Options & Real Options

Valuation of managerial fexibility

What is an option?

Call option

The right to purchase an asset, at a previously agreed upon price, during a certain time period.

Put option

The right to sell an asset, at a previously agreed upon price, during a certain period.

Rights and obligations

	Buyer of the option	Seller of the option
Call option	Right to buy the asset acquired by the payment of the option premium	Obligation to sell the asset assumed by receiving the premium
Put option	Right to sell the asset acquired by the payment of the option premium	Obligation to buy the asset assumed by receiving the premium

Some basic terminology

- Expiration date: the time when the option becomes void and can no longer be exercised.
- Exercise date: the time period during which the purchaser may notify the seller the he is exercising its option
- Exercise price: the price at which the option gives its holder the right to buy (call) or to sell (put) the underlying asset.
- Premium: the price at which the option (call or put) may be aquired.

Some basic terminology

- American option: gives investors the right to exercise the contract at any time up to the expiration date.
- European option: gives investors the right to exercise the contract only at the expiration date
- Warrants

Terminologia

- Datas de vencimento e datas de exercício;
- Opções americanas e opções europeias;
- *Warrants;*
- Liquidação com entrega física ou por equivalente monetário;
- Anular a posição;
- Prémio = preço da opção;
- Preço de exercício.

Option Value

- There are two components in the value of an option:
  - Intrinsic value;
  - Time value.

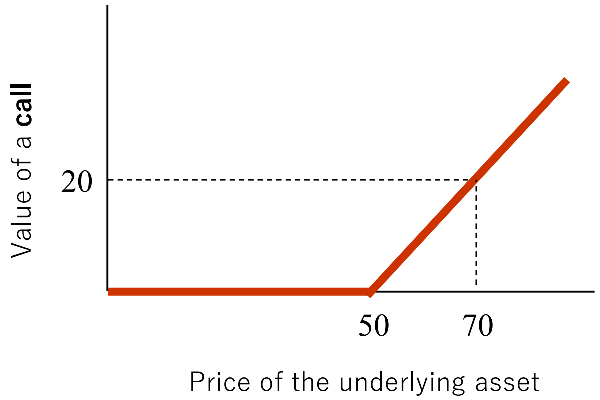
Intrinsic Value

- The intrinsic value of an option is equal to the difference between its exercise price and price of the underlying asset at that moment.
- Assuming the underlying asset is currently priced at $50 the intrinsic values of an option with exercise price of $50 is:

Price of underlying asset	25	35	45	55	65	75
Call	0	0	0	5	15	25
Put	25	15	5	0	0	0

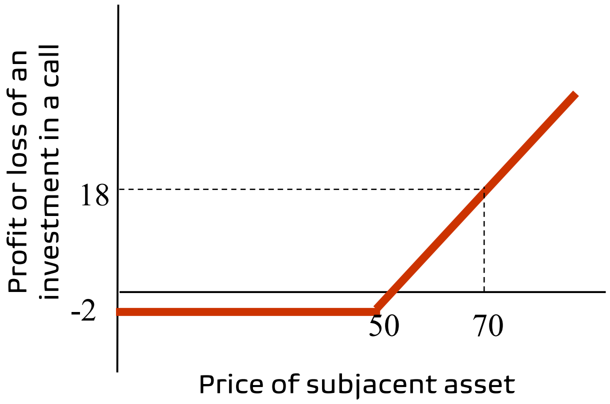
Intrinsic Value

Intrinsic value of a call with exercise price of $50



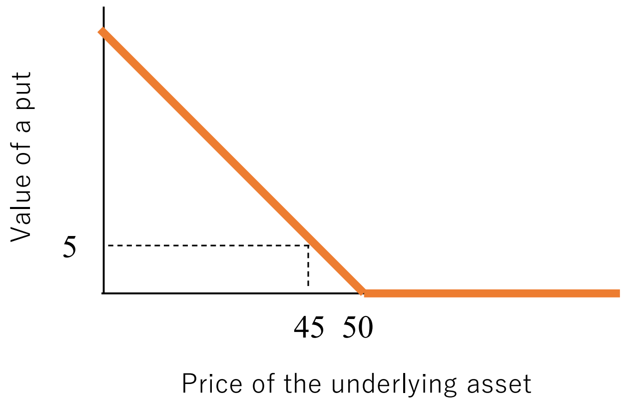
Intrinsic Value

Profit or loss of the investment in a call with exercise price of $50 and premium of $2 on the exercise date



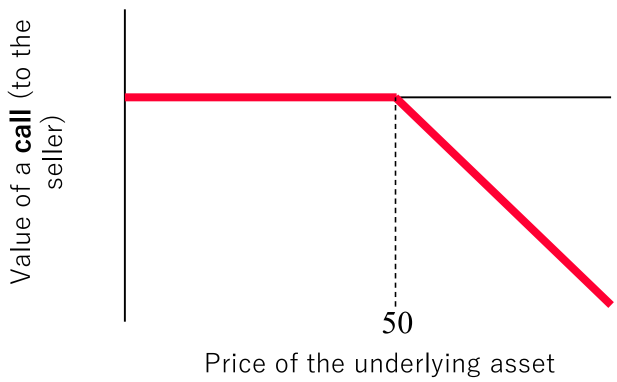
Intrinsic Value

Intrinsic value of a put with exercise price of $50



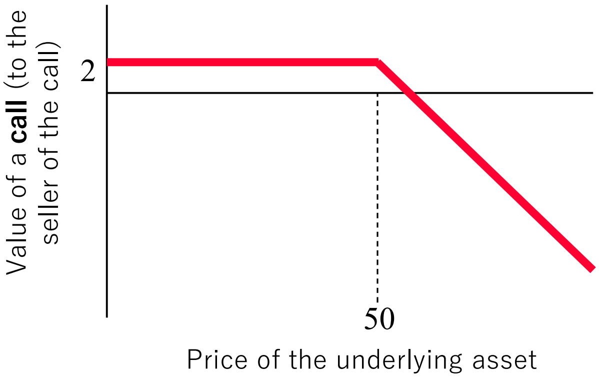
Intrinsic Value

Intrinsic value of a call with exercise price of $50 to its seller.



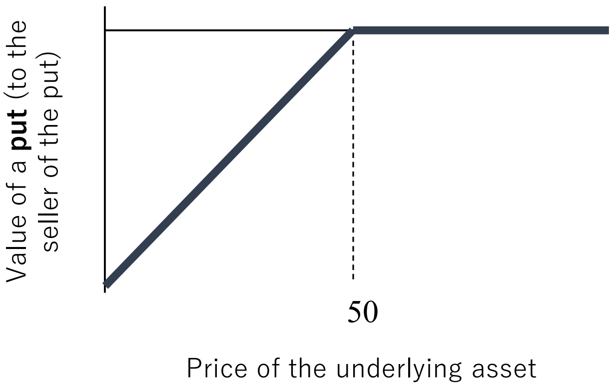
Intrinsic Value

Profit or loss of a **call** to its seller with exercise price of $50 and premium of $2.



Intrinsic Value

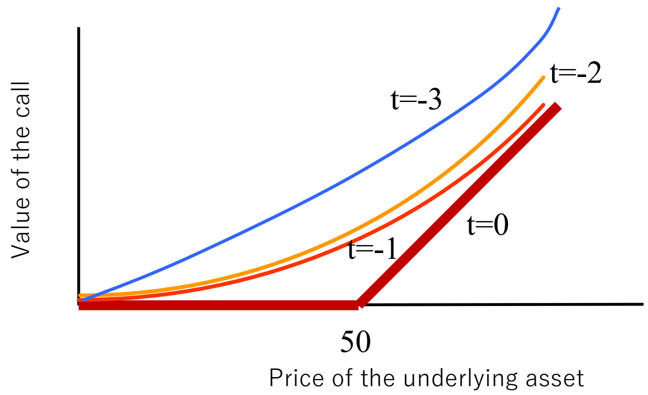
Intrinsic value of a put to the seller with exercise price of $50.



Temporal Value

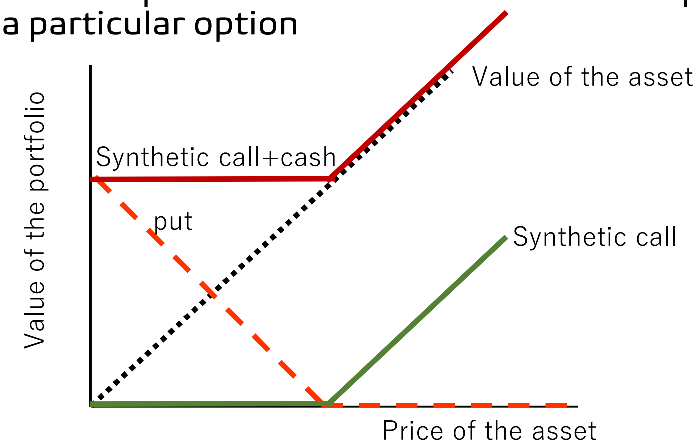
Option value = Intrinsic value + Temporal value

Temporal value = value of being able to make a decision later



Synthetic Options

A synthetic option is a portfolio of assets with the same payoff and risk profile of a particular option



Put-Call Parity

$$p + P_a = c + E / e^{kT}$$

p = value of the put

c = value of the call

P_a = price of the underlying asset

E = exercise price of the put and the call

k = risk free interest rate

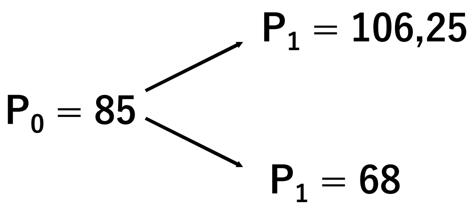
T = time to maturity of the put and the call

Put-Call Parity

- Synthetic call: an example
  - Replication of a call with
    - Maturity: 6 months
    - Exercise price: $85
  - Six months risk free interest rate: 2.5%
  - Price of underlying asset:
    - Now: $P_0 = 85$
    - After 6 months: $P_1 = 68$ or $P_1 = 106{,}25$

Put-Call Parity

- Synthetic call: an example



Value of the call on its maturity	$P_1 = 68$	$P_1 = 106{,}25$
	0	21,25

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price
    - Buy 5/9 of the underlying asset
    - Borrow $36.86

	$P_1 = 68$	$P_1 = 106{,}25$
5/9 of the underlying asset	37,78	59,03
Loan interest and amortization	−37,78	−37,78
Portfolio value	0	21,25

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price

Value of the call = value of 5/9 units of the asset – value of the loan

= 85 x 5/9 – 36,86

= 10,36

But,
how was the ratio 5/9 arrived at?

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price

It was arrived at through the ***call delta*** (hedge-ratio):

$$Delta = \frac{\text{Amplitude of the possible prices of the call}}{\text{Amplitude of the possible prices of the u. asset}}$$

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price

Delta (hedge-ratio):

$$Delta = \frac{P_{call}^{Highest} - P_{call}^{Lowest}}{P_{asset}^{Highest} - P_{asset}^{Lowest}}$$

Hedge ratio: proportion of the underlying asset to include in the portfolio with payoff and risk profile similar to the call

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price

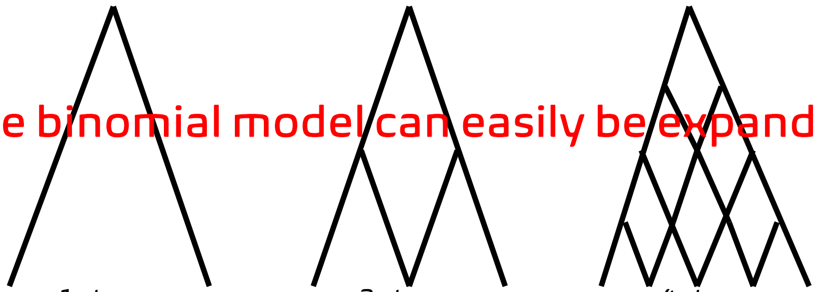
Delta (hedge-ratio):

$$Delta = \frac{P_{call}^{Highest} - P_{call}^{Lowest}}{P_{asset}^{Highest} - P_{asset}^{Lowest}}$$

$$Delta = \frac{21{,}25 - 0}{106{,}25 - 68} = \frac{5}{9}$$

Put-Call Parity

- Synthetic call: an example
  - Buy the underlying asset and borrow a percentage of its price



1 step
(2 possibilites)

2 steps
(3 possibilities)

4 steps
(5 possibilities)

etc. etc.

Black-Scholes Pricing Model

<u>The Black-Scholes Option Pricing Model</u>

$$V_c = N(d_1)P - \frac{E}{e^{kt}} N(d_2)$$

The most elegant equation in the
history of math!

Black-Scholes Pricing Model

The second most voted was:

$$E = mc^2$$

Black-Scholes Pricing Model

The Black-Scholes Option Pricing Model

$$V_c = N(d_1)P - \frac{E}{e^{kt}} N(d_2)$$

$$d_1 = \frac{\ln\left(P / Ee^{-kt}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Black-Scholes Pricing Model

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The Black-Scholes Option Pricing Model

$$V_c = N(d_1)P - \frac{E}{e^{kt}} N(d_2)$$

Delta P_{asset} PV(E) $N(d_2)$

Loan

$$d_1 = \frac{\ln\left(P/Ee^{-kt}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$