Final Examination of
Discrete Mathematics BS - 103

First Question (16 Points)

- Calculate the value of $(12 \bmod 5) + h(15) - 3\lfloor -4.7 \rfloor + 2\lceil 0.3 \rceil$.
- Solve the 1^{st} order difference equation: $a_n = a_{n-1}$, with $a_1 = 4$.
- Change the lower index of the sum $\sum_{k=1}^{n} a_{n-k}$ to start with $k = 0$.
- Find the <u>check digit</u> of the opposite bar code :

4 710088 41255

Second Question (16 Points)

- Find a numerical value of $\sqrt{50}$ approximated to four digits.
- Solve the recurrence relation $P_n = a + s\,P_{n-1}$ of the economic model where, a and s are parameters depend on the model.
- If $f(x) = x^2$, Find the $\lim_{n \to \infty} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$.
- Give three different examples of odd functions.

Third Question (20 Points)

- Calculate the numerical value of the summation: $\sum_{k=1}^{k=800}(2k + 1)$.
- Department consists of 10 men and 5 women. How many ways to conform a committee consists of 4 persons provided that at least 2 men are selected?
- Find Taylor series expansion (just 4 terms) for the function: $f(x) = \cos x$ about $x = 0$.
- Choose the correct answer:

① $\lfloor x \rfloor = \lceil x \rceil + 1$ for all $x \in \cdots$	$\{\mathbb{Z}, \mathbb{Q} - \mathbb{Z}, \mathbb{R}\}$
② The function $f(x) = \sin(3x)$ is	{even, odd}
③ If $f(x) = \lfloor x + 1 \rfloor$, then $10 \bmod 5 - 0.5\, f(-1.5)$ equals	{1, 2, 0}
④ A tree with 3 −vertices has exactely .... edges	{2, 4, 8}
⑤ Every relation is a function?	{Yes, No}

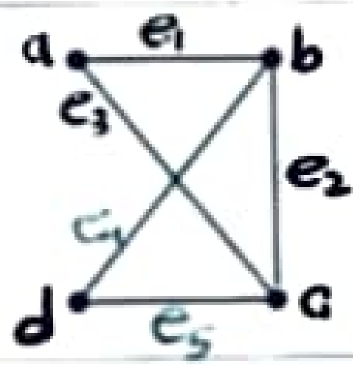
الإمتحان يقع في صفحتين من نفس الورقة.

Fourth Question (28 Points)

- **Construct the tree of the following mathematical expression:**

$$\left((2 \times x) + (3 - (4 \times x))\right) + (x - (3 \times 11))$$

- **For the opposite diagraph:**

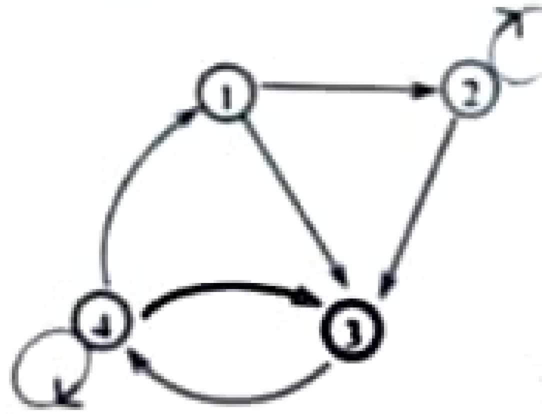


(1) Find the adjacency matrix.

(2) Find the Laplacian matrix.

(3) Find the incident matrix.

- **For the opposite graph:**



(1) Write the ordered pairs for the relation *R*

(2) Construct a linked list representation.

VERT, TAIL, HEAD and NEXT.

The End.

Dr/ Yasser M. Hamada

Final 2018

First Question :

1- Calculate Value of

$$(12 \bmod 5) + h(15) - 3\lfloor -4.7 \rfloor + 2\lceil 2.3 \rceil$$

Sol. :- 15 mod 11

$$2 + 4 - 3(-5) + 2(1) = 23$$

2- $a_n = a_{n-1}$ $a_1 = 4$

$a_2 = a_1 = 4$

$a_3 = a_2 = 4$

$a_4 = a_3 = 4$

⋮

$a_n = 4$ #

3- $K = 1$ to $j = 0$ $\therefore K = 1 + j$ at Lower at $K = 1$

$\sum_{K=1}^{K=n} a_{n-k}$ to $K = 0$ at upper : at $K = n$

$\therefore n = j + 1$

$j = n - 1$

$\because$ Sum start from $K = 1$ to $K = n$

Lower ↓ upper ↓

$K = 1$ $K = n$

$k = 1 + j$ $K = 1 + j$

Put $K = 1$ $\therefore j = 0$ # $j = n - 1$ #

$$\sum_{j=0}^{j=n-1} a_{n-(j+1)} = \sum_{j=0}^{n-1} a_{n-j-1} \quad \#$$

(4) $x_{13} = 10 - \left[(x_1 + 3x_2 + x_3 + 3x_4 + \cdots + 3x_{12}) \bmod 10 \right]$

$= 10 - \left[(4 + 3(7) + 1 + 3(0) + 0 + 3(8) + 8 + 3(4) + 1 + 3(2) + 5 + 3(3)) \right] \bmod 10$

$= 10 - (91 \bmod 10) = 9$

Second Question :

1- $\sqrt{50} = (49+1)^{1/2} = \left[49\left(1 + \frac{1}{49}\right) \right]^{1/2}$ $\quad \because \left| \frac{b}{a} \right| < 1$

$\therefore \sqrt{50} = \sqrt{49} \left[1 + \frac{\left(\frac{1}{2}\right)}{1!} \left(\frac{1}{49}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!} \left(\frac{1}{49}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!} \left(\frac{1}{49}\right)^3 + \cdots \right] \simeq 7.071$

الحل في الكتاب
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2- $P_n = a + sP_{n-1}$

$= a + s(a + sP_{n-2}) = a + as + s^2 P_{n-2}$

$= a + as + s^2(a + sP_{n-3}) = a + as + as^2 + s^3 P_{n-3}$

$\vdots$

$a + as + as^2 + \cdots + as^{k-1} + s^k P_{n-k}$

Put $n - k = 0$ initial value $\therefore n = k$

$= \underbrace{(a + as + as^2 + \cdots + as^{n-1})}_{\text{Sum of } n \text{ terms of geometric series}} + s^n P_0$

$S_n = \frac{a(r^n - 1)}{r - 1}$, $r = \frac{a_{n+1}}{a_n} = \frac{as^n}{as^{n-1}} = s$ $\quad$ كنا ممكن نوجدها بمجرد التعويض

$a = a$

$P_n = \frac{a(s^n - 1)}{s - 1} + s^n P_0$ #

(3) if $f(x) = x^2$ Find the $\lim\limits_{n \to 1} \sum\limits_{k=1}^{n} f\left(\frac{k}{n}\right)$

$\because f(x) = x^2$

$\therefore f\left(\frac{k}{n}\right) = \left(\frac{k}{n}\right)^2 = \frac{k^2}{n^2}$

$$\lim_{n \to 1} \sum_{k=1}^{n} \frac{k^2}{n^2} = \lim_{n \to 1} \frac{1}{n^2} \sum_{k=1}^{n} k^2$$

لازم نفكها باستخدام القوانين بدلالة (n) عشان نقدر نوجد لها النهاية

$$= \lim_{n \to 1} \left[\frac{1}{n^2} \left(\frac{n(n+1)(2n+1)}{6} \right) \right]$$

Put $n = 1$

$$= \frac{2(3)}{1 \times 6} = 1 \quad \#$$

(4) $f(x) = x$ is odd function

$f(x) = x^3$ is odd function

$f(x) = \sin x$ is odd function

$f(x) = x^3 + 5x$ is odd function

$f(x) = \sin x + x^5$ is odd function

Third Question

1- Calculate the numerical value of the summation

$$\sum_{K=1}^{K=800}(2K+1) = 2\sum_{K=1}^{800} K + \sum_{K=1}^{800} 1$$

$$= 2\left[\frac{800(800+1)}{2}\right] + 1(800) = 641600 \#$$

شركة او مؤسسة تحتوي على 10 men & 5 women

2- كم عدد الطرق التي يمكن فيها تكوين لجنة من 4 اشخاص

بشرط ان يكون اللجنة دي فيها على الاقل رجلين

$$C^{10}_2\, C^5_2 + C^{10}_3\, C^5_1 + C^{10}_4\, C^5_0 = 1260$$

احتمال يكون رجلين من 10 رجال و 2 ست من اصل 5	احتمال يكون 3 رجال من اصل 10 وست واحدة من 5	احتمال يكونوا كلهم رجال ولا يوجد ست واحدة

(3) $f(x) = \cos x$ about $x = 0$

$f(x) = \cos x$	$f(0) = 1$
$f'(x) = -\sin x$	$f'(0) = 0$
$f''(x) = -\cos x$	$f''(0) = -1$
$f'''(x) = \sin x$	$f'''(0) = 0$
$f^{(4)}(x) = \cos x$	$f^{(4)}(0) = 1$
$f^{(5)}(x) = -\sin x$	$f^{(5)}(0) = 0$
$f^{(6)}(x) = -\cos x$	$f^{(6)}(0) = -1$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\,(x)^n}{n!}$$

$$\therefore \cos x = \frac{1}{0!} + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \frac{0}{5!}x^5 + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad \#$$

لأنه طالب 4 حدود ∴ لازم نوجد الحدود تكون غير صفرية

يعني لازم نوجد 4 terms

(4)

1- $\mathbb{Q}-\mathbb{Z}$

2- odd

3- 0

4- 2

5- No

1- كل rational ماعدا الجزء الـ integer فيها

لأن الناتج فيه كل الأعداد الـ integer ايضاً اعداد ratio

3- $10 \bmod 5 - 0.5\, f(-1.5)$

$= -0.5\, f(-1.5) = -0.5 \lceil -1.5+1 \rceil$

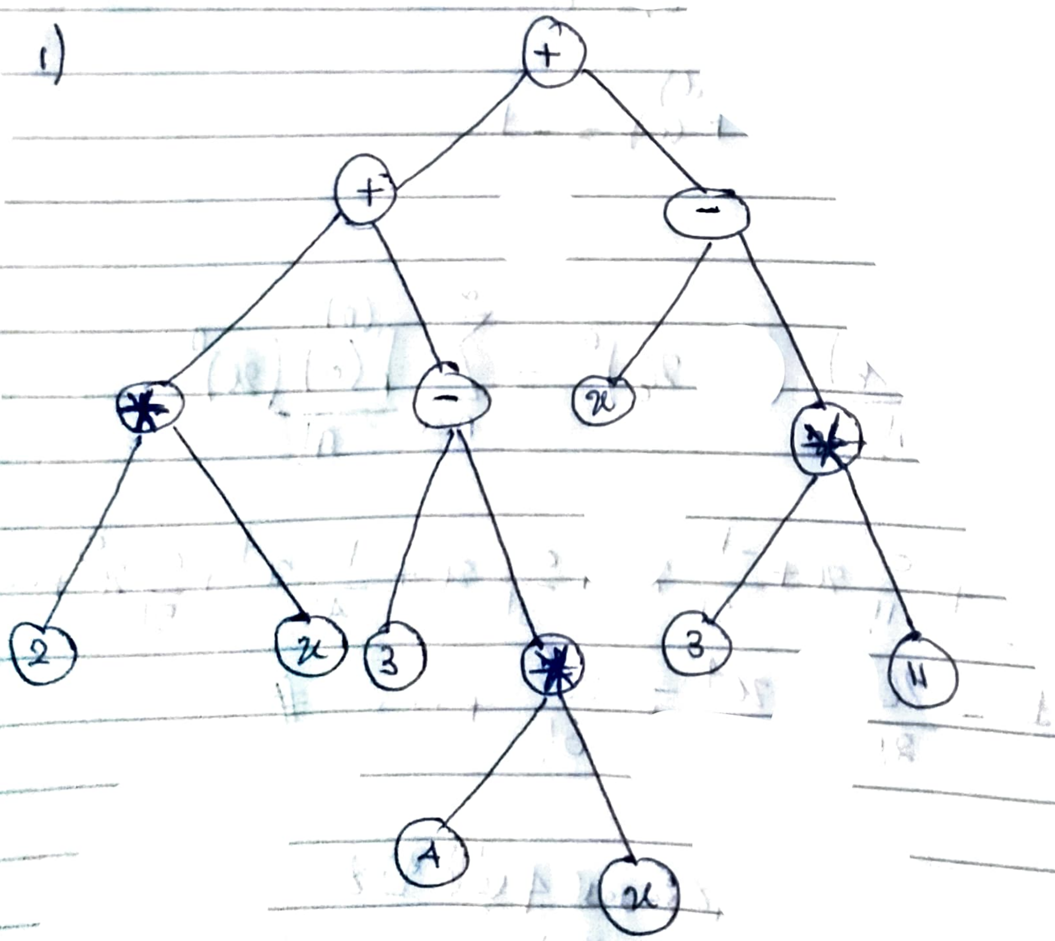
$= -0.5 \lceil -.5 \rceil$

$=$ zero

4- A tree with n vertices has exactly n-1 edges

Fourth Question

1)



(2)

(1) Find incident Matrix

بحط 1 عند كل عنصر والمسارات E منه

$$
\begin{array}{c|c}
 & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \end{matrix} &
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
$$

(2) بحط 1 بين كل two Vertex بينهم relation

لو مفيش علاقة بين two vertex Zero

$$
\begin{array}{c|c}
 & \begin{matrix} a & b & c & d \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \end{matrix} &
\begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

(3) LalPaCiean Matrix

$$
\begin{array}{c|c}
 & \begin{matrix} a & b & c & d \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \end{matrix} &
\begin{bmatrix}
2 & -1 & -1 & 0 \\
-1 & 3 & -1 & -1 \\
-1 & -1 & 3 & -1 \\
0 & -1 & -1 & 2
\end{bmatrix}
\end{array}
$$

1- القطر الرئيسي عبارة عن edge لكل vertex

2- باقي الـ Matrix عبارة عن 0, -1

3- لو في علاقة بين two vertex نكتب (-1)

مفيش علاقة نكتب Zero

(3)

(1) $R = \{(2,2),(4,4),(1,2),(1,3),(2,3),(3,4),(4,1),(4,3)\}$. #

ملحوظه حل السؤال الاخير

ملغي 😊