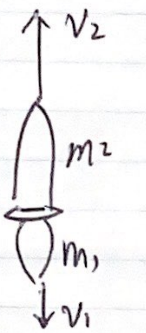
$\angle A$: $\frac{\sin A}{a} = \frac{\sin c}{c}$ ①

$A = \sin^{-1}\left(\frac{a \sin\angle C}{c}\right)$

$= \sin^{-1}\left(\frac{0.9 \sin 30°}{0.616}\right)$

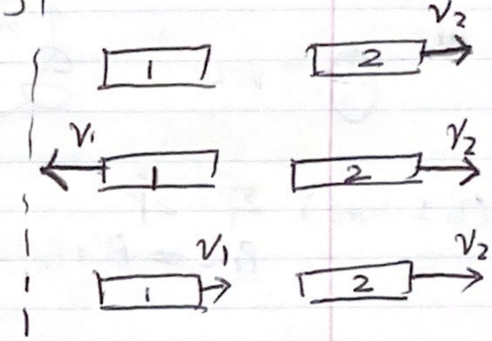
$= 46.9°$

$v_2' = 2.1$ m/s [47° S of E] ① ①



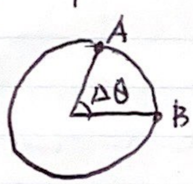
ex, ⟶ has an intial velociey |

outcom



==⇒ Rotational Motion ⇐==

- Angular position:



$\Delta\theta = \theta_B - \theta_A$ ($\Delta\theta$ unit: rad)

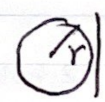
⇑

angular position

- Angular velocity: Recall $\vec{v} = \frac{\vec{d}}{t}$

$\therefore \vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}$ (unit rads/s)

↑ angular velocity

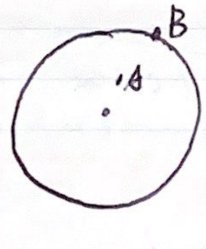
tangential velocity ⇒  ← the velocity on circumference / road distance

$\because \theta \times r = d$

$\therefore \vec{\omega} \times r = \vec{v}$ ⇒ $\left(\frac{\Delta\vec{\theta} \times r}{\Delta t} = \frac{\Delta d}{\Delta t} = \vec{v}\right)$

$\therefore \vec{v} = \vec{\omega} \cdot r$

~ problem:



$\vec{\omega}_A = \vec{\omega}_B$

$\because \theta$ is the same

$\vec{v}_A < \vec{v}_B$

$\therefore r_A < r_B$

$\therefore \vec{\omega}_A \cdot r_A < \vec{\omega}_B \times r_B$

①

— Other concepts:

$T = \frac{\text{time}}{\text{cycles}}$ (s/rad or s) $f = \frac{\text{cycles}}{\text{time}}$ (Hz)

$\therefore T = \frac{1}{f}$ $\boxed{\omega = 2\pi \cdot f}$ $\boxed{\omega = \frac{2\pi}{T}}$

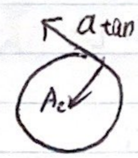
— Angular acceleration:

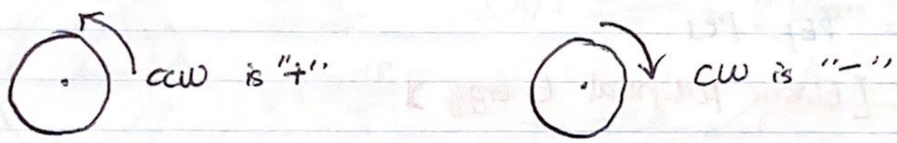
recall $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

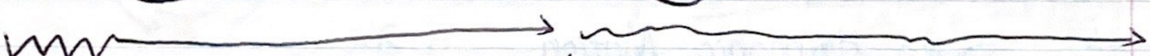
$\therefore \vec{\alpha} = \frac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$ (rads/s²)

$A_c = \frac{v^2}{R} = \frac{(\vec{\omega} R)^2}{R} = \omega^2 R$ $\therefore A_c = \omega^2 R$ as well.

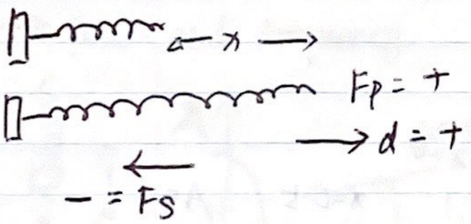
$\left\{ \begin{array}{l} \text{tangential acceleration or } a_{tan} \text{ or } a \\ = \hat{a} = \alpha \cdot R \end{array} \right.$







Hooke's Law & spring



$\boxed{F_p = kx}$

Restoring force $\boxed{F_s = -kx}$

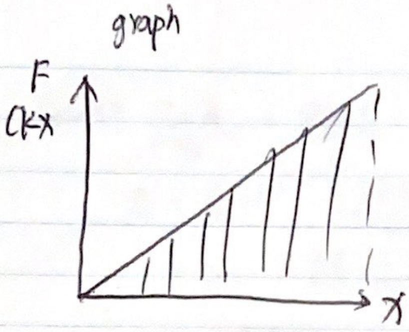
ex. A force of 200N stretches a spring by 4m. What is the value of spring constant?

$F_p = kx$ $\therefore k = \frac{F_p}{x} = \frac{200N}{4m} = 50$ [N/m unit of k]

Work required to stretch a spring:

$W = F \cdot d$ however: the force is inconsistent
$= F \cdot x$

$\therefore \downarrow$



Area = work

$\therefore W = \frac{1}{2} F \cdot x$

$\because F = kx$

$\therefore \boxed{W = \frac{1}{2}kx^2}$ (or $= \cancel{\frac{1}{2}k(\Delta x)}^2$

$\frac{1}{2}k\Delta x^2 / (x_F^2 - x_i^2)$

ex. How much work is required to stretch a spring by 75cm ($K = 250 N/m$)

$W = \frac{1}{2}kx^2$

$\therefore W = \frac{1}{2} \times 250 \times (0.75)^2$

$= 70.3 J$

~ Elastic Potential Energy: ⁜

$W = \Delta E_{ps} / \Delta U_s$

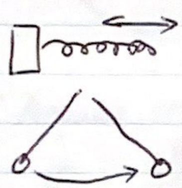
$W = \frac{1}{2}k\Delta x^2$

$W = PE_f - PE_i$

$\therefore \frac{1}{2}k(x_F^2 - x_i^2) = PE_F - PE_I$

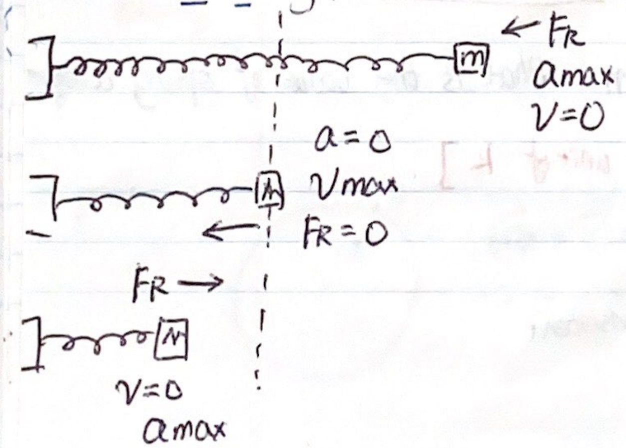
$\therefore PE_s = \frac{1}{2}kx^2$ [Elastic potential Energy]

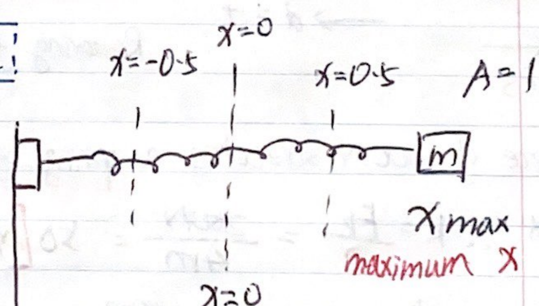
Simple Harmonic Motion



the time that takes to complete one cycle is always the same.

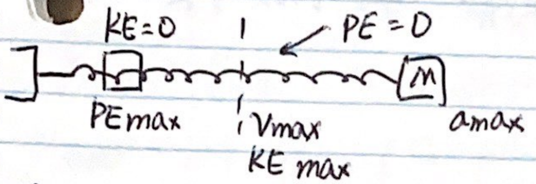
→ Velocity / Instant Acceleration





maximum x is A

Derivation: ⇓



$KE = \frac{1}{2}mv^2$

$PE = \frac{1}{2}kx^2$

①

$ME = KE + PE$

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

when the spring is fully stretched

ME = PE, If at middle,

ME = KE

$\therefore \cancel{\frac{1}{2}}kA^2 = \cancel{\frac{1}{2}}mv^2$

$\therefore v_{max} = \sqrt{\frac{k}{m}} \cdot A$

$\boxed{\therefore v_{max} = \sqrt{\frac{k}{m}} \cdot A}$

② $F_R = kx$

$ma = kx$

$\boxed{a = \frac{kx}{m}}$

$\boxed{\frac{F_R}{m} = a}$

$a_{max} = \frac{kA}{m}$

③ To Find the general velocity in this motion.

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$kA^2 = mv^2 + kx^2$

$\frac{k(A^2 - x^2)}{m} = v^2$

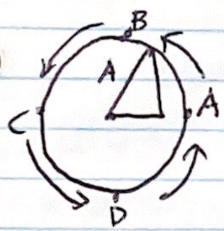
$v^2 = \frac{k}{m}A^2(1 - \frac{x^2}{A^2})$

$\rightarrow v(x) = \pm\sqrt{\frac{k}{m}} \cdot A(1 - x^2/A^2)$

$v(x) = \pm v_{max} \cdot A \div x^2/A^2)$

$\boxed{v(x) = \pm v_{max}(1 - x^2/A^2)}$

⟶ Relationship of T in springs:



$\vec{v} = \frac{d}{t}$

$v = \frac{2\pi R}{T}$

$\because R = A$

$\therefore v = \frac{2\pi A}{T}$

$\therefore vT = 2\pi A$

$T = \frac{2\pi A}{v}$

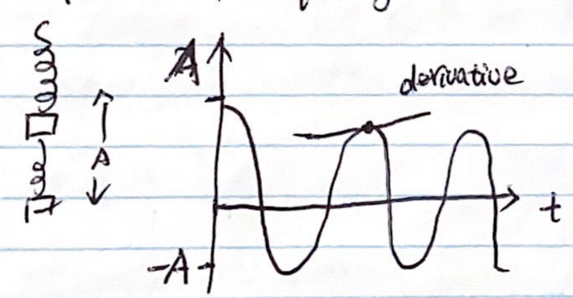
$\because v_{max} = \sqrt{\frac{k}{m}} \cdot A$

$\therefore A = \sqrt{\frac{m}{k}} \cdot v_{max}$

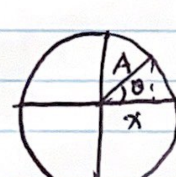
$\boxed{\therefore T = 2\pi\sqrt{\frac{m}{k}}}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

→ Period and Frequency



~~x(t) = x cos~~



$\cos\theta \times A = x$

$\therefore x = A\cos\theta$

$\because d = vt,$

$\theta = \omega \times t$

$\therefore x = A\cos(\omega \times t)$

$\because \omega = 2\pi \times f$

$\therefore x = A\cos(2\pi f t)$

$\therefore x(t) = A\cos(2\pi f t)$

$\therefore v(t) = \frac{d}{dx} A\cos(2\pi f t)$

$\therefore v(t) = A[-\sin(2\pi f t)] \cdot 2\pi f$

take the derivitive of the function, gives you the instantaneous velocity

Given $v(t) = A[-\sin(2\pi f t)]\, 2\pi f$, and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; $A = V_{max}\cdot\sqrt{\frac{m}{k}}$

$\therefore v(t) = V_{max}\cdot\cancel{\sqrt{\frac{m}{k}}}\cdot[-\sin(2\pi f t)]\cdot\cancel{2\pi f}\times\cancel{\frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

$\therefore v(t) = V_{max}[-\sin(2\pi f t)]$

$\therefore V_{inst} = -V_{max}\sin(2\pi f t)$

↑ take the derivative of Vinst, gives you instantaneous acceleration

$a(t) = -V_{max}\cdot\cos(2\pi f t)\cdot 2\pi f$

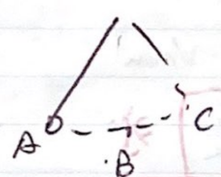
$= -\sqrt{\frac{k}{m}}\cdot A\cos(2\pi f t)\cdot\cancel{2\pi}\cdot\frac{1}{\cancel{2\pi}}\sqrt{\frac{k}{m}}$

$= -\sqrt{\frac{k^2}{m^2}}\, A\cos(2\pi f t)$

$\therefore a_{inst}(t) = -\frac{k}{m}A\cos(2\pi f t)$ $\because ma = kA$ $\therefore a = \frac{kA}{m}$

$\therefore a_{inst}(t) = -a_{max}\cos(2\pi f t)$

Pendulum



$\boxed{T = 2\pi\sqrt{\frac{\mathcal{L}}{g}}}$

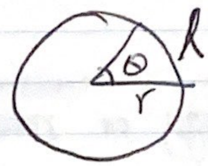
$\mathcal{L}$ is the length
g is the gravatation

$g_{Earth} = 98\ m/s^2$

$\boxed{\therefore f = \frac{1}{2\pi}\sqrt{\frac{g}{\mathcal{L}}}}$

Angular Kinematics / Rotational Motion

Angular Quantities: Positions: $\theta = \frac{l}{r}$



$360^\circ = 2\pi$ rads ($\pi = 180^\circ$ rads)

ex. A 28 cm wheel, rotate 90°, find linear displacement

$\frac{\pi}{2} \times 28 = 43.98$ cm

Angular ~~position~~ displacement:

$\Delta\theta = \theta_f - \theta_i$

Angular velocity $\boxed{\vec{\omega}} = \frac{\Delta\theta}{\Delta t}$ (unit: rads/s)

Angular acceleration: $\vec{\alpha} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\Delta\omega}{\Delta t}$ (rads/s²)

Conversion:

$d = \theta \times r$ $v = r \cdot \omega$ $a = r \cdot \alpha$

Equations for constant angular acceleration.

$\omega = \omega_i + \alpha t$ / $\rightarrow$ $\vec{v}_f = \vec{v}_i + \vec{a}t$

$\theta = \frac{1}{2}(\omega_i + \omega_f) \times t$ $\rightarrow$ $\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f) \times t$

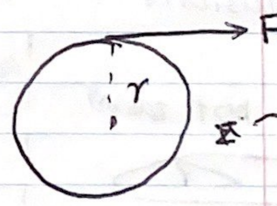
$\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $\rightarrow$ $\vec{d} = v_i \times t + \frac{1}{2}\vec{a}t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\rightarrow$ $v_f^2 = v_i^2 + 2\vec{a}\vec{d}$

— Torque: $\tau = r_\perp \cdot F$ or $F_\perp \cdot r$ $F = m\overset{r \cdot \alpha}{a}$ $\alpha = r \cdot a$

$\Sigma F = m\vec{a}$ (I = moment of Inertia)

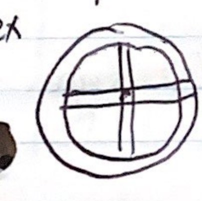
$\Sigma\tau = I\alpha$ $\therefore \tau = \underbrace{mr^2}_{I} \alpha$



$\tau = F_\perp \cdot r$

— complicated Inertia:

ex



$= I_{hoop} + I_{stick} + I_{stick}$ (Add the Inertia together)

$=$

Energy

Linear

$E_K = \frac{1}{2}mv^2$

Angular

$\boxed{E_{Krot} = \frac{1}{2}I\omega^2}$

Mechanical Energy or rotational ~~A~~ objects } In real life

$E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$

Angular momentum.

Linear

$\vec{p} = mv$

$F \cdot \Delta t = \Delta \vec{p}$

Angular.

$\boxed{L = I\omega}$

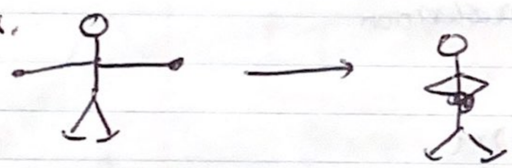
$\boxed{\tau \cdot \Delta t = \Delta L}$ ← Impulse momentum theorem

Law of conservation of Angular Momentum

$L = L'$

$I\omega = I'\omega'$

ex.

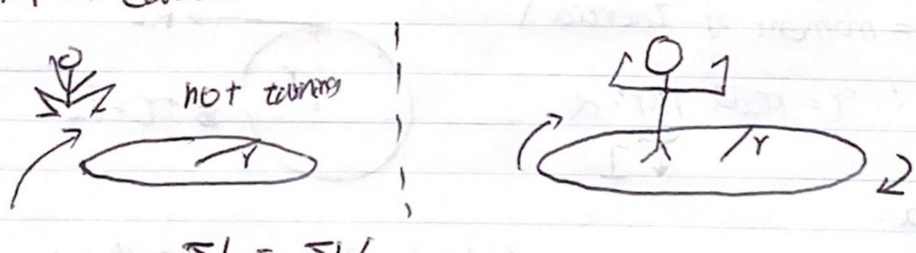


$I = Mr^2$ $\qquad I = Mr^2$

$r' < r$ $\quad \therefore I' < I$) since $L = L$

$\therefore \omega' > \omega$ $\qquad \therefore I\omega = I'\omega'$

AP ∠ Collisions



$\Sigma L = \Sigma L'$

$L_g + \cancel{L_w}^{\,0} = L_{(g+w)}$ the Inertia is a complex shape.