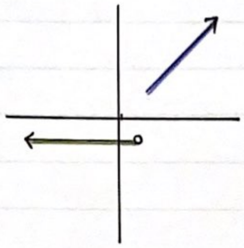
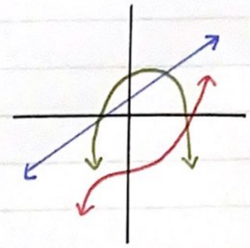
Unit 1 - Function

Quiz - Wed Sep. 21 Pretest - Mon. Sep. 26 Test - Mon. Oct. 3



$$f(x) = \begin{cases} x & \text{where } x \geq 1 \\ -1 & \text{where } x < 1 \end{cases}$$



Linear: ex. $y = 3x + 1$

Quadratic: ex. $y = -(x-1)^2 + 2$

Cubic: ex. $f(x) = x^3 - 3$

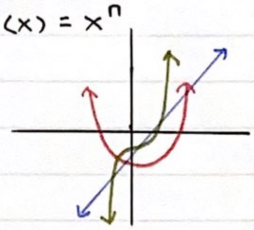
Power function: $f(x) = x^n$

ex. $f(x) = x^0 = 1$

$g(x) = x^1 = x$

$h(x) = x^2$

$k(x) = x^3$

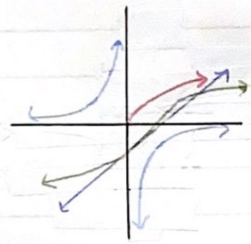


$g(x) = x^1 = x$

$h(x) = x^{\frac{1}{2}}$

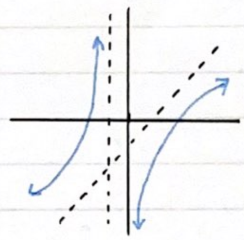
$k(x) = x^{\frac{1}{3}}$

$m(x) = x^{-1}$



Rational Function: $f(x) = \frac{P(x)}{Q(x)}$

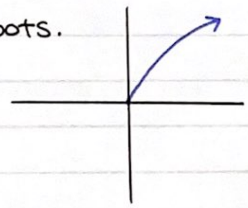
ex. $y = \frac{x^2 - 4}{x + 1}$



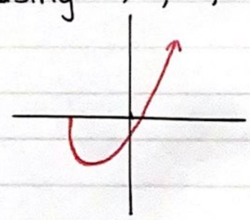
Algebraic function:

- Functions that can be constructed from polynomials using $+, -, \times, \div$ and extracting roots.

ex. $y = \frac{x}{2} + 3\sqrt{x}$

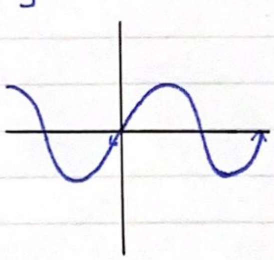


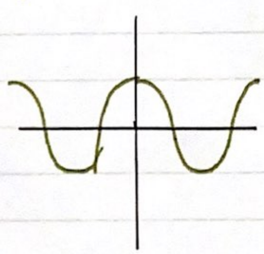
ex. $y = x\sqrt{x^3 + 64}$

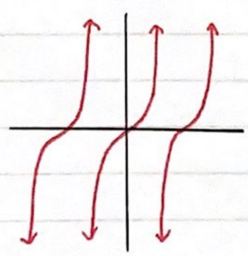


Trig:

$y = \sin x$ $y = \cos x$ $y = \tan x$ # 1 radian $= \frac{180°}{\pi}$



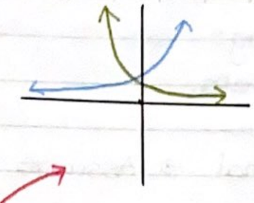




Exponential Functions: $y = a^x$

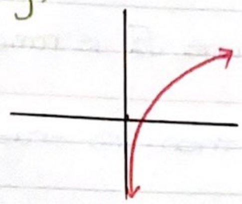
ex. $y = 2^x$ (increasing)

$y = (\frac{1}{4})^x$ (decreasing)



Logarithmic functions:

ex. $\log_3(x)$



reflection of exponential functions.

Transcendental functions:

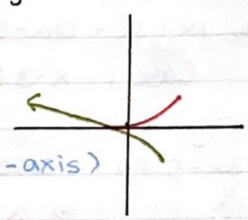
- Functions that are not algebraic: ex. Trig, log, exponential ...

Absolute Value:

ex. $y = |\sqrt{3-x} - 2|$

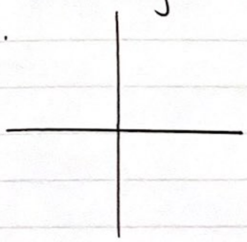
guide: $\sqrt{3-x} - 2 \rightarrow \sqrt{-(x-3)} - 2$ (2↓, 3→, refl. Y-axis)

$|\,| \rightarrow$ reflect below x-axis



Combining Function by graph:

ex.



ex. $f(x)$: ID $x \in \mathbb{R} \mid x > 3$ $\qquad g(x)$: ID $x \in \mathbb{R} \mid -7 < x < 20$

↳ $(f+g)(x)$: ID $x \in \mathbb{R} \mid 3 < x < 20$

ex. $m(x)$: ID $x \in \mathbb{R} \mid x \geq 7$ $\qquad n(x)$: ID $x \in \mathbb{Z} \mid -50 < x < 27$

a) $(mn)(x) \rightarrow x \in \mathbb{Z} \mid 7 \leq x < 27$ $\qquad$ b) $(\frac{m}{n})(x) \rightarrow x \in \mathbb{Z}$ $7 \leq x < 27$ or $0 < x < 27$

ex. $f(x) = \sqrt{x-1}$ $\qquad g(x) = \frac{2}{x}$

$f(g(x)) = \sqrt{(\frac{2}{x}) - 1} = \sqrt{\frac{2}{x} - 1}$ $\qquad g(f(x)) = \frac{2}{\sqrt{x-1}}$ $\qquad g(g(x)) = \frac{2}{(\frac{2}{x})} = x$

ID $f(g(x))$ is all values of x which (I) are are in ID of $g(x)$, & (II) for which the output $g(x)$ is in the ID of $f(x)$.

ex. $f(x)$ has ID: $x \in \mathbb{R} \mid -9 < x < 21$. $g(x) = \sqrt{x}$. Find ID of $f(g(x))$

ID of $g(x)$: $x \in \mathbb{R} \mid x \geq 0$

∴ ID of $f(g(x))$ is $x \in \mathbb{R} \mid 0 \leq x < 21$

$f(x) = \sqrt[3]{x-2}$ $\qquad g(x) = x - 2$ $\qquad f(x) = h(g(x))$ $\qquad f(x) = h(g(x))$

46 # 3,5,16,18,20,23,30,32,37,39,49,53 1.1, 1.2

General problem solving:

1. Understand the problem.
2. Think of a plan.

ex. Proof $\sqrt{2}$ is irrational → Assume $\sqrt{2}$ is rational → $\sqrt{2} = \frac{a}{b}$ → $(b\sqrt{2})^2 = a^2$

↳ $2b^2 \neq a^2$

Induction: show if true for x, then it's true for x+1. & show it's true for x=1.

3. Carry out the plan.
4. Look back.

ex. solve $|x-3| + |3x-5| = 10$.

Case 1 $x-3 \geq 0$

true only $x \geq 3$

case 1a $3x-5 \geq 0$

↳ $x \geq \frac{5}{3}$

↳ $x-3+3x-5=10$

⇒ $4x = 18$ $x = \frac{9}{2}$

Case 1b $3x-5<0$

↳ $x < \frac{5}{3}$

↳ $x-3-3x+5=10$

→ $-2x = 8$ $\cancel{x=-4}$

Case 2 $x-3<0$

true only $x<3$

case 2a $3x-5 \geq 0$

↳ $(x \geq \frac{5}{3})$

↳ $-(x-3)+3x-5=10$

$2x=12$ $\cancel{x=6}$

case 2b $3x-5<0$

↳ $(x<\frac{5}{3})$

$-(x-3)-(3x-5)=10$

$-4x=2$ $x=-\frac{1}{2}$

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