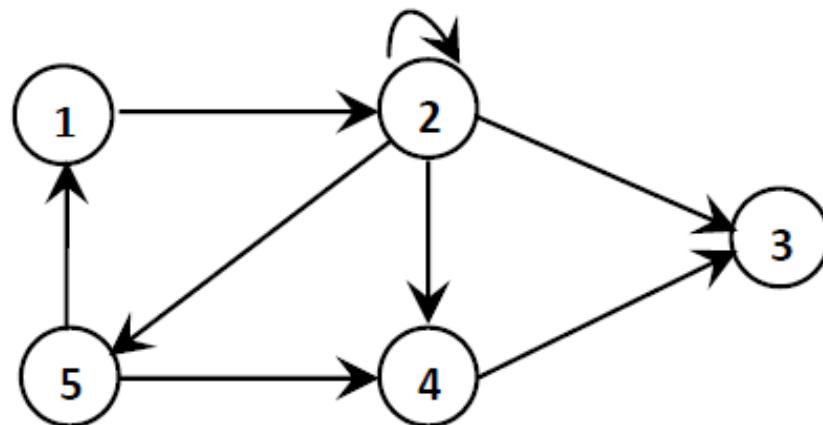


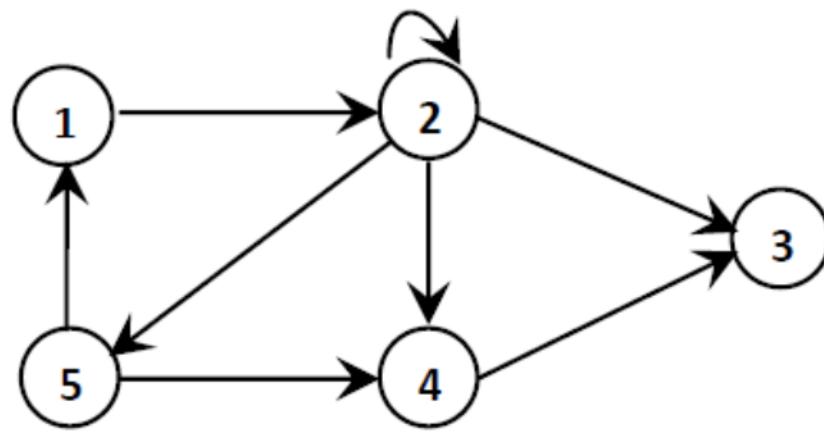
# Lecture 4

# Paths in Relations and Diagraphs

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- A path is succession of edges.
- The length of a path is "the number of edges" in the path

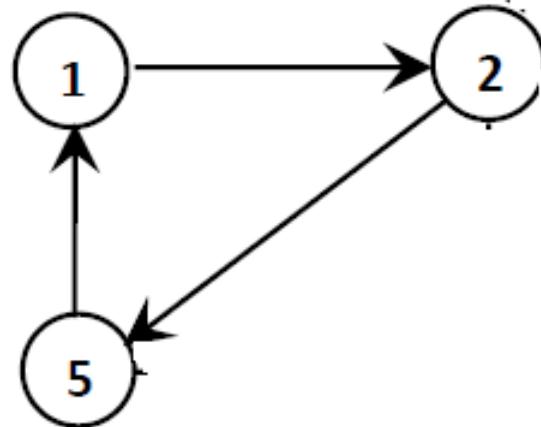




- The path  $\pi_1: 1, 2, 5, 4, 3$  is a path of length 4
- The path  $\pi_2: 1, 2, 5, 1$  is a path of length 3 (three edges).
- The path  $\pi_3: 2, 2$  is a path of length 1 from vertex 2 to itself.

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A path that begins and ends at the same vertex is called a **cycle**.



$$xR^n y$$

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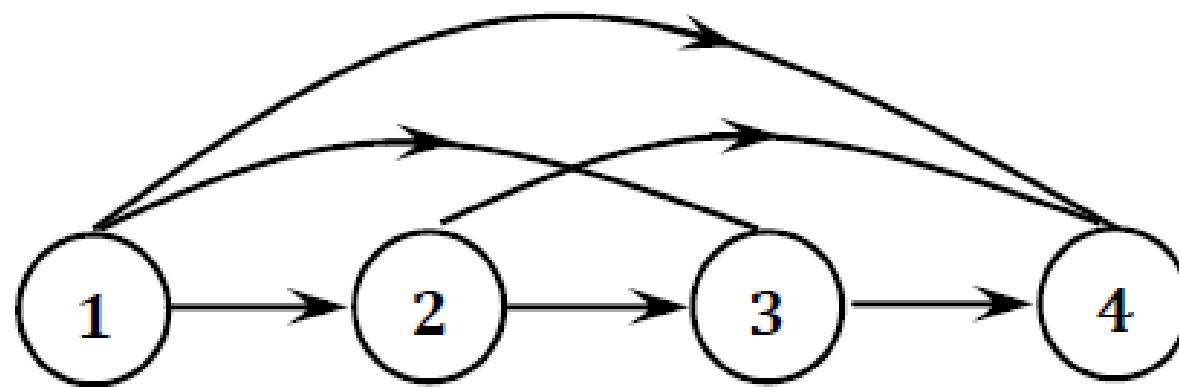
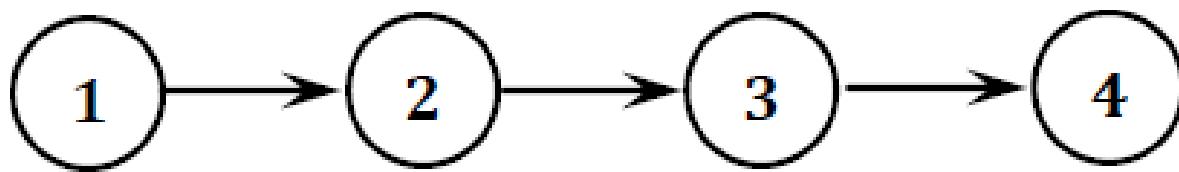
n is positive integer

We define a relation  $R^n$  as follows:

$xR^n y$  means that there is a path of length **n** from x to y in R.

$R^\infty$

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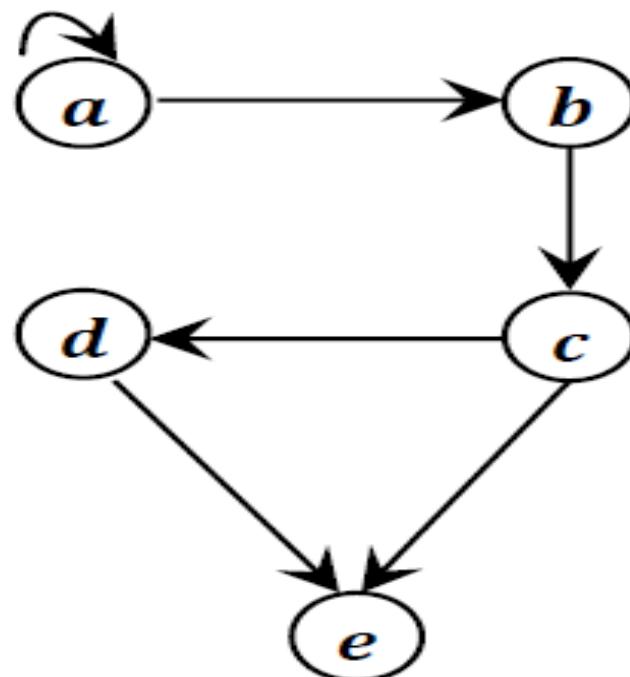
Let  $A = \{a, b, c, d, e\}$  and

$$R = \{(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)\}$$

Compute

①  $R^2$

②  $R^\infty$



$a R^2 a$  since  $a R a$  and  $a R a$

$a R^2 b$  since  $a R a$  and  $a R b$

$a R^2 c$  since  $a R b$  and  $b R c$

$b R^2 e$  since  $b R c$  and  $c R e$

$b R^2 d$  since  $b R c$  and  $c R d$

$c R^2 e$  since  $c R d$  and  $d R e$

Hence  $R^2 = \{(a, a), (a, b), (a, c), (b, e), (b, d), (c, e)\}$

■

$R^o = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, d), (c, e), (d, e)\}$

---

## Transitive closure

Transitive Closure  $R_1 = R^{\infty}$

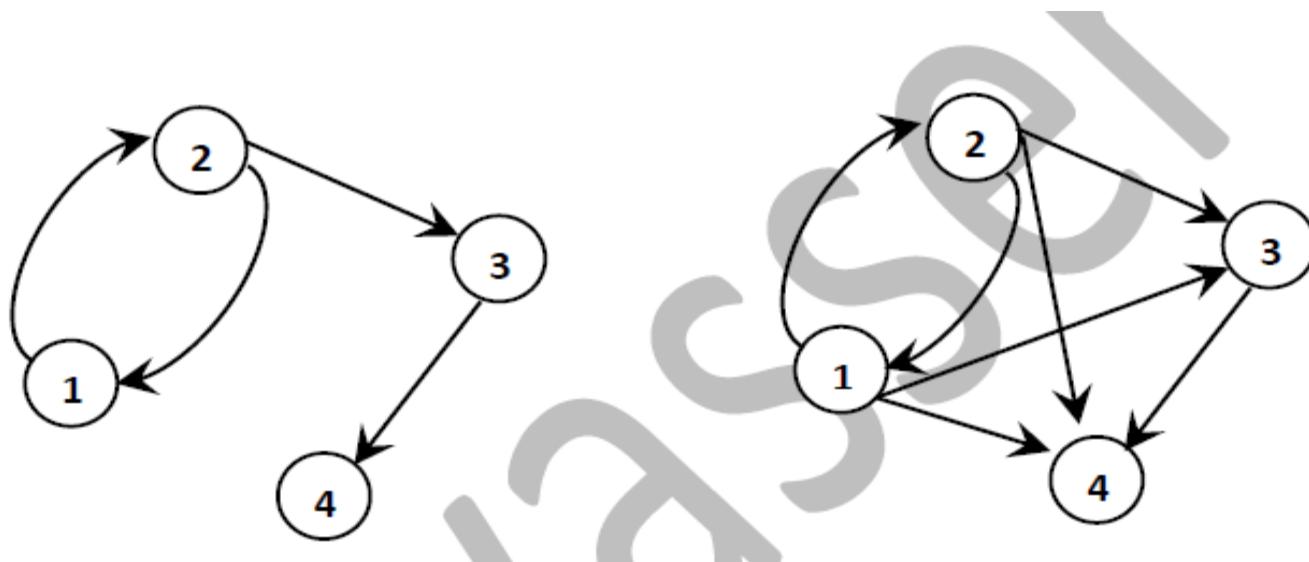
$(c,d) \dots (d,c) \rightarrow (c,c)$

$(d,c) \dots (c,d) \rightarrow (d,d)$



## Example

Let  $A=\{1,2,3,4\}$  and  $R=\{(1,2),(2,3),(3,4),(2,1)\}$ , find the transitive closure of  $R$ .



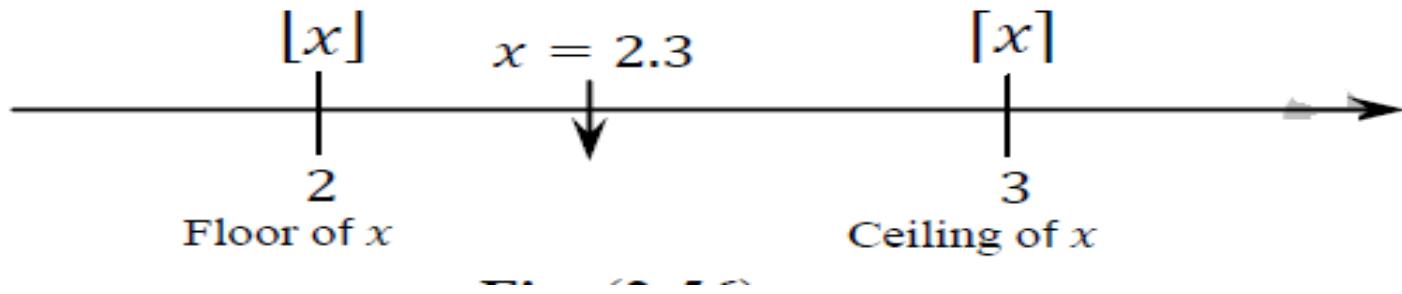
$$R^\infty = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\}$$

---

# Special Types of Functions

# Floor and Ceiling Functions

Let  $x$  be a real number



---

The *floor* of a real number  $x$ , denoted  $\lfloor x \rfloor$ , is the greatest integer less than or equal to  $x$

$$\lfloor 2 \rfloor = 2,$$

$$\lfloor \pi \rfloor = 3,$$

$$\lfloor -2.5 \rfloor = -3$$

---

The *ceiling* of a real number  $x$ , denoted by  $[x]$ , is the least integer greater than or equal to  $x$

$$[2]=2,$$

$$[\pi]=4,$$

$$[-2.5]=-2$$

$$\lfloor 2.3 \rfloor = 2$$

$$\lfloor 0.3 \rfloor = 0$$

$$\lfloor -2.3 \rfloor = -3$$

$$\lceil 2.3 \rceil = 3$$

$$\lceil 0.3 \rceil = 1$$

$$\lceil -2.3 \rceil = -2$$

$$\lceil \sqrt{5} \rceil = 3$$

$$\lceil -0.2 \rceil = 0$$

$$\lceil -4 \rceil = -4$$

$$\lceil 5 \rceil = 5$$

$$\lceil 5 \rceil = 5$$

$$\lceil 0 \rceil = 0$$

---

□  $\lceil x \rceil = \lfloor x \rfloor$ ,  $\forall x \in z$  (integer number)

□  $\lceil x \rceil = \lfloor x \rfloor + 1$ ,  $\forall x \notin z$  and x is rational ( $\mathbb{Q}$ ).

□ The functions  $f(x) = \lceil x \rceil$  and  $f(x) = \lfloor x \rfloor$  map from  $\mathcal{R}$  to  $z$  is **onto** but **not one to one**

---

## Mod Function

Define the function  $f(x,y)=x \bmod y$  as the remainder when  $x$  is divided by  $y$

where  $x$  is any integer ( $x \in \mathbb{Z}$ )

$y$  is positive natural number ( $y \in \mathbb{N}$ )

---

---

$f:(x,y) \rightarrow (x \bmod y)$

**domain**  $\{(x,y) : x \in \mathbb{Z}, \text{ and } y \in \mathbb{Z}^+\}$

**codomain**  $W = \mathbb{N} \cup \{0\}$

**range** is the interval  $[0, y)$ .

The mod function **onto** but **not one to one**,

$1 \bmod 12 = 1$  and also  $13 \bmod 12 = 1$ .

---

If  $x \bmod y = r$ , then  $0 \leq r < y$

---

$$7 \bmod 4 = 3$$

$$\frac{7}{4} = 1 + \frac{3}{4}$$

$$5 \bmod 1 = 0$$

$$\frac{5}{1} = 5 + \frac{0}{1}$$

$$5 \bmod 6 = 5$$

$$\frac{5}{6} = 0 + \frac{5}{6}$$

$$25 \bmod 7 = 4$$

$$\frac{25}{7} = 3 + \frac{4}{7}$$

$$16 \bmod 4 = 0$$

$$\frac{16}{4} = 4 + \frac{0}{4}$$

$$0 \bmod 4 = 0$$

$$\frac{0}{4} = 0 + \frac{0}{4}$$

## when x is negative

$$x \bmod y = y - [|x| \bmod y]$$

$$-26 \bmod 7 = 7 - [26 \bmod 7] = 7 - 5 = 2.$$

$$-16 \bmod 3 = 2$$

$$3 - (16 \bmod 3) = 2$$

$$-5 \bmod 1 = 1$$

$$1 - (5 \bmod 1) = 1$$

$$-5 \bmod 5 = 5$$

$$5 - (5 \bmod 5) = 5$$

## Theorem

If  $x \bmod y = r$ , then  $y$  divides  $x - r$ .

---

## Application

What day of the week will it be after 365 days from Wednesday?

## Solution

Since after 7 days the **Wednesday** will come again

$$365 \bmod 7 = 1$$

then Thursday is the required day

# Barcodes



## International Standard Book Number (ISBN)

ISBN 978-0-306-40615-7



Formally, the ISBN-10 check digit calculation is:

$$x_{10} = [(x_1 \times 1) + (x_2 \times 2) + (x_3 \times 3) + \cdots + (x_9 \times 9)] \bmod 11$$

### Example-

Find the *check digit* for an ISBN-10 of 0 – 306 – 40615 –  $x_{10}$ ?

### Solution

$$\begin{aligned}x_{10} &= [(0 \times 1) + (3 \times 2) + (0 \times 3) + (6 \times 4) + (4 \times 5) + (0 \times 6) + \\&\quad (6 \times 7) + (1 \times 8) + (5 \times 9)] \bmod 11 = 145 \bmod 11 = 2\end{aligned}$$

□ Formally, the ISBN-13 check digit calculation is:

$$x_{13} = 10 - [(x_1 + 3x_2 + x_3 + 3x_4 + x_5 + \cdots + 3x_{12}) \bmod 10]$$

### Example

Find the check digit for an ISBN-13 of

978-0-306-40615- $x_{13}$

$$x_1 + 3x_2 + x_3 + 3x_4 + x_5 + \cdots + 3x_{12} =$$

$$= (9 \times 1) + (7 \times 3) + (8 \times 1) + (0 \times 3) + (3 \times 1) + (0 \times 3) + (6 \times 1)$$

$$+ (4 \times 3) + (0 \times 1) + (6 \times 3) + (1 \times 1) + (5 \times 3) =$$

$$= 9 + 21 + 8 + 0 + 3 + 0 + 6 + 12 + 0 + 18 + 1 + 15 = 93$$

$$x_{13} = 10 - (93 \bmod 10) = 10 - 3 = 7$$

■

Thus, the check digit is 7, the ISBN 978-0-306-40615-7.

## Hash Function

If the computer memory cell is indexed from 0 to 10, then we have 11 places to store and retrieve numbers.

|     |   |   |     |    |     |     |   |     |   |    |
|-----|---|---|-----|----|-----|-----|---|-----|---|----|
| 165 |   |   | 102 | 15 | 258 | 137 |   | 558 |   | 76 |
| 0   | 1 | 2 | 3   | 4  | 5   | 6   | 7 | 8   | 9 | 10 |

$$h(n) = n \bmod 11$$

---

$$h(165)=0,$$

$$h(102)=3,$$

$$h(15)=4,$$

$$h(3)=3$$

$$h(76)=10$$

---

---

The Hash function

$$h(x): \mathcal{N}(\text{natural numbers}) \rightarrow \{0, 1, \dots, 10\}$$

It is **not one to one**

$$137 \neq 258 \rightarrow h(137) = h(258) = 5$$

but it is **onto**

---

## Example

Find the value of:  $2h(258) - 4(93 \bmod 10)$

## Solution

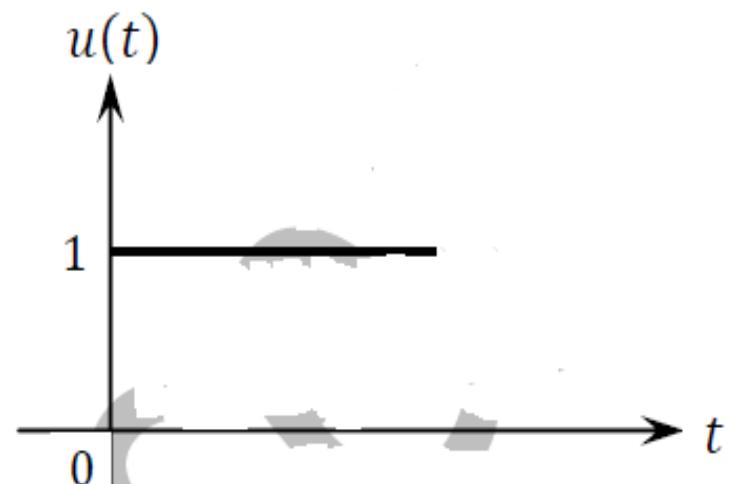
$$2h(258) - 4(93 \bmod 10) = 2(5) - 4(3) = -2$$

## Try to

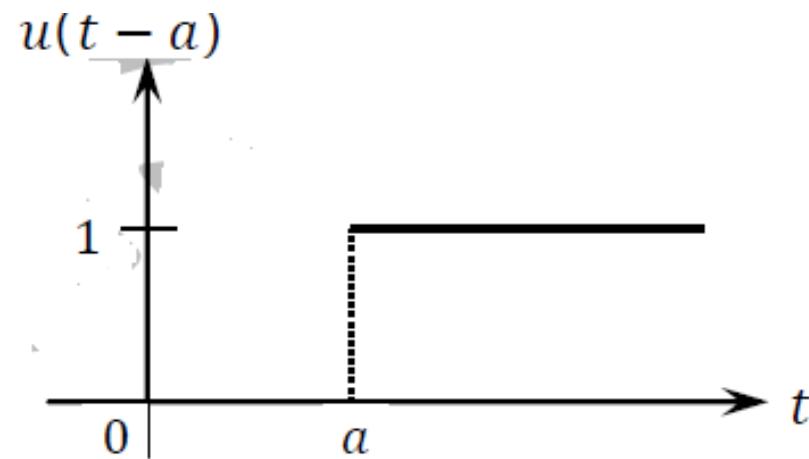
find the value of:  $2h(122) + 5 [2.7] - 2[-2.7] - 5 \times (10 \bmod 7)$

# Unit Step Function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

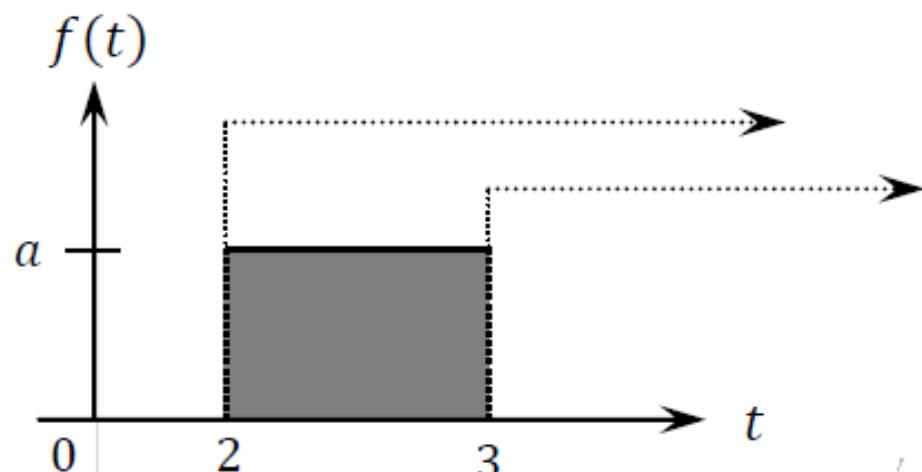


$$u(t - a) = \begin{cases} 0 & t - a < 0 \\ 1 & t - a \geq 0 \end{cases}$$



## Example-

Graph the function:  $f(t) = a$ ,  $2 < t < 3$ ; express the function  $f(t)$  in terms of unit step functions.



$$f(t)=a[u(t-2)-u(t-3)]$$

---

### Example-

Express the function  $g(t)$  in terms of unit step functions;

$$g(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$$

$$g(t) = t u(t) - t u(t-1) + u(t-1)$$

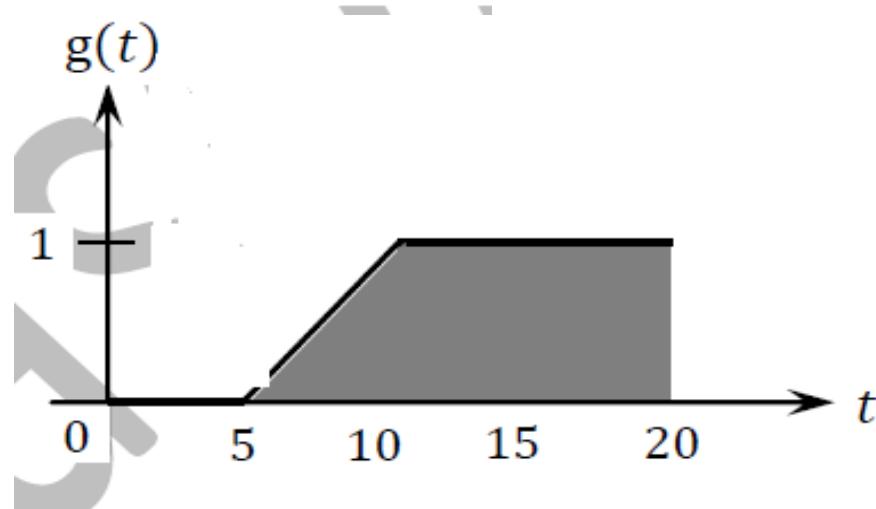
$$u(t) = 1 \quad 0 \leq t < 1$$

$$g(t) = t - (t-1) u(t-1)$$

### Example-38

Express the function  $g(t)$  in terms  
of unit step functions;

$$g(t) = \begin{cases} 0 & 0 < t < 5 \\ (t - 5)/5 & 5 \leq t < 10 \\ 1 & t \geq 10 \end{cases}$$



---

$$g(t) = 0 \times [u(t - 0) - u(t - 5)] + \left(\frac{t-5}{5}\right) \times [u(t - 5) - u(t - 10)] + 1 \times [u(t - 10)]$$

$$g(t) = \left(\frac{t-5}{5}\right) u(t - 5) - \left(\frac{t-5}{5}\right) u(t - 10) + u(t - 10)$$

$$g(t) = \left(\frac{t-5}{5}\right) u(t - 5) - \left(\frac{t-5}{5} - 1\right) u(t - 10)$$

$$g(t) = \left(\frac{t-5}{5}\right) u(t - 5) - \left(\frac{t-10}{5}\right) u(t - 10)$$

■