

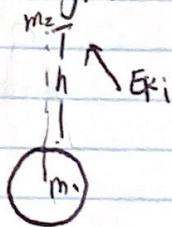
$$\begin{aligned}\Sigma E_i &= \Sigma E_f \\ E_{pi} + E_{ki} &= E_{pf} + E_{kf} \\ E_{pi} + E_{ki} &= 0 \\ -\frac{GMm}{r} + \frac{1}{2}mv^2 &= 0\end{aligned}$$

$$v = \sqrt{\frac{2GMp}{r}}$$

$$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.38 \times 10^6}}$$

$$v = 11.2 \text{ km/s}$$

Determine v_f of a piece of a rocket launch straight up if comes back down from a height of 405 km. Ignore air friction.



$$\Sigma E_i = \Sigma E_f$$

$$E_{pi} + E_{ki} = E_{pf} + E_{kf}$$

$$E_{pi} - E_{pf} = E_{kf}$$

$$-\frac{GMm}{r} - (-\frac{GMm}{r}) = \frac{1}{2}mv_f^2$$

$$-GM(\frac{1}{r_i} - \frac{1}{r_f}) = \frac{1}{2}v_f^2$$

$$\sqrt{+2GM(\frac{1}{r_f} - \frac{1}{r_i})} = v_f$$

$$v_f = 2.73 \times 10^3 \text{ m/s}$$

Momentum

Inertia is generally described as an object resistant to motion.

Momentum is the tendency to continue moving.

$$\vec{p} = m\vec{v}$$

momentum is a vector

unit: (kg · m/s)

$$\Delta \vec{p} = \Delta(m\vec{v}) \quad \begin{cases} \Delta p = \Delta m \cdot \vec{v} \\ \Delta p = m \cdot \Delta \vec{v} \\ \Delta p = \Delta m \Delta \vec{v} \end{cases}$$

Impulse !!!

$$\boxed{\vec{J} = \vec{F} \Delta t} \quad \text{or} \quad \vec{J} = \vec{F}_{\text{net}} \Delta t \quad (\text{unit: N s})$$

momentum.

$$(\text{unit: km/s}^2 \cdot \text{s} = \text{kg m/s})$$

[have same units]

Impulse Momentum Theorem

$$\boxed{\vec{F}_{\text{net}} \Delta t = \Delta \vec{p}}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{m \cdot v}{t} = m \cdot a$$

ex. A 5.0 kg mass is accelerated from rest to 3.5 m/s in 2.5 s. Find the net force acting on the force.

① $v_f = v_i + at$ $a \times m = 1.4 \times 5 = 7 \text{ N}$

$$3.5 = 0 + a \times 2.5$$
$$a = 1.4 \text{ m/s}^2$$

② $F_{\text{net}} \cdot \Delta t = \Delta p$

$$F_{\text{net}} \cdot \Delta t = m \cdot \Delta v$$

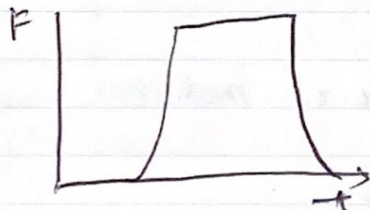
$$F_{\text{net}} \cdot \Delta t = m(v_f - v_i)$$

$$F_{\text{net}} \cdot 2.5 = 5(3.5 - 0)$$

$$F_{\text{net}} = 7 \text{ N}$$

~~ex. A 13 kg ball is~~

Graph F vs t



← A is impulse

$$\boxed{\text{Area} = \Delta \text{momentum} / \text{impulse}}$$

Law of conservation of Momentum

In a closed isolated system, momentum is conserved.

$$\Delta \vec{P}_{\text{total}} = 0$$

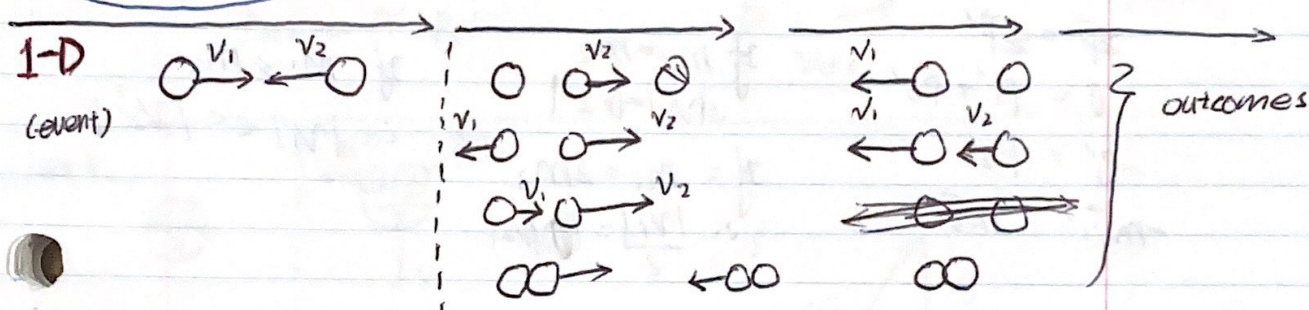
$$\Delta \vec{P}_1 + \Delta \vec{P}_2 = 0$$

$$\vec{P}_{1f} + \vec{P}_{2f} - \vec{P}_{1i} - \vec{P}_{2i} = 0$$

$$\vec{P}_{1f} + \vec{P}_{2f} = \vec{P}_{1i} + \vec{P}_{2i}$$

$$\therefore \Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$\text{or } \Sigma \vec{P} = \Sigma \vec{P}'$$



Elastic Collision: Kinetic energy and Momentum are both conserved.

Objects bounce off one another [Not realistic, no loss of energy (ex sound and heat energy)].

Inelastic Collision: Kinetic energy lost due to sound/heat. Momentum is conserved. Objects bounce off one another [Most Cases]

Perfectly Inelastic: Kinetic energy is not conserved. Momentum is conserved. Objects stuck together.

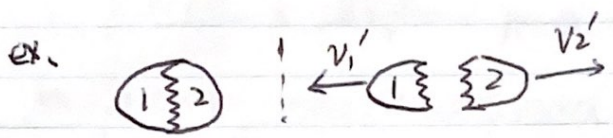
ex. A 3.5 kg cart moves to the right with a velocity of 3.5 m/s and collides with a 4.5 kg cart moving to the left at 1.5 m/s. If the 3.5 kg cart continues to move to the right at 0.5 m/s. What is the final velocity of cart 2?



Handwritten note: *Handwritten*

$$\begin{aligned}\Sigma \vec{P} &= \Sigma \vec{P}' \\ \vec{P}_1 + \vec{P}_2 &= \vec{P}_1' + \vec{P}_2' \\ m_1 \vec{v}_1 + m_2 \vec{v}_2 &= m_1 \vec{v}_1' + m_2 \vec{v}_2' \\ \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 - m_1 \vec{v}_1'}{m_2} &= \vec{v}_2'\end{aligned}$$

$$\frac{35 \times 35 + 45 \times (-1.5) - 35 \times (0.5)}{45} = \vec{v}_2' \quad \therefore \vec{v}_2' =$$

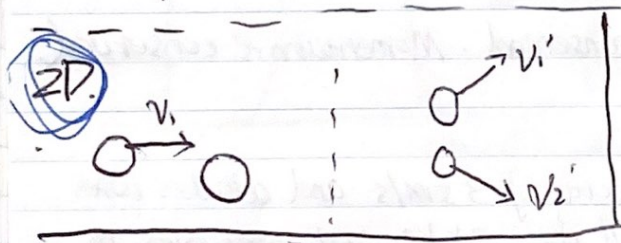
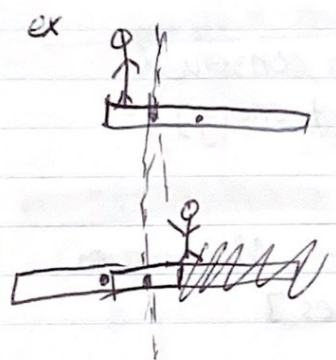


$$\begin{aligned}\Sigma \vec{P} &= \Sigma \vec{P}' \\ 0 &= \vec{P}_1' + \vec{P}_2' \\ -\vec{P}_1' &= \vec{P}_2' \\ -m_1 \vec{v}_1' &= m_2 \vec{v}_2'\end{aligned}$$

If $m_1 = m_2$
 $\therefore |\vec{v}_1'| = |\vec{v}_2'|$

If $m_1 > m_2$
 $\therefore |\vec{v}_1'| < |\vec{v}_2'|$

If $m_1 = 2m_2$
 $\therefore \frac{|\vec{v}_1'|}{2} = |\vec{v}_2'|$

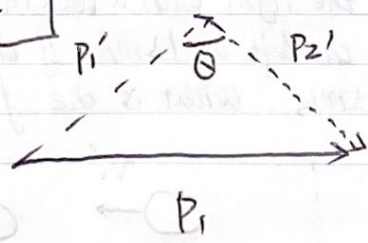


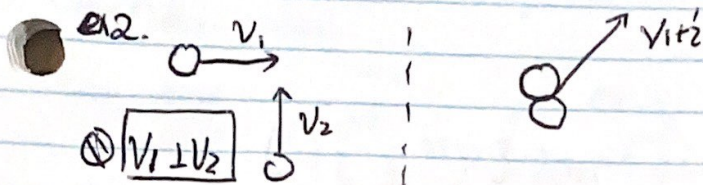
$\theta = 90^\circ$ for elastic collision only

$\theta \neq 90$ in inelastic collision.

$$\begin{aligned}\Sigma \vec{P} &= \Sigma \vec{P}' \\ \vec{P}_1 &= \vec{P}_1' + \vec{P}_2'\end{aligned}$$

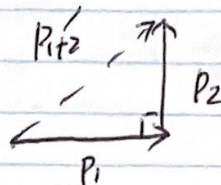
2D vector sum



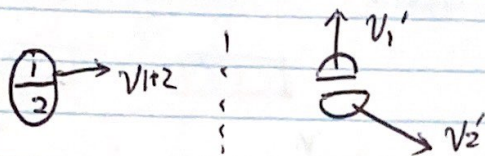


$$\Sigma \vec{P} = \Sigma \vec{P}'$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_{1+2}'$$



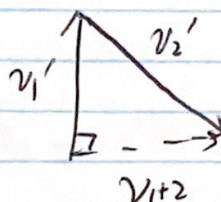
ex 3.



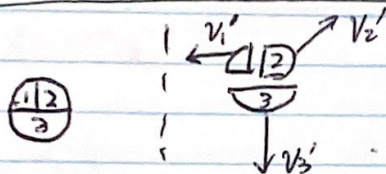
$(v_1 \perp v_{1+2})$

$$\Sigma \vec{P} = \Sigma \vec{P}'$$

$$P_{1+2} = P_1' + P_2'$$

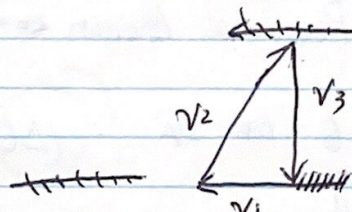


ex 4.

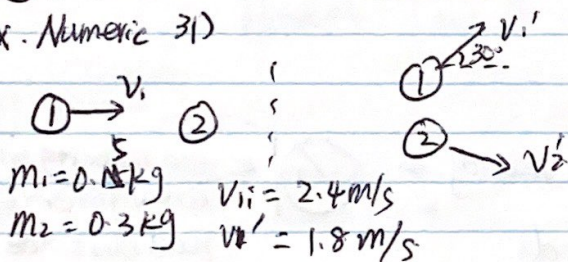


$$\Sigma \vec{P} = \Sigma \vec{P}'$$

$$\vec{P}_{1+2+3} = \vec{P}_1' + \vec{P}_2' + \vec{P}_3'$$



ex. Numeric 31)



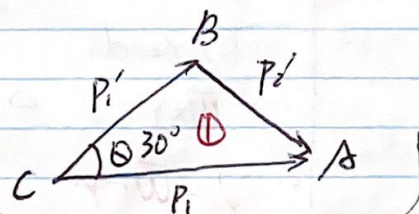
$m_1 = 0.5 \text{ kg}$
 $m_2 = 0.3 \text{ kg}$
 $v_{1i} = 2.4 \text{ m/s}$
 $v_{2i} = 1.8 \text{ m/s}$

① $\Sigma \vec{P} = \Sigma \vec{P}'$

② $\vec{P}_1 = \vec{P}_1' + \vec{P}_2'$

③ $|P_1| = m|v_1|$
 $= 0.5 \times 2.4$
 $= 1.2 \text{ kg m/s}$

④ $|P_1'| = m|v_1'|$
 $= 0.5 \times 1.8$
 $= 0.9 \text{ kg m/s}$



⑤ $|P_1'| = m|v_1'|$
 $= 0.5 \times 1.8$
 $= 0.9 \text{ kg m/s}$

⑥ $P_2' = m v_2'$
 $v_2' = \frac{P_2'}{m} = \frac{0.616}{0.3} = 2.05 \text{ m/s}$

$$\angle A = \frac{\sin A}{a} = \frac{\sin C}{c} \quad (1)$$

$$A = \sin^{-1} \left(\frac{a \sin C}{c} \right)$$

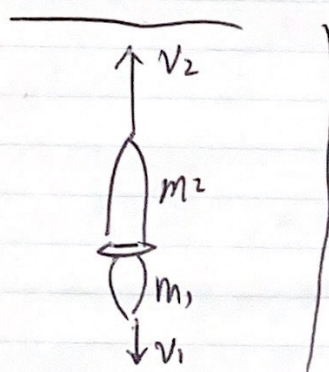
$$= \sin^{-1} \left(\frac{0.4 \sin 30^\circ}{0.616} \right)$$

$$= 46.9^\circ$$

$$v_2' = 2.1 \text{ m/s } [47^\circ \text{ S of E}]$$

(1)

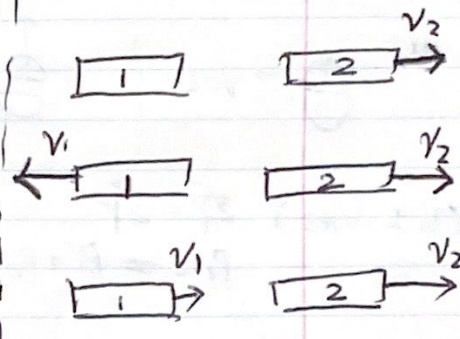
(1)



ex, \rightarrow has an initial velocity

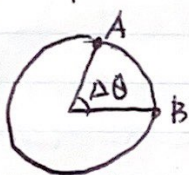


outcome



\Rightarrow Rotational Motion \Leftarrow

- Angular position:




$$\Delta \theta = \theta_B - \theta_A \quad (\Delta \theta \text{ unit: rad})$$

\uparrow
angular position

- Angular velocity: Recall $\vec{v} = \frac{d\vec{r}}{dt}$

$$\therefore \vec{\omega} = \frac{\Delta \theta}{\Delta t} \quad (\text{unit: rad/s})$$

\uparrow angular velocity

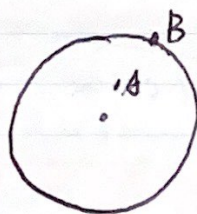
tangential velocity \Rightarrow  \leftarrow the velocity on circumference / rad distance

$$\therefore \theta \times r = d$$

$$\therefore \vec{\omega} \times r = \vec{v} \Rightarrow \left(\frac{\Delta \theta \times r}{\Delta t} = \frac{\Delta d}{\Delta t} = \vec{v} \right)$$

$$\therefore \vec{v} = \vec{\omega} \cdot r$$

~ problem:



$$\vec{\omega}_A = \vec{\omega}_B$$

$\therefore \theta$ is the same

$$\vec{v}_A < \vec{v}_B$$

$$\therefore r_A < r_B$$

$$\therefore \vec{\omega}_A \cdot r_A < \vec{\omega}_B \cdot r_B$$