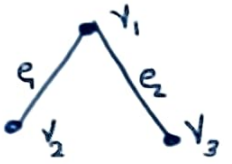
First Question

Final 2021

(1) True

(2) True → Geometric $|r| = \frac{1}{10} = 0.1 < 1$ Conv. #

(3) False



n Vertices ⟶ n-1 edges

(4) False

Let $j = 3$ $\quad k = K + 2 \Rightarrow K = j - 2$

$$\sum_{k=1}^{k=n} a_{k-1}\, a_{n-k} = \sum_{j-2=1}^{j-2=n} a_{(j-2)-1}\, a_{n-(j-2)}$$

$$= \sum_{j=3}^{j=n+2} a_{j-3}\, a_{n-j+2}$$

(5) True

(6) False

$f(x) = \lceil x + 1 \rceil$

$f(-1.5) = -1$ الأكبر

-2 -1.5 -1

12 mod 4 - .5 $f(-1.5)$

0 -1 $\neq 0$

7) خلص

(8) False

(9) False

$12 \bmod 5 = 2$

$h(15) = 15 \bmod 11 = 4$

$\lfloor -4.7 \rfloor = -5$

$\lceil .3 \rceil = 1$

$\Rightarrow \quad 2 + 4 - 3(-5) + 2(1) = 23$

$\left(\frac{6}{10^n}\right) = \frac{6}{10}, \frac{6}{(10)^2}, \frac{6}{(10)^3}, \ldots$

(10) False

Choose The Correct answer

(1) odd (ملغي)

(2) مش عايزلوا (للحل هتحتاجوا نظريات الكوين)

(3) (2)

$3\lceil 1.8 \rceil + 2\lfloor -1.1 \rfloor = 3(2) + 2(-2) = 6 - 4$

(4) 2

(5) مش عايزلوا

(6) مش عايزلوا

(7) R

(8) 7

$h(137) = 137 \bmod 11 = 5$

$12 \bmod 5 = 2$

$\lceil -1 \rceil = -1$

$5 + 2 - (-1) = 7$

(9) $[-4, 0]$

~~$-\infty < 3x - 1 < \infty$~~

$-1 \leq \sin(3x-1) \leq 1$ $\times -2$

$2 \geq -2\sin(3x-1) \geq -2$ -2

$0 \geq -2\sin(3x-1) - 2 \geq -4$

$f(x) \in [-4, 0]$

(10) Bijective

Second Question

(1) ملغي

(2)

$$-t\left[u(t-0)-u(t-1)\right]+t\left[u(t-1)-u(t-2)\right]+t^2\left[u(t-2)-u(t-3)\right]$$

$$= -t\,u(t)+u(t-1)\underbrace{(t+t)}_{2t}+u(t-2)(t^2-t)-t^2u(t-3)$$

(3) $\boxed{k=0}$

ممكن نسميها متغيرات تاني عشان مش نلغبط بين القديمة اللي بتساوي 1 والجديدة اللي هو عايز يبدأ الـ Sum بيها اللي بتساوي = صفر

$\boxed{j=0} \Rightarrow k = j+1$

$$\circledast \quad \sum_{k=1}^{k=n} a_{n-k} = \sum_{j+1=1}^{j+1=n} a_{n-(j+1)}$$

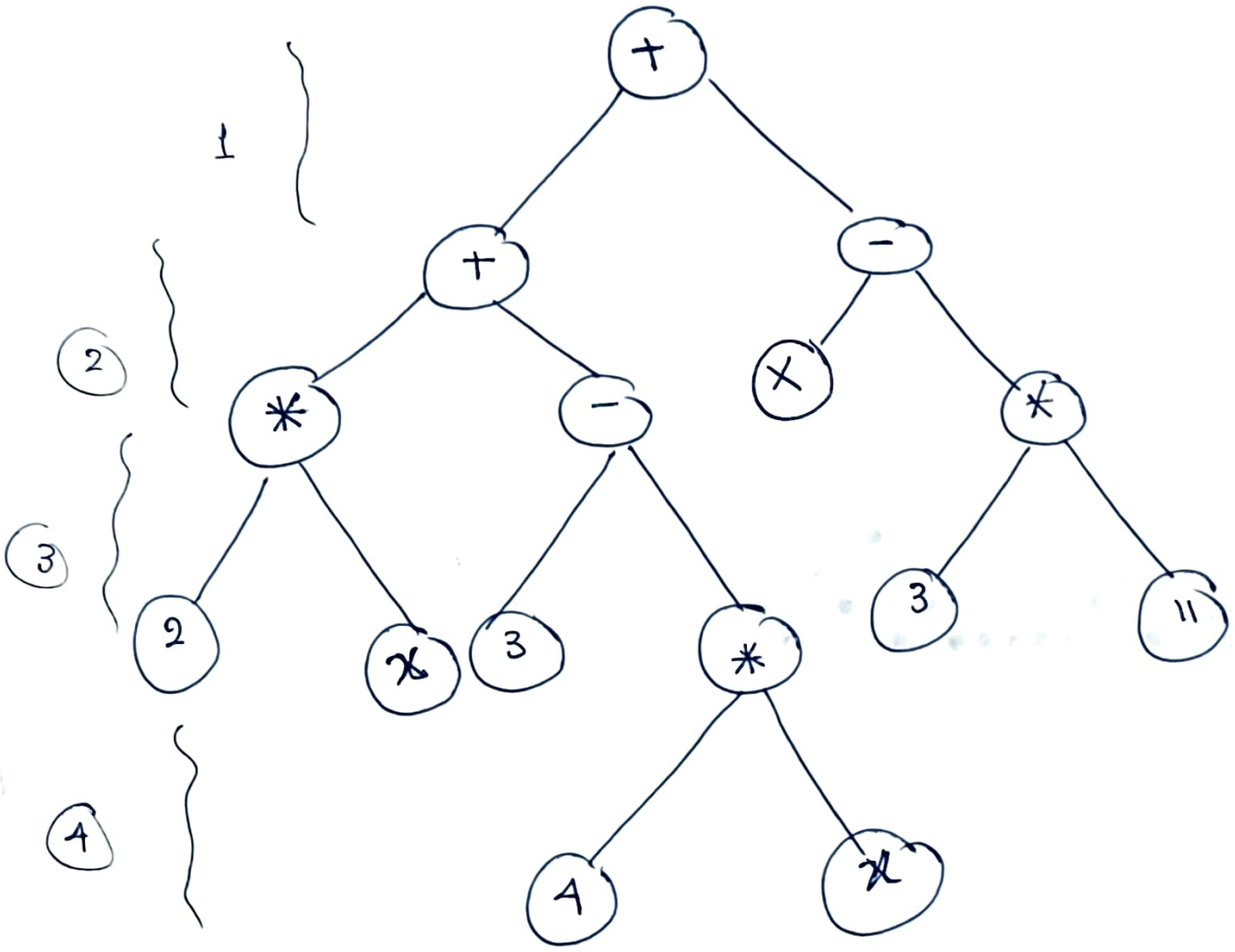
$$= \sum_{j=0}^{j=n-1} a_{n-j-1} \quad \#$$

ممكن نرجع نغير اسم الـ j بالـ k بما انه هو كان طالب منا الـ Summation الأول تبدأ بالـ $\boxed{k=0}$ مينفعش اقوله متغيرات يعني كل اللي هعمله هشيل اسم j فقط بـ k $\rightarrow \boxed{j=k}$

$$\boxed{\sum_{k=0}^{k=n-1} a_{n-k-1}} \quad \#$$

Third Question

(1) $\left[(2 \times x) + (3 - (4 \times x))\right] + (x - (3 \times 11))$



height = 4

2) use Maclaurin Series

$$F(x) = \sin(2x) + \cos 2x$$

$$F(x) = \sin 2x + \cos 2x \Rightarrow F(0) = 1$$

$$F'(x) = 2\cos 2x - 2\sin 2x \Rightarrow F'(0) = 2$$

$$F''(x) = -4\sin 2x - 4\cos 2x \Rightarrow F''(0) = -4$$

$$F'''(x) = -8\cos 2x + 8\sin 2x \Rightarrow F'''(0) = -8$$

$$F(x) = F(0) + xF'(0) + \frac{x^2}{2!}F''(0) + \frac{x^3}{3!}F'''(0) + \cdots$$

$$F(x) = \underbrace{1}_{1} + \underbrace{2x}_{2} - \underbrace{4\frac{x^2}{2!}}_{3} - \underbrace{8\frac{x^3}{3!}}_{4} + \cdots$$

خال 4 حدود فقط

$$F(0.1) = 1 + 2(.1) - 4\frac{(.1)^2}{2!} - 8\frac{(.1)^3}{3!} + \cdots$$

$$\simeq 1.17866$$

3

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$L = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & -1 & -1 & 0 & 2 \end{bmatrix} \end{array}$$

$$I = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$