 1^2 + 2^2 + 3^2 + \dots + n^2$$

1) Prove that  $S_n$  at  $n=1$  is True

$$\text{L.H.S} = S_n = S_1 = 1^2 = \text{R.H.S} = \frac{1(2)(3)}{6} = 1 \quad \therefore \text{True at } n=1$$

2) Suppose that  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  is True

3) we need to Prove that

$$S_{n+1} = 1^2 + 2^2 + 3^2 + \dots + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

Term افزود

بیشتر ال  $n$  و  $n+1$

$$\begin{aligned}
 \text{L.H.S} &= 1^2 + 2^2 + 3^2 + \dots + n^2 + (n+1)^2 \\
 &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \\
 &= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)[n(2n+1) + 6(n+1)]}{6} \\
 &= \frac{(n+1)[2n^2 + 7n + 6]}{6} = \frac{(n+1)(n+2)(2n+3)}{6} = \text{R.H.S}
 \end{aligned}$$

ex. 2: Prove That

$7^n - 1$  is divisible by 6  $\forall n \in \mathbb{N}$

Proof:

$$S_n = 7^n - 1$$

1) at  $n=1$   $S_1 = 7^1 - 1 = 6$  is divisible by 6

2) Suppose that  $S_n = 7^n - 1$  is divisible by 6

3) we need to prove that at  $S_{n+1}$  is True

$$\begin{aligned}
 S_{n+1} &= 7^{n+1} - 1 = 7^n(7) - 1 + 6 - 6 \\
 &= 7^n(7) - 7 + 6 \\
 &= 7(7^n - 1) + 6
 \end{aligned}$$

المستقر على 6  
المستقر على 6  
المستقر على 6

Sum of two divisible  
by 6 are also divisible  
by 6

$\therefore$  Sum is divisible by 6

### Example 3:

Prove that

$x^{2n} - y^{2n}$  is divisible by  $x+y$   
 $\forall n \in \mathbb{N} \cup \{0\}$

Proof:

$$S_n = x^{2n} - y^{2n}$$

1) at  $n=1$

$$S = x^2 - y^2 = (x-y)(x+y)$$

$\therefore$  divisible by  $x+y$

2) Suppose that

$$S_n = x^{2n} - y^{2n} \text{ is divisible by } x+y$$

3) we need to prove that

$S_{n+1}$  is True

$$S_{n+1} = x^{2n+2} - y^{2n+2} \pm x^{2n} y^2 \mp x^2 y^{2n}$$

$$= x^{2n+2} - y^{2n+2} + x^{2n} y^2 - x^2 y^{2n}$$

$$= x^{2n} (x^2 + y^2) + y^2 (x^{2n} - y^{2n})$$

بقولنا ان  $n=1$  قبل  
 الصيغة على  $x+y$

افترضنا ان  
 الصيغة قبل  $n$

$\therefore S_{n+1}$  is divisible by  $x+y$

### Example 4:

Prove that  $n! \geq 2^{n-1} \quad n \in \mathbb{N}$

Proof:

$$S_n = n! \geq 2^{n-1}$$

1) at  $n=1$

$$1! \geq 2^{1-1} \quad 1 \geq 1 \text{ True}$$

2) Suppose that  $S_n$  is true

3) we need to prove that  $S_{n+1}$  is True

$$S_{n+1} = (n+1)! \geq 2^{(n+1)-1}$$

$$\text{L.H.S} = (n+1)! = (n+1) n!$$

$$\geq (n+1) 2^{n-1}$$

$$\therefore n \in \mathbb{N}$$

$\therefore$  افتد رقم الصيغة  $n$  صحيح

$\therefore$  لو عوضنا بـ  $n+1$  في الصيغة

الـ L.H.S اكبر من الـ R.H.S

$$\therefore (n+1)! \geq (n+1) 2^{n-1}$$

$$\geq 2 \cdot 2^{n-1}$$

$$\geq 2^n$$

Examine 5:-

Prove that  $1 + 2n \leq 3^n \quad \forall n \in \mathbb{N}$

Proof:

$$S_n = 1 + 2n \leq 3^n$$

1) at  $n=1$   $S_1 = 1 + 2 \leq 3$

is True

2) Suppose that  $S_n$  is True

3) we need to prove that at  $n+1$  is True

$$S_{n+1} = 1 + 2(n+1) \leq 3^{n+1}$$

$$R.H.S = 3^{n+1} = 3^n \cdot 3$$

$$> (1 + 2n) \cdot 3$$

$$> 3 + 6n > 2n + 3$$

إذا كانت في القيمة الأكبر من

L.H.S أكبر من R.H.S

∴ لما القيمة تقل سيظل

R.H.S أكبر من قيمة L.H.S