

Mid-Term Exam (10/30) + Academic Assignments (20/30)

Mid-Term Exam of Discrete Math for Summer
Course (BSD-103)

Module-A

• Use mathematical induction to $x^{2n} - y^{2n}$ is divisible by $(x + y)$ for $n \in \{0, 1, 2, \dots\}$

- For the relation R on $X = \{1, 2, 3, 4\}$ defined by the rule $(x, y) \in R, x + 1 \leq y$.
- ① Find the relation R
- ② Give the graphical representation
- ③ Give the matrix representation of R ④ Identify that if R is reflexive, symmetric or transitive.

Second Question (15 Marks)

- Prove that if x^2 is even, then x is even

- Let $S_n = \{kn^2 | k = 1, 2, 3\}$. find $\bigcup_{n=1}^3 S_n$

- Simplify the following statement: $(p \wedge q) \vee (p \wedge \bar{q}) \vee (\bar{p} \wedge \bar{q})$

first Question

①

step 1: $S_n = x^{2n} - y^{2n}$

at $(n=1)$ $S_1 = x^2 - y^2 = (x+y)(x-y)$
is divisible by $(x+y)$

at $(n=2)$ $S_2 = x^4 - y^4$
 $= (x^2 + y^2)(x^2 - y^2)$
 $= (x^2 + y^2)(x - y)(x + y)$
is divisible by $(x+y)$

$\therefore S_n = (x^{2n} - y^{2n})$ is divisible by $(x+y)$

step 2: at $(n+1)$

$$\begin{aligned} S_{n+1} &= x^{2n+2} - y^{2n+2} \\ &= x^2 x^{2n} - y^2 y^{2n} \\ &= x^2 x^{2n} - x^2 y^{2n} + x^2 y^{2n} - y^2 y^{2n} \\ &= x^2 (x^{2n} - y^{2n}) + y^{2n} (x^2 - y^2) \end{aligned}$$

By The first step $S_n = x^{2n} - y^{2n}$ is divisible by $(x+y)$

By The second step $S_{n+1} = x^2(x^{2n} - y^{2n}) + y^{2n}(x^2 - y^2)$
is also divisible by $(x+y)$

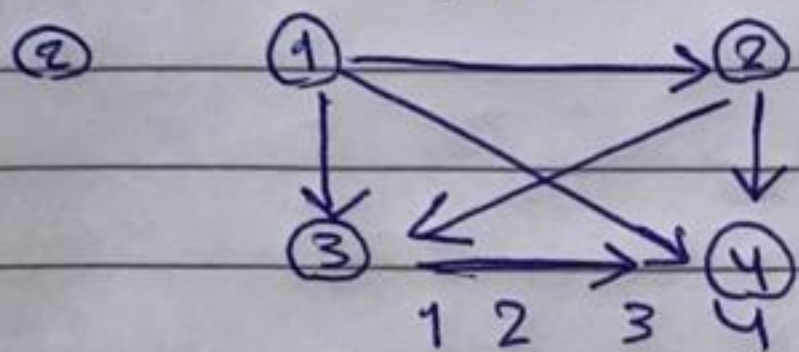
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② $X = \{1, 2, 3, 4\}$

R defined by The rule $(x, y) \in R$
 $x+1 \leq y$

① $R = \{(\underline{1}, \underline{2}), (\underline{1}, \underline{3}), (\underline{1}, \underline{4}), (\underline{2}, \underline{3}), (\underline{2}, \underline{4}), (\underline{3}, \underline{4})\}$



③

$$M_R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

④ R is not reflexive, not symmetric & Transitive

Date: _____

Subject: _____

second Question

①

 $P: x^2$ is even number $q: x$ is also even numberif $\bar{q}: x$ is odd number

$$x = 2k+1$$

$$x^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Then x^2 is odd numberThis mean That, if x^2 is even, Then x is even

$$\textcircled{2} \quad S_1 = \{k / k=1,2,3\} = \{1,2,3\}$$

$$S_2 = \{4k / k=1,2,3\} = \{4,8,12\}$$

$$S_3 = \{8k / k=1,2,3\} = \{8,16,24\}$$

$$\bigcup_{n=1}^3 S_n = S_1 \cup S_2 \cup S_3 = \{1,2,3,4,8,12,16,24\}$$

③ simplify $(P \wedge Q) \vee (P \wedge \bar{Q}) \vee (\bar{P} \wedge \bar{Q})$

$$(P \wedge Q) \vee (P \wedge \bar{Q}) \vee (\bar{P} \wedge \bar{Q})$$

$$\equiv P \wedge (Q \vee \bar{Q}) \vee (\bar{P} \wedge \bar{Q})$$

$$\equiv (P \wedge T) \vee (\bar{P} \wedge \bar{Q})$$

$$\equiv P \vee (\bar{P} \wedge \bar{Q})$$

$$\equiv (P \vee \bar{P}) \vee (P \vee \bar{Q})$$

$$\equiv T \vee (P \vee \bar{Q})$$

$$\equiv P \vee \bar{Q} \quad \#$$