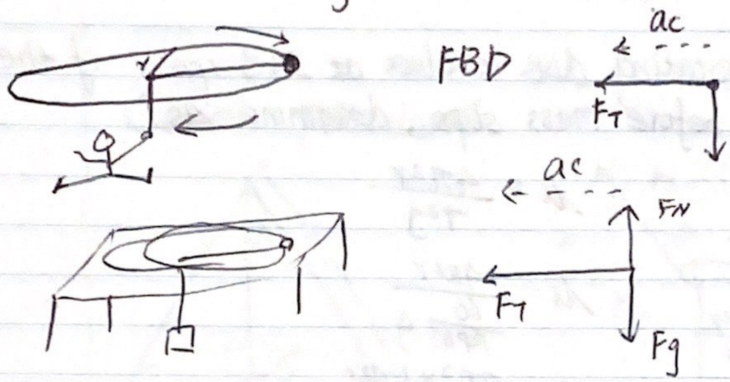
UCM part 2:

Dynamics of UCM
- FBD
- 2nd law (circle style)
- Determine $\vec{F}$, piece of $\vec{F}$ or combination of $\vec{F}$'s that provide F_c
- Choose appropriate version
- Solve

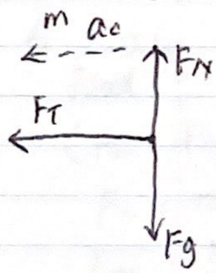
$T = \frac{60s}{r.p.m.}$ $v = \frac{2\pi r}{T}$

$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

Horizontal Geometry
- Only one $\vec{F}$ provides $\vec{F_c}$
- Fg does the same thing at all times.



ex. a small mass, $m = 0.50$ kg, revolves a frictionless table with a radius $r = 0.35$m. A large mass, $M = 0.75$kg, hang below table as shows. Determine T(period) of m.



$F_c = ma_c$

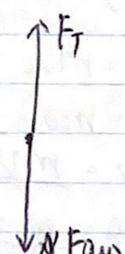
$F_T = \frac{m4\pi^2 r}{T^2}$

$T = \sqrt{\frac{m4\pi^2 r}{F_T}}$

$T = \sqrt{\frac{0.50 \times 4\pi \times 0.35}{F_T}}$

need F_T

M



$\Sigma F = ma(0)$

~~$F_T = M$~~

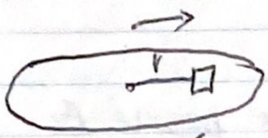
$\therefore F_T - Mg = 0$

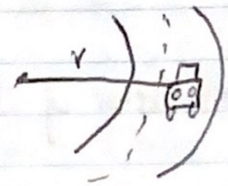
$\therefore F_T = Mg$

$\therefore T = \sqrt{\frac{0.5 \times 4\pi \times 0.35}{0.75 \times 9.8}}$

$T = 0.55s$

II. Rotating Disk / car rounding flat curve (Determine

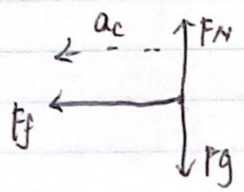




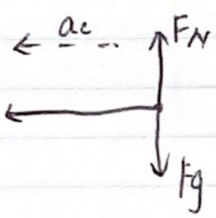
(static friction is the crucial element).

[the larger the "r", the larger centripetal force required]

FBD.



ex. ~~A rotating~~ A mass on a rotating disk revolves at 33.3 rpm. if the maximum radius 0.155m before mass slips, determine μ_s.



$\vec{F_c} = ma_c$

$F_f = \frac{m4\pi^2 r}{T^2}$

$\mu F_N = \frac{m4\pi^2 r}{T^2}$

$\mu mg = \frac{m4\pi^2 r}{T^2}$

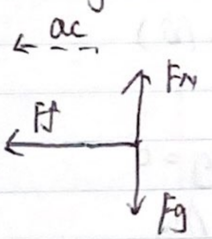
$\therefore \mu = \frac{4\pi^2 r}{T^2 g}$

$\therefore \mu = \frac{4\pi^2 r}{\frac{60}{RPM} g}$

$\therefore \mu = \frac{4\pi^2 \times 0.155}{(60/33.3)^2 9.8}$

$\therefore \mu = 0.192$

ex2. Safe max v around flat curve of radius r.



$\vec{F_c} = ma_c$

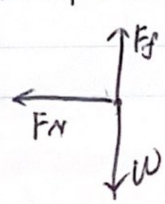
$F_f = ma_c$

$\mu F_N = \frac{mv^2}{r}$

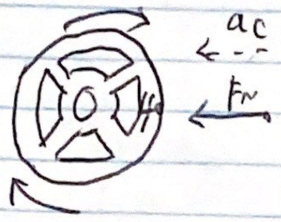
$\mu mg = \frac{mv^2}{r}$

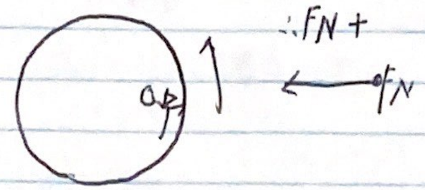
$v = \sqrt{\mu gr}$

III. Rotating Cylinder ($F_N = F_c$)



Space ship





$v\uparrow \therefore a_c \uparrow$

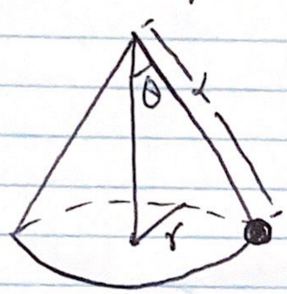
$\therefore F_N \uparrow$

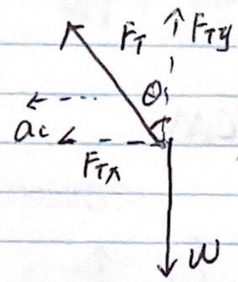
ex. $r = \cancel{?}\ 0.75\,m$ $\qquad a_c = \frac{4\pi^2 r}{T^2}$

$T = 0.30\,s$

$a_c = ?$ $\qquad a_c = \frac{4\pi^2 \times 0.75}{0.30^2} = 329\ m/s^2$

Conical pendulum





$F_{Tx} = F_c$

A 1.0 kg mass is swung around a 35.0° cone at a 1.0m long string. Find T.

$(F_c = F_{Tx})$ $\qquad$ need r

$r = l\sin\theta$

$F_c = ma_c$

$F_{Tx} = \frac{m4\pi^2 r}{T^2}$ $\qquad$ need F_{Tx} : $F_{Tx} = F_T \sin\theta$

$\cancel{F_{Tx}}\ T^2 = \frac{m4\pi^2 r}{F_{Tx}}$ $\qquad$ need F_T : $\Sigma F_y = 0$

$\therefore F_{Ty} = W$

$F_T\cos\theta = mg$

$\therefore F_T = \frac{mg}{\cos\theta}$

$$T^2 = \frac{m4\pi^2 \ell \sin\theta}{\frac{mg}{\cos\theta} \times \sin\theta}$$

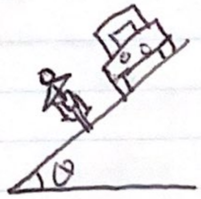
$$T^2 = \frac{m4\pi^2 \ell}{\frac{mg}{\cos\theta}}$$

$$T^2 = \frac{4\pi^2 \ell \cos\theta}{g}$$

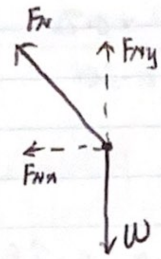
$$\therefore T = \sqrt{\frac{4\pi^2 \times 1.0 \cos 35^\circ}{9.8}}$$

$$\therefore T = 1.82\,s$$

Banked Curve

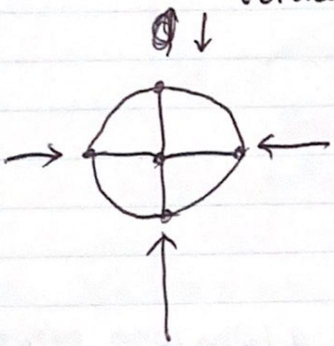


FBD

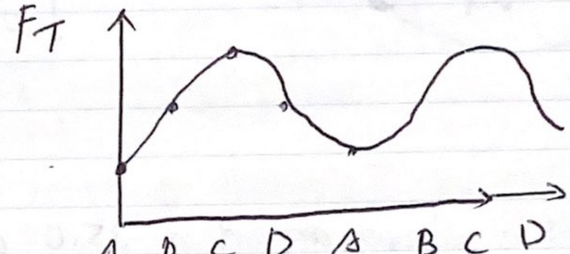


$F_{Nx} = F_c$

Vertical UCM

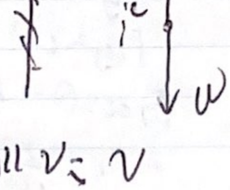


Tension



☆ Ball on String

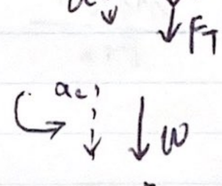
bottom



All $v \approx v$

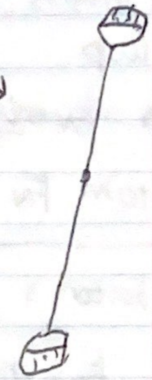
At max

top



At F_T min

ex2. Bucket on a string: FBD of water

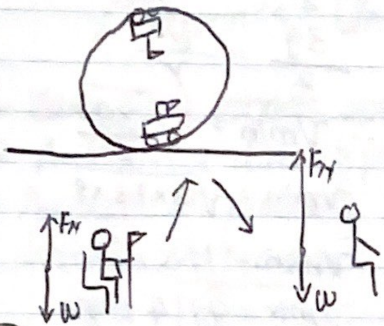


bottom: a_c ↑ ↑F_N ↓w
At all v

top: a_c ↓ ↓w ↓F_N
At all $v \neq$

↓ a_c ↓w graph ↘
At min

ex3. Loop de Loop.

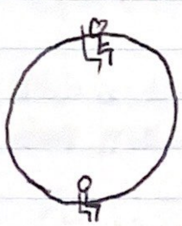


At bottom
a_c ↑ ↑F_N ↓w
At all v

At top
All $v \neq v_{min}$
a_c ↓ ↓w ↓F_N

F_N or $F_c = 0$
at min
a_c ↓ ↓w --

ex4. Ferris Wheel



At bottom
a_c ↑ ↑F_N ↓w
At all v

At top:
a_c ↓ ↑F_N ↓w
At all $v \neq v_{max}$

$F_N = 0$
At max v
a_c ↓ ↓w

ex1. $r = 2000\,m$, 71kg man feel lighter, upward F is 200N

FBD ↑F_N ↓F_g ↓a_c

$$\vec{F_c} = m a_c$$
$$\Sigma \vec{F_c} = \frac{m v^2}{r}$$
$$W - F_N = \frac{m v^2}{r}$$
$$\sqrt{\frac{r(W - F_N)}{m}} = v$$
$$v = 118\ m/s$$

ex. A driver wants to do a loop de loop stands on 9.0m radius vertical loop.

At top $F_N = \frac{W}{2}$

At bottom $F_N = 3W$

V_{max} (at bottom)

$\uparrow F_N$

$\downarrow W$

$F_c = ma_c$

$\Sigma F = \frac{mv^2}{r}$

$3W - W = \frac{mv^2}{r}$

$2mg = \frac{mv^2}{r}$

$\sqrt{2gr} = v_{max}$

$v_{max} = 13.3\ m/s$

$v_{max} = 47.8\ km/h$

v_{min} (at top)

FBD

$\downarrow W$ $\downarrow F_N$ $\downarrow a_c$

$F_c = ma_c$

$\Sigma F = \frac{mv^2}{r}$

$F_N + W = \frac{mv^2}{r}$

$\frac{3W}{2} = \frac{mv^2}{r}$

$\frac{3mg}{2} = \frac{mv^2}{r}$

$\frac{3g}{2} = \frac{v^2}{r}$

$\therefore v_{min} = \sqrt{\frac{3gr}{2}}$

$v_{min} = \sqrt{\frac{3 \times 9.8 \times 9}{2}}$

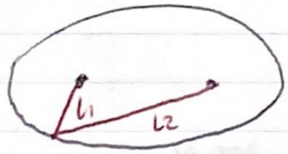
$v_{min} = 11.5\ m/s$

v_{min} 41.4 km/h

Kepler's Laws

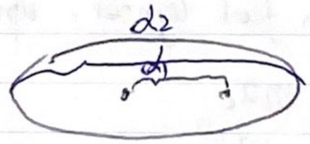
1st Law: Law of Ellipses:
All planets move in elliptical orbits with the sun at one focus.

foci:

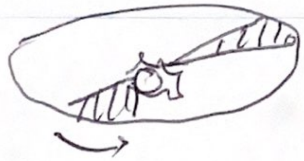


$l_1 + l_2$ is constant around the ellipses.

Measure of how ellipitical a shape is
measure the ratio of l (Eccentricity)



Kepler's 2nd Law: Law of areas: A line joining any planet to the sun sweeps out equal areas in equal time



Kepler's Third Law : Law of periods.

The square of the period of a planet is proportional to the cube of the semi - major axis of its orbit.

$T^2 \propto r^3$

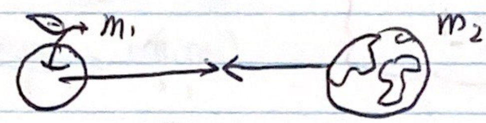
Constant C, we have an equation:

$T^2 = Cr^3$ $\quad (C = Y^2/AU^3)$

Newton's Universal Law of Gravitation

1. Every mass attracts every other mass
2. Attraction is directly proportional to the product or their masses.
3. Attraction in inversely proportional to the square of the distance between their center.

$$F_G = \boxed{\frac{Gm_1m_2}{r^2}}$$



(G = universal constant)

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

ex. What is the force of gravitational attraction between two students if their masses are 55kg and 60 kg and they 0.50 metres apart?

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67\times10^{-11})\ Nm^2/kg^2\ (55kg)(60kg)}{(0.50m)^2}$$

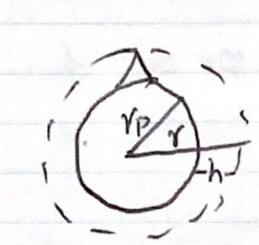
$$= 8.8\times10^{-7}\ N$$

Gravitational force field

$$F_G = \left(\frac{Gm_1}{r^2}\right)m_2$$

g

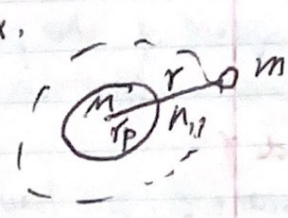
Satillite Motion:



radius of the orbit, r
radius of the planet, r_p
altitude of orbit h

$r = r_p + h$

ex.



$$F_c = ma_c$$

$$\frac{Gm_1m_2}{r^2} = ma_c$$

$$\frac{Gm_1\cancel{m_2}}{r^2} = \frac{\cancel{m_2}v^2}{r}$$

$$\boxed{\sqrt{\frac{GM}{r}} = v}$$

ex. Determine v of ISS in LEO

$h = 408$ km

$M = 5.98 \times 10^{24}$ kg

$r_p = 6.38 \times 10^6$ m

$v =$ __ m/s $v =$ __ km/s

$$F_c = ma_c$$

$$\frac{Gm_1\cancel{m_2}}{r^2} = \frac{\cancel{m}v^2}{r}$$

$$\sqrt{\frac{GM}{r}} = v = \sqrt{\frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{(4.08 \times 10^5 + 6.38 \times 10^6)}}$$

$$v = 7.67 \times 10^3 \text{ m/s}$$

$$F_c = ma_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}a_c$$

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\sqrt{\frac{4\pi^2 r^3}{GM}} = T$$

$$2\pi\sqrt{\frac{r^3}{GM}} = T$$

$$= \sqrt{\frac{4\pi^2[(6.38 \times 10^6)^3 + 408000]^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$

~~$= 6.71 \times 10^{-7}$ s~~

~~$= 8.2 \times 10^{-4}$ s~~

$=$

$$\sqrt{\frac{m^3}{\frac{Nm^2}{kg^2} \times kg}} \Rightarrow \sqrt{\frac{\cancel{kg}m/s^2}{\cancel{kg^2}} \times \cancel{kg}}$$

$= \sqrt{s^2}$

$= s$

Mass of the Planet

$F_c = ma_c$

$\frac{GMm}{r^2} = ma_c$

$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}4\pi^2 r}{T^2}$

$\boxed{M = \frac{4\pi^2 r^3}{GT^2}}$

Determine the mass of sol

$T = 3.16 \times 10^7\ s$

$r = 1.50 \times 10^{11}\ m$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

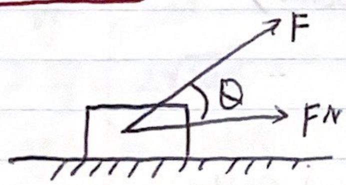
$M = \frac{4\pi^2 (1.50 \times 10^{11})^3}{6.67 \times 10^{-11} \cdot (3.16 \times 10^7)^2}$

$M = 2 \times 10^{30}$ [A = 1.98×10^{30}]

sidereal Time (23h, 56 min 04s)
[86100s]

Work:

$\boxed{W = F \cdot d}$ (Force is parellel to d) $\boxed{F\cos\theta \cdot d}$ (Not parellel)



work is a scalor but it can be "+" and "−"

Units: Joule (J) $J = kgm^2/s^2$

Work: Machanical transfer of energy.
(Giving energy