Suez Canal University

Faculty of Computers & Informatics

First Semester; First Level

Date:28-11-2020; Time: **1.5- Hours**

Module(A)

Mid-Term Examination 2020-2021

Discrete Mathematics (BS-103)

Module(A)

First Question (12.5- Marks)

❶ Choose the correct sign "✓ or ×" for the followings:

[1]	$P \vee T$ is always true iff at least one of p or q is true.	(… … …)
[2]	$p \wedge \bar{p}$ is always false	(… … …)
[3]	$\forall x \in \mathcal{R}, p(x): (x^2 - 1) = (x - 1)(x + 1)$	(… … …)
[4]	The domain of a function is contained in the codomain	(… … …)
[5]	$p \to q \equiv \bar{p} \to \bar{q}$	(… … …)
[6]	$\overline{\bar{p} \vee F} = p$	(… … …)
[7]	$\bar{p} \vee q \equiv \overline{p \wedge \bar{q}}$	(… … …)
[8]	$P \wedge T \equiv T$	(… … …)
[9]	$\forall x \in \mathcal{R}, \quad p(x): x^2 > x$	(… … …)
[10]	Let $P(x, y, z): xy < x + z + 1$, then $p(x, x, x)$ is always true $\forall x \in z$	(… … …)
[11]	$\exists x \in \mathcal{R}, p(x): x^2 - 5x + 6 = 0$	(… … …)
[12]	If p is $4 \geq 2$ and q is "$5 \leq 2$" then $p \oplus q$ is true	(… … …)
[13]	$(A \subset B) \wedge (B \subset A) \Leftrightarrow A = B$	(… … …)
[14]	The relation RUR^{-1} refers to reflexive closure.	(… … …)
[15]	$\overline{p \wedge F} = T$	(… … …)

❷ Choose the correct answer for the following statements:

[1]	The identity function, $f: \mathcal{R} \to \mathcal{R}^+, f(x) = x$ is	{one to one, onto, both}
[2]	If $f: \mathcal{R} \to \mathcal{R}^+, f(x) = 2^{x^2}$, then f is not	{one to one, onto, both}
[3]	The range of the function $f(x) = 3\cos(2x - 1)$ is	$\{[-1, 1], [-3, 3], [-2, 2]\}$
[4]	For the exponential function $f: \mathcal{R} \to \mathcal{R}^+, f(x) = a^x, a \in \cdots$	$\{\mathcal{N}, z^+, z^+ - \{1\}\}$
[5]	The domain of the function $f(x) = \text{Log}(x)$ is ….	$\{\mathcal{R}, \mathcal{R}^+, \mathcal{R}^-\}$
[6]	If p: 2 is a positive integer and q: is $\sqrt{2}$ is a rational number, then $p \wedge q$ is true.	{True, False}
[7]	$A - \overline{B} = \cdots$	$\{A \cup B, A \cap B, A - B\}$
[8]	$A \oplus B = (A \cup B) - (A \cap B)$	{True, False}
[9]	If $R_2 = R \cup R^{-1}$, then R_2 should be ….	{reflexive, transitive, symmetric}
[10]	The domain of the function $f(x) = 3\cos(2x - 1)$ is	$\{\mathcal{R}^+, \mathcal{R}\}$

Solve behind the page if necessary.

Second Question (12.5- Marks)

❶ Behind this paper, use the **laws of logic** to show that the statement: $(p \wedge \bar{q}) \wedge (\bar{p} \vee q) \wedge r$ is always false.

❷ Prove by contradiction that if $a^2 - 2a + 7$ is even, then a is even.

Third Question (15- Marks)

❶ Use indirect proof to prove that if x^2 is odd, then x is odd

❷ If $X = \{2, 3, 4\}$ and $Y = \{4, 5, 6, 8\}$,

① define the relation R from X to Y which defined by X divides Y,

② give the matrix of the relation R relative to the ordering 3, 4, 2 and $\{5, 6, 8, 4\}$,

③ show that if R is reflexive, symmetric or transitive,

④ if ③ is not satisfied use the closure concept to make it reflexive, symmetric, transitive.

(انتهت الأسئلة)

Solve behind the page if necessary.

Mid-Tearm Module A

Question 1 :

1) True

2) True

3) True $\quad (x-1)(x+1) = x^2 + x - x - 1 = x^2 - 1$

4) False

5) False $\quad$ Since $\quad F \to T \equiv T$, but $\quad T \to F \equiv F$ $\quad$ not equivalent

6) True $\quad \overline{\bar{P} \vee F} \equiv \bar{\bar{P}} \wedge \bar{F} \equiv P \wedge T \equiv T$

7) True $\quad \overline{P \wedge \bar{q}} \equiv \bar{P} \vee \bar{\bar{q}} \equiv \bar{P} \vee q$

8) False

9) False $\quad \left(\frac{1}{2}\right)^2 > \frac{1}{2} \qquad \left(\frac{1}{4}\right) > \frac{1}{2}$ false

10) False $\quad x^2 - 2x - 1 < 0$

$x_1 = 1-\sqrt{2}, \quad x_2 = 1+\sqrt{2}$

$$\begin{array}{ccccc} ++++ & & --- & & +++ \\ & 1-\sqrt{2} & & 1+\sqrt{2} & \end{array}$$

$P(x)$ is True at $x \in \,]1-\sqrt{2}, 1+\sqrt{2}[$

$P(x)$ is false at $x \in R - \,]1-\sqrt{2}, 1+\sqrt{2}[$

11) True $\quad$ Since at $x = 2, 3 \quad P(x)$ is True

$\therefore$ for some $x \in R \quad P(x) = 0$

12) True $\quad P : 4 \geq 2 \equiv T$

$q : 5 \not\geq 2 \equiv F$

$P \oplus q \equiv T$

13) True

14) False Symmetric closure

15) ~~True~~

$\overline{P \wedge F} \equiv \bar{P} \vee \bar{F} \equiv \bar{P} \vee T \equiv T$

$\overline{P \wedge F} \equiv \overline{F} \equiv T$

2

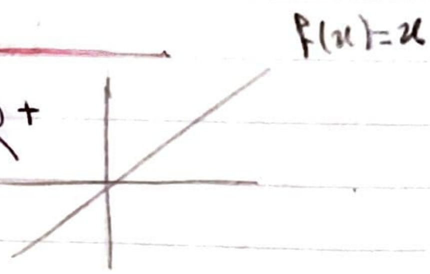
1) both

one-to-one

onto since Codomain $= R^+$

والدالة متصلة يعني كلها صور لعناصر x

في R^+ كلها



2) one-to-one

$x = \pm 1 \qquad f(x) = 2$

$x = \pm 2 \qquad f(x) = 2^4$, ...

3) $[-3, 3]$

4) Z^+ - [1 ٤]

5) R^+

6) False $\quad P$: 2 Positive $\equiv T \qquad q$: $\sqrt{2}$ rational $\equiv F$

$P \wedge q \equiv$ False

7) $A \cap B$

8) True

9) Symmetric

10) R

Question 2:

1)

$$(p \cap \bar{q}) \cap (\bar{p} \vee q) \wedge r \equiv$$

$$\overline{\overline{(p \cap \bar{q})}} \cap (\bar{p} \vee q) \cap r \equiv$$

$$\overline{(\bar{p} \vee q)} \cap (\bar{p} \vee q) \wedge r \equiv$$

$$F \wedge r \equiv F$$

2) $P: a^2-2a+6$ is even

q : a is even

by Contradiction $\bar{q}$: a is odd

$a = 2k+1 \qquad k = 0, 1, 2, \ldots$

$a^2-2a+6 = (2k+1)^2 - 2(2k+1) + 6$

$= 4k^2 + 4k + 1 - 2(2k+1) + 6$

$= 4k^2 + 4k + 1 - 4k - 2 + 6$

$= 4k^2 + 5$

$= 4k^2 + 4 + 1$

$= 4(k^2+1) + 1$

or $2(2k^2+2) + 1$

$\therefore a^2-2a+6$ is odd

This is Contradiction since we suppose a^2-2a+6 is even

$\therefore$ a is even

Question 3:

use indirect Proof:

if x^2 is odd then x is odd

Proof

indirect Proof $\bar{q} \longrightarrow \bar{p}$

let p : x^2 is odd

$\bar{p}$: x^2 is even

q : x is odd

$\bar{q}$: x is even

$\bar{q}$: x is even $\quad x = 2k \quad k = 0, 1, 2, \ldots$

$x^2 = (2k)^2 = 4k^2 = 2\underbrace{(2k^2)} \quad k = 0, 1, 2, \ldots$

let $2k^2 = M$ integer

$x^2 = 2M$ is even

$\bar{q} \longrightarrow \bar{p} \equiv p \rightarrow q$

2)

1) $R = \{(2,4),(2,6),(2,8),$
$(3,6),$
$(4,4),(4,8)\}$

2) Y →
X ↓

$$\begin{array}{c|cccc} & 5 & 6 & 8 & 4 \\ \hline 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \end{array}$$

1) *) not reflexive

since $(2,2),(3,3),(5,5),(6,6),(8,8) \notin R$

$xRx \ \forall x \in X, Y$

*) not symmetric

since $(2,4),(2,6),(2,8),(3,6),(4,8) \in R$

but $(4,2),(6,2),(8,2),(6,3),(8,4) \notin R$

if $xRy \in R$

then $yRx \in R$

$\forall x, y \in X, Y$

if xRy & yRz then xRz

Transitive $\forall\, x, y, z \in X, Y$

$$(2,4) \begin{cases} (4,4) \longrightarrow (2,4) \in R \\ (4,8) \longrightarrow (2,8) \in R \end{cases}$$

$(2,6)$, $(2,8)$, $(3,6)$ ملهاش زوج يكملها

$(4,4)$ & $(4,8) \longrightarrow (4,8) \in R$

* $\therefore$ Transitive

لازم لو وجدنا $(x,y),(y,z)$ في العلاقة

لازم نلاقي تالتهم

reflexive
Closure ~~symmetric~~ $xRx \quad \forall x \in set$

$R_1 = R \cup \Delta$

$= \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8)\}$

$\cup \{(2,2),(3,3),(5,5),(6,6),(8,8)\}$

$R_1 = \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8),(2,2),(3,3),(5,5),$
$(6,6),(8,8)\}$

Closure symmetric

$R_1 = R \cup R^{-1}$

$= \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8)\}$

$\cup \{(4,2),(6,2),(8,2),(6,3),(8,4)\}$

$R_1 = \{(2,4),(2,6),(2,8),(3,6),(4,4),(4,8),(4,2),(6,2)$
$(8,2),(6,3),(8,4)\}$