

Mathematical induction

الاستدلال الرياضي

عند دليل اي بطرق الاستدلال الرياضي

$S_n \rightarrow$ Statement يقرئنا

1) Prove that S_n at $n=1$ is True

2) Suppose that statement S_n is True

3) we need to Prove that S_{n+1} is True

ex: 1

Prove that

$$\text{Proof: } 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{let } S_n = 1^2 + 2^2 + 3^2 + \dots + n^2$$

1) Prove that S_n at $n=1$ is True



$$\text{L.H.S} = S_1 = 1^2 = \text{R.H.S} = \frac{1(2)(3)}{6} = 1 \therefore \text{True at } n=1$$

2) Suppose that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ is True

3) we need to Prove that

$$S_{n+1} = 1^2 + 2^2 + 3^2 + \dots + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

Term متعدد

b3, n JS Class

$n+1$

$$\begin{aligned}
 \text{L.H.S} &= \underbrace{1^2 + 2^2 + 3^2 + \cdots + n^2}_{n(n+1)(2n+1)} + (n+1)^2 \\
 &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = (n+1) \left[\frac{n(2n+1)}{6} + (n+1) \right] \\
 &= (n+1) \left[\frac{2n^2 + 7n + 6}{6} \right] = \frac{(n+1)(n+2)(2n+3)}{6} = \text{R.H.S}
 \end{aligned}$$

ex. 2: Prove That

$7^n - 1$ is divisible by 6 $\forall n \in \mathbb{N}$

Proof:

$$S_n = 7^n - 1 \quad \text{لما يتحقق}$$

1) at $n=1$ $S_1 = 7^1 - 1 = 6$ is divisible by 6

2) suppose that $S_n = 7^n - 1$ is divisible by 6

3) we need to prove that at S_{n+1} is True

$$\begin{aligned}
 S_{n+1} &= 7^{n+1} - 1 = 7(7^n - 1) + 6 - 6 \\
 &= 7(7^n - 1) + 6
 \end{aligned}$$

$$\begin{aligned}
 &= 7(7^n - 1) + 6 \quad \text{Sum of two divisible} \\
 &\quad \text{by 6 are also divisible} \\
 &\quad \text{by 6}
 \end{aligned}$$

\therefore Sum is divisible by 6

Example 3:

Prove that

$x^{2n} - y^{2n}$ is divisible by $x+y$
 $\forall n \in \mathbb{N} \cup \{0\}$

Proof:

$$S_n = x^{2n} - y^{2n}$$

1) at $n=1$

$$S = x^2 - y^2 = (x-y)(x+y)$$

 \therefore divisible by $x+y$

2) Suppose that

$S_n = x^{2n} - y^{2n}$ is divisible by $x+y$

3) we need to prove that

$\Leftrightarrow S_{n+1}$ is true

$$\begin{aligned} S_{n+1} &= x^{2n+2} - y^{2n+2} \\ &= x^{2n+2} - y^{2n+2} + x^{2n}y^2 - x^{2n}y^2 \\ &= x^{2n}(x^2 + y^2) + y^2(x^{2n} - y^{2n}) \end{aligned}$$

نهايات $n=1$ تقبل
 $x+y$ في المقدمة

أصناف نهاياتها
الفائق قبل المقدمة

$\therefore S_{n+1}$ is divisible by $x+y$

Example 4:

Prove that $n! \geq 2^n$ $\forall n \in \mathbb{N}$

Proof:

$$S_n = n! \geq 2^{n-1}$$

1) at $n=1$
 $1! \geq 2^{1-1}$ $1 \geq 0$ True

2) suppose that S_n is true

3) we need to prove that
 S_{n+1} is true

$$S_{n+1} = (n+1)! \geq \frac{(n+1)!}{2}$$

$$\text{L.H.S} = (n+1)! = (n+1) \cdot A!$$

$$\geq (n+1) \cdot 2^{n-1} \leftarrow \text{استدلال}$$

$\therefore n \in \mathbb{N}$

أقل رقم له قيمة المقدمة

لذلك فإنها هي الأكبر قيمة

R.H.S \geq L.H.S \therefore L.H.S \leq R.H.S

$$\therefore (n+1)! \geq (n+1) \cdot 2^{n-1}$$

$$\geq 2 \cdot 2^{n-1}$$

$$\geq 2^n$$

ناتج المقدمة

examite 5:-

Prove that $1+2n \leq 3^n \quad \forall n \in \mathbb{N}$

Proof:

$$S_n = 1+2n \leq 3^n$$

1) at $n=1$ $S_1 = 1+2 \leq 3$

is True

2) Suppose That

S_n is True

3) we need to prove that at $n+1$
is True

$$S_{n+1} = 1+2(n+1) \leq 3^{n+1}$$

~~$\Rightarrow 1+2(n+1) \leq 3^{n+1}$~~

$$\text{R.H.S} = 3^{n+1} = 3 \cdot 3^n$$

$$> (1+2n) \cdot 3$$

$$\geq 3 + 6n \geq 2n+3$$

الحالات في القسم الأكبر

1 L.H.S < R.H.S

لما القسم الأكبر نقل كيس خارج

L.H.S < R.H.S