ECE 270: Computer Methods in ECE

Assignment #3

Understanding Sequences

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1 Statement of the Problem

We need to understand the behavior of a few given sequences.

2 Description of Solution

1. **Reverse**

   (a) A numerical example of the actual numbers of sequnce x and y:

   $x : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

   $y : 9, 8, 7, 6, 5, 4, 3, 2, 1, 0$

   (b) Generalized example:

   $x : x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$

   The sequence y is simply the reverse order of sequence x.

   $y : y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9$

   (c) The relationship between the two sequences is:

   $y_0 = x_9$

   $y_1 = x_8$

   $y_2 = x_7$

   $y_3 = x_6$

   $y_4 = x_5$

   $y_5 = x_4$

   $y_6 = x_3$

   $y_7 = x_2$

   $y_8 = x_1$

   $y_9 = x_0$

   A general expression to represent this relationship for 10 elements is:

   $y_i = x_{9-i} \qquad i = 0, 1, 2....9$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n terms is necessary.

$x : x_0, x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_0, y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}, y_n$

Each Element of y can be written i terms of x:

$y_0 = x_n$

$y_1 = x_{n-1}$

$y_2 = x_{n-2}$

...

$y_{n-2} = x_2$

$y_{n-1} = x_1$

$y_n = x_0$

A general expression to represent this relationship for n elements is:

$y_i = x_{n-i} \qquad i = 0, 1, 2, ...n$

2. **Subsample**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$

$y : 30, 60, 90$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y simply takes every third element of the sequence x.

$y : y_1, y_2, y_3$

(c) The relationship between the two sequences is:

$y_1 = x_3$

$y_2 = x_6$

$y_3 = x_9$

A general expression to represent this relationship for 10 elements is:

$y_i = x_{3i} \qquad i = 1, 2....10$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}, y_n$

Each Element of y can be written i terms of x:

$y_1 = x_3$

$y_2 = x_6$

...

$y_{n-2} = x_{3n-6}$

$y_{n-1} = x_{3n-3}$

$y_n = x_{3n}$

A general expression to represent this relationship for n elements is:

$y_i = x_{3i} \qquad i = 1, 2, ..., n$

3. **Shift Right**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$

$y : 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y shifts the sequence x by implementing a 0 as a first element.

$y : y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}$

(c) The relationship between the two sequences is:

$y_0 = 0$

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

$y_4 = x_4$

$y_5 = x_5$

$y_6 = x_6$

$y_7 = x_7$

$y_8 = x_8$

$y_9 = x_9$

$y_{10} = x_{10}$

A general expression to represent this relationship for 10 elements is:

$y_i = x_i \qquad i = 1, 2....10 \qquad y_0 = 0$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_0, y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}, y_n$

Each Element of y can be written i terms of x:

$y_0 = 0$

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

...

$y_{n-2} = x_{n-2}$

$y_{n-1} = x_{n-1}$

$y_n = x_n$

A general expression to represent this relationship for n elements is:

$y_i = x_i \qquad i = 1, 2, ...n \qquad y_0 = 0$

4. **Shift Right by 2**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$

$y : 0, 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y shifts the sequence x by 2 implementing a 0 as a first two element.

$y : y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}$

(c) The relationship between the two sequences is:

$y_0 = 0$

$y_1 = 0$

$y_2 = x_1$

$y_3 = x_2$

$y_4 = x_3$

$y_5 = x_4$

$y_6 = x_5$

$y_7 = x_6$

$y_8 = x_7$

$y_9 = x_8$

$y_{10} = x_9$

$y_{11} = x_{10}$

A general expression to represent this relationship for 10 elements is:

$y_i = x_{i-1} \qquad i = 2....10 \qquad y_0 = 0,\ y_1 = 0$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_0, y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}, y_n$

Each Element of y can be written i terms of x:

$y_0 = 0$

$y_1 = 0$

$y_2 = x_1$

$y_3 = x_2$

$y_4 = x_3$

...

$y_{n-2} = x_{n-2}$

$y_{n-1} = x_{n-1}$

$y_n = x_n$

A general expression to represent this relationship for n elements is:

$y_i = x_{i-1} \qquad i = 2, ...n \qquad y_0 = 0,\ y_1 = 0$

5. **Repeat**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 0, 1, 2$

$y : 0, 1, 2, 0, 1, 2, 0, 1, 2$

(b) Generalized example:

$x : x_1, x_2, x_3$

The sequence y repeats the sequence x, $k = 3$ times.

$y : y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9$

(c) The relationship between the two sequences is:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

$y_4 = x_1$

$y_5 = x_2$

$y_6 = x_3$

$y_7 = x_1$

$y_8 = x_2$

$y_9 = x_3$

A general expression to represent this relationship for 3 elements is:

$y_i = x_{1+i\%3} \qquad i = 1....9$

(d) However the sequences x can have any number of elements n and y can have any number of repetitions k.

Therefore, a generalized expression for n, k terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{kn-2}, y_{kn-1}, y_{kn}$

Each Element of y can be written i terms of x:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_2$

...

$y_{kn-2} = x_{n-2}$

$y_{kn-1} = x_{n-1}$

$y_{kn} = x_n$

A general expression to represent this relationship for n elements is:

$y_i = x_{1+i\%k} \qquad i = 1, ...n.$

6. **Up Sample**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 0, 1, 2$

$y : 0, 0, 1, 1, 2, 2$

(b) Generalized example:

$x : x_1, x_2, x_3$

The sequence y repeats the each element of the sequence x, $k = 1$ time(s).

$y : y_1, y_2, y_3, y_4, y_5, y_6$

(c) The relationship between the two sequences is:

$y_1 = x_1$

$y_2 = x_1$

$y_3 = x_2$

$y_4 = x_2$

$y_5 = x_3$

$y_6 = x_3$

A general expression to represent this relationship for 3 elements is:

$y_i = x_{\lceil \frac{i}{2} \rceil} \qquad i = 1....6$

(d) However the sequences x can have any number of elements n and y can have any number of repetitions k.

Therefore, a generalized expression for n, k terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{(k+1)n-2}, y_{(k+1)n-1}, y_{(k+1)n}$

Each Element of y can be written i terms of x:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_2$

...

$y_{(k+1)n-2} = x_{n-2}$

$y_{(k+1)n-1} = x_{n-1}$

$y_{(k+1)n} = x_n$

A general expression to represent this relationship for n elements is:

$y_i = x_{\lceil \frac{i}{k+1} \rceil} \qquad i = 1, ...n.$

7. **Partial Sums**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$y : 1, 3, 6, 10, 15, 21, 28, 36, 45, 55$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y is the partial sum of the previous elements of x.

$y : y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}$

(c) The relationship between the two sequences is:

$y_1 = x_1$

$y_2 = x_1 + x_2$

$y_3 = x_1 + x_2 + x_3$

$y_4 = x_1 + x_2 + x_3 + x_4$

$y_5 = x_1 + x_2 + x_3 + x_4 + x_5$

$y_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

$y_7 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

$y_8 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

$y_9 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$

$y_{10} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

A general expression to represent this relationship for 10 elements is:

$y_i = x_i + x_{i+1} \qquad i = 1....10$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n terms is necessary.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}, y_n$

Each element of y can be written i terms of x:

$y_1 = x_1$

$y_2 = x_1 + x_2$

$y_3 = x_1 + x_2 + x_3$

...

$y_{n-2} = x_1 + x_2 + x_3 + ... + x_{n-4} + x_{n-3} + x_{n-2}$

$y_{n-1} = x_1 + x_2 + x_3 + ... + x_{n-3} + x_{n-2} + x_{n-1}$

$y_n = x_1 + x_2 + x_3 + ... + x_{n-2} + x_{n-1} + x_n$

A general expression to represent this relationship for n elements is:

$y_i = x_i + x_{i+1} \qquad i = 1, ...n.$

8. **Delete**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$y : 1, 2, 3, 4, 6, 7, 8, 9, 10$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y a random element x_5 from x.

$y : y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9$

(c) The relationship between the two sequences is:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

$y_4 = x_4$

$y_5 = x_6$

$y_6 = x_7$

$y_7 = x_8$

$y_8 = x_9$

$y_9 = x_{10}$

A general expression to represent this relationship for 10 elements is:

$$y_i = \begin{cases} x_i & \text{if } i = 0, 1, 2, 3, 4 \\ x_{i+1} & \text{if } i = 5, 6, 7, 8, 9 \end{cases}$$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n, k terms is necessary.Where k is the subscript of the deleted element.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{n-2}, y_{n-1}$

Each Element of y can be written i terms of x:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

...

$y_{k-1} = x_{k-1}$

$y_k = x_{k+1}$

..

$y_{n-1} = x_n$

$y_n = x_{n+1}$

A general expression to represent this relationship for n elements is:

$$y_i = \begin{cases} x_i & \text{if } i = 0, 1, 2, ..., k-1 \\ x_{i+1} & \text{if } i = k, ..., n \end{cases}$$

9. **Insert**

(a) A numerical example of the actual numbers of sequnce x and y:

$x : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$y : 1, 2, 3, 4, 5, 13, 6, 7, 8, 9, 10$

(b) Generalized example:

$x : x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$

The sequence y inserts a random element y_6 into x.

$y : y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}$

(c) The relationship between the two sequences is:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

$y_4 = x_4$

$y_5 = x_5$

$y_6 = 13$

$y_7 = x_6$

$y_8 = x_7$

$y_9 = x_8$

$y_{10} = x_9$

$y_{11} = x_{10}$

A general expression to represent this relationship for 10 elements is:

$$y_i = \begin{cases} x_i & \text{if } i = 0, 1, 2, 3, 4, 5 \\ x_{i-1} & \text{if } i = 6, 7, 8, 9, 10, 11 \end{cases}$$

(d) However the sequences x can have any number of elements n. Therefore, a generalized expression for n, k, p terms is necessary. Where K is the subscript of the inserted element and p is it's value.

$x : x_1, x_2, x_3, ..., x_{n-2}, x_{n-1}, x_n$

$y : y_1, y_2, y_3, ..., y_{n-1}, y_n, y_{n+1}$

Each Element of y can be written i terms of x:

$y_1 = x_1$

$y_2 = x_2$

$y_3 = x_3$

...

$y_{k-1} = x_{k-1}$

$y_k = p$

$y_{k+1} = x_k$

..

$y_n = x_{n-1}$

$y_{n+1} = x_n$

A general expression to represent this relationship for n elements is:

$$y_i = \begin{cases} x_i & \text{if } i = 0, 1, 2, ..., k \\ x_{i-1} & \text{if } i = k + 1, ..., n \end{cases}$$