Mass of the Planet

$F_c = ma_c$

$\frac{GMm}{r^2} = ma_c$

$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}4\pi^2 r}{T^2}$

$\boxed{M = \frac{4\pi^2 r^3}{GT^2}}$

Determine the mass of sol

$T = 3.16 \times 10^7 \, s$

$r = 1.50 \times 10^{11} \, m$

$G = 6.67 \times 10^{-11} \, Nm^2/kg^2$

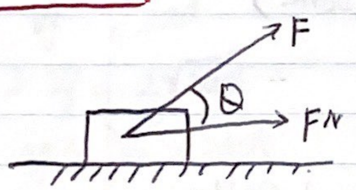
$M = \frac{4\pi^2 (1.50 \times 10^{11})^3}{6.67 \times 10^{-11} \cdot (3.16 \times 10^7)^2}$

$M = 2 \times 10^{30}$ [A = 1.98×10^{30}]

sidereal Time (23h, 56 min 04s)
[86100s]

Work:

$\boxed{W = F \cdot d}$ (Force is parellel to d) $\boxed{F\cos\theta \cdot d}$ (Not parellel)



work is a scalor but it can be "+" and "−"

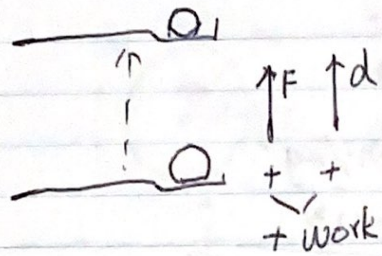
Units: Joule (J) $J = kgm^2/s^2$

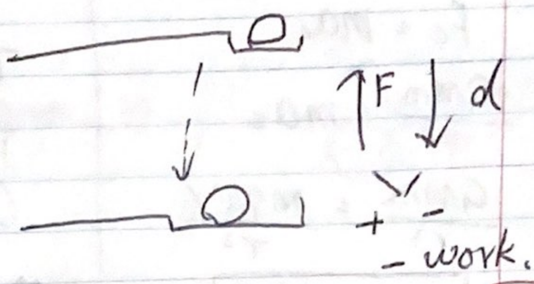
Work: Machanical transfer of energy.

(Giving energy

"+ work" "− work"

Lifting a ball = (hand's perspection) Lowering the ball





$\boxed{\text{Power:}}$ is the rate of doing work units: J/s watts, w $\boxed{\left(\frac{W}{t}\right)}$

$(F \cdot d/t = F \cdot v)$ $\boxed{\text{Another method}}$

horsepower: 1hp = 746 Watts.

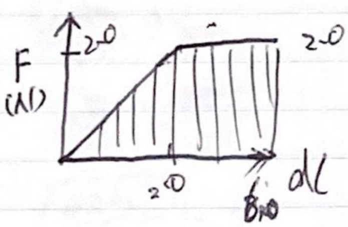
Becareful of the language.

Efficiency: How effective of ~~time e~~ how effiecently work, energy or power to used in accompolishing a given task.

$$\text{Efficiency} = \frac{\text{work/energy/power out}}{\text{work/energy/power in}} \times 100$$

$$\text{Labour efficiency} = \frac{\text{hours worked}}{\text{hours paid}} \times 100$$

Force vs ~~displa~~ distance graph (area is equal to work)



$\boxed{W = \text{Area}}$

$$\frac{(6.0+4.0)}{2} \times 2.0 = 1.0\text{ J}$$

$\boxed{\text{Energy}}$ the ability to do work

energy is a scalor and is measured in Joules. (kgm^2/s^2)

Work-Energy Theorem : Work = Δ Energy.

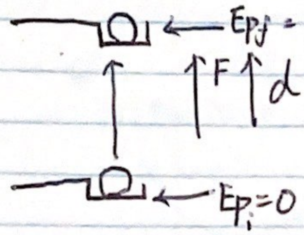
Ways to represent energy. (E) (E_p potential) (E_k kinetic)
or E U K

Mechanical energy = potential + kinetic.

Potential Energy: Stored energy.

Work done lifting a ball

$E_p \propto Mhg$



$W = F \cdot d$

$W = mg \cdot d$

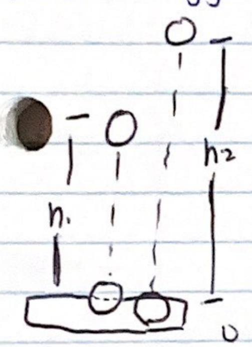
$W = \Delta E_p = E_{pf} - \cancel{E_{pi}}^{0} = E_{pf}$

$W = mhg$

Kinetic Energy: Energy of motion.

$E_k \propto M$

$E_k \propto v^2$ > $E_k \propto mv^2$



$h_1 = 200m$

$h_2 = ?$

$v_2 = 2v_1$

$v_f^2 = \cancel{v_i^2} + 2ad$

$v_f = \sqrt{2ad}$

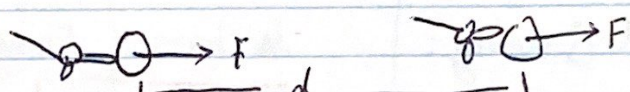
$2 \cdot v_f = 2\sqrt{2ad}$

$v_f^2 = \cancel{v_i^2}^{0} + 2ad$

$4 \cdot \cancel{2a}d_1 = \cancel{2a}d_2$

$d_2 = 4d_1$

Work done accelerating the ball



$W = Fd$

$F = ma$

$W = \cancel{F}mad$

$v_f^2 = v_i^2 + 2ad$

$\frac{v_f^2}{2} = ad$

$W = \frac{mv_f^2}{2}$

$\therefore W = \Delta E_k$

$= E_{kf} - \cancel{E_{ki}}^{0}$

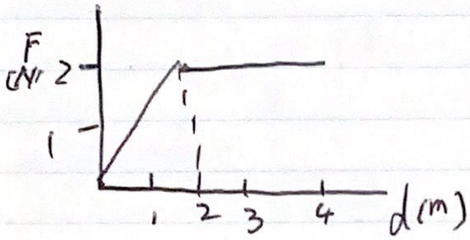
$= \frac{1}{2}mv^2$

Mechanical Energy

— Sum of an object's potential and kinetic energy.

$\Sigma E = E_k + E_p$

ex. Ted pushes a stationary 2.0 kg ball along the ground with a variable force as shown. Find v_f.



i) $W = \text{Area}$

$W = \frac{1}{2} b \cdot h + l \cdot w$

$W = \frac{1}{2} \times 2 \times 2 + 2 \times 2$

$W = 6\,J$

ii) $W = \Delta E = \Delta E_k$

$= E_{kf} - \cancel{E_{ki}}\ 0$

$= E_{kf}$

$W = \frac{1}{2} m v_f^2 = \cancel{\frac{1}{2} \times 2\,kg \times v^2}$

$\therefore \sqrt{\frac{2W}{m}} = v_f$

$= \sqrt{\frac{2 \times 6}{2}} = v_f$

$v_f = \sqrt{6} = 2.45\,m/s$

Conserved Quantity:

- mass
- energy
- momentum
- charge.

Mechanical Energy

$\boxed{E = E_K + E_P}$

In a closed, isolated system, mechanical energy is conserved.

$\Delta E = 0$

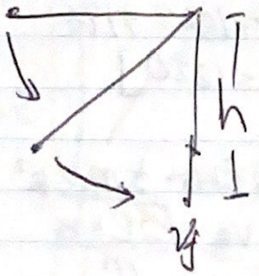
$E_f - E_i = 0$

$E_{ki} + E_{pi} = E_{kf} + E_{pf}$

or $\begin{cases} \Sigma E_i = \Sigma E_f \\ E_i = E_f \end{cases}$

$(\Sigma E_A = \Sigma E_B = \Sigma E_C$

Pendulum ①



$\Sigma E_i = \Sigma E_f$

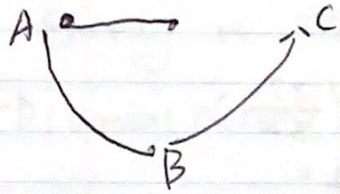
$\Sigma E_{pi} + \cancel{E_{ki}}^{0} = \Sigma \cancel{E_{pf}}^{0} + E_{kf}$

$E_{pi} = E_{kf}$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^2$

$\sqrt{2gh_i} = v_f$

②



$\Sigma E_B = \Sigma E_C$

$\cancel{E_{pB}}^{0} + E_{kB} = E_{pc} + \cancel{E_{kc}}^{0}$

$E_{kB} = E_{pc}$

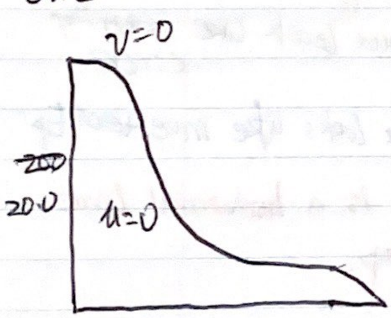
$\frac{1}{2}\cancel{m}v_B^2 = \cancel{m}gh_c$

$\frac{v_B^2}{2g} = h_c$

$\therefore \frac{(\sqrt{2gh_A})^2}{2g} = h_c$

$\therefore h_c = h_A$

ex2.



$\Sigma E_i = \Sigma E_f$

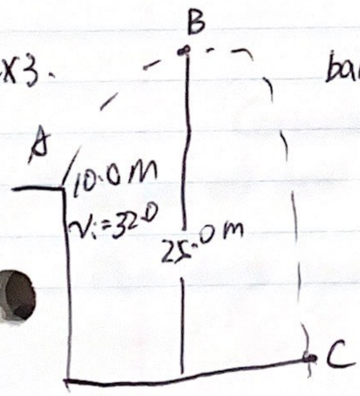
$E_{pi} + \cancel{E_{ki}}^{0} = \cancel{E_{pf}}^{0} + E_{kf}$

$E_{pi} = E_{kf}$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^2$

$v_f = ?$ $\sqrt{2gh_i} = v_f = 19.8\ m/s$

ex3.



ball = 2.0 kg Determine:

	h	v	E_p J	E_k J	ΣE
A	10	32.0	196J	1024J	1220J
B	25	~~32.0~~ 27.0	490J	730J	1220J
C	0	34.9	0	~~1220~~ 1220J	1220J

① $E_{PA} = mgh_A$

$= 2.0 \times 9.8 \times 10 = 196 J$

② $E_{KA} = \frac{1}{2} m v_A^2$

$= \frac{1}{2} \times 2.0 \times 32^2$

$= 1024 J$

③ $\Sigma E = E_{KA} + E_{PA}$

$= 1024 + 196$

$= 1220 J$

④ $E_{PB} = mgh_B$

$= 2 \times 9.8 \times 25$

$= 490 J$

⑤ $E_{KB} = \frac{1}{2} m v_B^2$ or

$(\Sigma E_B = E_{KB} + E_{PB})$

$E_{KB} = E_B - E_{PB}$

$= 1220 - 490 = 730 J$

⑥ $E_{KB} = \frac{1}{2} m v_B^2$

$v_B = \sqrt{\frac{2E_{KB}}{m}}$

$v_B = 27.0 m/s$

⑧ $E_{CK} = \frac{1}{2} m v^2$

$\sqrt{\frac{2E_{CK}}{m}} = v$

$v = \sqrt{\frac{2 \times 1220}{2}}$

$v = 34.9 m/s$

32 m/s, θ ?!, 27 m/s

$\theta = \cos^{-1}\left(\frac{27}{32}\right)$

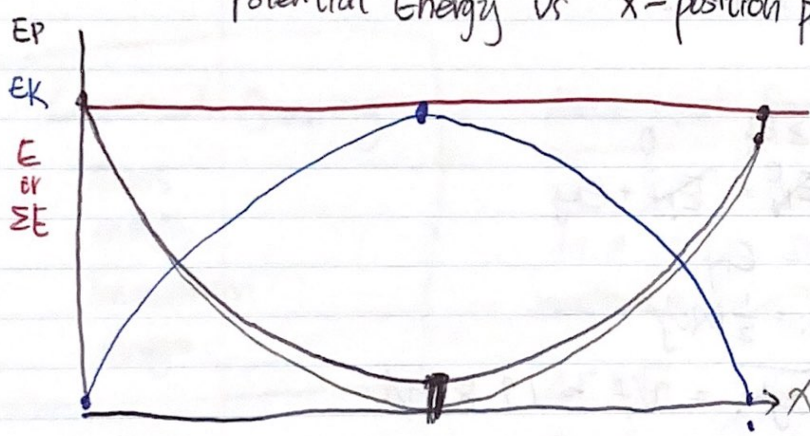
$\theta = 32.4°$

27, θ_2, 34.9

$\theta_2 = \cos^{-1}\left(\frac{27}{34.9}\right)$

$= 39.3°$

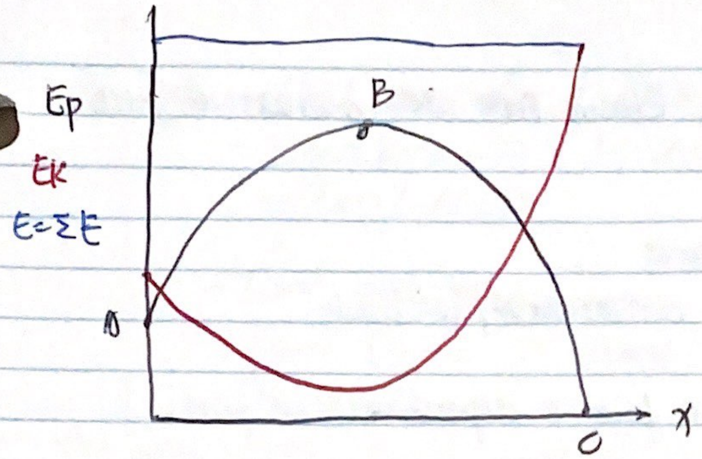
Potential Energy vs x-position plot.



Ep plot look like semi-circle

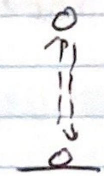
Ek plot looks like inverted Ep

ΣE or E is a horizontal line over top.

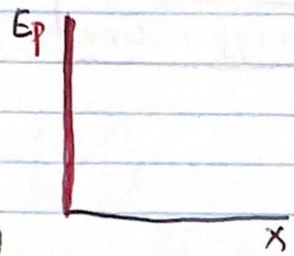


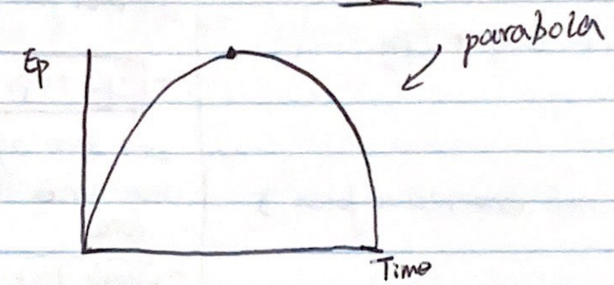
Energy vs Time plot

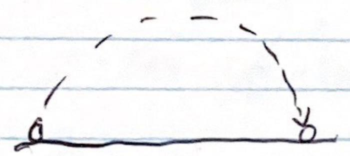
ex.



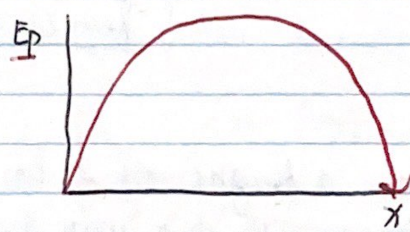
• 1-D (vertical motion)

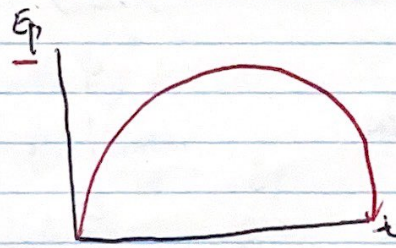






⇒





Conservative Force.

- You get all work against a conservative force. (→10J, ←10J)

- Work done against conservation force is independant of path.

- ex. gravity, Electrostatic Force, Elastic Force are conservative force.
  * ideal spring.

Non-Conservative Force

- You don't get work against a non-conservative force back.

- Work done against non-conservative force is dependant on path.
- friction (anything makes heat)

~ Non-conservative ~~Force~~ Work

$\Sigma E_i = \Sigma E_f$

$W_{nc} + E_{ki} + E_{pi} = E_{kf} + E_{pf}$

↑

negative
(work done by a non-conservative force)

another form $\Sigma E_i = \Sigma E_f$

- $\boxed{E_{ki} + E_{pi} = E_{kf} + E_{pf} + W_{nc}}$

work done against non-conservative force

- work lost to heat (friction)

ex. Example:

A 2.0kg ball is dropped from a height of 25.0m and hits the ground of 18.5m/s. Determine air friction as they way down.

$\Sigma E_i = \Sigma E_f$

$\cancel{E_{ki}}^{0} + E_{pi} = E_{kf} + \cancel{E_{pf}}^{0} + W_{nc}$

$\therefore E_{pi} = E_{kf} + W_{nc}$

$mgh_i = \frac{1}{2}mv_f^2 + W_{nc}$

$2.0 \times 9.8 \times 25 - \frac{1}{2} \times 2.0 \times (18.5)^2 = W_{nc}$

$147.75\,J = W_{nc}$

$W_{nc} = F_f \cdot d$

$\frac{W_{nc}}{d} = F_f$

$\frac{147.75}{25} = F_f$

$\boxed{F_f = 5.91N}$

$E_p = mgh$ – Used near surface of Earth, where g h and constant
– When $h=0$, can be assigned according to each problem/context.

– Used everywher is Universe including surface of Earth.
– Where $E_p = 0$ as agreed to by everyone everywhere in Universe

– this point where $E_p = 0$ must be equal distance from every other point in universe $\Rightarrow \infty$
– You must be able to reach this point in any direction

Work done lifting a ball at infinit distance from surface of Earth - - - -

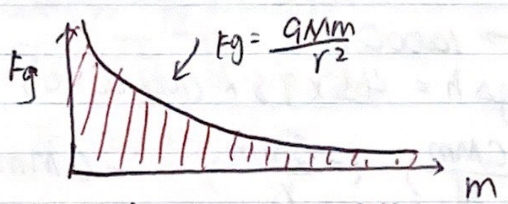
$\left.\begin{matrix} F\rightarrow \\ d\rightarrow \end{matrix}\right\} \rightarrow +W$

$W = \Delta E_p = \cancel{E_{pf}}^{\,0} - E_{pi}$

$W = -E_{pi}$

$+ = -(-)$

To find work done by a variable force, find area bounded by F vs d graph



$A = \int F(r)\,dr = \frac{GMm}{r}$

$W = \frac{GMm}{r} = -E_{pi}$

$\therefore E_p = -\frac{GMm}{r}$

Zero at infinity definition of Gravitational Potential Energy.

$E_p = -\frac{GMm}{r} = -\frac{Gm_1m_2}{r}$ $U_G = -\frac{Gm_1m_2}{r}$

~~$\Delta E_p = mg\Delta$~~

$\Delta U_g = mg\Delta y$

Ex. Determine E_p of a 4.5 kg mass at Earth's surface using both expressions

i) $E_p = mgh = 4.5\,kg \times 9.8 \times 0 = 0\,J$

ii) $E_p = -\frac{GMm}{r} = \frac{-6.67 \cdot 5.98 \times 10^{-11} \cdot 4.5}{6.38 \times 10^6} = -2.81 \times 10^8\,J$

→ move a 4.5 kg mass to infinity (work)

Binding Energy : work required to separate two masses to an infinite distance

$BE = -E_p$.

Ex. Determine work done to lift a 4.5 kg mass 100 m above Earth in two ways.

i) $W = \Delta E_p = mg\Delta h = 4.5 \times 9.8 (100 - 0) = 4410\,J$

ii) $W = \Delta E_p = \left(-\frac{GMm}{r_f}\right) - \left(-\frac{GMm}{r_i}\right) = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$= -6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 4.5 \times \left(\frac{1}{6.38 \times 10^6 + 100} - \frac{1}{6.38 \times 10^6}\right)$

$= 4409.52\,J$

Change 100 m → 100000 m

i) $W = \Delta E_p = mg\Delta h = 4.5 \times 9.8 \times (100000 - 0) = 4410000\,J$

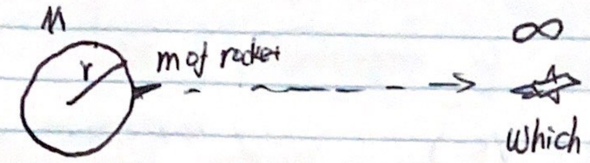
ii) $W = \Delta E_p = \left(-\frac{GMm}{r_f}\right) - \left(-\frac{GMm}{r_i}\right) = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$= -6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 4.5 \times \left(\frac{1}{6.38 \times 10^6 + 100000} - \frac{1}{6.38 \times 10^6}\right)$

$= \underline{4341540}\,J$

Use new expression:
launch, rocket
satellite
space,

Escape Velocity, speed required to escape a planet's gravitational field.



came to a stop.

which $E_k = 0 \,/\, E_p = 0$

$\Sigma E_i = \Sigma E_f$

$E_{pi} + E_{ki} = \cancel{E_{pf}}^{0} + \cancel{E_{kf}}^{0}$

$E_{pi} + E_{ki} = 0$

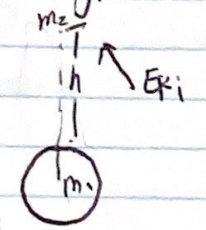
$-\frac{GM\cancel{m}}{r} + \frac{1}{2}\cancel{m}v^2 = 0$

$v = \sqrt{\frac{2GM_p}{r_p}}$

$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.38 \times 10^{6}}}$

$v = 11.2\ km/s$

Determine v_f of a piece of a rocket launch straight up if comes back down from a height of 405 km. Ignore air friction.



$\Sigma E_i = \Sigma E_f$

$E_{pi} + \cancel{E_{ki}}^{0} = E_{pf} + E_{kf}$

$E_{pi} - E_{pf} = E_{kf}$

$-\frac{GM\cancel{m}}{r} - \left(-\frac{GM\cancel{m}}{r}\right) = \frac{1}{2}\cancel{m}v_f^2$

$-GM\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{1}{2}v_f^2$

$\sqrt{+2GM\left(\frac{1}{r_f} - \frac{1}{r_f}\right)} = v_f$

$v_f = 2.73 \times 10^3\ m/s$

Momentum

Inertia is generally described as an object resistent to motion.

Momentum is the tendency to continue moving.

$\vec{p} = m\vec{v}$ momentum is a vector

unit: (kg·m/s)

$\Delta\vec{p} = \Delta(m\vec{v})$
$\begin{cases} \Delta p = \Delta m \cdot \vec{v} \\ \Delta p = m \cdot \Delta\vec{v} \\ \Delta p = \Delta m \Delta\vec{v} \end{cases}$