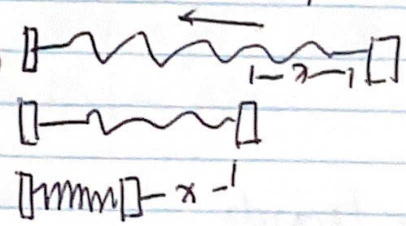
Simple Harmonic Motion (SHM)



$F_s \in x$

spring force is proportional to x.

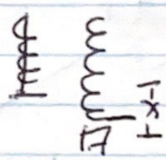
Hooke's Law.

$F_s = -kx$ k is spring constant (unit : N/m)

$F_{restoring} = -kx$ or kx

$F_s = F_g = mg$ / $F_s = F_{net} \rightarrow kx = ma$

ex. How to determine k?

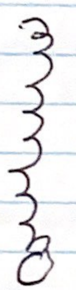


①
$F_s = kx$
$kx = mg$
$\frac{mg}{x} = k$
$\frac{0.2 \times 9.8}{0.065} = k$

$m = 0.2 kg$
$x = 0.065 m$

$\therefore k = 30.2 N/m$ [30.15 N/m]

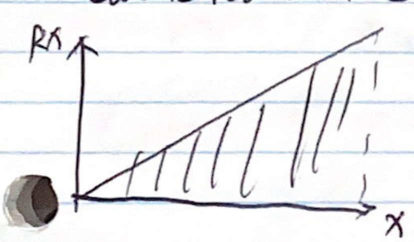
②



$m = ?$
$k = 30.15 N/m$
$x = 0.132 m$

$F_s = kx$
$kx = mg$
$\frac{kx}{g} = m$
$\frac{30.15 \times 0.132}{9.8} = m$
$m = 0.406 kg$

Elastic Potential Energy



$W = \Delta E_{es}$
$W = \frac{1}{2} \times kx \cdot x$
$= \frac{1}{2}kx^2$

$(\frac{1}{2}\Delta(kx^2))$

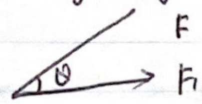
$\boxed{\text{or } U_s = \frac{1}{2}\Delta kx^2}$

Test:

Work: Mechanical transfer of Energy

$W \in \vec{P}$

$W = F_{\parallel} \cdot d$

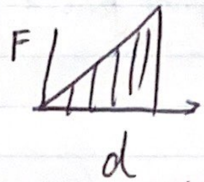


$W = F\cos\theta \cdot d$

$W_{net} = F_{net} \cdot d$

$W = \Delta E$

$W = \text{Area}$

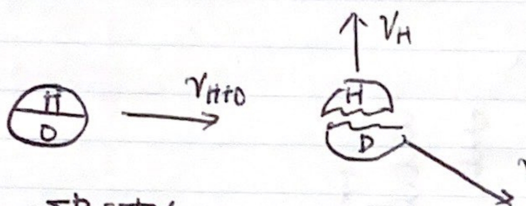


$W = \int F(x)\,dx$

$\text{power} = \frac{W}{t}$ or $F \cdot V$

$$\text{Efficiency} = \frac{\text{Work/power/Energy out}}{\text{Work/power/Energy in}} \times 100\%$$

ex 2.



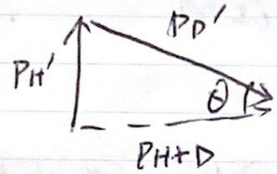
m of H = 1.0 kg $\vec{V}_H = 2.5\,m/s$

m of D = 1.5 kg $\vec{V}_{H+D} = 2.0\,m/s$

$\vec{V}_D = ?$

$\Sigma P = \Sigma P'$

$\vec{P}_{H+D} = P_H' + P_D'$



$|P_{H+D}| = (m_H + m_D) \times |\vec{V}_{H+D}|$

$= 2.5 \times 2$

$P_{H+D} = 5\,kgm/s$

$|P_H'| = m_H \cdot |\vec{V}_H|$

$= 1 \times 2.5 = 2.5\,kgm/s$

$|P_D'| = \sqrt{P_H'^2 + P_{H+D}^2}$

$= \sqrt{5^2 + 2.5^2}$

$= 5.59\,kgm/s$

$\angle\theta = \tan^{-1}\left(\frac{P_H'}{P_{H+D}}\right)$

$= \tan^{-1}\left(\frac{2.5}{5}\right)$

$= 26.6^\circ$

$V_D' = \frac{P_D'}{m_D}$

$= \frac{5.59}{1.5}$

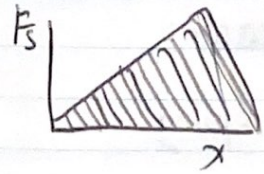
$= \cancel{3.67\,m/s}$ $3.73\,m/s$

$\therefore V_D = \cancel{3.67\,m/s}$ [26.6° S of E]

$3.73\,m/s$

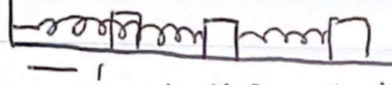
SHM so far: Me.

$|\vec{F_s}| = k|\vec{x}|$



$A = U_s = \frac{1}{2}kx^2$

Displacement:

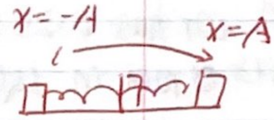


$x = -x$ $x=0$ $x = +x$

$\vec{a}$ → ← $\vec{a}$ acceleration is always pointing to the center.

$F = ma = -kx$

$\Rightarrow a = \frac{-kx}{m}$ (maximum acceleration at 'A' points



Maximum x is called A (amplitude)

Conservation of Energy: (Velocity)

$\frac{1}{2}kA^2$ is mechanical energy: $\cancel{E} = U + K = \frac{1}{2}kA^2$
($x = \pm A$ and $v = 0$)

$v_{formula} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

v_{max} when $x = 0$ $v = \sqrt{\frac{k}{m}}A$

acceleration:

since $-kx = ma$

$\therefore \boxed{a = \frac{-kx}{m}}$

Pendulum

Frequency & Periods:

Frequency $(f) = \frac{1}{T}$ Hz Period: $(T) = \frac{1}{f}$ (s)

$\omega = 2\pi/T$ / $\omega = 2\pi f$

$x = A\cos\theta = A\cos(\frac{2\pi}{T}t) = A\cos(2\pi f t) = A\cos(\omega t)$

position of x

period of simple Harmonic Oscalator:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}} \Leftarrow \text{spring}$$

Restoring force for pendulum $F_s = mg\sin\theta$

$$\boxed{T_{pendulum} = 2\pi\sqrt{\frac{L}{g\text{(gravitational field)}}}}$$

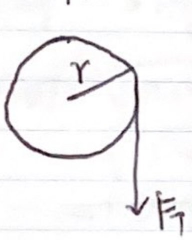
ex. A car has 0.25 m radius wheel, If the ~~work~~ wheel rotate a total of 575 rad in 15.0 seconds, find 1) ω b) v of car, c) d of car.

a) $\omega = \frac{\theta}{t} = \frac{575}{15.0} = 38.3$ rads/s

b) $v = r\cdot\omega$ $\quad v = 0.25 \times 38.3 = 9.58$ m/s

c) $d = r\cdot\theta$ - $d = 575 \times 0.25 = 144$ m

ex2, A 5.0kg disk of 30cm radius is initially at rest. A 75.0N tension is applied to a string wrapped around the disk. Find



~~a) α~~
~~b) ω~~ after

a) I b) α c) ω after 10s.

d) E_K stored after 10s.

a) $I = MR^2\cdot\frac{1}{2}$

$= \frac{1}{2}\times 5.0\times 0.3^2$

$= 0.225$ kgm²

b) $\Sigma\tau = I\alpha$

$\frac{\Sigma\tau}{I} = \alpha$

$\frac{F_T\cdot r}{I} = \alpha = \frac{75\times 0.3}{0.225} = 100$ rads/s²

c) $\omega_f = \omega_i + \alpha\cdot t$

$\omega_f = 0 + 100\times 10$

$\omega_f = 1000$ rad/s

d) $E_K = \frac{1}{2}I\omega^2$

$= \frac{1}{2}\times 0.225\times 1000^2$

$= 1.13\times 10^5$ J