Quiz 1

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"By the name of Allah (God) Almighty, herewith I pledge and that I have Solved quiz I by myself, didn't do any Cheating by any Means, didn't do plagiarism, and didn't accept anybody's help by any Means. I am going to accept all of the Consequences by any Means if it has proven that I have done any Cheating and/or plagiarism."

Surabaya, 27th March 2024

Helst

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1. Problem 1

• The title of the problem: Help Tohu

• Problem description:

Tohu wants to enter Moscow State University, and he must pass the math exam. He does not know math, and asks you to help him. The problem is to find the sum $S_n = a_1 + a_2 + \ldots + a_{n_0} \text{ the sequence } \{a_n\}_{0n} \text{ condition } \forall k \in \mathbb{N}: a_1 + 2a_2 + \ldots + ka_k = \frac{k+1}{k+2}.$

• Problem abstraction:

In this problem, we are asked to solve this problem by finding the output, which is the value of Sn.

- 1. Input:
- \circ T, the number of test cases (where 1≤T ≤ 20000 and T is a single integer).
- o Next T lines contains n (a single integer where 1≤n≤10^9)
- 2. Output:
- o For each n, output the value of Sn with 11 digits after the decimal point.
- 3. Constraints:
- o 1≤T≤ 20000

- o 1≤n≤10^9
- 4. Example

Input:

2

2

5

Output:

0.70833333333

0.73809523810

• Solution:

- o Algorithm Strategy & Observations:
- a. Implement a formula to calculate Sn for a given n based on the provided condition.
- b. Iterate through each test case.
- c. For each n, compute Sn using the implemented formula.
- d. Output the result for each test case with the required precision.

To solve the problem, we need to find the formula of Sn:

Given the condition based on the problem and we have to find the formula of Sn:

$$Sn = a1 + a2 + ... + an$$

$$k = a1 + 2a2 + ... + kak = k+1/k+2$$

• For S1:

S1 = a1

2/3 = a1

So, S1 = 2/3

• For S2 :

S2 = a1 + a2

3/4 = a1 + 2a2

3/4 = 2/3 + 2a2 (using substitution of equation from S1)

(3/4 - 2/3) / 2 = a2 then replace it with k (k = 2):

```
((k+1)/(k+2) - (k/(k+1))) / 2 = a2
```

0.5/((k+2)*(k+1)) = a2 then it can also be formed using fraction manipulation as

$$0.5/(k+1) - 0.5/(k+2) = a2$$

From the equation given above, we can also conclude that

S2 = 3/4 - a2 (using substitution of equation)

$$S2 = 3/4 - ((0.5/(k+1) - 0.5/(k+2)))$$

$$S2 = 3/4 + (0.5/(k+2)) - (0.5/(k+1))$$

From here we can determine the formula of Sn that also applies to k=3,4,5... by replacing k with n

$$Sn = 0.75 + (0.5/(n+2)) - (0.5/(n+1))$$

After that, can implement the formula of Sn to the program.

- Theory References and Concepts :
 - 1. Arithmetic Series Formula: Finding the sum of an arithmetic series. Formula of Sn = a1 + a2 + ... + an where a1 is the first term, an is the n-th term, and n is the number of terms.
 - 2.Substitution and Algebraic Manipulation: Substitution involves replacing one or more variables or expressions with other variables or expressions. Algebraic manipulation refers to the process of manipulating algebraic expressions to simplify them, solve equations, or prove identities. The provided equation is used to express a2 in terms of a1, and then S2 is calculated using the derived expression for a2.
 - 3.Generalization: It refers to the process of extending concepts, principles, or results from specific instances to more general cases. After finding S2, the formula is generalized to find Sn by replacing k with n in the derived expression.
- o Pseudocode:

Function main:

Declare integer variable T

INPUTT

For i = 0 to T - 1:

Declare integer variable N

INPUT N

Declare double variable answer

```
Set answer = answer = 0.75 + (0.5/(N+2))-(0.5/(N+1))
OUTPUT answer with 11 decimal places
```

Return 0

End Function

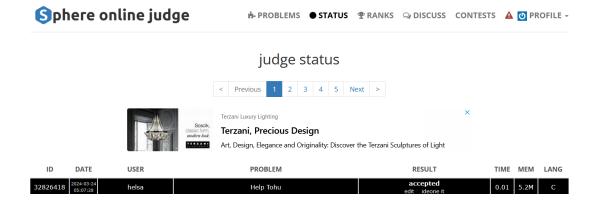
o Source Code:

```
#include <stdio.h>
int main () {
  int tohu;
  scanf("%d", &tohu);
  for(int i=0; i<tohu; i++) {
    int number;
    scanf("%d", &number);
    double answer;
  answer = 0.75 + (0.5/(number+2))-(0.5/(number+1));
  printf("%.11f\n",answer);
  }
  return 0;
}</pre>
```

o Code explanation:

The program starts by asking for an input (a single integer) called tohu (T). The program iterates "tohu" times and asks for a number (N). The program will output the result of Sn based on the input number using the formula.

o Evidence of accepted on SPOJ:



2. Problem 2

- The title of the problem: Amazing Factor Sequence
- Problem description:

Bhelu is the classmate of Bablu who made the Amazing Factor Sequence.

He felt jealous of his classmate and decides to make his own sequence. Since he was not very imaginative, he came up with almost the same definition just making a difference in f(n):

- a[0] = a[1] = 0.
- For n > 1, a[n] = a[n 1] + f(n), where f(n) is the sum of positive integers in the following set S
- $S = \{x \mid x < n \text{ and } n \% x = 0\}.$

Now, Bablu asked him to make a code to find f(n) as he already had the code of his sequence. So, Bhelu asks for a help since he doesn't know programming. The task is very simple, just find a[n] for a given value of $n < 10^6$.

Problem abstraction :

1. Input:

- First Line contains an integer T (<= 100), the number of test cases.
- \circ Next T lines contain a single positive integer N. (1 < N < 10⁶).

2. Output:

Single line containing a[n] i.e. n-th number of the sequence for each test case

3. Constraints:

- o T (<= 100)
- o (1 < N < 10⁶).

4. Example:

Input:

5 6

Explanation:

f(2) = 1 {1} f(3) = 1 {1} f(4) = 3 {1, 2} f(5) = 1 {1}

• Solution:

Algorithm Strategy & Observations :

The algorithm used in this problem is a modification of the Sieve of Erastothenes Algorithm, which is a well-known algorithm for finding all prime numbers up to a specified integer. However, in this case, instead of finding prime numbers, the algorithm is used to compute the AFS values for each integer up to a certain limit.

- ✓ The sequence a[n] can be computed using the formula a[n]=a[n-1]+f(n), where f(n) is the sum of positive integers that divide n (x % n = 0) and are less than n.
- ✓ The values of f(n) for $n \ge 2$ can be calculated efficiently by summing up the divisors of each number n.
 - a. Initialize an array a of size lim+10 to store the values of a[n].
 - b. Implement a function Iterate over ii from 2 to lim that :
 - For each ii, iterate over its multiples from 2i to lim.
 - Add ii to the corresponding positions in the array to calculate the sum of divisors.
 - Add 1 to the ith position in the array to account for f(i), effectively counting 1 as a factor of i.
 - c. Compute the cumulative sum of the array to get the values of a[n].
 - d. Read the number of test cases T.
 - e. For each test case, read the value of n and output a[n].
- o Theory References and Concepts:
- 1. Sum of Proper Divisors: Proper divisors are all positive divisors of a number excluding the number itself (sum of x where x % n = 0, x < n). For each number up to the limit (10^6 in this case), we need calculate the sum of its proper divisors.
- 2. Sieve of Eratosthenes Algorithm: This is an algorithm used to find all prime numbers up to a given limit. In this problem, it's used to calculate the sum of divisors of each number up to the limit and accumulate the sum of divisors for each number.

factors for each number in the given range using a function in the program (a[n] = a[n - 1] + $f(n)$).
o Pseudocode:
Constant lim = 1000000
ARRAY a[lim + 10]
FUNCTION solveAFS()
FOR i = 2 to lim
FOR j = 2*i to lim step i
a[j] += i
END FOR
FOR i = 2 to lim
a[i] += a[i-1]
END FOR
END FUNCTION
FUNCTION main ()
CALL solveAFS()
Declare INTEGER t,n
INPUT t
FOR i = 1 to t
INPUT n
OUTPUT a[n]
END FOR
END FUNCTION
o Source Code :

3. Recurrence relation: This recurrence relation is used to efficiently calculate the sum of

```
#include<bits/stdc++.h>
#define lim 1000000
using namespace std;
long long a[lim+10];
void solveAFS()
  for(int i=2; i<=lim; i++)
   for(int j = 2*i; j \le \lim_{j \to i} j = i)
     a[j] += i;
   a[i] += 1;
  for(int i = 2; i<=lim; i++)
    a[i] += a[i-1];
int main ()
  solveAFS();
  int t,n;
  scanf("%d",&t);
  for(int i=0; i<t; i++)
    scanf("%d",&n);
    printf("%lld\n",a[n]);
  }
  return 0;
```

o Code explanation:

The program starts by asking for an input (a single integer called t). The program iterates "t" times and asks for a number(n). The program will go back to the function called solveAFS using the algorithm and count the sum of proper divisors (f[n]) to output a[n].

o Evidence of Accepted on SPOJ:

