(8) $\forall x\,(x^2 - 1 > 0) \equiv$ False

False لأن مش $\forall$ (لكل قيم x) هتحقق الـ STatmenT

$x \in R$

at $x = 0$ $\quad (0)^2 - 1 > 0$ False

$x = 1$ $\quad 1 - 1 > 0$ False

$x = 2$ $\quad (2)^2 - 1 > 0$ True

$\boxed{10}$ For every positive integer n, if n is even then $n^2 + n + 19$ is prime

False

$2^2 + 2 + 19 = 25$ is not prime

$\boxed{17}$ Is the statement $(p \wedge q) \vee [\bar{p} \vee (p \wedge \bar{q})] \vee r$ from a Tautology, Contradiction or Contingency?

كلهم T — كلهم F — تحتوي على الاثنين

Solution $\quad (p \wedge q) \vee [(\bar{p} \vee p) \wedge (\bar{p} \vee \bar{q})] \vee r$

$(p \wedge q) \vee [T \wedge (\bar{p} \vee \bar{q})] \vee r$

$(p \wedge q) \vee [(\bar{p} \vee \bar{q})] \vee r$

$(p \wedge q) \vee \overline{(p \wedge q)} \vee r$

$= T \vee r = T$

$\therefore$ the statement from a Tautolgy

(21) use the law of logic To show that

$(p \to q) \wedge \bar{q} \to \bar{p}$ is a Tautology.

$(p \to q) \wedge \bar{q} \to \bar{p}$ ∵ $(p \to q) = \bar{p} \vee q$

∴ $\overline{[(p \to q) \wedge \bar{q}]} \vee \bar{p}$ from demorgen's law.

$\equiv \overline{[(\bar{p} \vee q) \wedge \bar{q}]} \vee \bar{p}$

$\equiv [\overline{(\bar{p} \vee q)} \vee \bar{\bar{q}}] \vee \bar{p} \quad \equiv [(p \wedge \bar{q}) \vee \bar{\bar{q}}] \vee \bar{p}$

$\equiv [q \vee (p \wedge \bar{q})] \vee \bar{p} \equiv [(q \vee p) \wedge (q \vee \bar{q})] \vee \bar{p}$

$\equiv [(q \vee p) \wedge T] \vee \bar{p} \equiv (q \vee p) \vee \bar{p}$

$\equiv (p \vee \bar{p}) \vee q = T \vee q = T$ ∴ Tautology.

(22) use the law of logic To show that

$(p \wedge \bar{q}) \wedge (\bar{p} \vee q) \wedge r$ always false.

$(p \wedge \bar{q}) \wedge (\bar{p} \vee q) \wedge r \equiv \overline{\overline{(p \wedge \bar{q})}} \wedge (\bar{p} \vee q) \equiv \overline{(\bar{p} \vee q)} \wedge (\bar{p} \vee q) \wedge r$

$\equiv F \wedge r = F$

Date: ______ no: ______

23 use the law of logic To show that $(p \to q) \leftrightarrow (\bar{q} \to \bar{p})$ Tautology.

$\because (p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$, $p \to q \equiv \bar{p} \vee q$

$\therefore \left[(p \to q) \to (\bar{q} \to \bar{p}) \right] \wedge \left[(\bar{q} \to \bar{p}) \to (p \to q) \right]$

$\equiv \left[\overline{(\bar{p} \vee q)} \vee (\bar{\bar{q}} \vee \bar{p}) \right] \wedge \left[\overline{(q \vee \bar{p})} \vee (\bar{p} \vee q) \right]$

$T \wedge T \equiv T$

5 $[p \wedge (p \to q)] \to q$

$\equiv p \wedge (\bar{p} \vee q)] \to q \equiv [(p \wedge \bar{p}) \vee (p \wedge q)] \to q$

$\equiv [F \vee (p \wedge q)] \to q \equiv (p \wedge q) \to q \equiv \overline{(p \wedge q)} \vee q$

$\equiv (\bar{p} \vee \bar{q}) \vee q = \bar{p} \vee T = T$

24 prove that the sum of any Two even numbers is an even numbers.

Direct proof:

let $a = 2k$ even num. $k = 0, 1, 2, 3, \ldots$

$b = 2m$ even num. $m = 0, 1, 2, 3, \ldots$

$a + b = 2k + 2m = 2(k+m) = 2L$ is even

proof by Contradiction

let $a = 2k$ even num.

$b = 2m$ even num.

let $a+b$ is odd num.

$a + b = 2k + 2m = 2(k+m) = 2L$ not odd

this is Contradiction

$\therefore a + b$ is even num.

(23) use indirect proof to prove that for some real number x, $\exists x$, $\left(\frac{1}{x^2+1} > 1\right)$ is false.

proof:

let $x \in R \equiv p$

let $\frac{1}{x^2+1} > 1 \equiv q$ is false

indirect proof: $\bar{q} \to \bar{p} \equiv p \to q$

$\therefore \bar{q} \equiv \frac{1}{x^2+1} > 1$ is True

$\bar{p} \equiv x \notin R$

طريقه الاثبات بـ indirect $\bar{q} \to \bar{p}$

∴ لازم نبدأ الاثبات من $\bar{q}$ الى ان نصل لـ $\bar{p}$

$\bar{q} = \frac{1}{x^2+1} > 1$ is True

مفيش رقم فى الـ R هيحقق

ان تربيعة اقل من Zero

حتى الـ Zero نفسه لو مربع

هيساوى الصفر وليس اقل منه

$\therefore 1 > x^2+1$ $\therefore x^2 < 0$

$\therefore x \notin R$

∴ بدأنا الاثبات بـ $\bar{q}$ ووصلنا لـ $\bar{p}$

$\therefore \exists x \in R \to \left(\frac{1}{x^2+1} > 1\right)$ false

(12) prove by contradiction that if $a^2 - 2a + 6$ is even then a is even.

let a is odd $\therefore a = 2k+1$, $k \in \mathbb{Z}$

$a^2 - 2a + 6 = (2k+1)^2 - 2(2k+1) + 6$

$= 4k^2 + 4k + 1 - 4k - 2 + 6$

$= 4k^2 + 5 = 4k^2 + 4 + 1 = 4(k^2+1) + 1$

$a^2 - 2a + 6$ is odd

this is contradiction

$\therefore$ if $a^2 - 2a + 6$ is even then a is even.

13 if $A = \{1,2\}$, $B = \{0,3\}$ and $C = \{1,4,5\}$ find

① $A \cap C = \{1,2\} \cap \{1,4,5\} = \{1\}$.

② $A \cup B = \{1,2,0,3\}$.

③ $A - B = \{1,2\}$

④ $A \oplus C = (A \cup C) - (A \cap C)$

$= \{5,1,2,4\} - \{1\} = \{2,4,5\}$

⑤ $A \times B = \{(1,0),(1,3),(2,0),(2,3)\}$

⑥ $A \times A = \{(1,1),(1,2),(2,1),(2,2)\} = A^2$

⑦ $B \times B = \{(0,0),(0,3),(3,0),(3,3)\} = B^2$

⑧ $B \times A = \{(0,1),(0,2),(3,1),(3,2)\}$

⑨ $A \times B \times C = \{(1,0,1),(1,0,4),(1,0,5)$
$(1,3,1),(1,3,4),(1,3,5)$
$(2,0,1),(2,0,4),(2,0,5)$
$(2,3,1),(2,3,4),(2,3,5)\}$

⑩ prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0,1,3,4,5\}$

L.H.S $= A \times (B \cup C) = \{1,2\} \times \{0,1,3,4,5\}$

$= \{(1,0),(1,1),(1,3),(1,4),(1,5)$
$(2,0),(2,1),(2,3),(2,4),(2,5)\}$

$\because A \times C = \{1,2\} \times \{1,4,5\}$

$= \{(1,1),(1,4),(1,5),(2,1),(2,4),(2,5)\}$

$\therefore$ R.H.S $= (A \times B) \cup (A \times C) = \{(1,0),(1,3),(2,0),(2,3),(1,1)$
$(1,1),(1,4),(1,5),(2,1),(2,4),(2,5)\}$

$\therefore$ L.H.S $=$ R.H.S