



Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date: 13-1-2020; Time: 3- Hours

(C) 15.2 = 6
E.S. 21 = 15



Final Examination of Discrete Mathematics BS - 103

الامتحان يقع في ورقة من صفحتين

First Question (10 Marks)

Choose the appropriate signs "✓" or "✗" for the following:

[1] The day of the week will it be after 200 days from Monday is Friday. ✓

[2] If $R_2 = R \cup R^{-1}$, then R_2 should be symmetric. ✗

[3] If $x \bmod y = r$, then y divides $x - r$

[4] The general term of the sequence: 3, 0.3, 0.03, ... is of the form: $\left(\frac{3}{10^n}\right)$ for $n \geq 1$

[5] A simple path is a path with no repeated vertices.

[6] The degree of a vertex in an in-directed graph is the number of edges incident with it.

[7] The Big O notation is used to give an upper bound of the running time of an algorithm.

[8] If $f: \mathbb{R} \rightarrow \mathbb{R}^+$, $f(x) = \sin(x^2)$, then f is one to one function. ✗

[9] If x is rational number, then $[x] = x + 1$.

[10] The number of ways to select two persons from a group of 4 persons is 6

Second Question (10 Marks)

Choose the correct answer

[11] The infinite series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ is

SS M T W T
0.75 1/2 1/4 1/8 1/16 1/32 1/64 1/128 1/256 1/512 1/1024 1/2048 1/4096 1/8192 1/16384 1/32768 1/65536 1/131072 1/262144 1/524288 1/1048576 1/2097152 1/4194304 1/8388608 1/16777216 1/33554432 1/67108864 1/134217728 1/268435456 1/536870912 1/1073741824 1/2147483648 1/4294967296 1/8589934592 1/17179869184 1/34359738368 1/68719476736 1/137438953472 1/274877906944 1/549755813888 1/1099511627776 1/2199023255552 1/4398046511104 1/8796093022208 1/17592186044416 1/35184372088832 1/70368744177664 1/140737488355328 1/281474976710656 1/562949953421312 1/1125899906842624 1/2251799813685248 1/4503599627370496 1/9007199254740992 1/18014398509481984 1/36028797018963968 1/72057594037927936 1/144115188075855872 1/288230376151711744 1/576460752303423488 1/1152921504606846976 1/2305843009213693952 1/4611686018427387904 1/9223372036854775808 1/18446744073709551616 1/36893488147419103232 1/73786976294838206464 1/147573952589676412928 1/295147905179352825856 1/590295810358705651712 1/1180591620717411303424 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1/2371421964192461965431441893372373505014251377653376 1/4742843928384923930862883786744747005028506755346752 1/94856878

Third Question (30 Marks)

① Given the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ to:

(1) prove by mathematical induction that $A^k = \frac{1}{2} \begin{bmatrix} 1+3^k & -1+3^k \\ -1+3^k & 1+3^k \end{bmatrix}$,

(2) solve the system of the recurrence relations in terms of the initial values x_0 and y_0

$$x_{n+1} = 2x_n + y_n$$

$$y_{n+1} = x_n + 2y_n$$

② Express the function $f(t)$ by the unit step function, where $f(t) = \begin{cases} t^3 + 2t^2 - 1 & 0 \leq t \leq 1 \\ 2t^2 - t - 1 & 1 < t \leq 2 \\ t - 1 & 2 < t \leq 3 \\ -1 & t > 3 \end{cases}$.

③ Prove that if n is odd, then $\left[\frac{n^2}{4}\right] = \frac{n^2+3}{4}$.

④ Construct the tree of the following expression: $(3 - (2 - (11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$

, and then find the height of the tree.

Fourth Question (30 Marks)

① Use Taylor series to approximate the function $f(x) = \sin(2x) + e^{5x}$ to just three terms and then use this to approximate the value $f(0.1)$.

② Change the lower index of the summation $\sum_{k=1}^{k=n} a_{k-1} a_{n-k}$ to start with $k = 3$.

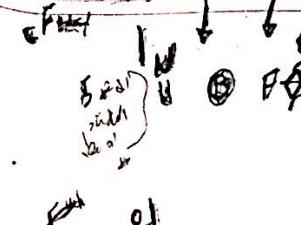
③ (a) From the opposite figure, complete:

(1) The adjacency matrix is $A =$

(2) The Laplacian matrix is $L =$

(3) The incident matrix is $I =$

(b) Find the degree of each vertex.



$$M_A = \begin{bmatrix} a & b & c & d \\ a & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 1 & 0 \\ d & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (a,a), (a,d)$$

④ Let $A = \{a, b, c, d\}$, and let R be a relation on A such that:

$$(a \sim a) \quad (a \sim b) \quad (a \sim c) \quad (a \sim d)$$

$$(b \sim a) \quad (b \sim b) \quad (b \sim c) \quad (b \sim d)$$

$$(c \sim a) \quad (c \sim b) \quad (c \sim c) \quad (c \sim d)$$

$$(d \sim a) \quad (d \sim b) \quad (d \sim c) \quad (d \sim d)$$

(1) Write the relation R as an ordered pairs,

(2) Construct a linked list representation, VERT, TAIL, HEAD and NEXT for the relation R .

(1,

(انتهت بالستة)

(٢) د. مصطفى عبد العزيز

(٣) أ.د.م/ ياسر محمد عبد العتيق

(٤) أ.د.م/ ياسر محمد عبد العتيق

2020

Q1

① $\frac{20^2}{7} = 28 \frac{4}{7}$
الباقي الأصغر من الترکار لا يعاد الحصص المدورة
 $6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0$ ✓

$$\begin{aligned} 20^2 &= 28 \frac{4}{7} \\ 202 - 28 \times 7 &= \end{aligned}$$

$$(2 + 200) \mod 7 = 6 \quad \text{الباقي}$$

② ✓

③ ✓

④ ✗ $\frac{3}{10^n} \quad n \geq 0$

⑤ ✗ $\frac{3}{10^n} \quad n \geq 0$ ✓

⑥ ✓

⑦ cusho

⑧ ✗

⑨ ✗ ~~.....~~ ✓

⑩ $4C_2 = 6$ ✓

⑪ ~~.....~~ $\frac{1}{2^m}$

⑫ ✗ ملاعنة

⑬ Symetric

$$f(x) = \underline{x^2 + ax}$$

$$x^2 + ax = x^2 - ax$$

$$f(-x) = \underline{x^2 - ax}$$

$$2ax = 0$$

$$\begin{cases} 2a = 0 \\ a = 0 \end{cases}$$

(15) Bijective

$$(16) 3 \lceil 1.5 \rceil + 2 \lfloor -1.5 \rfloor =$$

$$3 * 2 + 2 * (-2) = 2$$

(17) R^+

$$(18) -3 \times h(210) + 12 \bmod 5$$

$$\frac{210}{11} = 19 \text{ rest } 1$$

$$-3 \times 1 + 2 = (-1)$$

$$210 - 19 * 11 = 1$$

$$(19) -44 \bmod 4 = 0 \quad 44 \bmod 4 = 0$$

$$(20) {}^6C_2 = \frac{6!}{2! 4!} = 15$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Prove $A^K = \frac{1}{2} \begin{bmatrix} 1+3^K & -1+3^K \\ -1+3^K & 1+3^K \end{bmatrix}$

$K=1$ $A^1 = \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Let $K=n$

$$A^n = \frac{1}{2} \begin{bmatrix} 1+3^n & -1+3^n \\ -1+3^n & 1+3^n \end{bmatrix}$$

A is invertible \Leftrightarrow

$K=1$
 $K=m+1$

$$A^{m+1} = \frac{1}{2} \begin{bmatrix} 1+3^m & -1+3^m \\ -1+3^m & 1+3^m \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2 + 2 \times 3^m - 1 + 3^m & 1 + 3^m - 2 + 2 \times 3^m \\ -2 + 2 \times 3^m + 1 + 3^m & -1 + 3^m + 2 + 2 \times 3^m \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 + 3 \cdot 3^m & -1 + 3 \cdot 3^m \\ -1 + 3 \cdot 3^m & 1 + 3 \cdot 3^m \end{bmatrix}$$

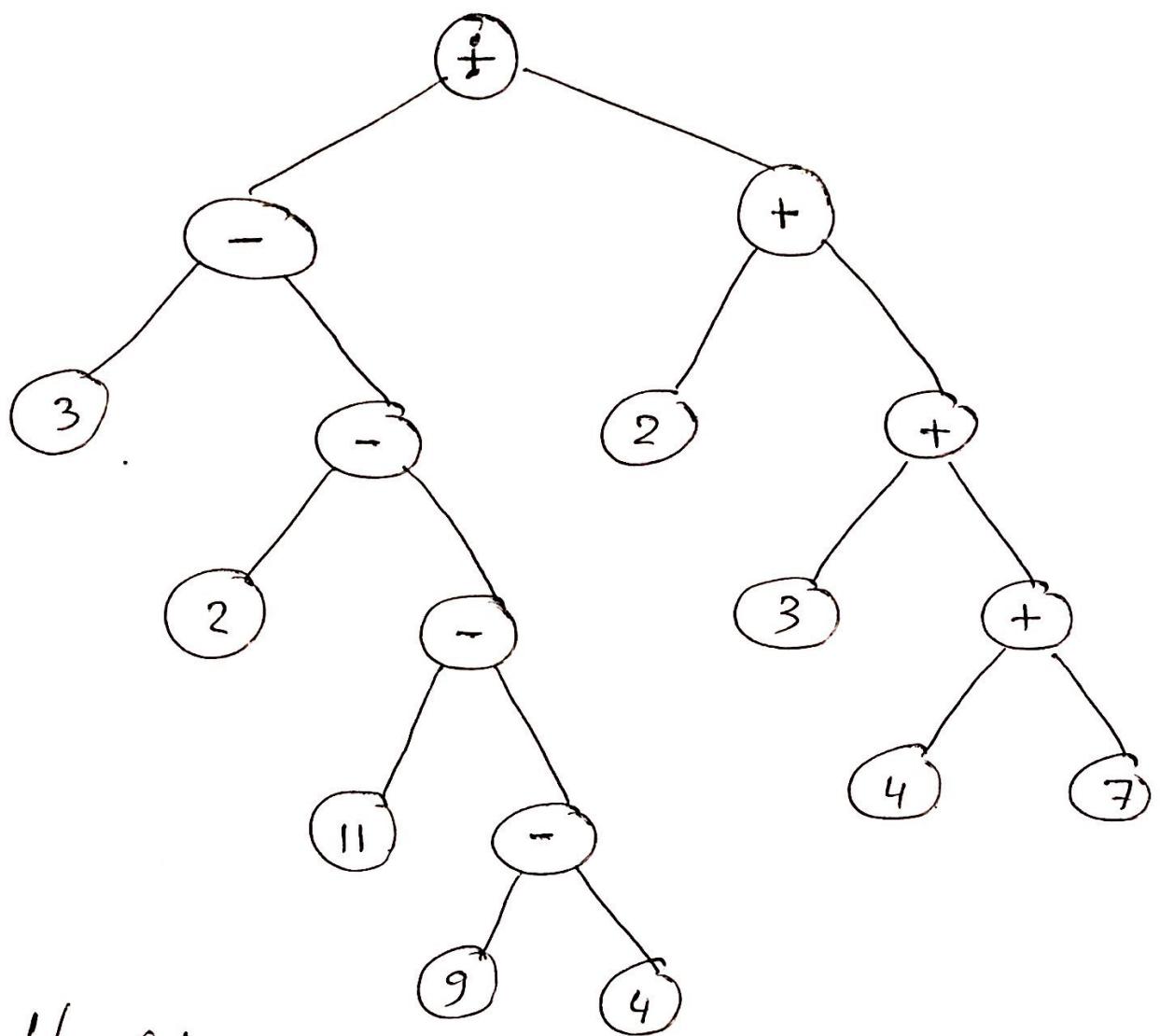
$$= \frac{1}{2} \begin{bmatrix} 1 + 3^{m+1} & -1 + 3^{m+1} \\ -1 + 3^{m+1} & 1 + 3^{m+1} \end{bmatrix}$$

(4) 3

(3)

$$② f(t) = \begin{cases} t^3 + 2t^2 - 1 & 0 \leq t \leq 1 \\ 2t^2 - t - 1 & 1 < t \leq 2 \\ t - 1 & 2 < t \leq 3 \\ -1 & t > 3 \end{cases}$$

$$\begin{aligned}
& (t^3 + 2t^2 - 1)u(t) + (2t^2 - t - 1)(u(t-1) - u(t)) \\
& + (t-1)(u(t-2) - u(t-3)) - u(t-3) \\
= & (t^3 + 2t^2 - 1)u(t) + (-t^3 - 2t^2 + 1)\cancel{u(t-1)} + (2t^2 - t - 1)\cancel{u(t-1)} \\
& + (-2t^2 + t + 1)u(t-2) + (t-1)u(t-2) + (-t+1)u(t-3) \\
& - u(t-3) \\
= & (t^3 + 2t^2 - 1)u(t) + (-t^3 - t)u(t-1) + (-2t^2 + 2t + 1) \\
& u(t-2) + (-t+1)u(t-3)
\end{aligned}$$



Height = 5

level $15 - 1$

$$f(x) = \sin(2x) + e^{5x}$$

(5)

$f(x) = \sin(2x) + e^{5x}$	$f(0) = 1$
$f'(x) = 2\cos 2x + 5e^{5x}$	$f'(0) = 7$
$f''(x) = -4\sin 2x + 25e^{5x}$	$f''(0) = 25$

$$f(x) = 1 + \frac{7}{1!}x + \frac{25}{2!}x^2 + \dots$$

$$\underline{f(0.1)} = 1 + 7 * 0.1 + \frac{25}{2!}(0.1)^2 = 1.7125$$

$$\sum_{K=1}^{K=m} a_{K-1} a_{m-K}$$

$$K=1$$

$$\delta=3$$

$$\delta = K+2 \rightarrow K = \delta - 2$$

$$\delta-2=n$$

$$\sum_{\delta-2=1} a_{\delta-2-1} a_{m-(\delta-2)}$$

$$\delta=m+2$$

$$\sum_{\delta=3} a_{\delta-3} a_{m-\delta+2}$$

$$\boxed{\sum_{K=3}^{K=m+2} a_{K-3} a_{m-K+2}}$$

④
③ adjacency

	a	b	c	d	e
a	0	1	1	1	1
b	1	0	1	0	0
c	1	1	0	1	1
d	1	0	1	0	1
e	1	0	1	1	0

Laplacian

	a	b	c	d	e
a	4	-1	-1	-1	-1
b	-1	2	-1	0	0
c	-1	-1	4	-1	-1
d	-1	0	-1	3	-1
e	-1	0	-1	-1	3

incident

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
a	1	0	0	0	1	0	1	1
b	1	1	0	0	0	0	0	0
c	0	1	1	0	0	1	0	1
d	0	0	1	1	0	0	1	0
e	0	0	0	1	1	1	0	0

(7)

$$\underset{\text{degree}}{\rightarrow} d(a) = 4$$

$$d(b) = 2$$

$$d(c) = 4$$

$$d(d) = 3$$

$$d(e) = 3$$
