Lecture 3

$\in$	belong to
$\notin$	not belong to
$=$	equality
$\subset$	Subset
$\emptyset$	Empty Set
S	Universal Set
$\cup$	Union
$\cap$	Intersection

Set: is a collection of objects, called elements of the set.

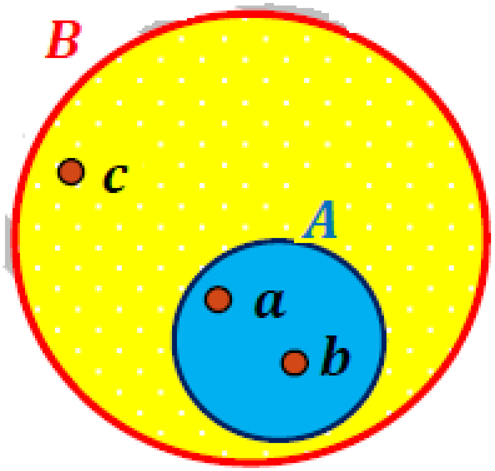
Set	Capital letter A, B
Element	small letter x, y

Equality: Two sets are equal if and only if they have the same elements

$A = B \equiv \forall x \, (x \in A \leftrightarrow x \in B)$.

Subset **A⊂B** or A contained in B

$$A \subset B = \{x : (x \in A) \rightarrow (x \in B)\}$$



Power Set: Is the collection of all possible subsets can be constructed

If A={1,2,3} ,then

$P(A)=\{\varphi,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},A\}$

if the number of elements of a set ***A*** is **n**,

the number of elements of the power set is 2^n.

Remark

If $A = \{\{1,3\},\{5\},2,6\}$, we observe that $\{1,3\} \in A$, because it represents an element of A, the element $1 \notin A$, but $2 \in A$.

Cardinal Number: Is the number of elements inside the set

If A={2,3,4}, then the Cardinal number is

$n(A)$=3

Example-30

List the elements of the following sets, where n is of type integer

① $\{n \mid n^2 = 1\}$ **Sol^n:** $\{1, -1\}$

② $\{n \mid n \text{ divides } 12\}$ **Sol^n:** $\{\mp 1, \mp 2, \mp 3, \mp 4, \mp 6, \mp 12\}$

③ $\{n^2 \mid n \in z\}$ **Sol^n:** $\{0, 1, 4, 9, 16, \ldots\}$

Union of sets

$$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$$

Intersection of sets

$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$$

Complement of sets

$$\bar{A} = \{x : (x \notin A) \wedge (x \in S)\}$$

<u>Difference of sets</u> or relative complement

$$A - B = \{x \mid (x \in A) \wedge (x \notin B)\}$$

$$A - B = A \cap \bar{B}$$

<u>symmetric difference</u>

$$A \oplus B = \{x \mid (x \in A) \oplus (x \in B)\}$$
$$A \oplus B = (A \cup B) - (A \cap B)$$
$$A \oplus B = (A - B) \cup (B - A)$$

1.7.2. Properties of sets

No.	The Rule	Property
❶	$(A \cap A) \equiv A$ $(A \cup A) \equiv A$	Idempotent Rule
❷	$(A \cap B) \equiv (B \cap A)$ $(A \cup B) \equiv (B \cup A)$	Commutative Rule
❸	$A \cap (B \cap C) \equiv (A \cap B) \cap C$ $A \cup (B \cup C) \equiv (A \cup B) \cup C$	Associatively Rule
❹	$\overline{A \cap B} \equiv \bar{A} \cup \bar{B}$ $\overline{A \cup B} \equiv \bar{A} \cap \bar{B}$	De-Morgan Rule
❺	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$	Distribution Rule

❻	$(A \cup S) \equiv S$ $(A \cup \emptyset) \equiv A$ $(A \cap S) \equiv A$ $(A \cap \emptyset) \equiv \emptyset$	Bound Rule
❼	$\overline{\overline{(p)}} \equiv p$	Double Negation
❽	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Rule
❾	$A \cup \bar{A} \equiv S$ $A \cap \bar{A} \equiv \emptyset$	Complement Rule
⑩	$\bar{\emptyset} = S$ and $\bar{S} = \emptyset$	0/1 Rule
❶❶	If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$	

Corollary

❶	If $A \cap B = \emptyset$,then A and B called disjoint sets.
❷	$(A \subset B) \wedge (B \subset A) \Leftrightarrow A = B.$
❸	$(A \subset B) \wedge (B \subset C) \Rightarrow A \subset C.$
❹	$\emptyset$ is a subset of any set i.e. $\emptyset \subset \{1\}$, $\emptyset \subset \{3, 4\}$, ...
❺	$\bar{A} = \bar{B} \quad \Leftrightarrow \quad A = B.$
❻	The **<u>symmetric difference</u>** between two sets A and B is $A \,\Delta\, B = A \oplus B = (A \cup B) - (A \cap B).$

Prove that:

❶ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Solution

❶ $\text{L.H.S} = \overline{A \cap B} \quad = \{x \mid (x \notin A \cap B)\}$

$= \{x \mid (x \notin A) \lor (x \notin B)\}$

$= \{x \mid (x \in \bar{A}) \lor (x \in \bar{B})\}$

$= \{x \mid x \in (\bar{A} \cup \bar{B})\} = \text{R.H.S}$

A	B	$\bar{A}$	$\bar{B}$	$A \cap B$	$\overline{A \cap B}$	$\bar{A} \cup \bar{B}$
1	1	0	0	1	0	0
1	0	0	1	0	1	1
0	1	1	0	0	1	1
0	0	1	1	0	1	1

Prove that:

❷ $A - \bar{B} = A \cap B$

❷ $\text{L.H.S} = A - \bar{B} = \{x \mid (x \in A) \wedge (x \notin \bar{B})\}$

$= \{x \mid (x \in A) \wedge (x \in B)\}$

$= \{x \mid x \in (A \cap B)\}$

A	B	$\bar{B}$	$A - \bar{B}$	$A \cap B$
1	1	0	1	1
1	0	1	0	0
0	1	0	0	0
0	0	1	0	0

Prove that:

❸ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

❸ $\text{L. H. S} = A \cap (B \cup C) = \{x | (x \in A) \wedge (x \in B) \vee (x \in C)\}$

$= \{x | [(x \in A) \wedge (x \in B)] \text{ or } [(x \in A) \wedge (x \in C)]\}$

$= \{x | [x \in (A \cap B)] \vee [x \in (A \cap C)]\}$

$= (A \cap B) \cup (A \cap C) = \text{R. H. S}$

						L.H.S.	**R.H.S.**
A	B	C	$A \cap B$	$A \cap C$	$B \cup C$	$A \cap (B \cup C)$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0
0	1	0	0	0	1	0	0
0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0

Example-32

Prove that: $(A \cap \bar{B}) = \emptyset \;\Leftrightarrow\; A \subset B$

Solution

First: Start from L.H.S. to R.H.S. with $(B \cup \bar{B}) = S$	**Second**: Start from R.H.S. to L.H.S. with $A \subset B$
$A \cap (B \cup \bar{B}) = A \cap S$	$A \cap \bar{B} \subset B \cap \bar{B}$
$(A \cap B) \cup (A \cap \bar{B}) = A$	$A \cap \bar{B} \subset \emptyset$
$(A \cap B) \cup \emptyset = A$	Then $A \cap \bar{B} = \emptyset$ ■
$(A \cap B) = A \rightarrow \; A \subset B$ ■	

Generalized Union and Intersection

$$\bigcup_{n=1}^{N} A_n = A_1 \cup A_2 \cup \ldots \cup A_N = \{x \mid \exists n\ (x \in A_n)\}$$

$$\bigcap_{n=1}^{N} A_n = A_1 \cap A_2 \cap \ldots \cap A_N = \{x \mid \forall n, x \in A_n\}$$

Example-33

Let $S_n = \{kn \,|\, k = 2, 3, 4, \dots\}$ be the set of multiples of n greater than n. Find the set of composite [not prime numbers] positive integers $\bigcup_{n=2}^{\infty} S_n$.

Solution

$$S_2 = \{2k | k = 2, 3, \dots\} = \{4, 6, 8, 10, \dots .\}$$
$$S_3 = \{3k | k = 2, 3, \dots\} = \{6, 9, 12, 15, \dots .\}$$
$$S_4 = \{4k | k = 2, 3, \dots\} = \{8, 12, 16, 20, \dots .\}$$
$$\bigcup_{n=2}^{\infty} S_n = S_2 \cup S_3 \dots \cup S_n = \{4, 6, 8, 9, 10, 12, 14, 15, 16, \dots\} \quad \blacksquare$$