ENGR-3000:

Renewable Energy, Technology, and Resource Economics

Supplemental Lecture: Average Power in the Wind

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World Learning SIT Study Abroad

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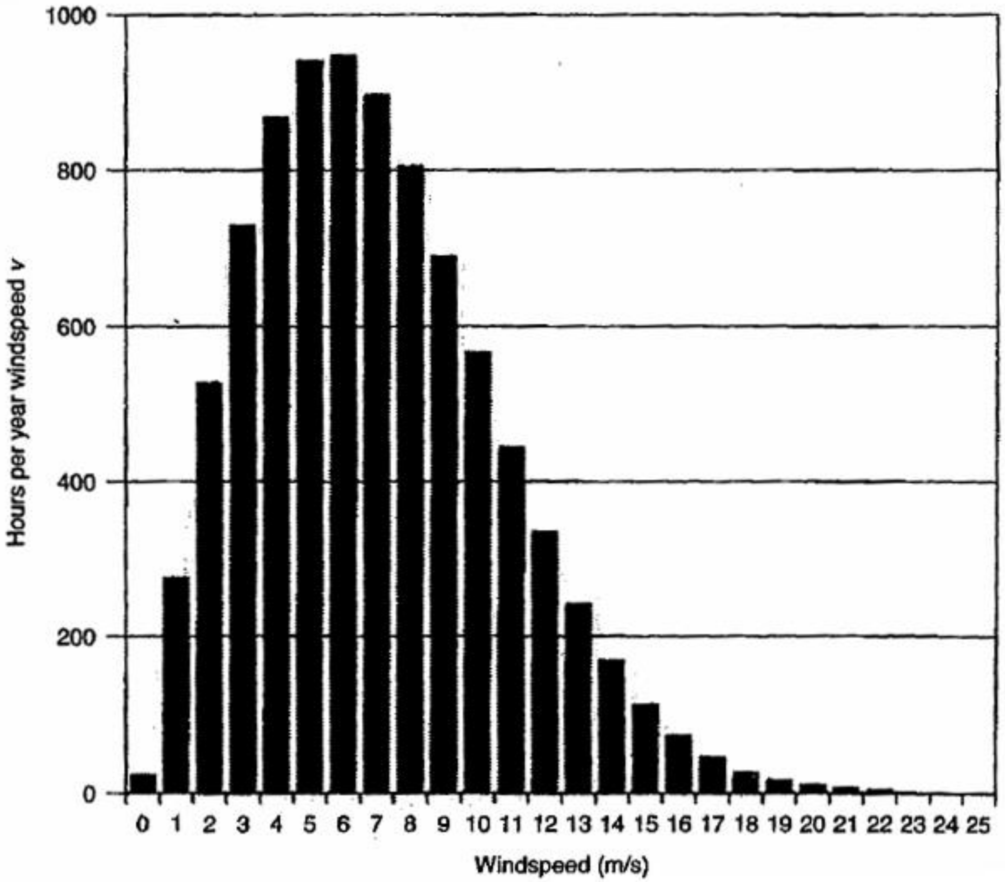
Wind Speed Frequency Distribution

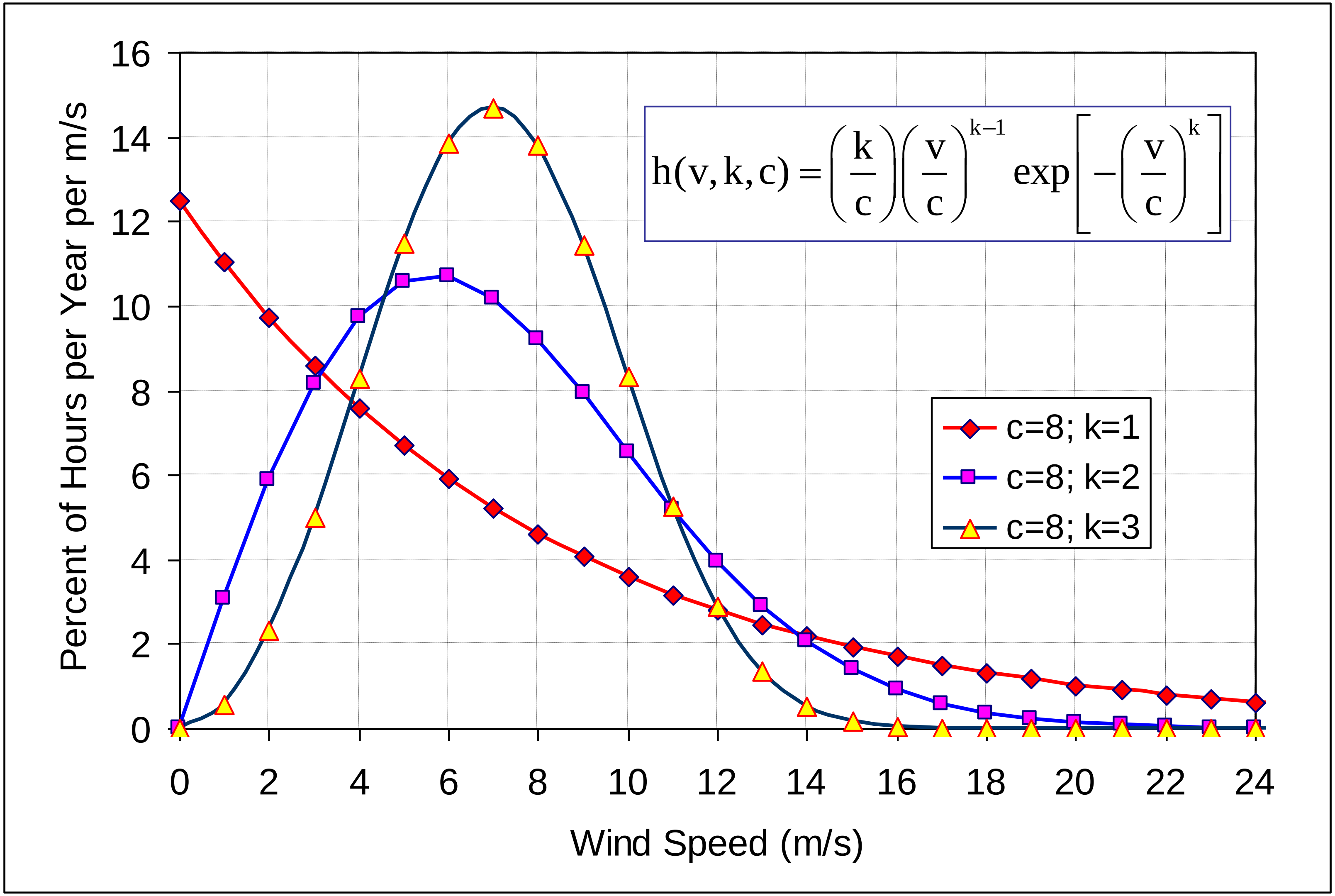
- Each vertical bar indicates how many hours/year the wind is blowing at that speed.
- The total adds up to 8760 hours.

v (m/s)	Hrs/yr
0	24
1	276
2	527
3	729
4	869
5	941
6	946
7	896
8	805
9	690
10	565
11	444
12	335
13	243
14	170
15	114
16	74
17	46
18	28
19	16
20	9
21	5
22	3
23	1
24	1
25	0
Total hrs	8,760



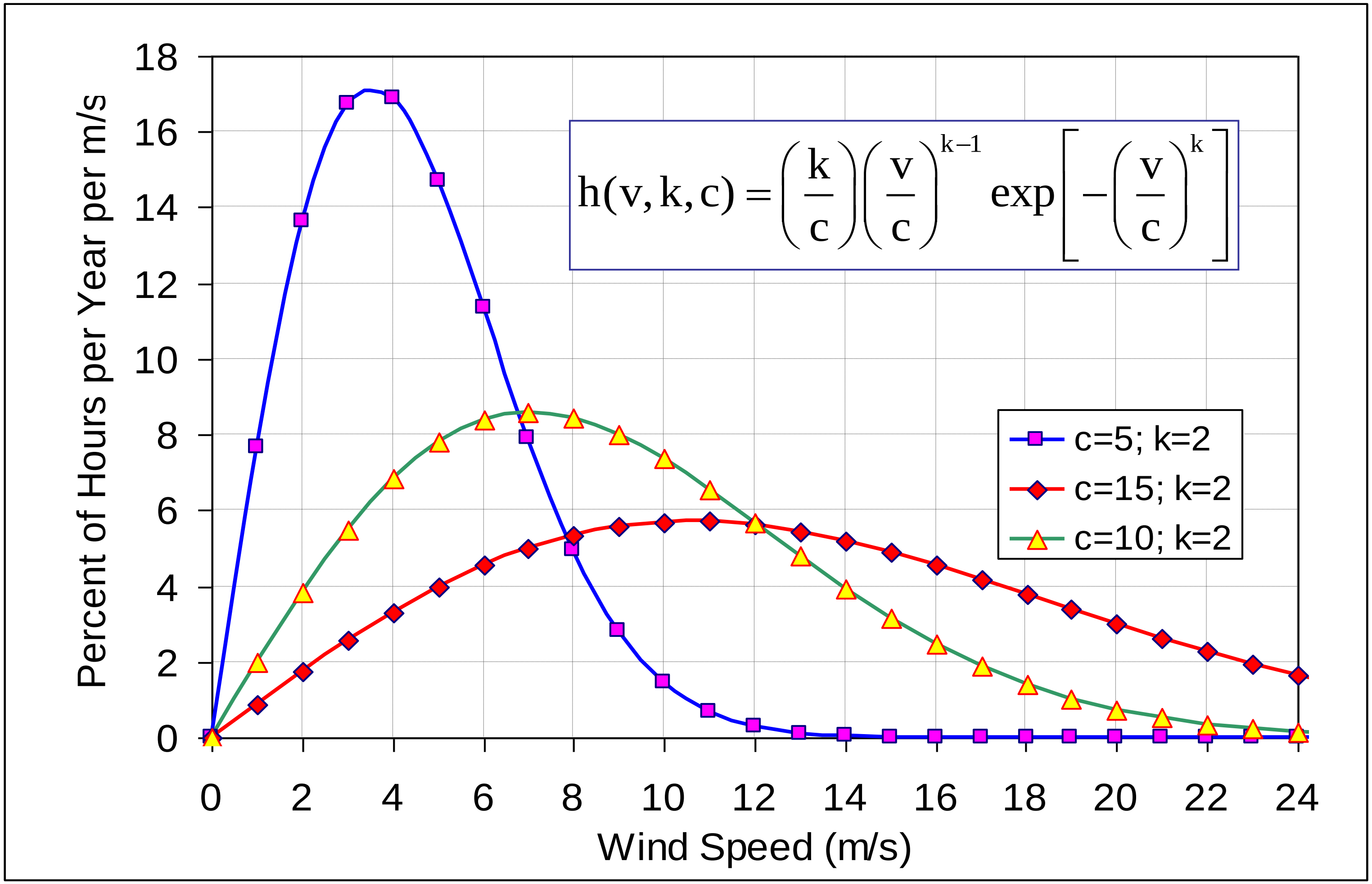
Wiebull Distribution

- For different values of the shape parameter k:



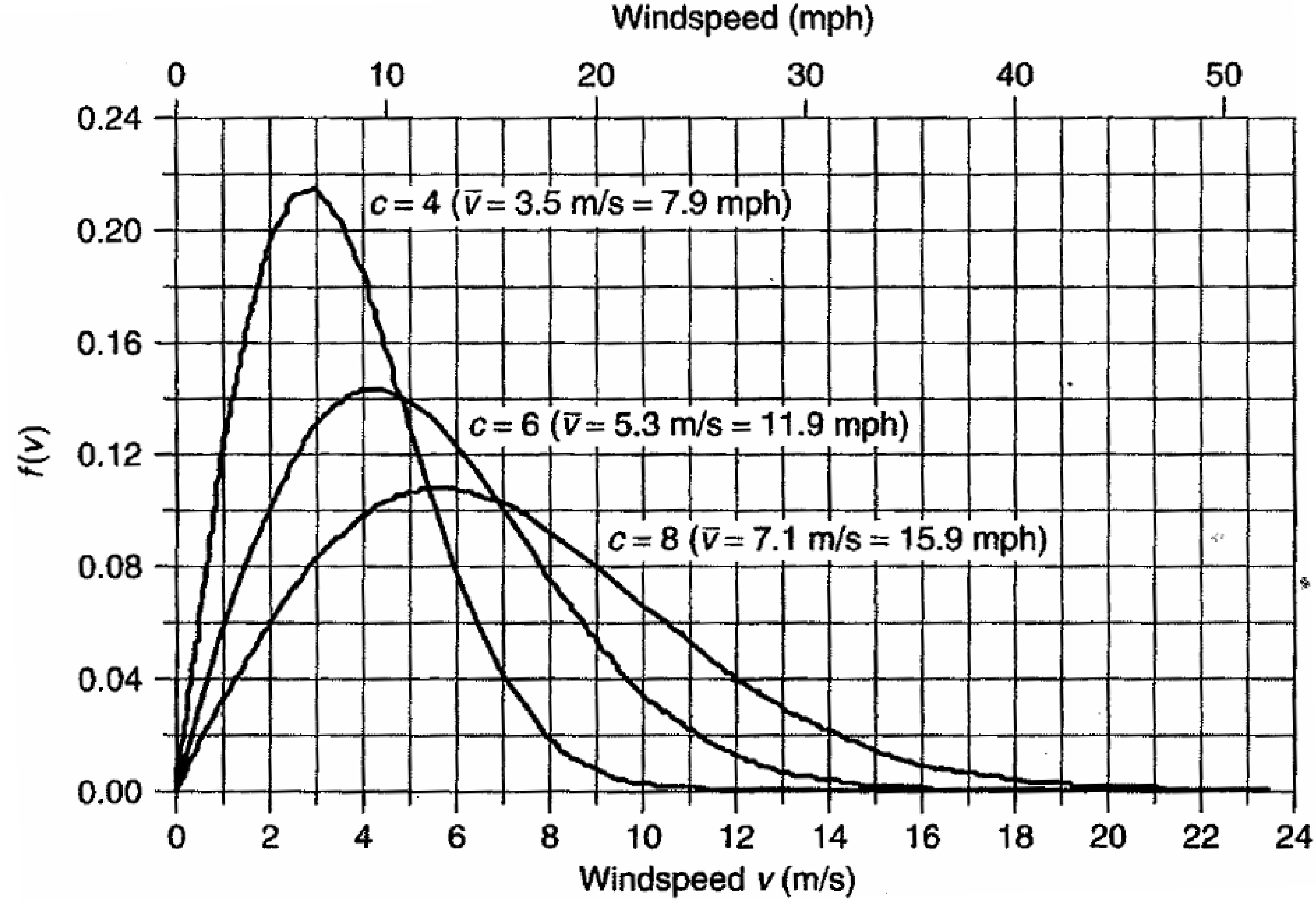
Wiebull Distribution

- For different values of the scale parameter c:



Rayleigh Disribution. (Weibull with k=2)

- Higher *c* implies higher average windspeeds



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Rayleigh p.d.f.

- When using a Rayleigh p.d.f., there is a direct relationship between average windspeed *v* and scale parameter *c*

$$v_{avg} = \bar{v} = \int_0^{\infty} v \cdot f(v)\,dv \qquad f(v) = \frac{2v}{c^2} \cdot e^{-\left(\frac{v}{c}\right)^2}$$

- The Rayleigh pdf into the integral for v_{avg} :

$$v_{avg} = \bar{v} = \int_0^{\infty} v \cdot \frac{2v}{c^2} \cdot e^{-\left(\frac{v}{c}\right)^2} dv \Rightarrow v_{avg} = \frac{\sqrt{\pi}}{2} c \cong 0.886 \cdot c$$

Thus our Rayleigh distribution function can be written as

$$f(v) = \frac{\pi v}{2\bar{v}^2} \cdot e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2}$$

Power in the Wind

$$\text{Power} = \frac{1}{2}\rho A V^3$$

- Where:
  - ρ = air density (kg/m^3)
  - Area (m^2)
  - V = upstream (undisturbed) wind velocity (m/s)
  - Power (Watts)

Rayleigh Statistics – Average Power in the Wind

- To figure out average power in the wind, we need to know the average value of the ***cube*** of velocity:

$$P_{avg} = \left(\frac{1}{2}\rho A v^3\right)_{avg} = \frac{1}{2}\rho A \left(v^3\right)_{avg}$$

- For a wind speed distrbution function f(v):

$$\left(v^3\right)_{avg} = \int_0^\infty v^3 \cdot f(v)dv$$

Rayleigh Statistics – Average Power in the Wind

- Assume the wind speed distribution is a Rayleigh distribution

$$f(v) = \frac{2v}{c^2} \cdot e^{-\left(\frac{v}{c}\right)^2} \qquad \bar{v} = \frac{\sqrt{\pi}}{2} c \qquad f(v) = \frac{\pi v}{2\bar{v}^2} \cdot e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2}$$

$$\left(v^3\right)_{avg} = \int_0^\infty v^3 \cdot f(v) dv = \int_0^\infty v^3 \cdot \frac{2v}{c^2} \cdot e^{-\left(\frac{v}{c}\right)^2} dv = \frac{3}{4} c^3 \sqrt{\pi}$$

Rayleigh Statistics – Average Power in the Wind

- This is $(v^3)_{avg}$ in terms of c, but we can

$$\left(v^3\right)_{avg} = \frac{3}{4} c^3 \sqrt{\pi}$$

$$c = \frac{2}{\sqrt{\pi}} v_{avg}$$

- Then we have $(v^3)_{avg}$ in terms of v_{avg} :

$$\Rightarrow \quad \left(v^3\right)_{avg} = \frac{6}{\pi} \left(v_{avg}\right)^3$$

Rayleigh Statistics – Average Power in the Wind

- With ***Rayleigh assumptions***, we can write the $(v^3)_{avg}$ in terms of v_{avg} , and the expression for average power in the wind is just

$$P_{avg} = \frac{6}{\pi} \cdot \frac{1}{2} \rho A \left(v_{avg} \right)^3$$