Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 28-11-2020; Time: **1.5- Hours**

Module(B)

Mid-Term Examination 2020-2021

Discrete Mathematics (BS-103)

Module(B)

First Question (12.5- Marks)

❶ Choose the correct sign "✓ or ✗" for the followings:

[1]	$\neg[\forall x\ P(x)] \equiv \exists x\ \neg P(x)$	(… … …)
[2]	$p \wedge \bar{p}$ is always true	(… … …)
[3]	$\forall x \in \mathcal{R}, p(x): (x^3 - 1) = (x-1)(x^2 + x + 1)$	(… … …)
[4]	The domain of a function is contained in the codomain	(… … …)
[5]	If n is an integer then $(\forall\ n)(3n + 1 > n)$	(… … …)
[6]	$\overline{p \to q} = p \wedge \bar{q}$	(… … …)
[7]	If $R: X \to Y$ is a relation, then its inverse is $R^{-1}: Y \to X$	(… … …)
[8]	$P \wedge T \equiv T$	(… … …)
[9]	The relation "less than" is reflexive	(… … …)
[10]	Let $P(x,y,z): xy < x + z + 1$, then $p(x, x, x)$ is always false $\forall x \in z$	(… … …)
[11]	$\exists x \in \mathcal{R}, \quad p(x): \dfrac{x^2 - 1}{2x + 1} \geq 0$	(… … …)
[12]	If p is $4 \geq 2$ and q is "$5 \leq 2$" then $p \oplus q$ is false	(… … …)
[13]	The range of the exponential function is $\mathcal{R}$	(… … …)
[14]	The relation $R \cup R^{-1}$ refers to reflexive closure.	(… … …)
[15]	$\overline{p \wedge F} = T$	(… … …)

❷ Choose the correct answer for the following statements:

[1]	The Logarithmic function $f(x) = \mathrm{Log}_a(x), f: \mathcal{R}^+ \to \mathcal{R}$, is	{one to one, onto, both}
[2]	If $f: \mathcal{R} \to \mathcal{R}^+, f(x) = 2^{x^2}$, then f is not	{one to one, onto, both}
[3]	The range of the function $f(x) = 5 \sin(2x - 1) + 2$ is	$\{[-1,1], [-3,7], [-3,3]\}$
[4]	For the exponential function $f: \mathcal{R} \to \mathcal{R}^+, f(x) = a^x, a \in \cdots$	$\{\mathcal{N}, z^+, z^+ - \{1\}\}$
[5]	The range of the function $f(x) = \mathrm{Log}(x)$ is ….	$\{\mathcal{R}, \mathcal{R}^+, \mathcal{R}^-\}$
[6]	$1 + \cot^2(x) = \cdots$	$\{cosec^2(x),\ sec^2(x)\}$
[7]	The domain of $y = \tan(x)$ is $\mathcal{R} - \{\ldots + k\pi\},\ \ k \in z$	$\{\frac{\pi}{2}, \pi,\ 2\pi\}$
[8]	$A - \bar{B} = \cdots$	$\{A \cup B, A \cap B, B - A\}$
[9]	$A \oplus B = (A \cup B) - (A \cap B)$	{True, False}
[10]	The domain of the function $f(x) = 5 \sin(2x - 1) + 2$ is	$\{\mathcal{R}^+, \mathcal{R}, [-1,1]\}$

Solve behind the page if necessary.

Second Question (12.5- Marks)

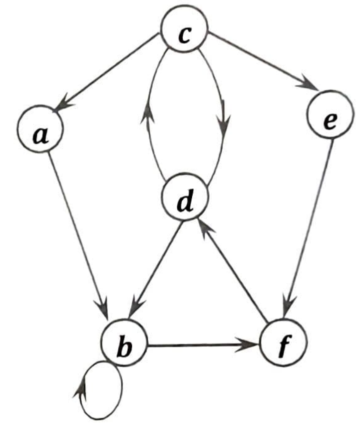
❶ Behind this paper, prove that $\sinh^{-1} x = \mathrm{Ln}\left(x + \sqrt{x^2 + 1}\right)$

❷ Use the laws of logic to simplify: $\overline{p \to (p \wedge q)}$

Third Question (15- Marks)

① If $x = 2$, then $3x - 5 \neq 10$. Prove that this statement is true by contradiction.

② From the opposite diagraph



❶ Write R as an ordered pairs,

❷ List all paths of length 2 starting from vertex c,

❸ Find the symmetric closure of R,

❹ Find the reflexive closure of R,

❺ Find the matrix A for the diagraph (or the relation)

(انتهت الأسئلة)

Solve behind the page if necessary.

Module (B)

Question 1:

(1) True

(2) False $p \wedge \bar{p} \equiv F$

(3) True $(x-1)(x^2+x+1) = x^3 + x^2 + x - x^2 - x - 1$

$= x^3 - 1$

(4) False Range of fun. is Contained in the Codomain

(5) False $n \in \mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$

at $n = -1$ $-3 + 1 > -1$

$-2 > -1$ False

(6) True $\overline{p \to q} \equiv \overline{\bar{p} \vee q} \equiv \bar{\bar{p}} \wedge \bar{q} \equiv p \wedge \bar{q}$

(7) True

(8) False $p \wedge T \equiv p$

(9) False

since less than

$(1,2), (1,3), (1,4), \ldots$

$(2,3), (2,4), (2,5), \ldots$

$\vdots$

all ordered Pairs $\in R$

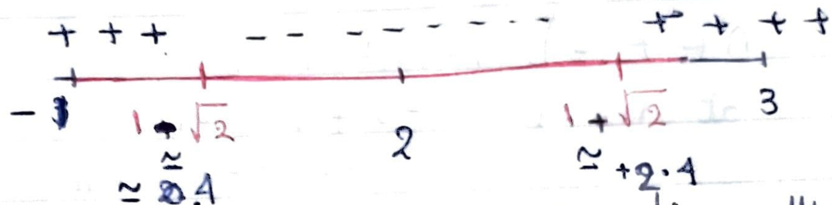
but $(1,1), (2,2), (3,3), \ldots \notin R$

$\therefore$ R not reflexive

10 False

$$P(x,y,z) : xz < x + z + 1$$
$$P(x,x,x) : x^2 < 2x + 1$$
$$x^2 - 2x - 1 < 0 \longrightarrow (*)$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = 1 \pm \sqrt{2}$$



في الفترة اللي هتخلي المعادلة (*) اقل من Zero يعني
تحقق المتباينة

at $x = (-1)$
$(-1)^2 - 2(-1) - 1 = 2 > 0$ مبتحققش

at $x = 2$
$(2)^2 - 2(2) - 1 = -1 < 0$ بتحقق

at $x = 3$
$(3)^2 - 2(3) - 1 = 2 > 0$ مش بتحقق

$\therefore$ $P(x)$ is True at $x \in]1-\sqrt{2}, 1+\sqrt{2}[$
$P(x)$ is False at $x \in R -]1-\sqrt{2}, 1+\sqrt{2}[$

الاجابة False لانها مش دايماً False لكل قيم x

11) True

since $\exists x \in R$

لبعض قيم الـ x

اللى بتنتمى للأعداد الحقيقة

$\therefore$ at $x=0$ $\quad \frac{x^2-1}{2x+1} = \frac{-1}{1} = -1 \ngeq 0$ مش هتتحقق

at $x=2$ $\quad \frac{(2)^2-1}{2(2)+1} = \frac{4-1}{4+3} = \frac{3}{7} \geq 0$ متحقق

$\therefore$ بعض القيم لـ x تتحقق مش شرط كل القيم

12) False $\quad p: 4 \geq 2 \quad p \equiv T$

$q: 5 \leq 2 \quad q \equiv F$

$T \oplus F \equiv T$ هنا بتطلع True لو الاتنين مختلفين

13) False $\quad D_F = R^d \quad R_F = R^+$

14) False $\quad R \cup R^{-1}$ refers to symmetric closure

15) $\quad \overline{P \wedge F} \equiv \overline{F} \equiv T$

True

Question 2:

1) both

2) one-to-one

3) $[-3, 7]$

4) $Z^+ - [1\ 4]$

5) R

6) $Cosec^2(x)$

7) $\frac{\pi}{2}$

8) $A \cap B$

9) True

10) R

$\sin^2 x + \cos^2 x = 1$

$\div \sin^2 x$

$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\boxed{1 + \cot^2 x = \csc^2 x}$

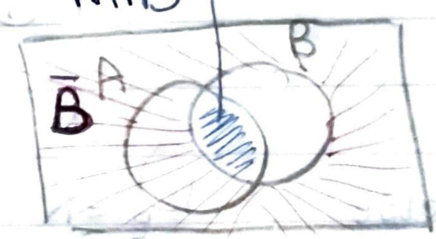
$\div \cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$

$\boxed{\tan^2 x + 1 = \sec^2 x}$

داخل A ولكنه ليس داخل $\bar{B}$

$A \cap B$



$A - \bar{B}$: موجود في A مش موجود في $\bar{B}$

∴ الموجود في A ومش موجود في $\bar{B}$

هو الجزء التقاطع

① Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$

let $\sinh^{-1} x = y$

$x = \sinh y$

$$x = \frac{e^{y} - e^{-y}}{2}$$

$$2x = e^{y} - e^{-y} \qquad \times e^{y}$$

$$2xe^{y} = e^{2y} - 1$$

$$e^{2y} - 2xe^{y} - 1 = 0 \qquad \text{let } e^{y} = z$$

$$z^2 - 2xz - 1 = 0 \qquad z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2x) \pm \sqrt{4x^2 - 4}}{2}$$

$$z_{1,2} = \frac{2x \pm \sqrt{4(x^2-1)}}{2} = \frac{2x \pm 2\sqrt{x^2-1}}{2} = \frac{2\left(x \pm \sqrt{x^2-1}\right)}{2}$$

$$z_{1,2} = x \pm \sqrt{x^2-1}$$

Substitute z by e^{y}

$$e^{y} = x \pm \sqrt{x^2-1}$$

$$y = \ln\left(x \pm \sqrt{x^2-1}\right)$$

$$= \ln\left(x + \sqrt{x^2-1}\right)$$

Since $\ln$(negative value) undefined

(2) use Laws of Logic to simplify

$$\overline{P \rightarrow \overline{(P \cap q)}} \qquad \text{use } P \rightarrow q \equiv \bar{P} \vee q$$

$$\equiv \overline{\bar{P} \vee \overline{(P \cap q)}} \qquad \text{using deMorgen's Law}$$

$$\equiv \bar{\bar{P}} \cap \overline{\overline{(P \cap q)}} \equiv P \cap (P \cap q)$$

$$\equiv (P \cap P) \cap q \equiv P \cap q \ \#$$

Question Three:

1) if $x=2$ then $3x-5 \neq 10$

Prove that this statement is Tiere by Contraduction

Proof:

let P: $x=2$

q: $3x-5 \neq 10$

$\bar{q}$: $3x-5=10$

Start with $\bar{q}$

$3x-5=10 \rightarrow 3x=15 \rightarrow x=5$

This Contradiction with $P=2$

2) 1) $R = \{ (c,a), (c,d), (c,e), (a,b), (d,b), (d,c), (b,f)$
$(e,f), (f,d), (b,b) \}$

2)
$\pi_1 : c, a, b$
$\pi_2 : c, d, c$
$\pi_3 : c, d, b$
$\pi_4 : c, e, f$
$\pi_5 : c, d, c$

3) Symmetric closure of R

$$R_1 = R \cup R^{-1}$$

$= \{ (c,a), (c,d), (c,e), (a,b), (d,b)$
$(d,c), (b,f), (e,f), (f,d), (b,b) \}$

$\cup$

$\{ (a,c), (d,c), (e,c), (b,a), (b,d)$
$(c,d), (f,b), (f,e), (d,f), (b,b) \}$

$= \{ (a,c), (c,a), (c,d), (d,c), (c,e), (e,c), (a,b), (b,a)$
$(d,b), (b,d), (d,c), (c,d), (b,f), (f,b), (e,f)$
$(f,e), (f,d), (d,f), (b,b) \}$

4) reflexive closure of R

$R_1 = R \cup \Delta$

$= \{(c,a),(c,d),(c,e),(a,b),(d,b),$
$(d,c),(b,f),(e,f),(f,d),(b,b)\}$

$\cup$

$\{(a,a),(b,b),(c,c),(d,d),(e,e),(f,f)\}$

$R_1 = \{(c,a),(c,d),(c,e),(a,b),(d,b),(d,c),(b,f),(e,f)$
$(f,d),(b,b),(a,a),(c,c),(d,d),(e,e),(f,f)\}$

$$
\begin{array}{c|c}
 & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$