# Special Types of Fun.:

1- Floor & Ceiling Fun.

2- Mod Fun.

3- hash Fun.

4- unit step Fun.

## 1) Floor and Ceiling Fun.:

Let X be a real num.
then X Lies between two integers Called Floor & Ceiling

Floor: $\lfloor x \rfloor$ is greatest int. Less than or equal to $x$

اكبر رقم صحيح اقل منه او يساوي $x$ ←

Ceiling: $\lceil x \rceil$ is Least int. greater than or equal to $x$

اقل رقم صحيح اكبر منه او يساوي $x$ ←

note

1) if $x$ is integer, then $\lceil x \rceil = \lfloor x \rfloor = x$

For example: $\lceil 2 \rceil = \lfloor 2 \rfloor = 2$

2) if $x$ is ~~integer~~ not integer, then $\lceil x \rceil = \lfloor x \rfloor + 1$

3) if $f(x) = \lceil x \rceil$ or $\lfloor x \rfloor$ then $f: R \longrightarrow Z$
is onto but not one-to-one

6) Comiute each the following

(i) $\lceil -2.78 \rceil$ Ceiling $= -2$

(ii) $\lceil 2.78 \rceil + \lfloor -17.3 \rfloor$

$= 3 + -18 = -15$

(iii) $|-3| - 2\lfloor -17.2 \rfloor$

absaute value

$3 - 2(-18) = 39$

Date: 11

Subject:

ex: Prove that if $n$ is odd

Then $\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$

Proof:

$$L.H.S = \left\lceil \frac{n^2}{4} \right\rceil = \left\lceil \frac{(2k+1)^2}{4} \right\rceil = \left\lceil \frac{4k^2+4k+1}{4} \right\rceil$$

$$= \left\lceil k^2+k+\frac{1}{4} \right\rceil$$

الدالة اللي بتكتب ⌈ ⌉ بتاخذ اي رقم مش صحيح

$k^2$ عدد صحيح ، $k$ عدد صحيح بس $\frac{1}{4}$ كسر وليس صحيح

لازم نخلي الربع $\left(\frac{1}{4}\right) = 1$

مثلاً $\lceil 97.25 \rceil = 98$

يعني .25 اللي هو ربع $\left(\frac{1}{4}\right)$

زودناها كلها وصلت الى (1)

$97 + 1$

integer $k^2+k$

$$\therefore L.H.S = k^2+k+1$$

$$R.H.S = \frac{n^2+3}{4}$$

$$= \frac{(2k+1)^2+3}{4} = \frac{4k^2+4k+1+3}{4}$$

$$= \frac{4k^2+4k+4}{3} = k^2+k+1$$

## 2- Mod Fun. :-

$$\boxed{f(x,y) = x \bmod y}$$

الداله معناها x فيها كام y والباقي هو الـ $r$ = remainder

as the remainder when $x$ is divided by $y$

where $x$ : any integer $x \in \mathbb{Z}$

$y$ : natural num. $y \in \mathbb{N}$

* الداله دي بتعبر عن الباقي عندما $x$ تتقسم بواسطه $y$

$$\boxed{f = x \bmod y = r} \quad \text{remainder}$$

$$\boxed{0 \leq r < y}$$

note

1- $x$ is divided by $y$ ⇒ $\frac{x}{y}$

2- $x$ divided $y$ ⇒ $y/x$

3- if $x$ is Positive

if $x < y$

$\therefore r = x$

if $x = y$

$\therefore r = 0$

$x \geq y$

① هنفضل نطرح الـ $y$ من $x$ الى ان نصل لرقم اقل من $y$ يكون هو ده الـ remainder

طويلة شوية للحل

②

Ex:

20 mod 9

$20 \div 9 = 2.2$

$20 - (9 \times 2) = 2$

له ده الـ

$r = 2$

4- if $x$ is negative

$$r = y - [|x| \bmod y]$$

7) Let f be the mode 10 fun.

مقياس mod 10

Compute :

1) $f(417) = 417 \text{ mod } 10$

$f(417) = 7$

$417 \div 10 = 41.7$

$417 - (10 \times 41)$

$= 7$

2) $f(38) = 38 \text{ mod } 10 = 8$

$38 \div 10 = 3.8$

$38 - (10 \times 3) = 8$

3) $f(253) = 253 \text{ mod } 10$

$= 3$

$253 \div 10 = 25.3$

$253 - (10 \times 25)$

$= 3$

4) $-2 f(41) + 2$

$f(41) = 41 \text{ mod } 10$

$= 1$

$41 \div 10 = 4.1$

$41 - (10 \times 4) = 1$

$-2(1) + 2 = 0$

Date  Subject

(19) let $f$ be the function from $X = [0, 1, 2, 3, 4]$ to $X$ defined by $f(x) = 4x \bmod 5$ write $f$ as a set of ordered pairs.

Is $f$ one-to-one or onto?

## Solution

at $x=0$ $f(0) = 0 \bmod 5$ $f(0) = 0$

$0 \div 5 = 0$
$0 - (5 \times 0) = 0$

at $x=1$ $f(1) = 4 \bmod 5$ $f(1) = 4$

$4 \div 5 = 0.8$ صحيح $0$
$4 - (5 \times 0) = 4$

at $x=2$ $f(2) = 8 \bmod 5$ $f(2) = 3$

$8 \div 5 = 1.6$ صحيح $1$
$8 - (5 \times 1) = 3$

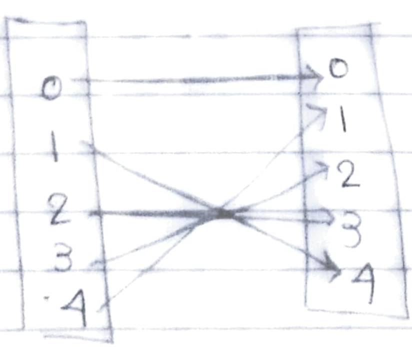
at $x=3$ $f(3) = 12 \bmod 5$ $f(3) = 2$

$12 \div 5 = 2.4$ صحيح $2$
$12 - (5 \times 2) = 2$

at $x=4$ $f(4) = 16 \bmod 5$ $f(4) = 6$

$f(x) = [(0,0), (1,4), (2,3), (3,2), (4,6)]$

$16 \div 5 = 2.4$
$16 - (5 \times 2) = 6$
هناك خطأ
٥ اكبر عدد اللي
هو الـ mod
∴ هنطرح 5 هتبقا
1




one-to-one & onto

20) Let $f$ a function on $X = \{0,1,2,3,4,5\}$ defined by $f(x) = 4x \bmod 6$

Write $f$ as a set of ordered Pairs

IS $f$ one-to-one or onto ?

Solution:

at $x = 0$

$f(0) = 0 \bmod 6$

$f(0) = 0$

at $x = 1$

$f(1) = 4 \bmod 6$ | $4 \div 6 = 0.666 = 0.$ صحيح

$f(1) = 4$ | $4 - (0 \times 6) = 4$

at $x = 2$

$f(2) = 8 \bmod 6$ | $8 \div 6 = 1.333 = 1$ صحيح

$f(2) = 2$ | $8 - (6 \times 1) = 2$

at $x = 3$

$f(3) = 12 \bmod 6$ | $12 \div 6 = 2$

$f(3) = 0$ | $12 - (6 \times 2) = 0$

at $x = 4$

$f(4) = 16 \bmod 6$ | $16 \div 6 = 2.666$ صحيح $2$

$f(4) = 4$ | $16 - (6 \times 2) = 4$

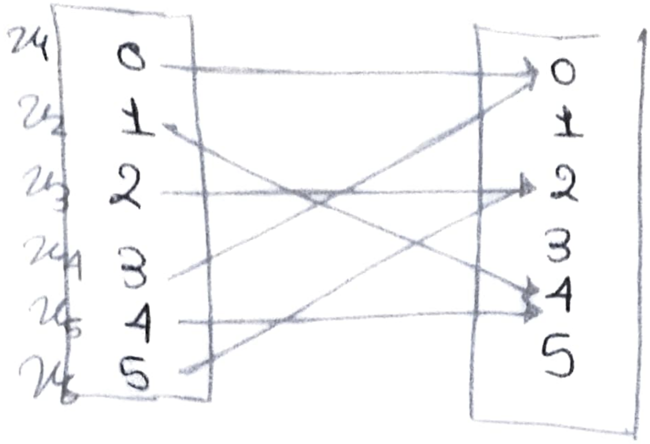
at $x = 5$

$f(5) = 20 \bmod 6$ | $20 \div 6 = 3.33$ صحيح $3$

$f(5) = 2$ | $20 - (6 \times 3) = 2$

$f = \{(0,0), (1,4), (2,2), (3,0), (4,4), (5,2)\}$

$$f = \{(0,0),(1,4),(2,2),(3,0),(4,4),(5,2)\}$$




① Function: since

① كل عناصر ال X لهم صور

② صور كل X وحيدة

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② not one-to-one

since $x_1 = 0 \neq x_2 = 3$

$f(x_1) = f(x_2) = 0$

and

$x_1 = 1 \neq x_2 = 4$

$f(x_1) = f(x_2) = 4$

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③ not onto since not all elements of $Y$ are images

$y_1 = 1$, $y_2 = 5 \in Y$ but $y \neq f(x)$

Date:

Subject:

20) $f(x) = \lceil x \rceil$, and $g(x) = \lfloor x \rfloor$ be the Ceiling and Floor fun

Find Values of the Following.

1) $10 - 3f(2.3)$

$f(2.3) = \lceil 2.3 \rceil = 3$

$10 - 3(3) = 1$

2) $-3g(-2.1) + 0.5f(0.3)$

$g(-2.1) = \lfloor -2.1 \rfloor = -3$

$0.3) = \lceil 0.3 \rceil = 1$

$3(-3) + 0.5 = 9.5$

) $0 \bmod 3 - f(-1.3)$

$0 \bmod 3 = 0$

$f(-1.3) = \lceil -1.3 \rceil = -1$

4) $2g(2.7) - 0.5f(-0.3)$

$g(2.7) = \lfloor 2.7 \rfloor = 2$

$f(-0.3) = \lceil -0.3 \rceil = 0$

$2(2) - 0.5(0) = 4$

5) $-10 \bmod 3 + g(2.3)$

$-10 \bmod 3 = 1$

$3 - (10 \bmod 3) = 2$

$10 \div 3$
$10 - (3 \times 3)$
$= 1$

$g(2.3) = \lfloor 2.3 \rfloor = 2$

$2 + 2 = 4$

6) $|10| + 0.5f(-0.3)$

$10 + 0.5 \lceil 0.3 \rceil$

$10 + 0.5(0) = 10$

apliCation :-

* BarCodes

اخر رقم في ال ( BarCodes ) يسمى ( Check digit )

بيطلب منك في الامتحان توجد الـ Check digit

او تعملـ Check اذا كان الرقم صح ولا غلط

الـ BarCode اللي مكون من 13 رقم ازاى اوجد

الرقم الـ 13 ؟

$$x_{13} = 10 - \left[ (x_1 + 3x_2 + x_3 + 3x_4 + \cdots + 3x_{12}) \bmod 10 \right]$$

Date: Barcode Subject:

32) Verify that the check digit is Correct for the following ISBN:

① ISBN 978-0674-02795-4

قانون الباركود

$$x_{13} = 10 - \left[ (x_1 + 3x_2 + x_3 + 3x_4 + \cdots + 3x_{12}) \bmod 10 \right]$$

الزوجي مضروب في 3 والفردي 8

هو عايز يشوف $x_{13}$ ده صح ولا غلط فهنرجع نشوف كتب لكل واحد

① $x_{13} = 10 - [(9 + 7(3) + 8 + 0(3) + 6 + 7(3) + 4 + (0)(3) + 2 + 7(3) + 9 + 5(3)) \bmod 10]$

$= 10 - [116 \bmod 10]$

$= 10 - 6 = 4$

ISBN 978-0-674-02795-4

هو هو

$\therefore x_{13} = 4$ is right

(2) ISBN 978-0-9713718-4-2

$$x_{13} = 10 - \left[ (x_1 + 3x_2 + x_3 + 3x_4 + \text{———} + 3x_{12}) \bmod 10 \right]$$

$$= 10 - \left[ (9 + 3(7) + 8 + (0)(3) + 9 + 7(3) + 1 + 3(8) + 7 + (3)1 + 8 + 4(3)) \bmod 10 \right]$$

$$x_{13} = 10 - (108 \bmod 10)$$

$$108 \div 10 = 10.8$$

$$10.8 - 10 = 8$$

$$x_{13} = 10 - 8 = 2$$

(3) $$x_{13} = 10 - \left[ (9 + 3(1) + 8 + 0(3) + 6 + 7(3) + 4 + 0(3) + 2 + 7(3) + 9 + 5(3)) \bmod 10 \right]$$

$$x_{13} = 10 - (116 \bmod 10)$$

$$116 \div 10 = 11.6$$

$$11.6 - 11 = 6$$

$$x_{13} = 10 - 6 = 4$$

3- hash fun. :-

$$h(n) = n \bmod 11$$

نفس طريقة Mod fun. باختلاف ان هنا قيمه y دايماً ثابته

وتساوي ← 11

4- unit step fun. :-

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

35 Express fun. f(t) in terms of unit step fun.

1 $$f(t) = \begin{cases} t^3 & 0 < t < 1 \\ 3 & t > 1 \end{cases}$$

$$f(t) = t^3 \left[ u(t-0) - u(t-1) \right] + 3 \left[ u(t-1) - \underbrace{u(t-\infty)}_{\text{zero}} \right]$$

$$= t^3 u(t) - t^3 u(t-1) + 3u(t-1)$$

$$= t^3 \underbrace{u(t)}_{1} + u(t-1)\left[3 - t^3\right]$$

$$= t^3 + u(t-1)(3 - t^3)$$

② $$f(t) = \begin{cases} 0 & t < 1 \\ t^2 + 7 & t \geq 3 \end{cases}$$

$$f(t) = 0\left[u(t-1)\right] + (t^2+7)\left(u(t-3)\right]$$
$$= (t^2+7)\,u(t-3)$$

③ $$f(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 5 \\ 2t & 5 < t < 8 \\ t^2/7 & t \geq 8 \end{cases}$$

$t > 1$
$u(t) = 1$

$$f(t) = \cancel{0\left[u(t-0)\right]} + 2\left[u(\cancel{t}-0) - u(t-5)\right]$$
$$+ 2t\left[u(t-5) - u(t-8)\right] + t^2/7\left[u(t-8)\right]$$

$$= 2\,u(\cancel{t}) + u(t-5)\left[2t-2\right] + u(t-8)\left[\frac{t^2}{7} - 2t\right]$$
$1$