The Little CPPler

v0.1.8

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Chapter 1

What to expect from this book

This is NOT a reference book, this is a teaching/practicing book. It is for anybody who is willing to learn C++ by practicing. This book is also an unusual book. It has taken inspiration (copied) from a great teaching book for the Scheme programming language: "The little schemer".

There is a saying in, uh, somewhere in instructional courses, it goes like: Learn like you work, i.e., you should learn in the same way that you will at some point work. For example, if you wanna be a chef then cook like a chef, not just learn the recipes! Or if you are an student of Engineering, learn to solve problems in your area like you will do in your work, not just apply the equations for the exercise.

Learning is rewarding but it is hard work! If you don't feel that it takes effort to learn, (you can recall everything) then you are not learning, you are just applying (which is not bad, it is necessary, but it's NOT learning).

Chapter 2

Prerequisites

Tools

To follow this book you will need two tools:

- A text editor
- A compiler

or, alternatively, you could install a tool that integrates those two tools into one, and IDE.

Examples of IDE's are: Code::Blocks (Dev-C++) (for windows and linux), Visual Studio (for windows), NetBeans or Eclipse (for windows, linux and macOS), Xcode (for macOS).

The rest of this guide will suppose you are using Code::Blocks.

Installing Code::Blocks Dev-C++

Go to http://www.codeblocks.org/, clic on Downloads, then clic on "Download the binary release", and select the download that contains `mingw` and `setup` in the name.

Once you have download it, install it like any other windows program.

Using the tools

I'm gonna assume that you have installed CodeBlocks Dev-C++ in your Windows computer. The procedure shown here is similar in another Operating Systems like Linux or iOS.

Open Code::Blocks and clic on the menu "File", then "New", and finally "Empty file". A space to write text should appear in the middle of the window with the name "Untitled1".

Write in the text space the following:

```
#include <iostream>

int main()
{
  std::cout << "Hello World!" << std::endl;
  return 0;
}
```

Once you have entered the text without any typos save it in a new folder with the name `helloworld.cc`. Note: you could get stuck if there is some typo on the text, please revise you don't have any typos.

Now clic on the menu “Build” and then the option “Build”. If you go now to the folder where you saved `helloworld.cc`, you will find a file called `helloworld.exe`[1]

If you double clic the created file, you should see a window appear and dissapear in a fraction of a second. To actually see what that window was, go back to Code::Blocks and clic in the menu “Build” and then the option “Run”.

Now, you should see a black window open. In it, you will see the line:

```
Hello World!
```

followed by some lines telling you how long did the program take to run.

Congratulations! You have compiled and executed your first program!

Now that everything is working, why don’t you try fiddling with the file, change the message, or add more, repeat some lines, or something. Have fun.

[1] You may not see a file with that name, you may see a file without the extension `.exe`, only `helloworld`. That’s ok, windows by default hides extensions. `.exe` is the extension that tells windows when a file is an executable, a program. If you want to see the extension of files under windows, search on the internet “displaying file extensions in windows”.

Chapter 3

How to read this book

Yeah, you should learn first how to read this book! :P

The book is divided into three blocks, each block is divided into 4 (or 5) sub-blocks. Each (sub-)block is dependent in the previous (sub-)blocks, this means that you need to read sequentially to understand what is any block about.

Each subblock is composed of several (10 to 20) exercises. Each exercise is divided into two columns, the left column is a question and the answer is in the second column.

Because the answers to the exercises are in the same page as the questions, I recommend you to cover the answer and try to answer by yourself, when you are sure of the answer (or you don't know how to answer) look at the right column for the proposed answer.

There are some little tasks in each subblock to perform, tasks are here exercises without answers, they are possible to solve with the material covered to the point were they are presented.

Chapter 4

Block 1: Basics

Getting to know C++(11)

000.
What do you think the following code will output after compiling and running it?

```
#include <iostream>

int main()
{
  std::cout << "Hello World!" << std::endl;
  return 0;
}
```

The output is:

```
Hello World!
```

001.
Now, what if you compile this other file:

```
#include <iostream>

int main()
{
  std::cout << "Hello World!" << std::endl;
  std::cout << "I'm a program example and I'm "
            << "in English."
            << std::endl;
  return 0;
}
```

The output is:

```
Hello World!
I'm a program example and I'm in English.
```

Pay close attention to the output, there are three sentences surrounded by quotation marks ("), but there are only two lines in the output. Why?

002.

If we change our example slightly (notice the added semicolon (;) at the end of line 6) what do you think it will happen?

```
#include <iostream>

int main()
{
  std::cout << "Hello World!" << std::endl;
  std::cout << "I'm a program example and I'm ";
            << "in English."
            << std::endl;
  return 0;
}
```

Well, it doesn't compile! We get an error similar to:

```
:7:13: error: expected expression
            << "in English."
            ^
```

It is telling us that it was expecting something (a `std::cout` for example) before `<<`.
Try removing or adding random characters (anywhere) to the example and you will find that the compiler just admits a certain arrangement of characters and not much more. But, why? Well, the compiler just understands the grammar of C++ as we just understand the grammar of our human languages. Going a little further with the analogy, we can understand the grammar of any human language (its parts (verbs, prepositions, …) and how are they connected) but we can only understand the meaning (semantics) of those languages we have studied (or our mother tongues).

003.

Does the following program compiles. If yes, what is its output?

```
#include <iostream>

int main()
{
  std::cout << "Hello World!" << std::endl;
  std::cout << "I'm a program"
            << std::endl
            << "example and I'm"
            << std::endl
            << "in English."
            << std::endl;
  return 0;
}
```

Yep, it in fact compiles, and its output is:

```
Hello World!
I'm a program
example and I'm
in English.
```

Notice how `std::endl` puts text in a new line, that's in fact its whole job.

Task for home: Write a working C++ program and add or remove some character to it so it fails to compile. Ask a fellow classmate to try to find the error and explain it

---

004.

Well that's getting boring. What if we try something different for a change. What is the otput of this program:

```
#include <iostream>

int main()
{
  std::cout << "Adding two numbers: "
            << 2 + 3
            << std::endl;
  return 0;
}
```

Nice[†]

```
Adding two numbers: 5
```

[†]this is a footnote, read all of them, they may tell you little things that the main text won't.

005.
Let's try something a little more complex[†]

```
std::cout
  << "A simple operation between " << 3
  << " " << 5 << " " << 20 << ": "
  << (3+5)*20 << std::endl;
```

```
A simple operation between 3 5 20: 160
```

[†]Here you can see only a snippet of the whole code. The complete code the snippet represents can be found in the source accompaning this book.

From now on, all code will be given on snippets for simplicity but remember that they are that, snippets, uncomplete pieces of code that need your help to get complete.

006.
What is the purpose of `<< " " <<` in the code?

`<< " " <<` adds an space between the numbers otherwise the output would look weird.

007.
What if we remove all spaces from the last example?

```
std::cout
  << "A simple operation between " << 3
  << 5 << 20 << ": "
  << (3+5)*20 << std::endl;
```

```
A simple operation between 3520: 160
```

It looks aweful, doesn't it? Spaces are important as formatting!

008.
Let's try it now multiline:

```
std::cout
  << "A simple operation between " << std::endl
  << 3 << std::endl
  << 5 << std::endl
  << 20 << ": " << std::endl
  << (3+5)*20 << std::endl;
```

```
A simple operation between
3
5
20:
160
```

009.
Did you noticed that we only used a `std::cout`?
What is then the output of the code below?

```
std::cout
  << "A simple operation between " << std::endl;
std::cout << 3 << std::endl;
std::cout << 5 << std::endl;
std::cout << 20 << ": " << std::endl;
std::cout << (3+5)*20 << std::endl;
```

```
A simple operation between
3
5
20:
160
```

Yeah, it's the same as before!
Notice how the semicolon (`;`) indicates the ending of a statement in code. The code above could all be written in a single line (and not in 6 lines) and it would output the same:[†]

```
std::cout << ... << (3+5)*20 << std::endl;
```

Remember every `std::cout` is always paired with a semicolon (`;`) which indicates the ending of its effects, like a *dot* indicates the ending of a sentence or paragraph.

[†]sorry, the single line is too long to show in here all at once.

010.

What happens if you try to compile and run this faulty code?

```
std::cout
  << "A simple operation between " << std::endl;
std::cout << 3 << std::endl;
std::cout << 5 << std::endl
std::cout << 20 << ": " << std::endl;
std::cout << (3+5)*20 << std::endl;
```

It fails to compile because there is a semicolon missing in the code!
The error shown by the compiler is actually[†] useful here, it is telling us that we forgot a ;!

```
:8:30: error: expected ';' after expression
  std::cout << 5 << std::endl
                               ^
                               ;
1 error generated.
```

[†]Why "actually"? Well, you will find that most of the time errors thrown by the compiler are hard to understand. It is often something that we programmers need to learn to do. We learn to understand the confusing error messages compilers give us.

011.

But what if we want to not input 5 or 20 twice?
What is the output of the code below?

```
int num_1 = 3;
int num_2 = 5;
int num_3 = 20;

std::cout
  << "A simple operation between "
  << num_1 << " "
  << num_2 << " "
  << num_3 << ": "
  << (num_1+num_2)*num_3
  << std::endl;
```

The output is:

```
A simple operation between 3 5 20: 160
```

012.

what is the output if you change the value 5 for 7?

The output is:

```
A simple operation between 3 7 20: 200
```

013.

`num_1` is a **variable** and it allows us to save integers on it, you can try changing it's value for any number between -2147483648 and 2147483647[†]
What if we put -12 in the variable `num_1`?

[†]This numbers are based on a program compiled for a 32bit computer, the values may vary between different computers.

The output is:

```
A simple operation between -12 5 20: -140
```

Interlude: Variables

Now, it's time to explain what are **variables** and what happens when we write `int name = 0`.

`int name = 0` is equivalent to:[1]

```
int name;
name = 0;
```

The first instruction **declares** a space for an `int` in memory (RAM memory).

So, let's study the architecture of computers, mainly RAM and CPU. ***Topic to study on class. Sorry guys, I should've known it was too long to explain written, if I don't explain it to you, please tell me. Now, let's continue with the explanation***

`int name;` is telling the compiler to reserve (*declare*) some space that nobody else should use. This space can have any value we want. Because this space in memory could have been used by anybody else in the past, its value is *nondefined*, meaning that it can be anything. Therefore we use the next line `name = 0;` to save a zero in the space declared.

BEWARE! `name = 0;` is NOT an equation!

I repeat, `name = 0;` is NOT an equation!, you are **assigning** a value to a variable, you could easily assign many different values to a variable, though just the last one will stay in memory thereafter:

```
int name;
name = 0;
name = 12;
```

The procedure of *declaring* and then *assigning* a value to a variable is so common that the designers of the language have made a shortcut:

```
int name = 0;
```

Now, let's go back to the code!

[1] only for the simplest values `int`, `double`, ..., but not for objects. Objects are out of the scope of this book, but it is important to know they exist.

014.
What is the output of:

```
int var1 = 6;
int var2 = 3;
int var3 = 10;

std::cout
  << (var1+var2) * var3 - var2
  << std::endl;
```

The output is:

```
87
```

015.
What is the output of:

```
int var1 = 6;
int var2 = 3;

std::cout
  << (var1+var2) * var3 - var2
  << std::endl;

int var3 = 10;
```

Yeah, it doesn't compile, you are trying to use a variable *before* declaring it (asking for a space in memory to use it). The compiler gives you the answer:

```
:9:20: error: use of undeclared identifier 'var3'
  << (var1+var2) * var3 - var2
                   ^
1 error generated.
```

ORDER (of sentences) is key! It is not the same to say "Peter eats spaguetti, then Peter clean his teeth" than "Peter clean his teeth, then Peter eats spaguetti".
A program runs sequentially from the first line of code to the last

016.
What is the output of:

```
int var1 = 6;
int var2 = 3;
int var3 = 10;

var2 = var1*3;

std::cout
  << (var1+var2) * var3 - var2
  << std::endl;
```

The output is:

```
222
```

We can in fact assign to the variable (at the left of `=`) any `int` value result of any computation. In this case, the computation `var1*3` is assigned to `var2`

017.
What is the output of:

```
int var1 = 6;
std::cout << var1 << " ";
var1 = 20;
std::cout << var1 << " ";
var1 = -5;
std::cout << var1 << std::endl;
```

The output is:

```
6 20 -5
```

018.

What is the output of:

```
int var1 = 6;
std::cout << var1 << " ";
int var1 = 20;
std::cout << var1 << " ";
var1 = -5;
std::cout << var1 << std::endl;
```

It doesn't compile because you cannot ask for more space in memory (declare) with the same name variable, you gotta use a different name.†

†This is truth, you cannot use the same name twice in the same block of code, usually called *scope*. With scopes we can shadow a variable name, we will do it later.

Task for home: I have two variables in memory, what should I do if I want to swap their contents?, i.e., I want the content of the first variable (say `var1`) inside the second variable (say `var2`), and I want the value of `var2` to be inside `var1`.

Notice that the code below does NOT work, it doesn't swap the content of the variables.

```
int var1 = 4;
int var2 = 20;

var2 = var1;
var1 = var2;
```

What should I do?

019.

What is the output of:†

```
#include <cmath>
...
int var1 = 6;
int var2 = 3;
int var3 = var1 * pow(var2, 3);

std::cout << "var1 * pow(var2, 3) => "
          << var3 << std::endl;
```

†Notice the dots (...) in the code. This dots are just for notation, they aren't meant to be written in the file to be compiled. The dots represent a division between two different parts of code. You can see the code for this example in the source code of the book https://github.com/helq/the-little-cppler-book (take a look into the folder that says **code**).

The output is:

```
var1 * pow(var2, 3) => 162
```

Remember that
`int var3 = var1 * pow(var2, 3);`
is equivalent to

```
int var3;
var3 = var1 * pow(var2, 3);
```

020.

Till now we've seen just two operations (`*` and `+`), but there are plenty more:

```
int var1 = (2 + 18 - 6 * 2) * 5;
int var2 = var1 / 3;
int var3 = var1 % 3;

std::cout << "var1 => " << var1 << std::endl;
std::cout << "var2 => " << var2 << std::endl;
std::cout << "var3 => " << var3 << std::endl;
```

The output is:

```
var1 => 40
var2 => 13
var3 => 1
```

You probably know all operations here, but maybe not `%`. It is the *modulus* operation, or residue operation, and it symbolises the result of the residue of dividing integer numbers. For example, the result of dividing 50 by 3 can be written as:

$$50 = 3 \times 16 + 2$$

Where 50 is the dividend, 3 is the divisor, 16 the (integer) result, and 2 the modulus/remainder.
If an integer is divisible by another then the modulus of operating them must be 0, e.g., $21 = 7 \times 3 + 0$.

021.

What is the output of:

```
std::cout << 8 % 3 << " "
          << 8 % 2 << " "
          << 7 % 2 << " "
          << 17 % 1 << " "
          << 17 % 7 << " "
          << std::endl;
```

The output is:

```
2 0 1 0 3
```

Task for home: Write a program that outputs the steps to make a healthy breakfast.

022.

```
std::cout << 20 - 6 * 2 << " "
          << (20 - 6) * 2 << " "
          << 20 - (6 * 2) << " "
          << std::endl;
```

```
8 28 8
```

Notice how `20 - 6 * 2` does reduce[†] to 8 and not to 28! (i.e., $20-6\times 2 = 20-(6\times 2) \neq (20-6)\times 2$). Each operator has a specific precedence that indicates if it must be applied before another operator, `*` for example has a higher precedence than `+`.

[†] or compute

What if's

023.

What is the output of:[†]

```
double acction   = 9.8; // m/s^2 acceleration
double mass      = 3;   // kg    mass
double initial_v = 10;  // m/s   ini. velocity
double time      = 2.3; // s     time passed

double final_v = initial_v + acction * time;

std::cout << "Velocity after " << time
          << "s is: " << final_v << "m/s"
          << std::endl;

double momentum = mass * final_v;

std::cout << "Momentum after " << time
          << "s is: " << momentum << "kg*m/s"
          << std::endl;
```

[†] `double` allows us to declare "real" numbers (they are actually rational). And we can operate with them as we did with `int`'s

The output is:

```
Velocity after 2.3s is: 32.54m/s
Momentum after 2.3s is: 97.62kg*m/s
```

We are using here the equation $v = u + a * t$ to determine the final velocity of an object (in a line) after $2.3s$. The object starts with a velocity $2.3m/s$, has a constant acceleration of $9.8m/s^2$[†], and we know the weight of the object, so we can calculate too its momentum.

[†] free fall ;)

024.

What is the output of:

```
int var1 = 8;
if (var1 < 10) {
  std::cout << "var1 smaller than 10"
            << std::endl;
} else {
  std::cout
    << "var1 greater than or equal to 10"
    << std::endl;
}
```

The output is:

```
var1 smaller than 10
```

025.

What is the output of the code above if we change `var1`'s assignment from `8` to `19`?

The output is:

```
var1 greater than or equal to 10
```

Precisely, the second line runs but not the first.
The structure:

```
if (statement) {
  // true branch
} else {
  // false branch
}
```

runs the `true branch` if the statement `statement` evaluates to `true` otherwise it runs the `false branch`.

026.
What about the output of

```
int var1 = 10;
if (var1 == 7 + 3) {
  std::cout << "var1 is equal to 7 + 3"
            << std::endl;
} else {
  std::cout << "var1 does not equal 10"
    << std::endl;
}
```

Well, that's easy:

```
var1 is equal to 7 + 3
```

Note, `==` is the equality operation[†], and it compares any two statements as `var1` or `3+2*12` for equality.
As you expect it, there are many operations to compare between different statements, these are `==`, `<=`, `>=`, `<`, `>`, and `!=`.

[†]take care, don't confuse it with the assignment operator `=`!

027.
What is the output of:

```
int var1 = 2*2+2;
if (var1 != 7 + 3) {
  std::cout << "2*2+2 != 7+3" << std::endl;
} else {
  std::cout << "2*2+2 == 7+3" << std::endl;
}
```

The output is:

```
2*2+2 != 7+3
```

Yeah, `!=` is the unequality operator.

028.
What is the output of:

```
int var1 = 2*2+20;
if (var1 <= 7 + 3) {
  std::cout << "i never run :(" << std::endl;
} else {
  if (var1 > 23) {
    std::cout << "hey!!" << std::endl;
  } else {
    // nothing in this branch
  }
}
```

The output is:

```
hey!!
```

Usually, if we only care about the true branch of an `if` statement, then we simply ignore it. The code at left is equivalent then to:

```
int var1 = 2*2+20;
if (var1 <= 7 + 3) {
  std::cout << "i never run :(" << std::endl;
} else {
  if (var1 > 23) {
    std::cout << "hey!!" << std::endl;
  }
}
```

Task for home: I give you three numbers, each in a distinct variable, and what I want is you to print them in ascendent order, i.e., if I give you `3`, `20` and `-2`, you should print in terminal: `-2 3 20`. The code could start by:

```
int a = /*a number here*/;
int b = /*a number here*/;
int c = /*a number here*/;

// Your code goes here ;)
```

029.

It's possible to put more than one statement inside the true branch of the `if` statement. For example:

```
int var1 = 2*2+20;
if (var1 >= 7 + 3) {
  std::cout
    << "you are our visitor number 3889"
    << std::endl;
  int magicnumber = 999999;
  std::cout
    << "and your our winner! please "
    << "deposit " << magicnumber
    << " into our account and you will "
    << " receive 20x what you deposited "
    << std::endl;
}
std::cout
  << "Welcome to e-safe-comerce"
  << std::endl;
```

The output as you expected.

```
you are our visitor number 3889
and your our winner! please deposit 999999 into our a
Welcome to e-safe-comerce
```

030.

```
int magicnumber = -2;
if (2!=3) {
  magicnumber = 0;
} else {
  magicnumber = 42;
}
std::cout << "Every integer is a divisor of "
          << magicnumber << std::endl;
```

Of course, 0 can be divided by any number, because it can be written as $0 = k * n$ where $k = 0$ and n is an arbitrary number.

```
Every integer is a divisor of 0
```

Task for home: Write a program that outputs the steps to follow to make a healthy breakfast if there are multiple options for breakfast. You should be able to select your option for breakfast by setting a variable to one of many values (as many as you like).

For example:

```
int option = 3;

std::cout << "Good morning, Sir" << std::endl;
std::cout << "For today's breakfast you will need:" << std::endl;

if (option == 0) {
// Your code continues from here...
```

031.

What is the output of:

```
if (2!=3) {
  int magicnumber = 0;
} else {
  int magicnumber = 42;
}
std::cout << "Every integer is a divisor of "
          << magicnumber << std::endl;
```

It doesn't compile! You wanna know why? Well, keep guessing with the following exercises.

032.
Does this compile? If yes, then what is its output?

```
int magicnumber = -2;
if (2!=3) {
  int magicnumber = 0;
} else {
  int magicnumber = 42;
}
std::cout << "Every integer is a divisor of "
          << magicnumber << std::endl;
```

It does compile, and its output is:

```
Every integer is a divisor of -2
```

But wait, what? -2 is not divisible by any arbitrary integer, only by one and itself!

033.
Maybe, 2 and 3 are really the same thing.

```
int magicnumber = -2;
if (2==3) {
  int magicnumber = 0;
} else {
  int magicnumber = 42;
}
std::cout << "Every integer is a divisor of "
          << magicnumber << std::endl;
```

The output is:

```
Every integer is a divisor of -2
```

Uff, fortunatelly they are not the same, but why? What have we changed from before? What is the difference with the code that gives us a zero?

034.
Ok, let's stop with the silliness and try with an example that actually gives us some clue of the situation.

```
int num = 20;
std::cout << num << " ";
{
  int num = 42;
  std::cout << num << " ";
  num = 3;
  std::cout << num << " ";
}
std::cout << num << " ";
num = 0;
std::cout << num << std::endl;
```

Well, the code inside the brackets ({}) acts as if it was being run alone without the intervention of the code of the outside.

```
20 42 3 20 0
```

It is effectively as if this was the code being run:

```
int num = 20;
std::cout << num << " ";
{
  int var = 42;
  std::cout << var << " ";
  var = 3;
  std::cout << var << " ";
}
std::cout << num << " ";
num = 0;
std::cout << num << std::endl;
```

035.

Any time we enclose code between brackets (`{}`) we are defining a new **scope**. Any variable we declare inside a scope lives only in that scope, the variable *dies* once the scope is closed, this is the reason why this code

```
{
  int num = 42;
  std::cout << num << " ";
}
std::cout << num << std::endl;
```

doesn't compile. There is no `num` variable in the bigger scope when it wants to show it.
A rule about scopes is that they can access to variables from the outside, scopes that enclose them. For example this code, does indeed compiles:

```
int num = 42;
std::cout << num << " ";
{
  num = 1;
  std::cout << num << " ";
}
std::cout << num << std::endl;
```

What is its output?

You guessed it right! Given that `num` is a variable outside the inner scope, the inner scope can read and modify the variable content.

```
42 1 1
```

036.

What is the output of:

```
int num = 42;
std::cout << num << std::endl;
{
  int num = 1;
  std::cout << num << std::endl;
}
std::cout << num << std::endl;
{
  num = 1;
  std::cout << num << std::endl;
}
std::cout << num << std::endl;
```

```
42
1
42
1
1
```

Right, in the first inner scope, we declare a new space and shadow the access to the outside variable `num`, and in the other inner scope, we use the variable accessible from the outside scope.

037.

What is the output of:[†]

```
int a = 20;
int b = 21;
if (b-a > 0) {
  float num = -12.2;
  std::cout << num * 3 << std::endl;
} else {
  double num = -12.2;
  std::cout << num * -3 << std::endl;
}
```

[†]yet again another type of data. `float`s are like `double`s but they represent numbers with less precision. Well I haven't talked what precision is, forgive me, for the time being just assume that using `double` in your code is better than using `float`.

The output is:

```
-36.6
```

Task for home: Given an integer, print if it is divisible by 3 or not. *Tip*: the remainder of an integer (divisible by 3) divided by 3 is zero, e.g., $33 = 3 * 11 + 0$, thus the remainder of the division 33/3 is zero.

038.
What does this output:

```
int n = 15;
bool either = n<3;
if (either) {
  std::cout << ":P" << std::endl;
} else {
  std::cout << ":(" << std::endl;
}
```

The output is:

```
:(
```

with bool† we tell the compiler that it should interprete `either` as a boolean (either `true` or `false`).
This means that you can in fact store integer values inside a bool (i.e., `bool bad = 23;`) but it's consider bad practice and may result in undefined behaivor.

†True and False are the only two possible values that a statement can take in traditional logic, and so it does for us, there are only two options for bool. But modern computers are build on blocks of 32 or 64 bits and operations, therefore, with values of 32 and 64 bits are cheap to perform. Operations on single bits are not simple when you manipulate multiple bits, it is slightly more expensive to use a single bit to represent truth or false. Compilers usually use a block of memory (32 or 64 bits) to represent a boolean, the convention is for zero (all 32-64 bits in zero) to be false and anything else (eg, all bits in zero, or only one bit in zero, etc) to be true.

039.
Let's take a look at another type of variable, char:†

```
char apples = 23;
std::cout << "A small integer: " << (int)apples
          << std::endl;
```

†you can ignore the weird `(int)` thing for now, it is called **cast** if you are curious, we will get to them later on.

The output isn't surprising:

```
A small integer: 23
```

`char` may not seem different to `int`, but it is. `int`, depending on the system, has a size of 32 or 64, but `char` has always the same size 8 bits. With 8 bits we can represent 2^8 different states, that is 256 different numbers.

040.
What is the output of:

```
int i = 2;
std::cout << i << " ";
i = i + 1;
std::cout << i << " ";
i = i + 1;
std::cout << i << " ";
i = i * 3;
std::cout << i << std::endl;
```

The output is:

```
2 3 4 12
```

Task for home: The sum of a triangle's internal angles (in an euclidian space) is always equal to 180. Write a program, that given the three internal angles of a triangle, tells the user if the triangle is really a triangle or not (ie, it can be constructed):

Example:

```
int angle1 = /*a number here*/;
int angle2 = /*a number here*/;
int angle3 = /*a number here*/;

// Your code goes here
```

Note: All the angles in a triangle are always positive, never zero, but the user could input here negative integers. Check positiveness of numbers!

041.

Every compiler may define the size of `int`, `char`, `double`, ..., differently depending on the architecture. If you want to know how many bytes[†] are assigned to any variable type, you can use `sizeof`. An example of use:

```
std::cout << "A char   is " << sizeof(char)
          << " bytes" << std::endl;
std::cout << "A int    is " << sizeof(int)
          << " bytes" << std::endl;
std::cout << "A double is " << sizeof(double)
          << " bytes" << std::endl;
std::cout << "A float  is " << sizeof(float)
          << " bytes" << std::endl;
std::cout << "A bool   is " << sizeof(bool)
          << " bytes" << std::endl;
```

(Hint: if a byte is 8 bits, a `char` is 8 bits, how many bytes are a `char`?)

[†] one byte is 8 bits

This may look different in your computer, but mine runs on 64 bits, therefore `double` has 64 bits and `int` half of that.

```
A char   is 1 bytes
A int    is 4 bytes
A double is 8 bytes
A float  is 4 bytes
A bool   is 1 bytes
```

042.

What is the output of:

```
char a = 100;
char b = 20;
char c = a + b;
std::cout << "a : " << (int)a << std::endl
          << "b : " << (int)b << std::endl
          << "a+b : " << (int)c << std::endl;
```

Not surprising, that is the output.

```
a : 100
b : 20
a+b : 120
```

043.

What is the output of:

```
char a = 100;
char b = 30;
char c = a + b;
std::cout << "a : " << (int)a << std::endl
          << "b : " << (int)b << std::endl
          << "a+b : " << (int)c << std::endl;
```

The output is:[†]

```
a : 100
b : 30
a+b : -126
```

well, that's surprising! What the heck happened?
The answer lies in the 8 bit part that I was talking about. `char` can only hold 256 different numbers, but usually we use one of those bits to indicate the sign[‡], therefore we have left only 7 bits for the number. 2^7 is 128, so we can store 128 positive numbers and 128 negative numbers. If we did it naïvely we could represent the numbers from 0 to 127 with 7 bits plus one bit for the sign, but that's rarely used, we would be representing 0 in two ways +0 and -0, the answer is to count from 0 to 127 and from -128 to -1, i.e., we can lay down all the representable numbers by 8 bits in the following way: -128, -127, -126, ..., -2, -1, 0, 1, 2, ..., 125, 126, 127.
When you pass over the limit of what 7 bits[§] can store you need to go somewhere, and by convention that is going back to the first number, i.e., adding numbers one by one will lead you to the begining no matter where you start: $1+1=2$, $2+1=3$, ..., $125+1=126$, $127+1=-128$, $-128+1=-127$, ..., $-1+1=0$, $0+1=1$.

[†]This output may change depending on the compiler you are using, it could have happened that you don't see anything wrong at all. In that case, try changing `100` for `220`.

[‡]For more details look at "two's complement binary representation". Wikipedia's article https://en.wikipedia.org/wiki/Signed_number_representations#Two%27s_complement

[§]this is called overflow

044.

What is the output of:

```
char i = 126;
std::cout << (int)i << " ";
i = i + 1;
std::cout << (int)i << " ";
i = i + 1;
std::cout << (int)i << " ";
i = i + 1;
std::cout << (int)i << std::endl;
```

The output, as you may have easily guessed is:

```
126 127 -128 -127
```

Task for home: Given a number between 1 and 12 output the number of days in a month (where each month is represented by a number, 1 for January and 12 for December).

What if and if and if and …

045.

What is the output of:

```
int i = 0;
if (i<3) {
  std::cout << "There is no else statement";
  i = i * 2;
}
std::cout << std::endl;
```

The output is:

```
There is no else statement
```

Notice how we ignored the `else` statement, well that's ok, we can just do stuff for when something is true otherwise we don't do anything.

046.

What is the output of:

```
int i = 0;
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
std::cout << std::endl;
```

The output is:

```
0 1
```

047.

What is the output of:

```
int i = 0;
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
if (i<3) {
  std::cout << i << " ";
  i = i + 1;
}
std::cout << std::endl;
```

The output is:

```
0 1 2
```

048.

What is the output of the above if we change all appearances of `i<3` for `i<2` or `i<4`.

The outputs are:

```
0 1
```

for `i<2` and

```
0 1 2 3
```

for `i<4`.

049.
What is the output if we change `i<3` for `i<5`?

Well there are only 4 `if`s, the limit is 4 numbers printed on the screen:

```
0 1 2 3
```

050.
What should we do to print five numbers?

Well, there are many options. We could add a `std::cout << i << std::endl;` line after the last `if` statement, or we could copy an `if` (and this is what I wanted you to answer)

051.
What do you think this outputs?

```
int i = 0;
while (i<4) {
  std::cout << i << " ";
  i = i + 1;
}
std::cout << std::endl;
```

The output is:

```
0 1 2 3
```

Nice, no repeated code!

052.
What happens if we change `i<4` above for `i<5`?

Well, now we can see five numbers:

```
0 1 2 3 4
```

053.
What is the output of (notice the initialization):

```
int i = 1;
while (i<4) {
  std::cout << i << " ";
  i = i + 1;
}
std::cout << std::endl;
```

The output is:

```
1 2 3
```

054.
What is the output of:

```
int i = -2;
while (i<4) {
  std::cout << i*2 << std::endl;
  i = i + 1;
}
```

The output is:

```
-4
-2
0
2
4
6
```

055.
Something a little more elaborated:

```
int i = -2;
while (i<3) {
  std::cout << i << " squared is: " << i*i
            << std::endl;
  i = i + 1;
}
```

The output is:

```
-2 squared is: 4
-1 squared is: 1
0 squared is: 0
1 squared is: 1
2 squared is: 4
```

But I'm getting bored with all those `i = i + 1`, fortunatelly that operation is so common in day to day work that it can be written shorter: `i++`.

056.
What is the output of:

```
int i = -6;
while (i<=-2) {
  std::cout << i << " squared is: " << i*i
            << std::endl;
  i++;
}
```

```
-6 squared is: 36
-5 squared is: 25
-4 squared is: 16
-3 squared is: 9
-2 squared is: 4
```

Task for home: Print the sum of all cubed odd numbers from 21 to 53, i.e.,

$$\sum_{i=21}^{i\leq 53} i^3$$

057.
What is happenning here?

```
int a = 13;
int b = 6;
int c = 8;
while (b<a && b<c) {
  std::cout << "Both conditions met"
            << std::endl;
  b++;
}
```

```
Both conditions met
Both conditions met
```

058.
And here?

```
int a = 13;
int b = 6;
int c = 8;
while (b<a || b<c) {
  std::cout << "At least one condition met"
            << std::endl;
  b++;
}
```

```
At least one condition met
At least one condition met
At least one condition met
At least one condition met
At least one condition met
At least one condition met
At least one condition met
```

Well, `&&` is called the **and** operator, and `||` is called the **or** operator, they operate on `bool`s only.[†]

[†]Well, they can actually operate in any number, any number not zero is treated as `true` otherwise `false`. This means that `a && b` is a valid statement above, but it is a weird one, its result is `true`. Such expressions, even though totally legal, are discouraged.

Task for home: Write some code to print the even numbers from 20 to 0. The output should be:

```
20 18 16 14 12 10 8 6 4 2 0
```

059.

What is the output of:

```
bool a = 5 > 2;
bool b = true;
if (a==b) {
  std::cout << "5 > 2 is equal to true"
            << std::endl;
} else {
  std::cout << "bananas!" << std::endl;
}
```

```
5 > 2 is equal to true
```

As usual.

060.

What is the output of:

```
if ((5>2)==true) {
  std::cout << "5 > 2 is equal to true"
            << std::endl;
} else {
  std::cout << "bananas!" << std::endl;
}
```

It's basically the same thing from above, isn't it.

```
5 > 2 is equal to true
```

061.

We have printed only `int`s and `double`s to date, what would be the result of printing `bool`s?

```
std::cout << "true gets printed as " << true
          << std::endl;
std::cout << "But false gets printed as "
          << false << std::endl;
```

```
true gets printed as 1
But false gets printed as 0
```

Yep, `true` is represented as `1` always![†]
Moral of the fable: don't use boolean operations on variables that are not boolean! and don't use non-boolean operations on boolean variables!

[†]this is standard behaivor, because comparing `true` with `1` will result in `true` again, but it won't if you compare it to `3` even though `3` is treated as true if you use it in an `if` expression.

062.

Let's take a look at the *truth table* for `&&`:[†]

```
std::cout
  << "1 && 1 == " << (true  && true)
  << std::endl
  << "1 && 0 == " << (true  && false)
  << std::endl
  << "0 && 1 == " << (false && true)
  << std::endl
  << "0 && 0 == " << (false && false)
  << std::endl;
```

```
1 && 1 == 1
1 && 0 == 0
0 && 1 == 0
0 && 0 == 0
```

[†]the parenthesis around the boolean expressions are necessary, otherwise they would get `<<` confused.

063.
And the *truth table* for `!` (not/negation):

```
std::cout
  << "!1 == " << !true  << std::endl
  << "!0 == " << !false << std::endl;
```

```
!1 == 0
!0 == 1
```

All tables as you know them from logic, old stuff (actually, not that old, this tables were first used as tables in the 20th century, but they are so intuitive that one may think they're an older invention [†]).

[†]for some more history see wikipedia's article: *Truth table*.

064.
Now, what should be changed in the code above (for `&&`) to make the *truth table* for `||`. The output of the table should be:

```
1 || 1 == 1
1 || 0 == 1
0 || 1 == 1
0 || 0 == 0
```

And here is the code:

```
std::cout
  << "1 || 1 == " << (true  || true)
  << std::endl
  << "1 || 0 == " << (true  || false)
  << std::endl
  << "0 || 1 == " << (false || true)
  << std::endl
  << "0 || 0 == " << (false || false)
  << std::endl;
```

065.
There is an special operation on boolean variables[†] called $\oplus$ (**xor**), and can be written in C++ with `^`.
This is its truth table[‡]:

```
1 ^ 1 == 0
1 ^ 0 == 1
0 ^ 1 == 1
0 ^ 0 == 0
```

Write down the truth table for $(p \oplus q) \oplus q$.

[†]Actually, the operation is on numbers, but we can ignore that for the time being.

[‡]You can find the source code that prints this table on: https://github.com/helq/the-little-cppler-book/tree/master/code

p	q	$p \oplus q$	$(p \oplus q) \oplus q$
1	1	0	1
1	0	1	1
0	1	1	0
0	0	0	0

Notice how $p = (p \oplus q) \oplus q$!!
This characteristic is very important in cryptography, because one can use some secret key as q (one bit in this case), and we cypher a bit of information p by applying $p \oplus q$. We can get the result back if we apply *xor* again over the result we just got, and you know what? if you don't tell anybody the key, nobody could guess what the value of p originally was!

Task for home: Swaping two variables content is quite simple if one uses an auxiliary variable. But, it is indeed possible to swap the content of two variables without using any more memory, without an auxiliary variable! Your goal is to find the trick.

But, how could that be possible?

Tip: To make it simpler, try to swap the value of boolean variables using only assignations and the *xor* operation. Use the following code as a template, and remember the property of *xor*: $p = (p \oplus q) \oplus q$!

```
bool a = 1;
bool b = 0;

// .. your code goes here, use only `=` (assignment) and `^` (xor operation)
```

Task for home: Given a number find the sum of its digits, e.g., the sum of the digits of 850347 is $8+5+0+3+4+7 = 27$.

Syntactic sugar with `for`

066.

What should be the value of `c` in the code below for the code to output `0 1 2 3 4`?

```
int b = 10;
int c = ???;
int i = 0;

while ((i<b) && (i<c)) {
  std::cout << i << " ";
  i++;
}
std::cout << std::endl;
```

Precisely, it should be 5. Let's take a look at the code running line by line:

1. We declare and initialize the variables `b`, `c` and `i` with the values `10`, `5` and `0` respectively.
2. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, and both are truth ($0 < 10$ and $0 < 5$).
3. We print in the screen the value of `i` (0)
4. We increment the value of the variable `i`
5. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, and both are truth ($1 < 10$ and $1 < 5$).
6. We print in the screen the value of `i` (1)
7. We increment the value of the variable `i`
8. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, and both are truth ($2 < 10$ and $2 < 5$).
9. We print in the screen the value of `i` (2)
10. We increment the value of the variable `i`
11. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, and both are truth ($4 < 10$ and $3 < 5$).
12. We print in the screen the value of `i` (3)
13. We increment the value of the variable `i`
14. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, and both are truth ($4 < 10$ and $4 < 5$).
15. We print in the screen the value of `i` (4)
16. We increment the value of the variable `i`
17. We ask, is `i` content smaller than `b`?, and is `i` content smaller than `c`?, one is false ($5 < 10$ and $5 < 5$).
18. We get out of the while loop and go to the next line, thus we print at last a newline.

067.

Describe line by line what the following code does (and its output):

```
int b = 10;
int c = 5;
int i = 0;

while ((i<b) && (2*i<c)) {
  std::cout << i << " ";
  i++;
}
std::cout << std::endl;
```

The steps are:

1. We declare and initialize three `int` variables (`b`, `c` and `i` with `10`, `7` and `0` respectively).
2. We ask, is `i` content smaller than `b`?, and is `2*i` content smaller than `c`?, and both are truth ($0 < 10$ and $0 < 5$).
3. We print in the screen the value of `i` (0)
4. We increment the value of the variable `i`
5. We ask, is `i` content smaller than `b`?, and is `2*i` content smaller than `c`?, and both are truth ($1 < 10$ and $2 < 5$).
6. We print in the screen the value of `i` (1)
7. We increment the value of the variable `i`
8. We ask, is `i` content smaller than `b`?, and is `2*i` content smaller than `c`?, and both are truth ($2 < 10$ and $4 < 5$).
9. We print in the screen the value of `i` (2)
10. We increment the value of the variable `i`
11. We ask, is `i` content smaller than `b`?, and is `2*i` content smaller than `c`?, and one is false ($3 < 10$ and $6 < 5$).
12. We get out of the while loop and go to the next line, thus we print at last a newline.

The complete output is:

```
0 1 2
```

068.
What is the output of:

```
int imzero = 0;

while (imzero < 4) {
  std::cout << imzero << " ";
}
std::cout << std::endl;
```

The output is...:

```
0 0 0 0 0 0 0 0 0 0 0 0 0 ...
```

an unending sequence of zeros, so, what is wrong?
We forgot to add the statement `imzero++`! Without it, well, the variable `imzero` never changes its state/value, and we ask *ad infinitum* if zero is smaller than four. †

†I actually wrote faulty code like that several times in the writing of this book XD

069.
What should be the value of `c` in the code below for the code to output `0 2 4 6 8`?

```
int b = 7;
int c = ???;
int i = 0;

while ((i<b) || (i+1<c)) {
  std::cout << i << " ";
  i++;
  i++;
}
std::cout << std::endl;
```

Yep, either `c` should be either `10` or `11`.

070.
What is the output of:

```
std::cout << "3%2 == " << (3%2) << std::endl;
std::cout << "4%2 == " << (4%2) << std::endl;
std::cout << "0%2 == " << (0%2) << std::endl;
std::cout << "7%2 == " << (7%2) << std::endl;
std::cout << "2%2 == " << (2%2) << std::endl;
```

The output is:

```
3%2 == 1
4%2 == 0
0%2 == 0
7%2 == 1
2%2 == 0
```

Notice how the odd numbers all return `1`, and all even numbers return `0`. Why?

071.
What is the output of:

```
int a = 0;
while (a<=10) {
  if (a%2 == 0) {
    std::cout << a << " ";
  }
  a++;
}
std::cout << std::endl;
```

The output is:

```
0 2 4 6 8 10
```

Cool, we can skip things while we count up (or down), like –in this case– odd numbers.

072.
What is the output of:

```
int a = 0;
int b = 0;
while (a<=10) {
  if (a%2 == 0) {
    b = b + a;
  }
  std::cout << b << " ";
  a++;
}
std::cout << std::endl;
```

The output is:

```
0 0 2 2 6 6 12 12 20 20 30
```

073.
What should we change in the code above to print each number one time only?, i.e., what lines are needed to change or be moved (from the code above) to output this:

```
0 2 6 12 20 30
```

?

Yep, we move the printing statement (line) to inside the true branch of the `if` statement:

```
int a = 0;
int b = 0;
while (a<=10) {
  if (a%2 == 0) {
    b = b + a;
    std::cout << b << " ";
  }
  a++;
}
std::cout << std::endl;
```

Task for home: Given a number, print it in reverse. For example, given the kick start code:

```
int mynum = 12367;

// Your code continues here
```

It should print in screen:

```
76321
```

074.
Say now, we want something simple, like knowing what is $1+2+3+\cdots+10$ equal to. There are many ways to do this, for example:[†]

```
int sum = 1+2+3+4+5+6+7+8+9+10;
std::cout << "1+2+3+...+10 == " << sum
          << std::endl;
```

But, could we code it using a `while` loop?

[†]we may calculate it using the equation $\frac{10*11}{2}$, thus we could write `int sum = (10*11)/2`, but we are going to ignore that and try to use what we've been discussing to date to solve this problem.

Yep, there is, for example:

```
int sum = 0;
int i = 1;
while (i<=10) {
  sum = sum + i;
  i++;
}
std::cout << "1+2+3+...+10 == " << sum
          << std::endl;
```

Notice how this way of writing $1+2+\cdots+10$ allows us to sum up to any number not only 10! This little piece of code is very similar at what we do in maths when we write $\sum_{i=1}^{i=10} i$ instead of $1+2+\cdots+10$.

075.
And what if we wanted to calculate

$$\sum_{i=1}^{i=10} i^2 = 1^2 + 2^2 + \cdots + 10^2$$

What should we change from the code above?

Well, that's right, `sum = sum + i` changes for `sum = sum + i*i`.
The full code and its output:

```
int sum = 0;
int i = 1;
while (i<=10) {
  sum = sum + i*i;
  i++;
}
std::cout << "1^2 + 2^2 + 3^2 + ... + 10^2 == "
          << sum << std::endl;
```

```
1^2 + 2^2 + 3^2 + ... + 10^2 == 385
```

Task for home:

All the multiples of 3 or 5 smaller than 20 are 0, 3, 5, 6, 9, 10, 12, 15 and 18, and their sum is 78.[2]

What is the sum of all natural numbers smaller than 1000 which are multiples of 3 or 5? Write some code using a `for` or `while` loop to solve the problem.

076.
Writing code is often annoying because you can get errors that you didn't expect. The following code supposedly should print on the screen the numbers from 10 to 1, top-bottom. But it doesn't! What is wrong with the code?

```
int i = 10;
while (i<=1) {
  std::cout << i << " ";
  i--;
}
std::cout << std::endl;
```

Precisely, the condition `i<=1` is never met, all we need to do is to change "smaller than" for "larger than", i.e., change the condition for `i>=1`.

077.
Robert has written a weird piece of code, it has two loops! What is it doing? What is the output of the code below?

```
int i = 0;
while (i<=10) {
  int j = 0;
  while (j<=20) {
    std::cout << "*";
    j++;
  }
  std::cout << std::endl;
  i++;
}
```

Nice, it runs 10 times the same code, a code that prints twenty asterisks.

```
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
```

Task for home: Print the numbers from 1 to 9 in a diagonal, something like:

```
1
  2
```

[2] exercise extracted from project euler (https://projecteuler.net exercise 1)

```
   3
     4
       5
         6
           7
             8
               9
```

It is is easy to write something like (but it's boring):

```
std::cout << "1"      << std::endl;
std::cout << "  2"    << std::endl;
std::cout << "    3"  << std::endl;
```

The challenge is to use a **while** loop. Write the program using a **while** loop to print the diagonal.

078.

But I'm getting bored of writing so much code with the **while** loop. In fact, the pattern we have been using with while is so common that there is a shorter version of it, the **for**. Let's see if you can guess what the following code does:

```
for (int i=0; i<=10; i++) {
  for (int j=0; j<=20; j++) {
    std::cout << "*";
  }
  std::cout << std::endl;
}
```

```
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
```

It does the same as the previous code![†]

A **for** loop is syntactic sugar[‡] for a **while** loop, it makes convenienent and more explicit that we want to iterate over a value.

A **for** loop has four different parts:

```
for (A; B; C) {
  D;
}
```

and it's equivalent to the following **while** loop.

```
{
  A;
  while (B) {
    D;
    C;
  }
}
```

[†]It does the same, but the two pieces of code are semantically different if we put them in a different context, like inside a block of code which has already a variable named `i` in it.

[‡]This word will appear many times here onward, and it means, roughly: a way to write something in a simpler way. Similar to how we write LOL or WTF, and not "that made me laught so hard, man" or "seriously, that's weird as heck!".

Task for home: Write the multiplication table for any number I give you, for example, with some code like:

```
int num = 3;

for (int i=0; i<=10; i++) {
  // your code goes here
}
```

I want you to output:

```
3 * 0 == 0
3 * 1 == 3
3 * 2 == 6
3 * 3 == 9
3 * 4 == 12
3 * 5 == 15
3 * 6 == 18
3 * 7 == 21
3 * 8 == 24
3 * 9 == 27
3 * 10 == 30
```

Task for home: Write a program that prints a rectangle (made of characters `'#'`) of height `m` and width `n`, for example, for the values `m = 5` and `n = 12`, the program should print:

```
############
#          #
#          #
#          #
############
```

Task for home: Write a program that prints a triangle of base `n`, for example, the triangles created by the program for sizes of `n` equal to `5` and `9` are:

```
  #
 # #
#####
```

```
    #
   # #
  #   #
 #     #
#########
```

079.

Write the following **while** loop as **for** loop.

```
int top = 10;
int i = 0;
while (i<top) {
  std::cout << i * top + 1 << " ";
  i++;
}
std::cout << std::endl;
```

Feels good, isn't it? Simple exercises for a while.

```
int top = 10;
for (int i=0; i<top; i++) {
  std::cout << i * top + 1 << " ";
}
std::cout << std::endl;
```

080.

What is the output of the following code:

```
for (int i=0; i<18; i++) {
  if ((i%3!=0) && (i%7!=0)) {
    std::cout << i << " ";
  } else {
    std::cout << ". ";
  }
}
std::cout << std::endl;
```

The output is:

```
. 1 2 . 4 5 . . 8 . 10 11 . 13 . . 16 17
```

Numbers multiple of 3 or 7 are printed as dots (`.`), while all others are printed as themselves.

Task for home:

The fibonacci sequence is a sequence defined by:

$$fib(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = 1 \\ fib(x-1) + fib(x-2) & \text{if } x > 1 \end{cases}$$

Which gives us the sequence $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$. Notice how every number in the sequence (except the first two) is equal to the sum of both its predecesors.

Write a program that prints on the screen the first 20 fibonacci numbers using loops (either **while** or **for** loops).

Breaking the code into pieces

081.
What does the following code does?

```
double acction   = 9.8; // m/s^2 acceleration
double mass      = 3;   // kg    mass

double initial_v = 10;  // m/s   ini. velocity
double time      = 2.3; // s     time passed

double final_v = initial_v + acction * time;
double momentum = mass * final_v;
std::cout << "Momentum after " << time
          << "s is: " << momentum << "kg*m/s"
          << std::endl;

time = 4.2;
final_v = initial_v + acction * time;
momentum = mass * final_v;
std::cout << "Momentum after " << time
          << "s is: " << momentum << "kg*m/s"
          << std::endl;

time = 9;
final_v = initial_v + acction * time;
momentum = mass * final_v;
std::cout << "Momentum after " << time
          << "s is: " << momentum << "kg*m/s"
          << std::endl;
```

It prints the result of applying the same formula to different values. Its output is:

```
Momentum after 2.3s is: 97.62kg*m/s
Momentum after 4.2s is: 153.48kg*m/s
Momentum after 9s is: 294.6kg*m/s
```

You see all that repeated code! Wouldn't it be cool if we could just write that once and use it many times?

082.
Introducing **functions**. What do you think the following code will do?

```
#include <iostream>

void momentum(double initial_v, double time) {

  double acction   = 9.8; // m/s^2 acc
  double mass      = 3;   // kg    mass

  double final_v = initial_v + acction * time;
  double momentum = mass * final_v;
  std::cout << "Momentum after " << time
            << "s is: " << momentum << "kg*m/s"
            << std::endl;
}

int main() {
  momentum(10, 2.3);
  momentum(10, 4.2);
  momentum(10, 9);

  return 0;
}
```

Its output is the same as the code above, but we have eliminated all the repeated code.

```
Momentum after 2.3s is: 97.62kg*m/s
Momentum after 4.2s is: 153.48kg*m/s
Momentum after 9s is: 294.6kg*m/s
```

`momentum` is called a *function* (or procedure in old people talk terms), and it consists in a piece of code that can be run by calling it with the syntax `momentum(num1, num2)` (where `num1` and `num2` are either numbers or expressions that evaluate to numbers (`double`)).
Functions need to be defined outside `main` because `main` itself is a function! And functions cannot be defined[†] inside a function.

[†]actually, it is possible to define as many functions as one may like inside another function, but requires object oriented stuff to understand. For more info see "lambda functions".

083.

What should we be modify in the function `show_addition` for it to output the right answer?

```
#include <iostream>

void show_addition(int a, int b) {
  std::cout << a << " + " << b
            << " == " << a*b;
}

int main() {
  for (int i=4; i<=7; i++) {
    show_addition(i,3);
  }

  return 0;
}
```

Yeah, we are using the wrong operation, it shouldn't be `*`, it should be `+`. Also, we didn't add a newline after each new statement. The new function should be written as:

```
void show_addition(int a, int b) {
  std::cout << a << " + " << b
            << " == " << a+b
            << std::endl;
}
```

Notice how we do NOT use in `show_addition` the variable names we declared in `main`. Remember the scope thing? Well, each function has its own variables and they are invisible to all the other. When you call/run/invoque a function, you are copying the values of the variables in your scope to a newly created scope for the function.

084.

Now, what will this code output?

```
#include <iostream>

void add_two_nums(int a, int b, int c) {
  c = b + a;

  std::cout << "a == " << a << std::endl;
  std::cout << "b == " << b << std::endl;
  std::cout << "c == " << c << std::endl;
}

int main() {
  int first  = 4;
  int second = 3;
  int total  = -2;

  add_two_nums(first, second, total);

  std::cout
    << "total == " << total << std::endl;

  return 0;
}
```

The output is

```
a == 4
b == 3
c == 7
total == -2
```

weird, isn't it? No, it is not weird. When we call/run the function `add_two_nums` we copy the variables values to newly created variables `a`, `b` and `c`, which are only visible to the function `add_two_nums` and not to `main`.

085.

Have you noticed the `void` word[†] at the definition of the function?
`void` tells us that the function doesn't return any value as a result of its computation. There are other keywords that tell us when a function returns something, for example, the keyword `int`.
What, do you think, is the output of the following code:

```
#include <iostream>

int addtwice(int a, int b) {
  int c = 2 * (a + b);
  return c;
}

int main() {
  std::cout << addtwice(5, 3) << std::endl;
  return 0;
}
```

[†] `void` is actually a keyword, i.e., a word that is special for C++, nobody can use void as a name for a variable or function.

The output is:

```
16
```

When you call a function, you pass it some data (variables values), the function makes some calculation and returns, optionally, some result value.
Notice the similarities between a function definition in C++ and they representation in math. The function above can be written in "math" as:

$$f : (\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}$$
$$f(a, b) = 2(a + b)$$

Task for home: Write a function that takes two integers (`int`) and returns 6 if the sum of the integers is even otherwise -2, i.e., code the function $someScore : (\mathbb{Z} \times \mathbb{Z}) \to \mathbb{Z}$ in C++:

$$someScore(x, y) = \begin{cases} 6 & \text{if } x + y \text{ is even} \\ -2 & \text{otherwise} \end{cases}$$

086.

What is the output of:[†]

```
#include <iostream>

int sumToN(int n) {
  int total = 0;
  for (int i=0; i<=n; i++) {
    total += i;
  }
  return total;
}

int main() {
  if (sumToN(20) > 100) {
    std::cout << "0+1+2+..+20 > 100"
              << std::endl;
  } else {
    std::cout << "0+1+2+..+20 <= 100"
              << std::endl;
  }

  return 0;
}
```

[†] Note: the statement `total += i;` is equivalent to `total = total + i;`. There are several shourtcut versions for other operations like `*`, `-`, `/`, `%`: `*=`, `-=`, `/=`, `%=`.

The output is:

```
0+1+2+..+20 > 100
```

087.

Given the function `sumToN`:

```
1 |  int sumToN(int n) {
2 |    int total = 0;
3 |    for (int i=0; i<=n; i++) {
4 |      total += i;
5 |    }
6 |    return total;
7 |  }
```

we can analyse line by line and step by step (the **trace** of a program execution) of what happens when we call the function `sumToN(1)`:

```
sumToN(1)
 (n=1; L2)=> int total = 0;
 (n=1, total=0; L3)=> int i=0
 (n=1, total=0, i=0; L3)=> i<=n
 (n=1, total=0, i=0; L4)=> total += i
 (n=1, total=0, i=0; L3)=> i++
 (n=1, total=0, i=1; L3)=> i<=n
 (n=1, total=0, i=1; L4)=> total += i
 (n=1, total=1, i=1; L3)=> i++
 (n=1, total=1, i=2; L3)=> i<=n
 (n=1, total=1, L5)=> return total;
=> 1
```

where `(n = 3; L2)` tells us what is the value of `n` inside the function, and `L` indicates which line are we analysing (`3` and `2` in this case).

What is the *trace* of `sumToN(3)`?

```
sumToN(3)
 (n=3; L2)=> int total = 0;
 (n=3, total=0; L3)=> int i=0
 (n=3, total=0, i=0; L3)=> i<=n
 (n=3, total=0, i=0; L4)=> total += i
 (n=3, total=0, i=0; L3)=> i++
 (n=3, total=0, i=1; L3)=> i<=n
 (n=3, total=0, i=1; L4)=> total += i
 (n=3, total=1, i=1; L3)=> i++
 (n=3, total=1, i=2; L3)=> i<=n
 (n=3, total=1, i=2; L4)=> total += i
 (n=3, total=3, i=2; L3)=> i++
 (n=3, total=3, i=3; L3)=> i<=n
 (n=3, total=3, i=3; L4)=> total += i
 (n=3, total=6, i=3; L3)=> i++
 (n=3, total=6, i=4; L3)=> i<=n
 (n=3, total=6, L5)=> return total;
=> 6
```

088.

What is the output of:

```
#include <iostream>
#include <cmath>

double aproxTan(double angle) {
  return sin(angle) / cos(angle);
}

int main() {
  double angle = 0.2;
  double pi = 3.14159265358979;
  std::cout
    << "The tangent of " << angle << "pi "
    << "is aprox. " << aproxTan(pi*angle)
    << std::endl;

  return 0;
}
```

Yeah, we have a calculator on steroids:

```
The tangent of 0.2pi is aprox. 0.726543
```

089.
Does this code compile? (notice we changed `double angle` for `int angle`), if yes, what is the output?

```
#include <iostream>
#include <cmath> // for `sin` and `cos` functions

double aproxTan(double angle) {
  return sin(angle) / cos(angle);
}

int main() {
  int angle = 0.2;
  double pi = 3.14159265358979;
  std::cout
    << "The tangent of " << angle << "pi "
    << "is aprox. " << aproxTan(pi*angle)
    << std::endl;

  return 0;
}
```

Yeah, it does compile. And the result is:

```
The tangent of 0pi is aprox. 0
```

What is happening is that we are assigning a `double` (`0.2`) to an `int` (`angle`). In this process, the value gets converted to a value that can be stored in an `int`, meaning that we loose the fractional part of `0.2` leaving us with `0`.
Then `angle` gets converted (or **cast**) from an `int` to a `double`, because it is being operated with `pi` which is a variable that holds a `double`[†]

[†]General rule: the compiler detects if the operators in a binary operation are both of the same type, if they aren't, the compiler **casts** the value that contains the *less* information to a type with more information, e.g., `int` -> `double`, `int` -> `long`, `float` -> `double`

090.
What is the output of:

```
#include <iostream>
#include <cmath>

int aproxTan(int angle) {
  return sin(angle) / cos(angle);
}

int main() {
  double angle = 0.2;
  double pi = 3.14159265358979;
  std::cout
    << "The tangent of " << angle << "pi "
    << "is aprox. " << aproxTan(pi*angle)
    << std::endl;

  return 0;
}
```

The output is:

```
The tangent of 0.2pi is aprox. 0
```

No surprise here. `pi` and `angle` are variables of type `double`, when we operate with them we get a value of type `double`. But, once we pass the value to `aproxTan`, it gets cast to `int` because `aproxTan` receives `int`s, and returns `int`s. The value of tangent that gets computed is the integer part of `0.2 * pi`, i.e., `0`.

091.
What do you think the following outputs?[†]

```
std::cout
  << (int)5.3       << " "
  << (int)5.8       << " "
  << ((int)5.3)*2   << " "
  << (int)(5.3*2)   << " "
  << 3/2            << " "
  << (double)3/2    << " "
  << (double)(3/2) << std::endl;
```

The output is:

```
5 5 10 10 1 1.5 1
```

[†]We can force a cast by prepending the value to cast with *(type)*, where *type* can be any type of the many we have seen, for example, `(int)12.1` casts `12` (a `double` or `float`) into an `int`.

Task for home: Define a function (in C++) that takes a number n (a `double`) and returns the smallest integer m (an `int`)

such that $m \geq n$. For example:

```
smallestBig(5.3) == 6
smallestBig(8.9) == 9
smallestBig(2.0) == 2
```

092.

Files are sequences of bytes[†]. A byte contains 8 bits, which means that we have 2^8 different possibilities/states of a byte. We have `00000000`, but also `00000001`, and `10110001`, all representable as numbers in decimal notation (the first one in decimal notation is 0, the second is 1, and the third is $2^7 + 2^5 + 2^4 + 2^0 = 177$)[‡]

Do you remember which of the following types has size one byte?

`int`, `float`, `char`, and `double`.

Yeah, it was `char`, we can save up to 256 numbers in a char (from -128 to 127).

[†]Most of the time

[‡]If you want to know how precisely convert from binary to decimal, and viceversa, please visit Wikipedia's article: *Binary number*.

093.

We can represent each of these 256 possibilities in many ways, one of them is a decimal number, but other possibility is assigning arbitrarily a symbol to each. Somebody has already done the work for us here, and it is called ASCII. For example, the byte `01000001`, in decimal is 65, and the symbol given to it is: `'A'`. This means we can print a single character by saving 65 in a `char` variable and printing it:

```
char symbol = 65;

std::cout << symbol << std::endl;
```

The output as expected:

```
A
```

094.

Because when we print a `char` variable we print its ASCII symbol, then there is a simpler way to input the value to the variable, using single quotes (`'`).

This

```
char symbol = 'A';

std::cout << symbol << std::endl;
```

prints the same as above. What do you think this will print?

```
char symbol = 'A';

while (symbol <= 'Z') {
  std::cout << symbol;
  symbol++;
}
std::cout << std::endl;
```

All characters from `A` to `Z`! (as they're ordered in the Enghlish alphabet)

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
```

095.
Now let's try to print more characters (if you look at the ASCII table (online or somewhere), you will notice that not all characters are visible, therefore we will print just a subset of the characters):

```
char symbol = 32; // first "visible" character

while (symbol < 127) {
  std::cout << symbol;
  symbol++;
  if (symbol % 32 == 0) {
    std::cout << std::endl;
  }
}
std::cout << std::endl;
```

Well, a list of all characters available to us.[†]

```
 !"#$%&'()*+,-./0123456789:;<=>?
@ABCDEFGHIJKLMNOPQRSTUVWXYZ[\]^_
`abcdefghijklmnopqrstuvwxyz{|}~
```

[†]All symbols for the other 128 characters, which values are between 128 and 255, are machine dependent, they will render in the windows console but not anywhere else (If you want to know more, I recommend Wikipedia's articles on *Unicode* and *UTF-8*).

Task for home: Write a function that takes a `char` variable with a value between `A` and `Z`, and returns it in small case, for example:

```
char toSmallCase(char c) {
  // your code goes here
}

int main() {
  char capitalCase = 'M';
  char smallCase = toSmallCase(capitalCase);
  std::cout << capitalCase << std::endl;
}
```

And it should print:

```
m
```

Tip: Look at the ASCII values of each of the characters, for example, to print the value of `'R'` use:

```
std::cout << (int)'R' << std::endl
```

Note: A possible type for the function toSmallCase (if we were to write it more mathy) would be: $toSmallCase : \mathbb{ASCII} \to \mathbb{ASCII}$, which is telling us that it is a function that takes an ASCII value and returns an ASCII value.

096.
What is the output of:

```
#include <iostream>

int adding(int a, int b) {
  int c = a + b;
  return c;
  std::cout << "I'm never run :("
            << std::endl;
  c = c + 2;
}

int main() {
  for(int i=0; i<10; i++) {
    std::cout << adding(i,i) << ' ';
  }
  std::cout << std::endl;
  return 0;
}
```

```
0 2 4 6 8 10 12 14 16 18
```

Nothing written after **return** gets ever done! This is telling us a very important property of **return**: Any time **return** is called the function call ends and its output value is whatever the **return** had defined to compute.

097.
What is the output of:

```
#include <iostream>

int adding(int a, int b) {
  int c = a + b;
  return c;
  return a * 2;
  if (a<b*c==0) {
    return b;
  }
}

int main() {
  for(int i=0; i<10; i++) {
    std::cout << adding(i,i) << ' ';
  }
  std::cout << std::endl;
  return 0;
}
```

```
0 2 4 6 8 10 12 14 16 18
```

The same as before, the first time we meet a **return** the function gives back the value it is told to, in this case is `c`.

098.
What does the following outputs:

```
for(int i=0; i<10; i++) {
  for(int j=0; j<15; j++) {
    if(i%2 == 0) {
      std::cout << '#';
    } else {
      std::cout << '.';
    }
  }
  std::cout << std::endl;
}
```

```
###############
...............
###############
...............
###############
...............
###############
...............
###############
...............
```

Neat!
Notice the rule `i%2 == 0`, what is it doing?

099.

We can look at the statement/proposition in the `if` statement as a function that takes two numbers and returns `true` if they fulfill something, i.e., the type of `(i+j)%4 == 0` is $(\mathbb{Z} \times \mathbb{Z}) \to \mathbb{B}$[†]

```
for(int i=0; i<10; i++) {
  for(int j=0; j<15; j++) {
    if((i+j)%4 == 0) {
      std::cout << '#';
    } else {
      std::cout << '.';
    }
  }
  std::cout << std::endl;
}
```

[†]I'm abusing the math notation here to indicate that the result is a boolean, i.e., `true` or `false`. The set $\mathbb{B}$ is defined as the set that contains only two elements, namely: $\{\bot, \top\}$ ($\bot$ is `false`, and $\top$ is `true`)

```
#...#...#...#..
...#...#...#...
..#...#...#...#
.#...#...#...#.
#...#...#...#..
...#...#...#...
..#...#...#...#
.#...#...#...#.
#...#...#...#..
...#...#...#...
```

Neat! And so weird. So, I guess we can write very complex propositions inside the `if` to print arbitrary things.

100.

Let's take a look at a little more complex rule[†]:

```
#include <iostream>

bool surprise(int a, int b) {
  double y = a - 4.5;
  double x = (b - 7)/2;
  double rad = 3;
  bool circle = x*x + y*y < rad*rad;
  return circle;
}

int main() {
  for(int i=0; i<10; i++) {
    for(int j=0; j<15; j++) {
      if( surprise(i, j) ) {
        std::cout << '#';
      } else {
        std::cout << '.';
      }
    }
    std::cout << std::endl;
  }
  return 0;
}
```

[†]remember, this rule can be seen as a function of type is $(\mathbb{Z} \times \mathbb{Z}) \to \mathbb{B}$

```
...............
...............
....#######....
..###########..
..###########..
..###########..
..###########..
....#######....
...............
...............
```

What the heck is happening here?!

Notice how the function above can be written more concisely as:

$$surprise(a, b) := (a - 4.5)^2 + \left(\frac{b-7}{2}\right)^2 < 3^2$$

[†]

Tip: Go to http://pythotutor.com, you can copy and paste there the code and look for yourself what is the code doing step by step.

[†]Where have you seen such a formula before? Haven't you? Well, take a look at, it may refresh your memory ;) : https://en.wikipedia.org/wiki/%43%69%72cle#Equations

---

Task for home: Define a function $triangle : (\mathbb{Z} \times \mathbb{Z}) \to \mathbb{B}$ that when called in the loop

```
for(int i=0; i<10; i++) {
  for(int j=0; j<15; j++) {
    if( surprise(i, j) ) {
      std::cout << '#';
    } else {
      std::cout << '.';
    }
  }
```

```
  std::cout << std::endl;
}
```

prints a triangle, any triangle.

Play and experiment with many different functions. Some functions you can experiment with are:

$$p(x, y) := (x + y = 8)$$

$$q(x, y) := (x + 2 = y)$$

$$r_1(x, y) := p(x, y) \wedge q(x, y)$$

$$r_2(x, y) := p(x, y) \vee q(x, y)$$

$$s(x, y) := (x + y \leq 8) \vee (x + 2 \geq y)$$

Chapter 5

Block 2: Going further with functions and more space to play with

Answering with a question[1]

101.
The successor function is the function that returns the natural number that follows any natural number we give it, i.e., the successor function is $succ(x) = x+1$ where x is a natural number.
Can you write this function in C++?

Of course, and it's:

```
int succ(int x) {
  return x+1;
}
```

102.
What is the output of:

```
#include <iostream>

int succ(int x) {
  return x + 1;
}

int main() {
  std::cout << succ(0)
     << " " << succ(succ(0))
     << " " << succ(0)
     << " " << succ(succ(succ(0)))
     << " " << succ(succ(succ(succ(0))))
     << std::endl;
  return 0;
}
```

Not very surprisingly:

```
1 2 1 3 4
```

---

[1] Well, one of the ideas of recursive functions is that we can write an answer as a question.

103.

In maths we define addition (+) on natural numbers in the following way:[†]

1. We define there is a $0 \in \mathbb{N}$
2. We define that every natural number has a successor: $n \in \mathbb{N} \rightarrow (n+1) \in \mathbb{N}$
3. We define how to add two numbers

$$add(a,b) = \begin{cases} a & \text{if } b = 0 \\ succ(add(a,c)) & b = succ(c) \end{cases}$$

How would you write `add(a, b)` in C++?

[†]This method to define natural numbers is called "Peano Numbers", you can take a deeper look at them if you search in the wikipedia for "Peano axioms", highly recommended! It's a basic structure in maths and every mathematician knows it.

One possibility is:

```
int add(int x, int y) {
  if (y==0) {
    return x;
  } else {
    return succ( add(x, y-1) );
  }
}
```

Digression, this is NOT how the computer adds, but this is how in maths addition is defined.

104.

So, what is the output of:

```
#include <iostream>

int succ(int x) {
  return x + 1;
}

int add(int x, int y) {
  if (y==0) {
    return x;
  } else {
    return succ( add(x, y-1) );
  }
}

int main() {
  std::cout << succ(0)
     << " " << succ(succ(0))
     << " " << add( succ(succ(0)), 0 )
     << " " << add( succ(0), succ(0) )
     << " " << add( 0, succ(succ(0)) )
     << " " << add( 0, succ(succ(0)) )
     << " " << succ( add( 0, 2 ) )
     << " " << add( 10, 2 )
     << " " << add( 3, 21 )
     << std::endl;
  return 0;
}
```

Yeah, well, it seems just wasteful, so many additions and succesions to print just a couple of 2s:

```
1 2 2 2 2 2 3 12 24
```

105.

What is happenning with `add(succ(0), succ(succ(0)))`? (step by step)

A step by step of the execution of `add(succ(0), succ(succ(0)))`:

1. Before any function is executed, all its arguments are computed, so to run `add` we need to run first `succ(0)` and `succ(succ(0))` which return us the values `1` and `2`, respectively.
2. We call `add` with the values `1` and `2`
3. Inside the call of `add` we ask if `y` is equal to zero. It isn't, therefore we execute `succ( add(x, y-1) )`.
4. Before we can call `succ( add(x, y-1) )`, we need to find the value of the argument that we want to pass to `succ`. This means, we compute `add(x, y-1)`, and to compute it we need first `y-1`.
5. We call now `add` with the values `1` and `1`.
6. Inside the call of `add` we ask if `y` is equal to zero. It isn't (`y==1`), therefore we call once more `succ` with argument `add(x, y-1)` (for which we require `y-1`).
7. Inside the call of `add` we ask if `y` is equal to zero. It IS, finally, and we decide to return the value of `x`.
8. Now, we come back to the last call of the `add` function, where we compute the `succ` of `1`, which is `2`. And return the value to the last call.
9. We get back to the place where `add` was called, we apply the `succ` function and return its final value `3`.

A simpler way to represent what is happenening step by step when we call a function is to write each instruction executed line by line, a *trace*. So, the explanation above can be written as:

```
   add(succ(0), succ(succ(0)))

=> add(1, succ(succ(0)))
  (x=0)=> return x+1
  (x=0)=> return 0+1
  (x=0)=> return 1
=> add(1, succ(1))
  (x=1)=> return x+1
  (x=1)=> return 1+1
  (x=1)=> return 2
=> add(1, 2)
  (x=1, y=2)=> if (y==0) { return x; } else { return succ( add(x, y-1) ); }
  (x=1, y=2)=> return succ( add(x, y-1) )
  (x=1, y=2)=> return succ( add(1, 1) )
    (x=1, y=1)=> if (y==0) { return x; } else { return succ( add(x, y-1) ); }
    (x=1, y=1)=> return succ( add(x, y-1) )
    (x=1, y=1)=> return succ( add(1, 0) )
      (x=1, y=0)=> if (y==0) { return x; } else { return succ( add(x, y-1) ); }
      (x=1, y=0)=> return x
      (x=1, y=0)=> return 1
    (x=1, y=1)=> return succ( 1 )
      (x=1)=> return x+1
      (x=1)=> return 1+1
      (x=1)=> return 2
    (x=1, y=1)=> return 2
  (x=1, y=2)=> return succ( 2 )
    (x=2)=> return x+1
    (x=2)=> return 2+1
    (x=2)=> return 3
  (x=1, y=2)=> return 3
=> 3
```

Notice, how each time we call the function the values of the variables `x` and `y` are different, this is because their scope its different. **Everytime we call a function it creates a new scope!**

106.

As you can see, what we are doing is to answer a question with another question. We ask, "how much is 1 plus 3," and we answer, "the successor of 1 plus 2". We are solving problems by reframing them, by asking the same problem from a different angle.

But, how does it work? How is it possible to answer a question with another question?

Well, the trick is …

that the answer is not any question, it is a special question, a simpler question. The idea is to simplify our problem by asking something simpler.

A function that calls itself to solve a problem is named **recursive** and it's composed of two important parts:

- **A base case**: A base rule that determines the answer to the simplest case (above this would be $n + 0 = n$, the sum of any number with zero is the same number)
- **A recursive case**: A rule that simplifies the problem but answers it with the same function (e.g., $n+(k+1) = (n+k)+1$, the sum of two natural numbers is the same as the successor of the sum of the first number and the predecesor of the second).

107.

Do you remember the definition of fibonacci?

Well, it's this:

$$fib(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = 1 \\ fib(x-1) + fib(x-2) & \text{if } x > 1 \end{cases}$$

108.

Now, translate the fibonacci function into C++ code.

One possible C++ version is:

```
int fib(int x) {
  if (x==0) {
    return 0;
  } else if (x==1) {
    return 1;
  } else {
    return fib(x-1) + (x-2);
  }
}
```

Challenge: Can you think on other ways to write fibonacci's formula? using only an `if` statement?

109.

A recursive function is always defined by two equations: the **base case** and the **recursive case**.
When translating the cases into C++ we always get the structure (if the function takes and returns integer values):

```
int recursive_f(int x, int y) {
  if ( /* Base case */ ) {
    return /* base case result */;

  } else { // Recursive case
    return /* call to 'recursive_f' */;
  }
}
```

How is the following function coded into C++ using a recursive function:

$$fact(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ x * fact(x-1) & \text{otherwise} \end{cases}$$

Well, the function is the factorial of a number, often written in math as $n! = 1 \cdot 2 \cdot 3 \cdots n$.
And the code is:

```
int fact(int x) {
  if (x<=1) {
    return 1;
  } else {
    return x * fact(x-1);
  }
}
```

110.

What is the trace of running `fact(4)`?[†]

[†]same as to say, what is the trace of evaluating the function (in this case only)

```
fact(4)

  (x=4)=> if (x<=1) { return 1; }
          else { return x*fact(x-1); }
  (x=4)=> return x*fact(x-1);
  (x=4)=> return 4*fact(3);
    (x=3)=> if (x<=1) { return 1; }
            else { return x*fact(x-1); }
    (x=3)=> return x*fact(x-1);
    (x=3)=> return 3*fact(2);
      (x=2)=> if (x<=1) { return 1; }
              else { return x*fact(x-1); }
      (x=2)=> return x*fact(x-1);
      (x=2)=> return 2*fact(1);
        (x=1)=> if (x<=1) { return 1; }
                else { return x*fact(x-1); }
        (x=1)=> return 1;
      (x=2)=> return 2*1;
      (x=2)=> return 2;
    (x=3)=> return 3*2;
    (x=3)=> return 6;
  (x=4)=> return 4*6;
  (x=4)=> return 24;
=> 24
```

111.
We can write the evaluation of `fact(5)` more compactly:

$$\begin{aligned} fact(5) &= 5 \cdot fact(4) \\ &= 5 \cdot (4 \cdot fact(3)) \\ &= 5 \cdot (4 \cdot (3 \cdot fact(2))) \\ &= 5 \cdot (4 \cdot (3 \cdot (2 \cdot fact(1)))) \\ &= 5 \cdot (4 \cdot (3 \cdot (2 \cdot 1))) \\ &= 5 \cdot (4 \cdot (3 \cdot 2)) \\ &= 5 \cdot (4 \cdot 6) \\ &= 5 \cdot 24 \\ &= 120 \end{aligned}$$

How does the evaluation of `fib(4)` looks?

$$\begin{aligned} fib(4) &= fib(3) + fib(2) \\ &= (fib(2) + fib(1)) + fib(2) \\ &= ((fib(1) + fib(0)) + fib(1)) + fib(2) \\ &= ((1 + fib(0)) + fib(1)) + fib(2) \\ &= ((1 + 0) + fib(1)) + fib(2) \\ &= (1 + fib(1)) + fib(2) \\ &= (1 + 1) + fib(2) \\ &= 2 + fib(2) \\ &= 2 + (fib(1) + fib(0)) \\ &= 2 + (1 + fib(0)) \\ &= 2 + (1 + 0) \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

112.
Recursive functions are very powerful. They are as powerful as loops (`for` and `while`) to iterate over a series of numbers. Take a look at the following function:

```
void huh(int i, int n) {
  if(i>=n) {
    std::cout << "This is the end: " << n
              << std::endl;
  } else {
    std::cout << i << std::endl;
    huh(i+1, n);
  }
}
...
huh(4, 10);
```

What do you think it does when run with `huh(4, 10)`?

Yeah, it acts precisely as a `for` loop!

```
4
5
6
7
8
9
This is the end: 10
```

In fact, we can rewrite the function to work with a `for` loop and not recursion:

```
void huh(int i, int n) {
  for(int j=i; j<=n; j++) {
    std::cout << i << std::endl;
  }
  std::cout << "This is the end: " << n
            << std::endl;
}
...
huh(4, 10);
```

Task for home: Write a recursive function that adds the numbers from 1 to 10 without using any `for` or `while` loop.

113.
What does the following function outputs:

```
#include <iostream>

void print_i2n_squared(int i, int n) {
  if(i<=n) {
    std::cout << i*i << " ";
    print_i2n_squared(i+1, n);
  } else {
    std::cout << std::endl;
  }
}

int main() {
  print_i2n_squared(3, 5);
  print_i2n_squared(12, 21);
  return 0;
}
```

The output is:

```
9 16 25
144 169 196 225 256 289 324 361 400 441
```

114.
Write the function from the last example (`print_i2n_squared`) using a `for` loop and not recursion

```
#include <iostream>

void print_i2n_squared(int start, int n) {
  for(int i=start; i<=n; i++) {
    std::cout << i*i << " ";
  }
  std::cout << std::endl;
}

int main() {
  print_i2n_squared(3, 5);
  print_i2n_squared(12, 21);
  return 0;
}
```

115.
So, there seems to be a pattern. Look at the two pieces of code in the last two examples. Are they gonna do the same for every input `i` and `n`?
Can you write down the rules on how to convert a function like `print_i2n_squared` from a `for` loop to a recursive function?

Yes, they behave almost in the same way. Thought, depending on the compiler you're using and the options you've set it on, the recursive function may not work properly always.
And what are the rules, or the scheme, to translate from one to the other, well:
As a recursive function:

```
void f_rec(VARS) {
  if(ASSERTION) {
    MAIN ROUTINE
    f_rec(VARS on next step);
  } else {
    END ROUTINE
  }
}
```

And as a for or while loop:

```
void f_loop(VARS) {
  for( init vars; ASSERTION; VARS on next step)
    MAIN ROUTINE
  }
  END ROUTINE
}
```

Where `init vars` initialises the variables we are going to use inside the loop.

116.
But some functions do return something (i.e., their type is not `void` but `int`, `double`, …). For example:

```
int sum_i2n(int i, int n) {
  if(i<=n) {
    return i + sum_i2n(i+1, n);
  } else {
    return 0;
  }
}
```

What is this function doing? What is the output of:

```
int main() {
  std::cout << sum_i2n(1,10) << std::endl;
  std::cout << sum_i2n(1,10)*3 << std::endl;
  return 0;
}
```

Well, the function is adding all numbers from `i` to `n` (inclusively). The output is:

```
55
165
```

117.
Now, write the function above using a `for` loop and not recursion.

```
int sum_i2n(int i, int n) {
  int sum = 0;
  for(int j=i; j<=n; j++) {
    sum += i;
  }
  return sum;
}
```

118.
These kind of recursive functions (which return something rather than `void`) have a similar pattern to convert them from and to `for` (or `while`) loops.
Given a "`for` loop" function:

```
TYPE f_loop(VARS) {
  TYPE result = BASE_VALUE;
  for(init vars; ASSERTION; VARS on next step) {
    result = OPERATE( result, VARS );
  }
  return result;
}
```

where `OPERATE` may be one simple operation like `+` (or `*`), or it may be a more complex operation.
What is it equivalent recursive function?

We can write the "equivalent"[†] recursive function:

```
TYPE f_rec(VARS) {
  if( ASSERTION ) {
    return OPERATE( result, VARS on next step );
  } else {
    return BASE_VALUE;
  }
}
```

Congratulations! Now you have a new scheme to convert from a recursive function to a loop-function.

[†]dependend on the compiler and optimizations

Task for home: Write a recursive function that returns the number its given inverted. For example, if we call `reversed(23464)` it should return `46432`.

Take the following code as template:

```
#include <iostream>

int reversed(int n) {
  // Your code should go here
  return 0;
}

int main() {
  std::cout << 1234 << " reversed is " << reversed(1234) << std::endl;
  std::cout << 643 << " reversed times two is " << 2*reversed(643) << std::endl;
  return 0;
}
```

with output:

```
1234 reversed is 4321
643 reversed times two is 692
```

Note: you may use more than one function if necessary (in fact, it may be impossible to do it without auxiliary (additional) functions).

119.
Sometimes, it's easier to write a recursive function with an additional parameter (variable) which it's gonna hold the result of the recursive operation. For example, what does the following function do if it is called with the parameters `arecfun(1, 10, 0)`?

```
int arecfun(int i, int n, int acc) {
  if(i<=n) {
    return sum_i2n(i+1, n, acc+i);
  } else {
    return acc;
  }
}
```

Yep, this is precisely the same function we were playing with before. And it would return when called with the parameters `(1, 10, 0)` the value `55`.

Task for home: Write a recursive function that prints on the screen the first `20` numbers of the fibonacci sequence.

Task for home: Read, analyse (and execute) the following two equivalent functions:

Its `for` version.

```
int sum_i2n(int i, int n) {
  int sum = 0;
  for(int j=i; j<=n; j++) {
    sum += i;
  }
  return sum;
}
```

and, its recursive version.

```
int helper_f(int i, int n, int acc) {
  if(i<=n) {
    return sum_i2n(i+1, n, acc+i);
  } else {
    return acc;
  }
}

int sum_i2n(int i, int n) {
  return helper_f(i, n, 0);
}
```

Derive a set of rules, or a scheme, to convert a `for` loop-function into a recursive function[2].

---

[2]This kind of recursive functions is called "tail recursive functions", and can be usually be easily converted into `for` loops by the compiler, therefore, they are recommended over non-tail recursive function if they can be converted.

Grouping data together

120.
Write the following function in C++:

$$sum\_squared : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$(i, n) \mapsto \sum_{j=i}^{n} j^2$$

```
int sum_squared(int i, int n) {
  int result = 0;
  for(int j=i; j<=n; j++) {
    result += j*j;
  }
  return result;
}
```

121.
And what if we wanted to write a C++ function with the following type:

$$twothings : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$$
$$(i, n) \mapsto \left(\sum_{j=i}^{n} j^2, \prod_{j=i}^{n} j^2 \right)$$

Well, that's not possible right now. The only thing we may do is to write two functions, each one of them computes a single part of the output. Something like this:

$$twothings_1 : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$(i, n) \mapsto \sum_{j=i}^{n} j^2$$
$$twothings_2 : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$(i, n) \mapsto \prod_{j=i}^{n} j^2$$

122.
Breaking a function its components is feasible only sometimes. For example, take a look at the following function[†]:

$$fib : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$$
$$i \mapsto \begin{cases} (0,1) & \text{if } i \leq 0 \\ \textbf{let} \quad (a,b) = fib(i-1) & \\ \textbf{in} \quad (b, a+b) & \text{otherwise} \end{cases}$$

What is the result of $fib(5)$?

[†]**let in** lets us declare a variable (or many variables) to later use them in more computations, for example: **let** $x = 3+4$ **in** $y = x*x*20$ is the same as writing $y = (3+4)*(3+4)*20$.

Let's start with the smallest value 0, and go up till 5:

$$fib(0) = (0,1)$$

$$\begin{aligned} fib(1) &= \textbf{let } (a,b) = fib(0) \textbf{ in } (b, a+b) \\ &= \textbf{let } (a,b) = (0,1) \textbf{ in } (b, a+b) \\ &= (1, 0+1) \\ &= (1,1) \\ fib(2) &= \textbf{let } (a,b) = fib(1) \textbf{ in } (b, a+b) \\ &= \textbf{let } (a,b) = (1,1) \textbf{ in } (b, a+b) \\ &= (1, 1+1) \\ &= (1,2) \\ fib(3) &= \textbf{let } (a,b) = fib(2) \textbf{ in } (b, a+b) \\ &= (2,3) \\ fib(4) &= \textbf{let } (a,b) = fib(3) \textbf{ in } (b, a+b) \\ &= (3,5) \\ fib(5) &= \textbf{let } (a,b) = fib(4) \textbf{ in } (b, a+b) \\ &= (5,8) \end{aligned}$$

123.
What do you think will the following code output?

```
#include <iostream>

struct pair {
  int x;
  int y;
};

int main() {
  pair p;
  p.x = 12;
  p.y = 6;
  std::cout << p.x << " " << p.y << std::endl;
  return 0;
}
```

Well, nothing very surprising:

```
12 6
```

A ***struct*** allows us to declare and manipulate several blocks of memory as in one.

124.
We can access to each one of the individual parts of a variable declared as as *struct* with help of the dot (`.`) operator. Let's suppose that we define `pair` as:

```
struct pair {
  int x;
  int y;
};
```

What is the output of the following code[†]:

```
pair first;
pair second;

first.x = 12;
second.x = 45;
second.y = 3;
first.y = first.x * second.y;
second.y = 12;

std::cout << first.x << " "
          << first.y << std::endl
          << second.x << " "
          << second.x << std::endl;
```

[†]we're back inside `main` for simplicity

```
12 36
45 45
```

125.
What is very neat about **struct**s is that we can use them as any other type we have encountered to date. We can declare a `pair` which will reserve two spaces of memory of type `int` on the fly. We can even pass and get values of type `pair`, i.e., we can use `pair` as any other type we have seen, we can pass it to a function or return it from a function. For example:

```
pair fib(int i) {
  if(i<=0) {
    pair ret;
    ret.x = 0;
    ret.y = 1;
    return ret;
  } else {
    pair ret;
    pair ab = fib(i-1);
    ret.x = ab.y;
    ret.y = ab.x + ab.y;
    return ret;
  }
}
...
pair res;
res = fib(5);
std::cout << res.x << " "
          << res.y << std::endl;
```

What will be its output?

```
5 8
```

Yeah, this is precisely the *fib* function from above, we can now write functions with outputs that are not only `int`, `double`, `char`, `bool`, … `:D`

126.
The function above doesn't look too nice, it's quite long and cumbersome to read. So, we are going to apply some tricks to make it look nicer.
The first, is that we can in fact pass values directly to a `pair` when we declare it. Similar to how we write:

```
int a = 5;
```

and not:

```
int a;
a = 5;
```

We can write

```
pair p = {1, 2};
```

as a shorthand to:

```
pair p;
p.x = 1;
p.y = 2;
```

Rewrite the function above (`fib`) using this shorthand.

```
pair fib(int i) {
  if(i<=0) {
    pair ret = {0, 1};
    return ret;
  } else {
    pair ab = fib(i-1);
    pair ret = { ab.y, ab.x + ab.y };
    return ret;
  }
}
...
pair res;
res = fib(5);
std::cout << res.x << " "
          << res.y << std::endl;
```

Wow, that looks much better.

127.
Writing something like:

```
pair ret = {0, 1};
return ret;
```

seems wasteful. Why two lines for something like that? Fortunatelly, this is a common enough case to have a short version.

```
return pair{0, 1};
```

:D
Rewrite, again, the `fib` function from above to use this new simplification.

```
pair fib(int i) {
  if(i<=0) {
    return pair{0, 1};
  } else {
    pair ab = fib(i-1);
    return pair{ ab.y, ab.x + ab.y };
  }
}
...
pair res;
res = fib(5);
std::cout << res.x << " "
          << res.y << std::endl;
```

Well, that is nice.
We could even simplify the code further. For example, there is no need for brackets around a single sentence in an `if` branch; also, there is no need to have an `else` branch when the `if` branch calls a `return`.
This is how it looks like after some more simplifications:

```
pair fib(int i) {
  if(i<=0)
    return pair{0, 1};
  pair ab = fib(i-1);
  return pair{ ab.y, ab.x + ab.y };
}
```

Task for home: Write the function

$$twothings : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$$
$$(i, n) \mapsto \left(\sum_{j=i}^{n} j^2, \prod_{j=i}^{n} j^2 \right)$$

using the struct `pair`.

Take this as a template:

```
pair twothings(int i, int n) {
  // your code goes here
  return pair{0, 0};
}
```

More space to play with! Yay!

Task for home: Sort a given an array with 3 elements. For example:

```
int elems[] = {3, 20, -2};

// Your code goes here

std::cout << elems[0] << " " << elems[1] << " " << elems[2] << std::endl;
// -> It should output: `-2 3 20`
```

Task for home: Given an array and its length, your task is to sort the array in ascending order. For example:

```
int elems[] = {3, 20, -2, 3, 4, -30, -2, 0, 12, 5, -100};
int n_elems = 11;

// Your code goes here

std::cout << "{ ";
for (int i=0; i<n_elems; i++) {
  std::cout << elems[i] << " ";
}
std::cout << "}" << std::endl;
// -> It should output: `{ -100, -30, -2, -2, 0, 3, 3, 4, 5, 12, 20 }`
```

128.
What do you think this will output?

```
char helloagain[] =
    {'H', 'e', 'l', 'l', 'o', ' ',
    'A', 'g', 'a', 'i', 'n', ' ', ':', ')'};
int hellolen = 14;

for (int i=0; i<hellolen; i++) {
  std::cout << helloagain[i];
}
std::cout << std::endl;
```

Well, it is just like a **string**, just like if we had written `std::cout << "Hello Again :)" << std::endl;`:

```
Hello Again :)
```

129.
Good news everyone! What you see above is (almost) precisely how strings are saved on memory, they are sequences of bytes!
We could write the above code as:

```
char helloagain[] =
    {'H', 'e', 'l', 'l', 'o', ' ',
    'A', 'g', 'a', 'i', 'n', ' ', ':', ')', 0};

std::cout << helloagain << std::endl;
```

Notice that this time helloagain contains a final `0`. This is used to tell `std::cout` when to stop.

The output as we expected:

```
Hello Again :)
```

Or we could've written:

```
char helloagain[] = "Hello Again :)";
std::cout << helloagain << std::endl;
```

130.
What will this print?

```
char helloagain[] =
    {'H', 'e', 'l', 'l', 'o', ' ', 0, //<- added
    'A', 'g', 'a', 'i', 'n', ' ', ':', ')', 0};

std::cout << helloagain << std::endl;
```

Notice the added `0` after "Hello".

And the output is, uh, truncated:

```
Hello
```

The line only prints to the point where it finds the first zero. Alternatively, we could write:

```
char helloagain[] = "Hello\0 Again :)";
std::cout << helloagain << std::endl;
```

Chapter 6

Block 3: Pointers (the truth behind arrays) and some other useful stuff

TTTT

Chapter 7

Further reading

A web page for learning C++ from zero (but also a place to look for documentation): http://www.learncpp.com

A good reference book on C++: http://www.cppstdlib.com/

For a load of books on learning to code, visit: https://github.com/EbookFoundation/free-programming-books