Black-Scholes Pricing Model

The Black-Scholes Option Pricing Model

$$V_c = N(d_1)P - \frac{E}{e^{kt}}N(d_2)$$

Delta P_{asset} PV(E) $N(d_2)$

Loan

$$d_1 = \frac{\ln\left(P/Ee^{-kt}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

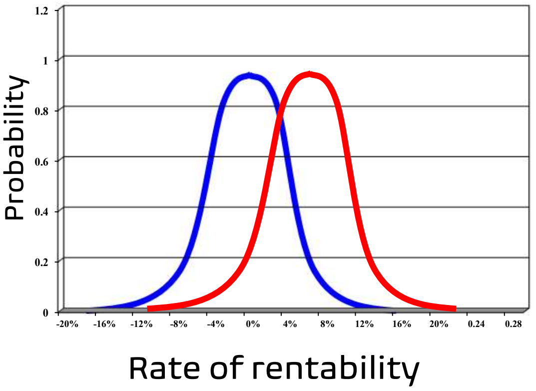
Black-Scholes Pricing Model

Components of the value of a call

1 – Market price of the underlying asset (P_{asset});
2 – Exercise price (E);
3 – Price volatility of underlying asset (sigma);
4 – Period of time until maturity (t);
5 – Time value of liquidity (risk free interest rate, *k*).

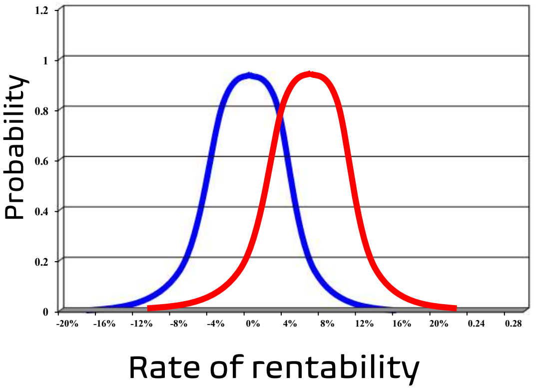
Market Risk & Firm Specific Risk

Which of these assets has more risk?



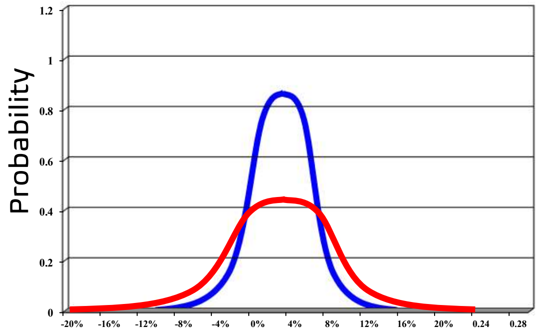
Market Risk & Firm Specific Risk

Both assets have the same total risk



Market Risk & Firm Specific Risk

Two assets with diferent total risk

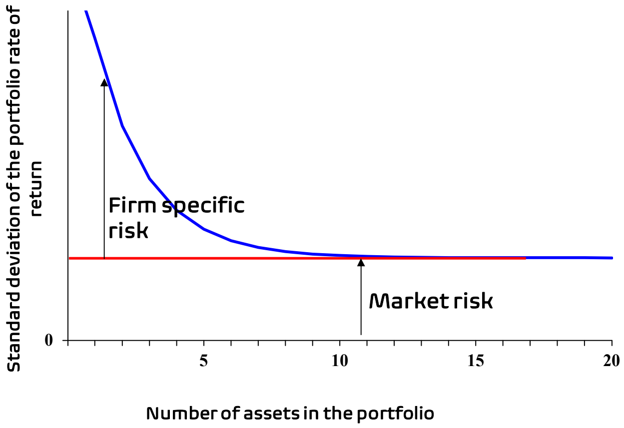


Rate of rentability

Market Risk & Firm Specific Risk

Total risk = Firm specific risk + Market risk

Market Risk & Firm Specific Risk



Market Risk & Firm Specific Risk

Total risk = Firm specific risk + Market risk

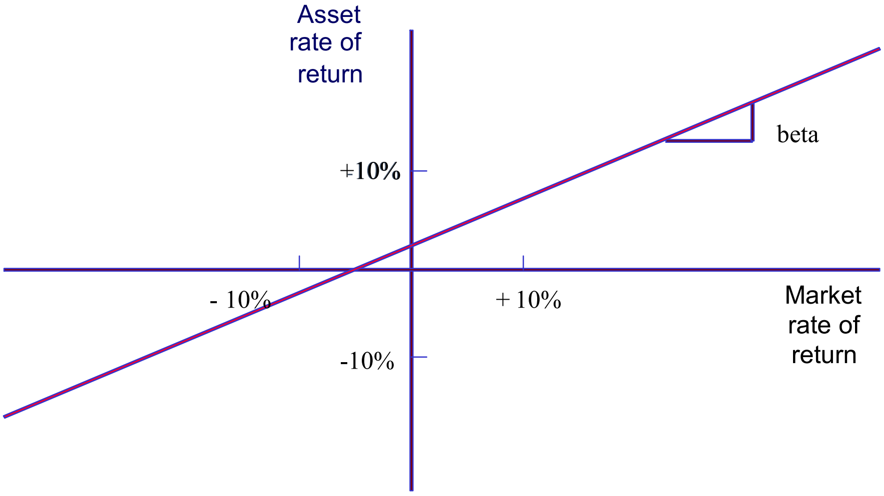
Through diversification, firm specific risk decreases and becomes irrelevant.

Market Risk & Firm Specific Risk

Total risk = Firm specific risk + Market risk

Market risk is measured with β, a measure of sensibility of the rate of return of the asset to changes in the rate of return of the market portfolio.

Market Risk & Firm Specific Risk

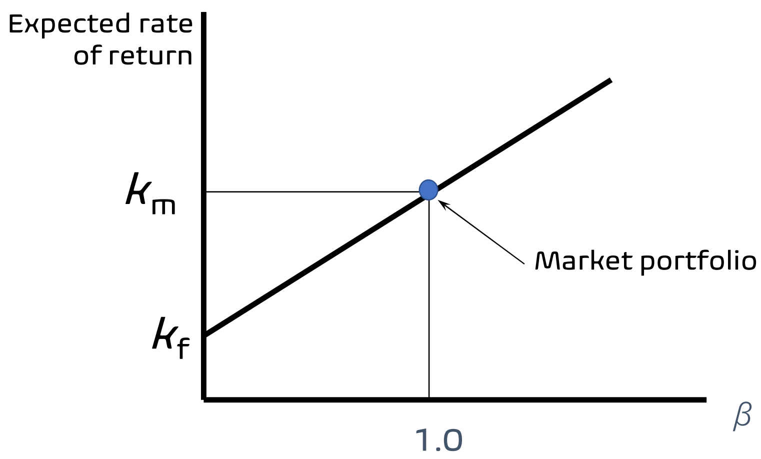


Market Risk & Firm Specific Risk

The asset's β is a function of the covariance between the rate of return of the asset and the rate of return of the market portfolio.

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Market Risk & Firm Specific Risk

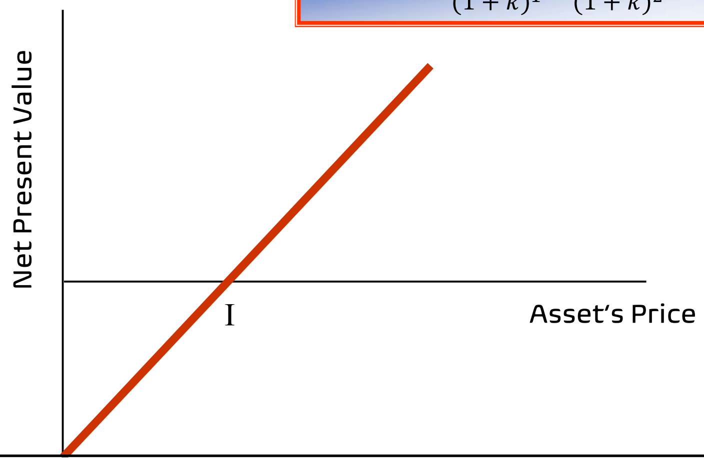


$$k_i = k_f + \beta_i \left(k_m - k_f \right)$$

Real Options

NPV=PV-Investment

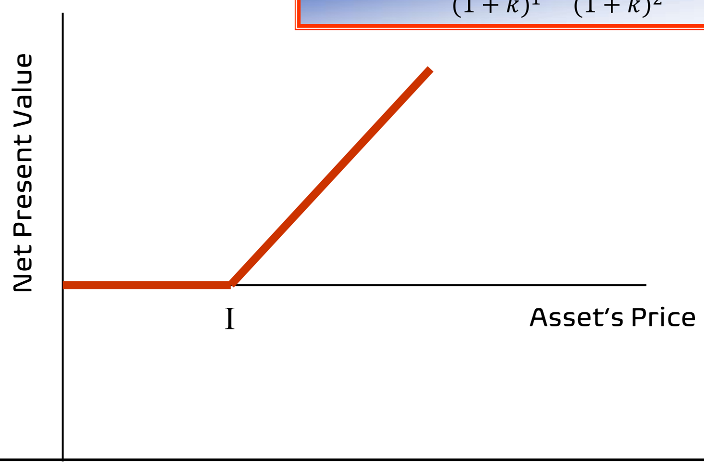
$$NPV = -I + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots.$$



Real Options

NPV=PV-Investment

$$NPV = -I + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots.$$



Real Options

A real option is the possibility, but not the obligation, to take some action during a certain time period at a predetermined cost.

Real Options

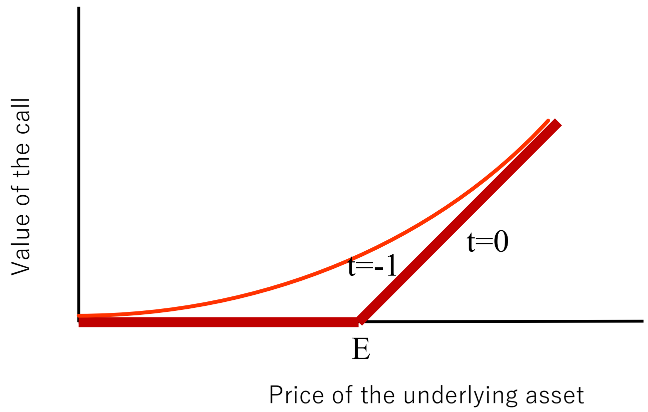
The possibility, but not the obligation, to take some action must have some value.

The NPV does not capture, or measure, this value.

Real Options

Option value = Intrinsic value + Temporal value

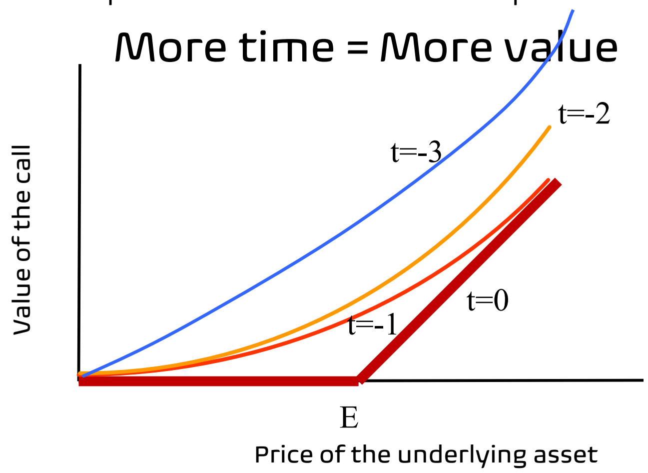
Temporal value = value of being able to make a decision later



Real Options

Option value = Intrinsic value + Temporal value

More time = More value



Real Options

There are 4 types or real options:

- The opportunity to make an expansion investment
- The opportunity to abandon an investment
- The opportunity to wait and make the investment later
- The opportunity to change the inputs or the outputs

Real Options

Strategic NPV = NPV + Value of all real options

Real Options

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Correspondence between financial and real options

Components of the value of a call

1 – Market price of the underlying asset (P_{asset}); → PV of future cash flows generated by the investment

2 – Exercise price (E); → Amount of the investment

3 – Price volatility of underlying asset (sigma); → Volatility of NPV

4 – Period of time until maturity (t); → Time until investment

5 – Time value of liquidity (risk free interest rate, *k*). → Risk free interest rate

Real Options

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An opportunity to make an expansion investment

- Risk free interest rate: K_f=10%
- Cost of capital: K_u=30%
- Standard deviation: σ=0.80
- Free cash flow:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
CAPEX	300	0	600	0	0	0	0
Operational cash flow	-100	120	100	450	600	420	0
Changes in WCR	0	40	100	10	130	-140	-140
Free Cash Flow	-400	80	-600	440	470	560	140
NB: Operational cash flow = EBIT(1-t) + Depreciation							

NPV= -149 IRR=20%

Real Options

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An opportunity to make an expansion investment

- Risk free interest rate: K_f=10%
- Cost of capital: K_u=30%
- Standard deviation: σ=0.80
- Free cash flow:

Initial investment

	Year 0	Year 1	Year 2	Year 3	Year 4
CAPEX	300	0	0	0	0
Operational cash flow	−100	120	300	210	0
Changes in WCR	0	40	100	−70	−70
Free cash flow	−400	80	200	280	70

NPV_0= -68 IRR_0=20%

Expansion investment

	Year 2	Year 3	Year 4	Year 5	Year 6
CAPEX	600	0	0	0	0
Operational cah flow	−200	240	600	420	0
Changes in WCR	0	80	200	−140	−140
Free cash flow	−800	160	400	560	140

NPV_2= -136 IRR_2=20%

Real Options

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Four steps to estimate Strategic NPV

1. Estimate Tobin's q:
   - $q = P$ / PV(E) = (664/1,3²)/(800/1,1²) = 0,5942
     (664=-I+NPV=800-136)
2. Calculate cumulative volatility:
   - standard deviation= 80%
   - t = 2
   - Cumulative volatility= 1,131
3. Using the Black-Scholes equation (Excel):
   - Value of the call in % of the value of the underlying asset: 0,2826
   - V_C = 0,2826 x 392,9 = 110,8
     (392.9 = 664/1.3²)
4. Strategic NPV = -68 + 110,8 = 42,8