ENGR-3000:
Renewable Energy, Technology, and Resource Economics

Introduction to Wind Power

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World Learning SIT Study Abroad

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An Introduction to Wind Power: Outline

- **Origins**
- **Resource Characterization: Measurements**
- **Energy Characterization**
  - **Kinetic Energy**

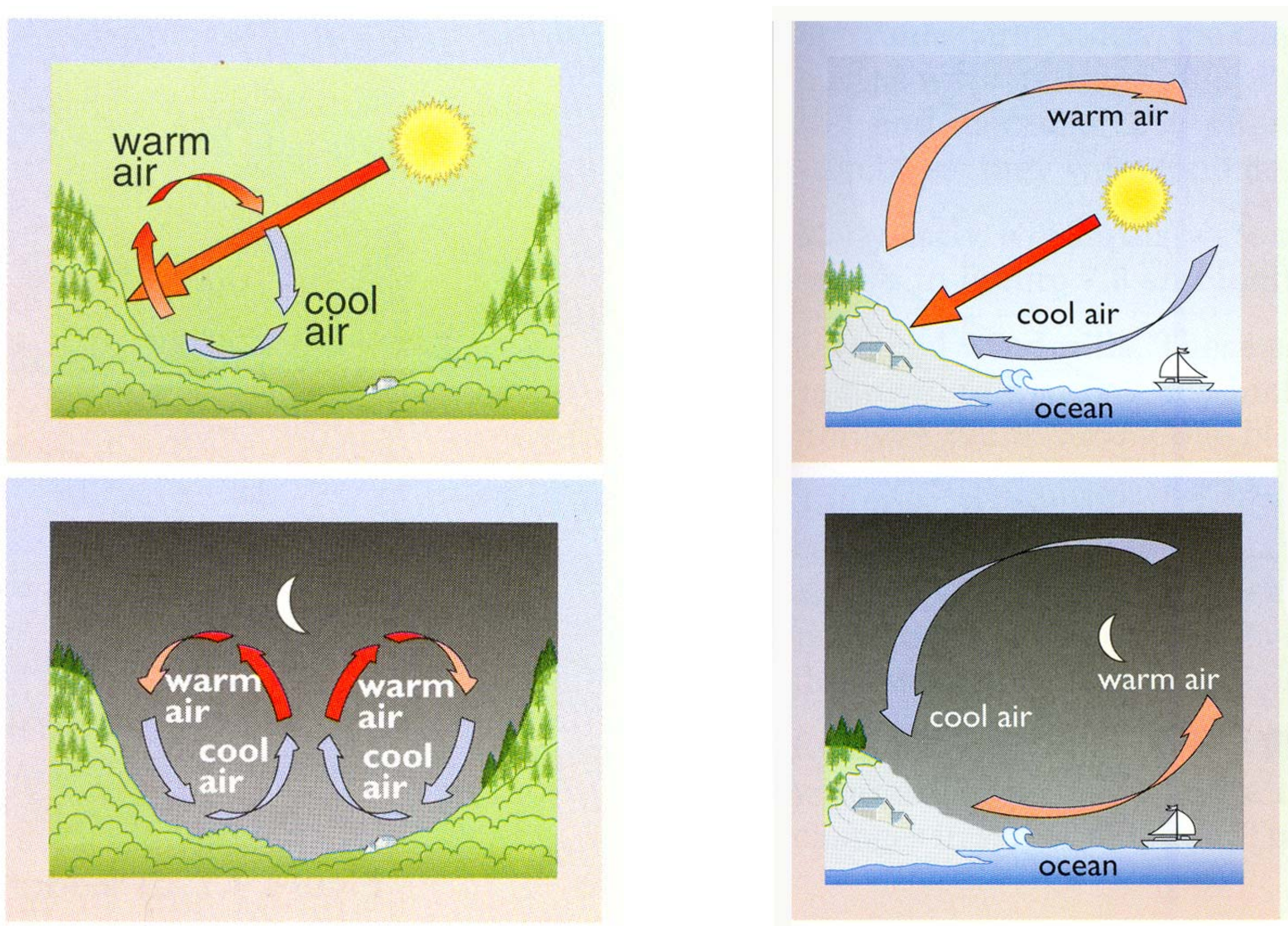
Wind Energy: Origins

- Winds are caused primarily by convection due to alternate heating and cooling cycles, combined with airflow around or over geographical features.

- Air flows between areas of high and low pressure. The rotation of the earth changes the direction of these airflow patterns:

Origins of Wind Power

- Winds due to convection caused by differential heating of the earths surface: essentially solar power



Global Wind Patterns

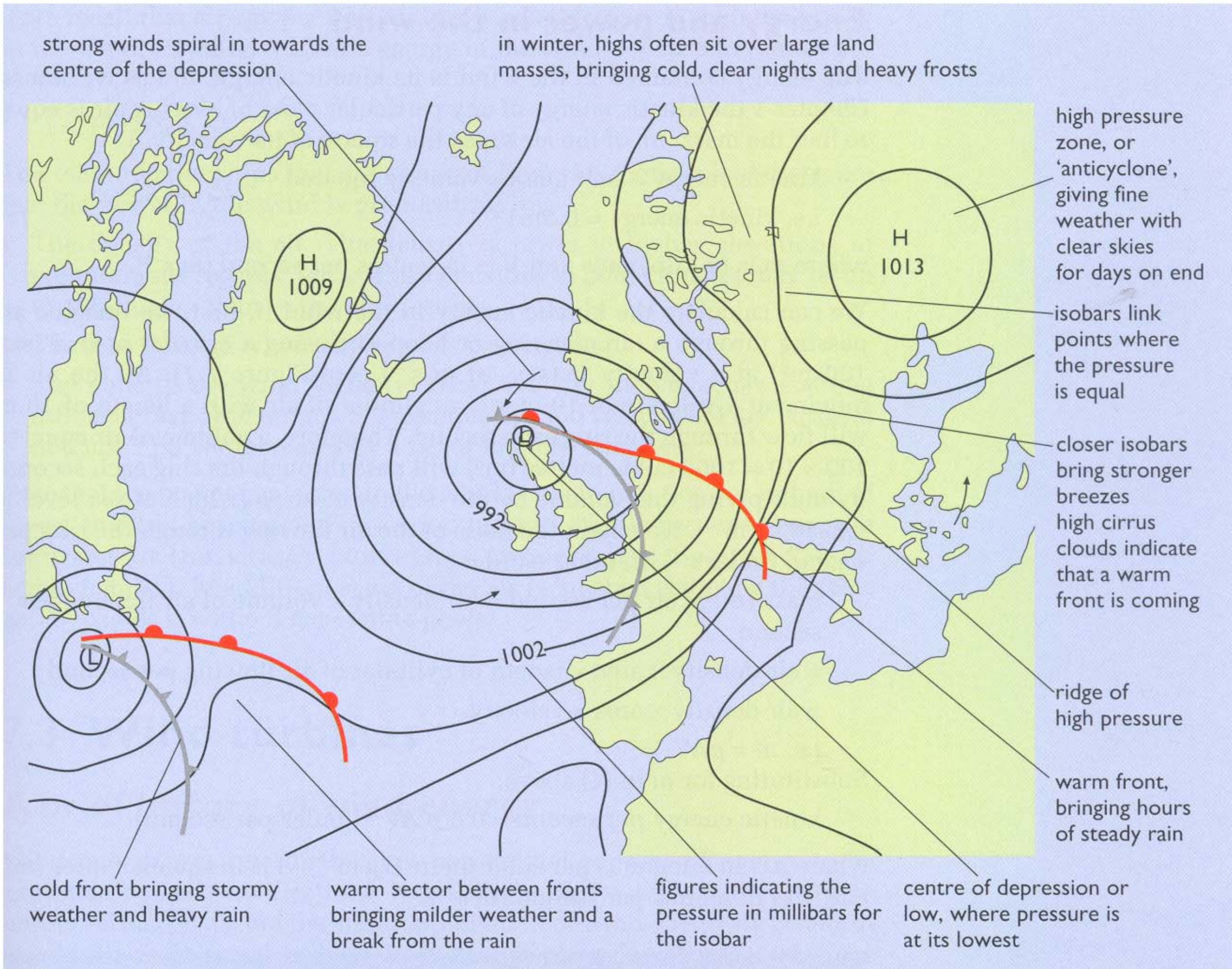
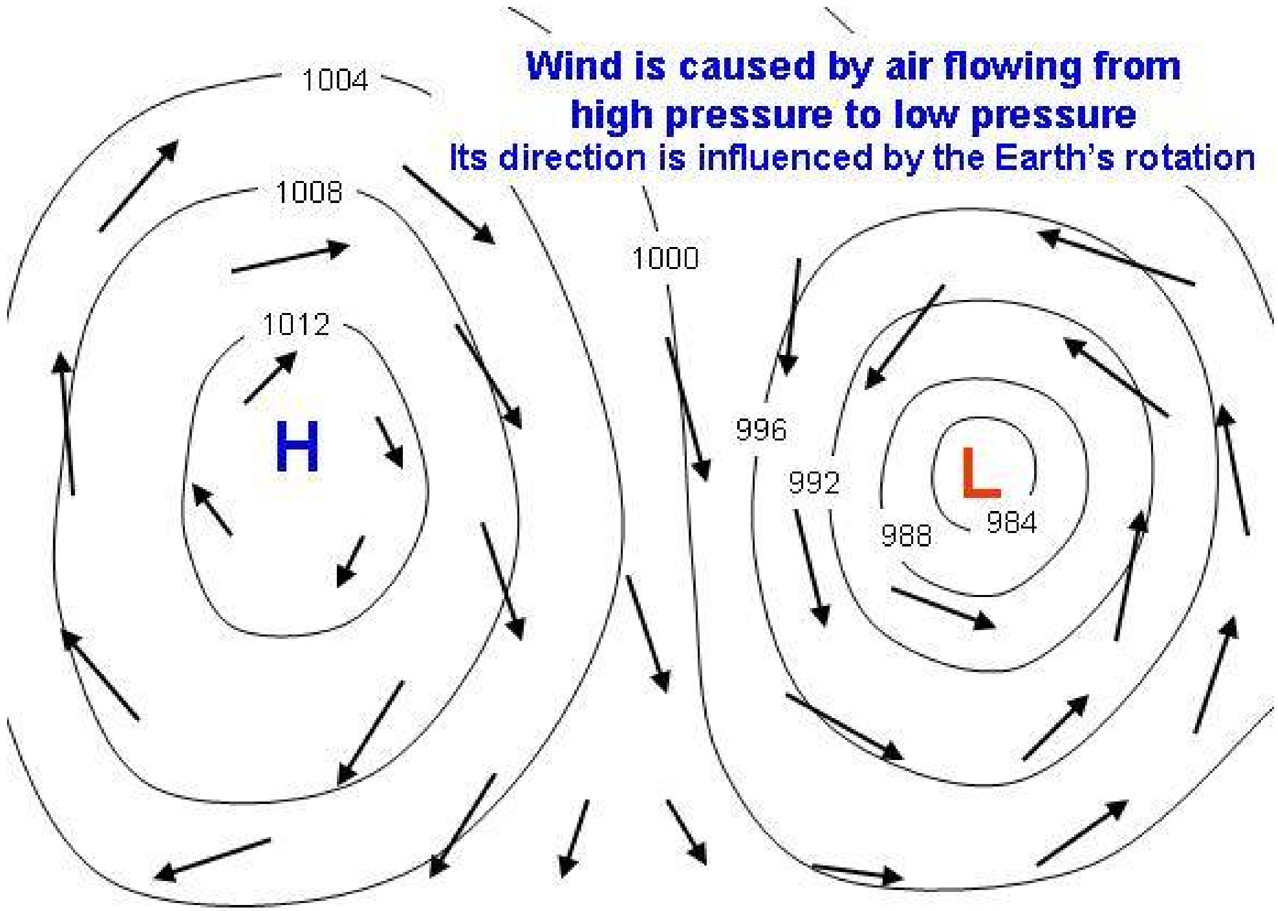
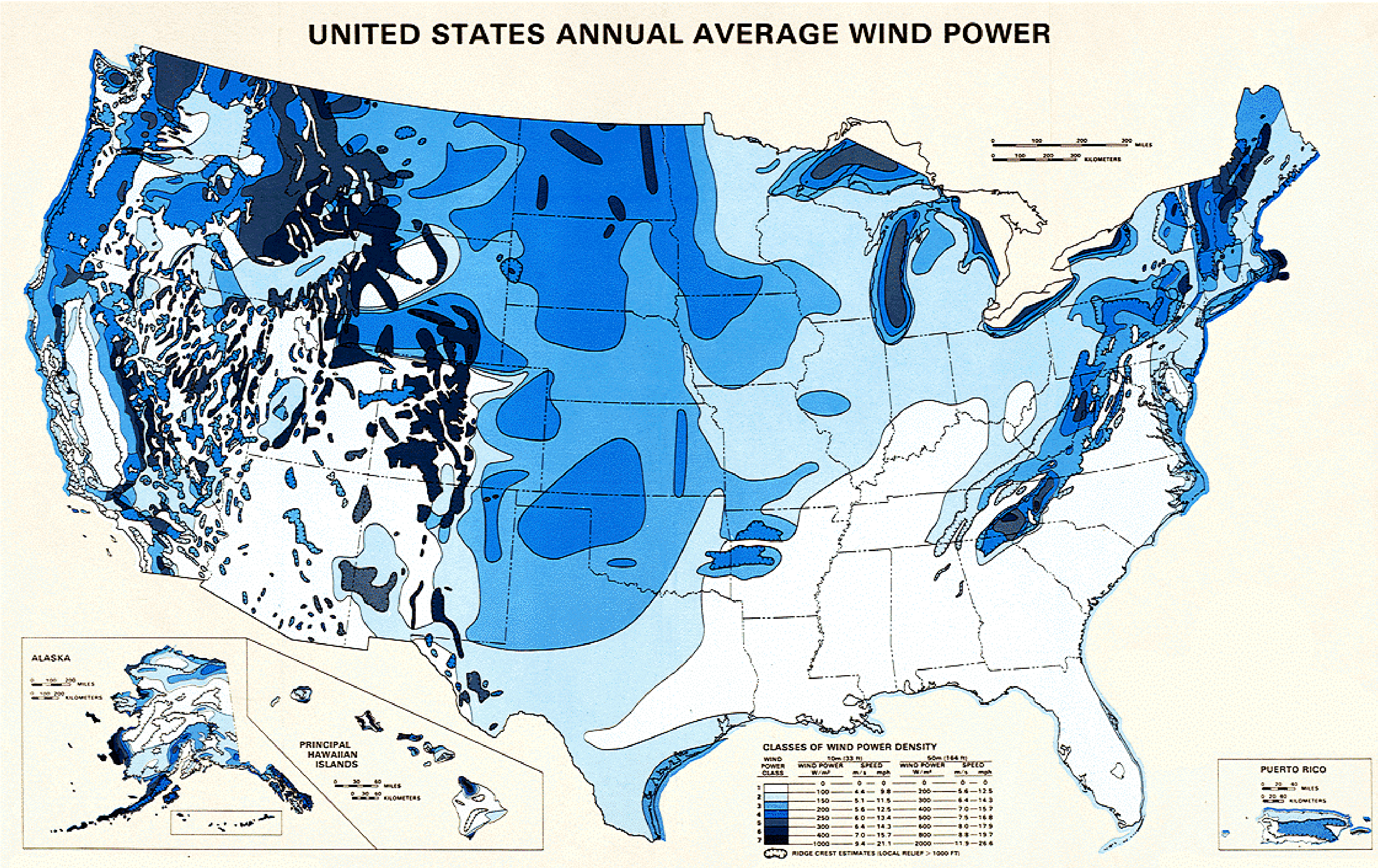


Figure 7.4 Typical weather map showing regions of high (H) and low (L) pressure

Air flows between areas of high and low pressure. The rotation of the earth changes the directon of these airflow patterns:



United States Annual Average Wind Power



NREL: World Energy Resource of the United States

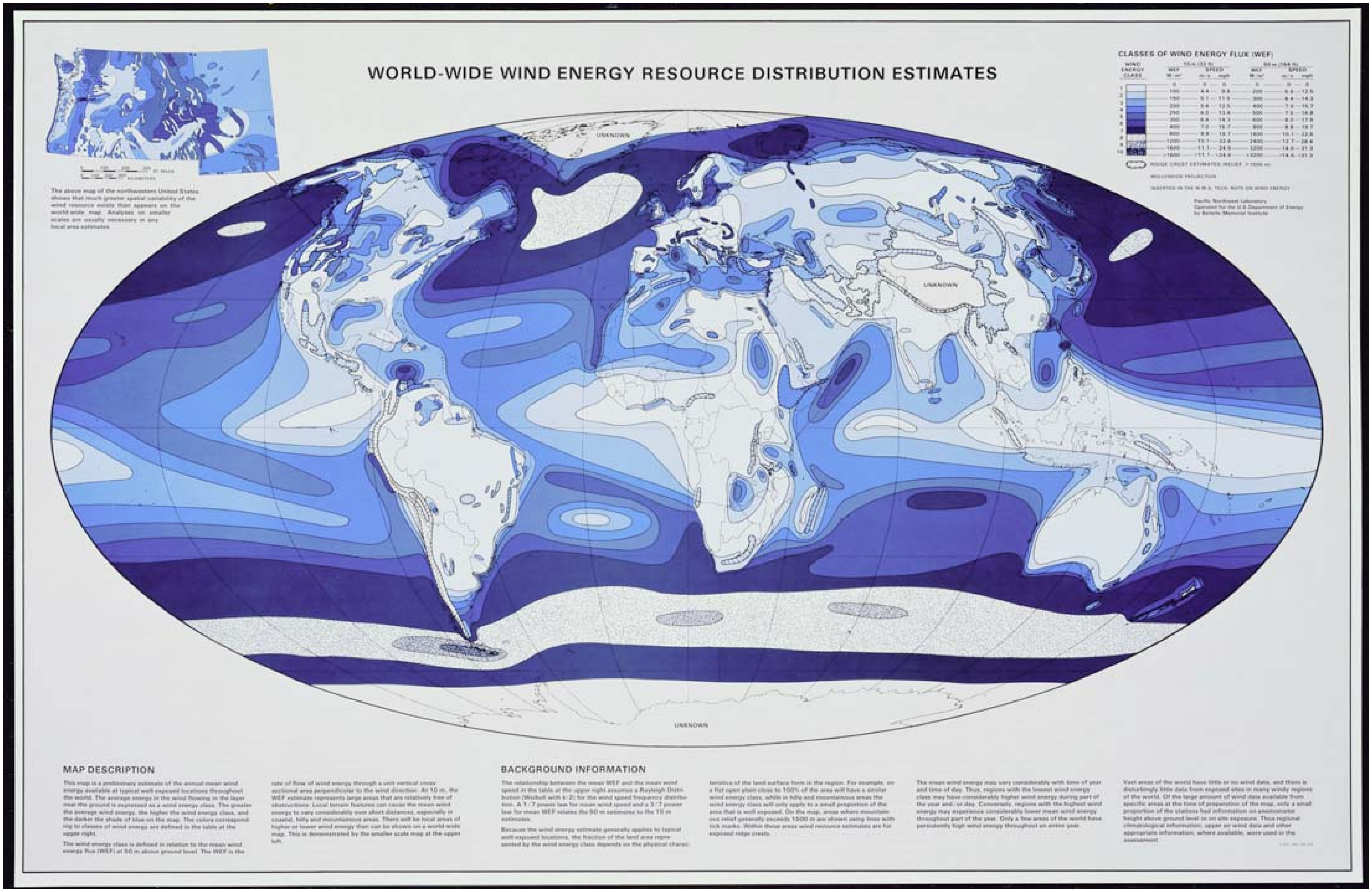
Classes of Wind Power Density

Classes of Wind Power Density

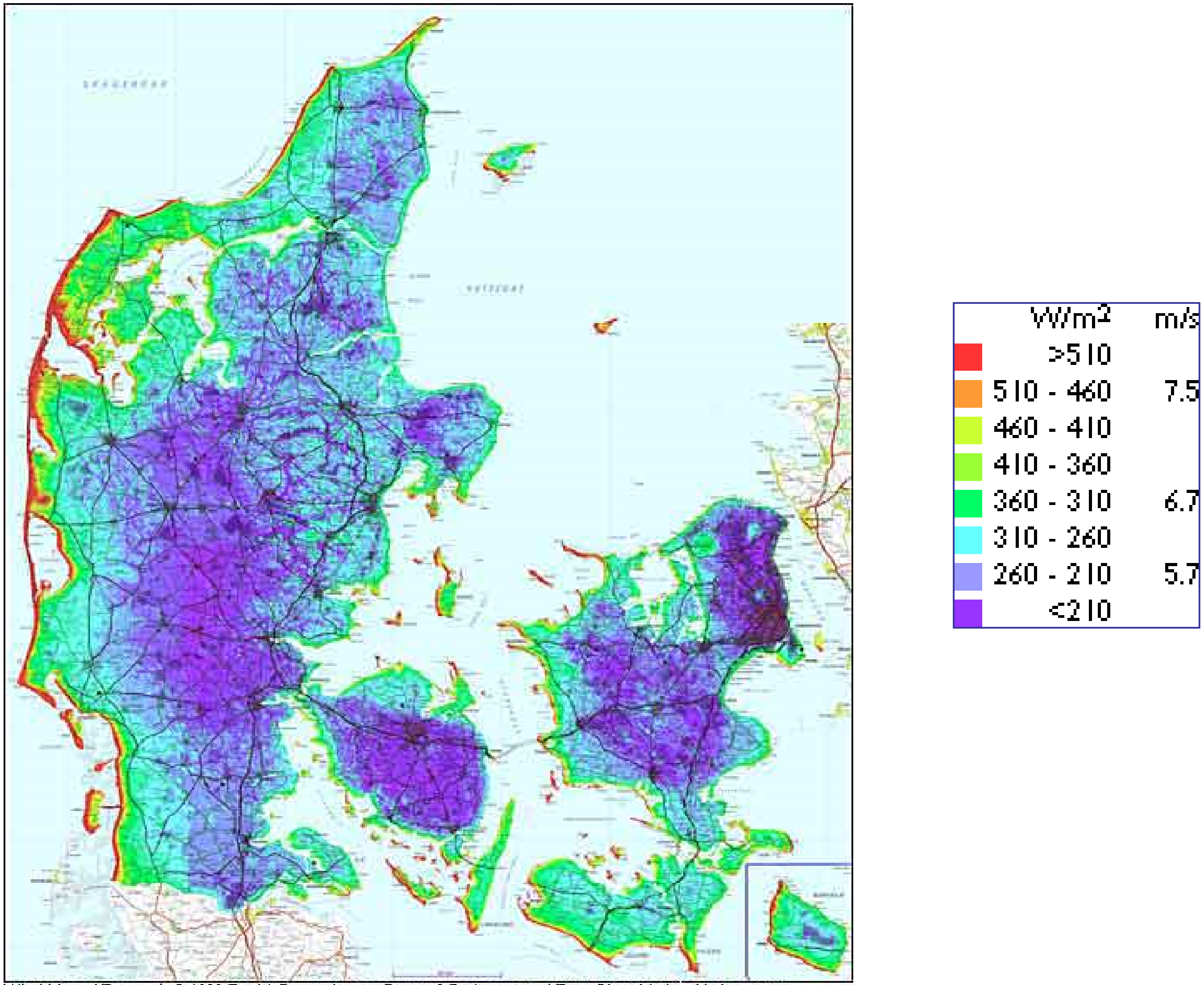
	30 m (98 ft)		50 m (164 ft)	
Wind Power Class	Wind Power Density (W/m²)	Wind Speed m/s (mph)	Wind Power Density (W/m²)	Wind Speed m/s (mph)
1	≤160	≤5.1 (11.4)	≤200	≤5.6 (12.5)
2	≤240	≤5.9 (13.2)	≤300	≤6.4 (14.3)
3	≤320	≤6.5 (14.6)	≤400	≤7.0 (15.7)
4	≤400	≤7.0 (15.7)	≤500	≤7.5 (16.8)
5	≤480	≤7.4 (16.6)	≤600	≤8.0 (17.9)
6	≤640	≤8.2 (18.3)	≤800	≤8.8 (19.7)
7	≤1600	≤11.0 (24.7)	≤2000	≤11.9 (26.6)

http://www.nrel.gov/wind/pdfs/22223.pdf

World-Wide Energy Resource Distribution Estimates



Wind Map of Denmark



Wind Map of Denmark © 1999 Danish Energy Agency, Energy & Environmental Data, Risoe National Laboratory

http://www.windpower.org/en/tour/wres/dkmap.htm

Impact of Elevation and Earth's Roughness on Windspeed

- There is a lot of friction in the first few hundred meters above ground – smooth surfaces (like water) are better
- Windspeeds are greater at higher elevations – tall towers are better
- Forests and buildings slow the wind down a lot
- Can characterize the impact of rough surfaces and height on wind speed

Impact of Elevation and Earth's Roughness on Windspeed

$$\frac{v}{v_0} = \left(\frac{H}{H_0} \right)^{\alpha}$$

- α = friction coefficient:
  - Typical value of α in open terrain is 1/7
  - For a large city, α = 0.4; for calm water, α = 0.1
- v = windspeed at height H
- v_0 = windspeed at height H_0 (H_0 is usually 10 m)

Impact of Elevation and Earth's Roughness on Windspeed

- Alternative formulation (used in Europe)

$$\frac{v}{v_0} = \frac{\ln(H / z)}{\ln(H_0 / z)}$$

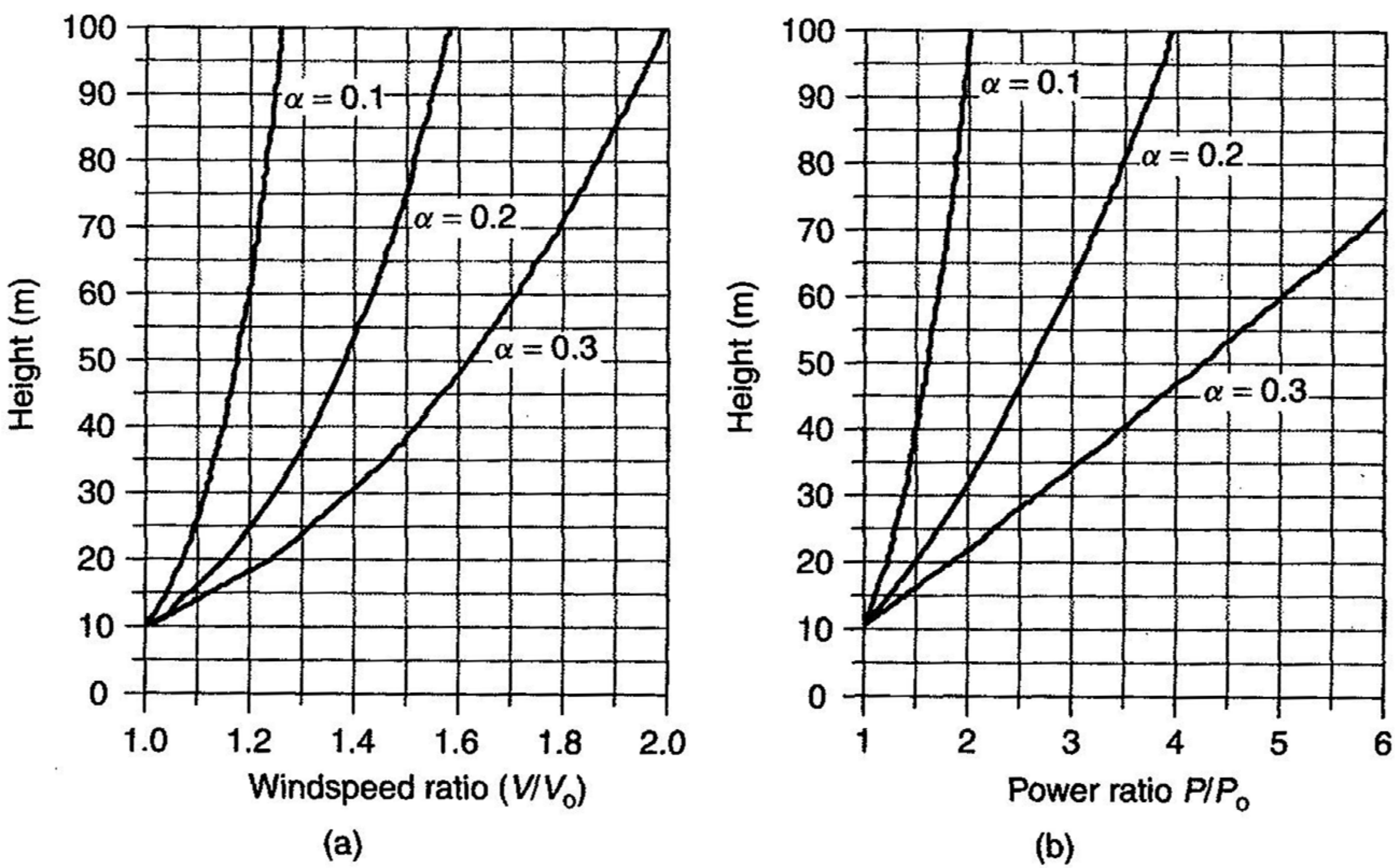
- *z* is the "roughness length"
- Note that these equations are just approximations of the variation in windspeed due to elevation and roughness– the best thing is to have actual measurements
  - Use anemometer tower

Roughness Lengths

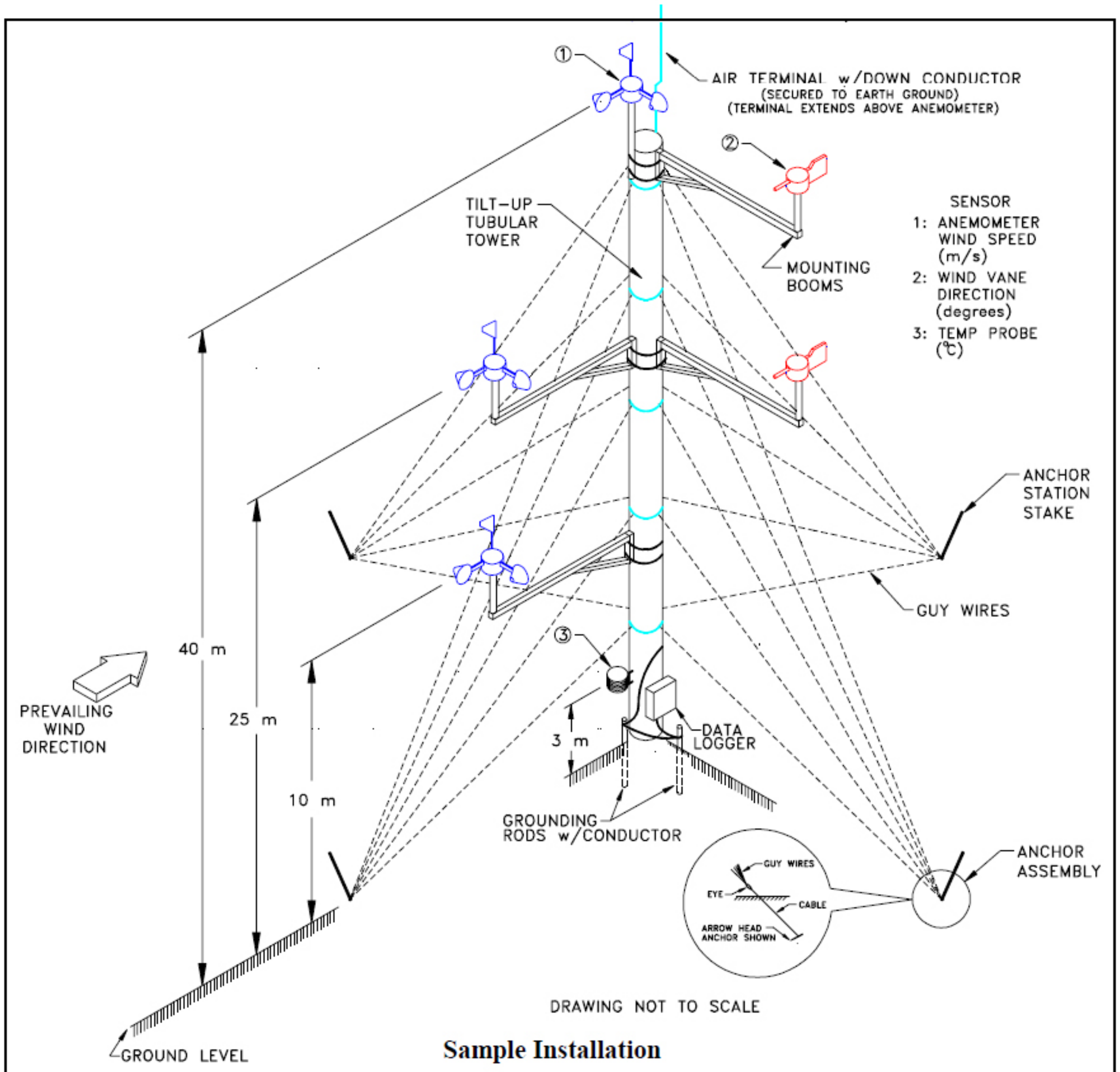
Category	Z_0 (m)		
1	0.0002	Sea	Open sea or lake
2	0.005	Smooth	featureless land surface
3	0.03	Open	level country with low vegetation
4	0.1	Roughly Open	cultivated or natural areas with low crops with occasional obstacles
5	0.25	Rough	Scattered obstacles at relative distances of 8 to 15 obstacle heights
6	0.5	Very Rough	Large obstacle groups separated by open spaces of about 8 obstacle heights
7	1	Skimming	landscape covered with similar-size objects, mature forests
8	2	Chaotic	City centers, mixture of low-rise and high-rise buildings, or forests with trees of varying height

Weiringa et. al., “New revision of Davenport Roughness Classification, 3EACWE 2001.

Impact of Elevation and Earth's Roughness on Windspeed

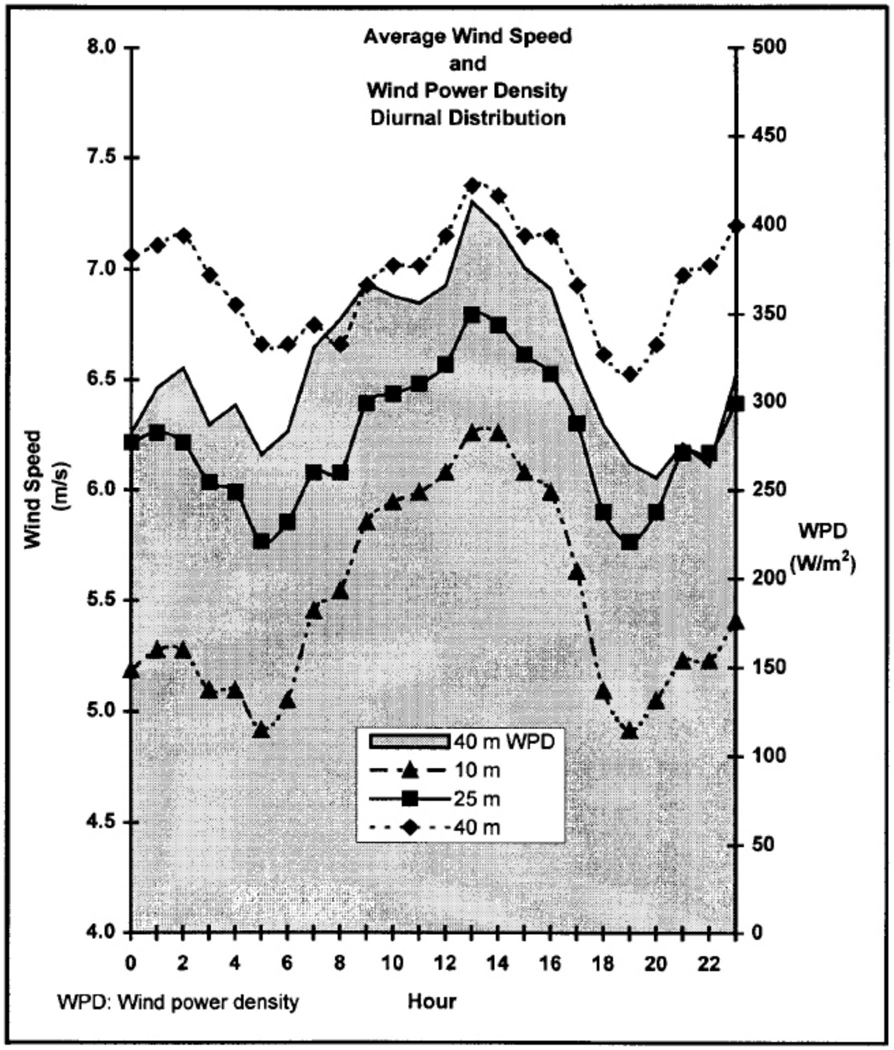


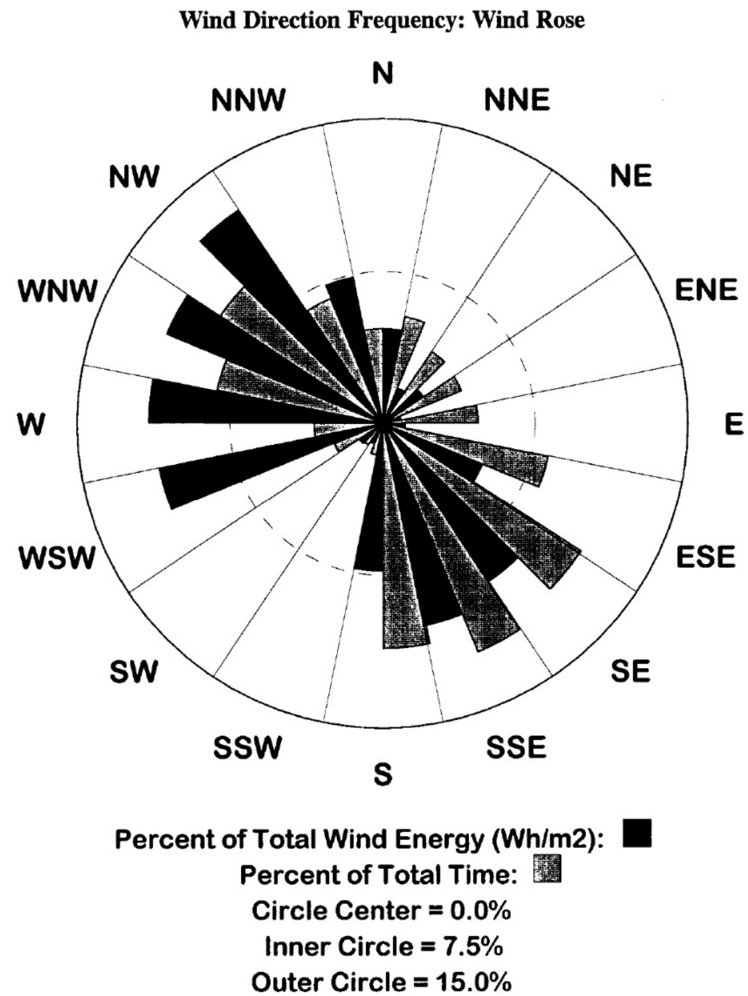
Measuring Wind



Sample Installation

Wind Measurements

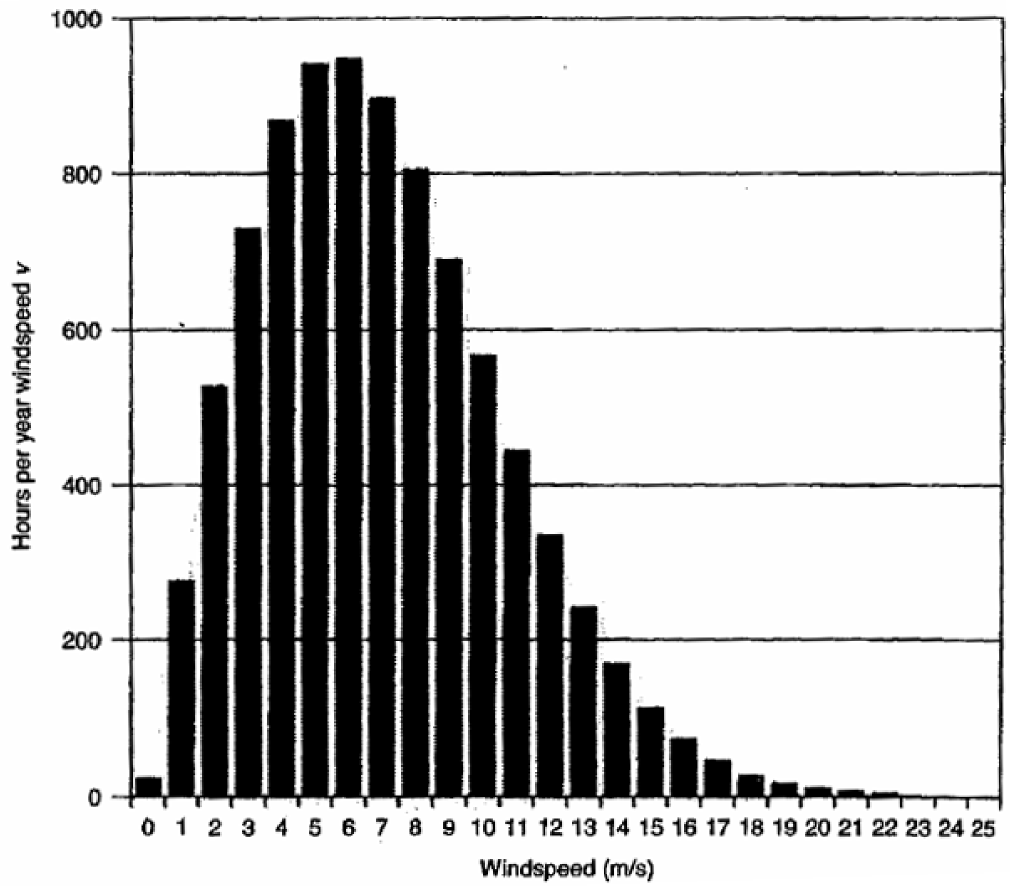




Wind Speed Frequency Distribution

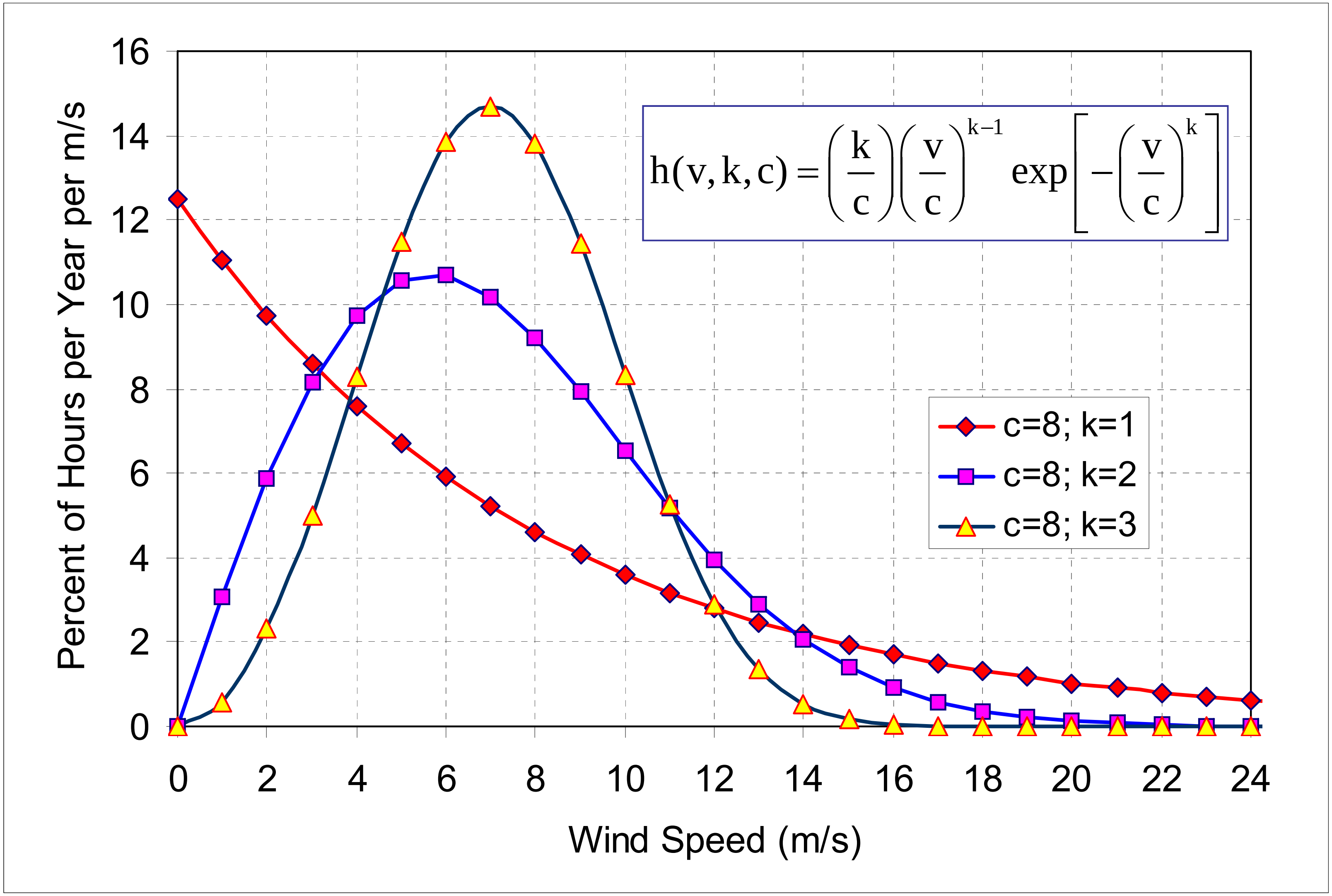
- Each vertical bar indicates how many hours/year the wind is blowing at that speed.
- The total adds up to 8760 hours.

v (m/s)	Hrs/yr
0	24
1	276
2	527
3	729
4	869
5	941
6	946
7	896
8	805
9	690
10	565
11	444
12	335
13	243
14	170
15	114
16	74
17	46
18	28
19	16
20	9
21	5
22	3
23	1
24	1
25	0
Total hrs	8,760



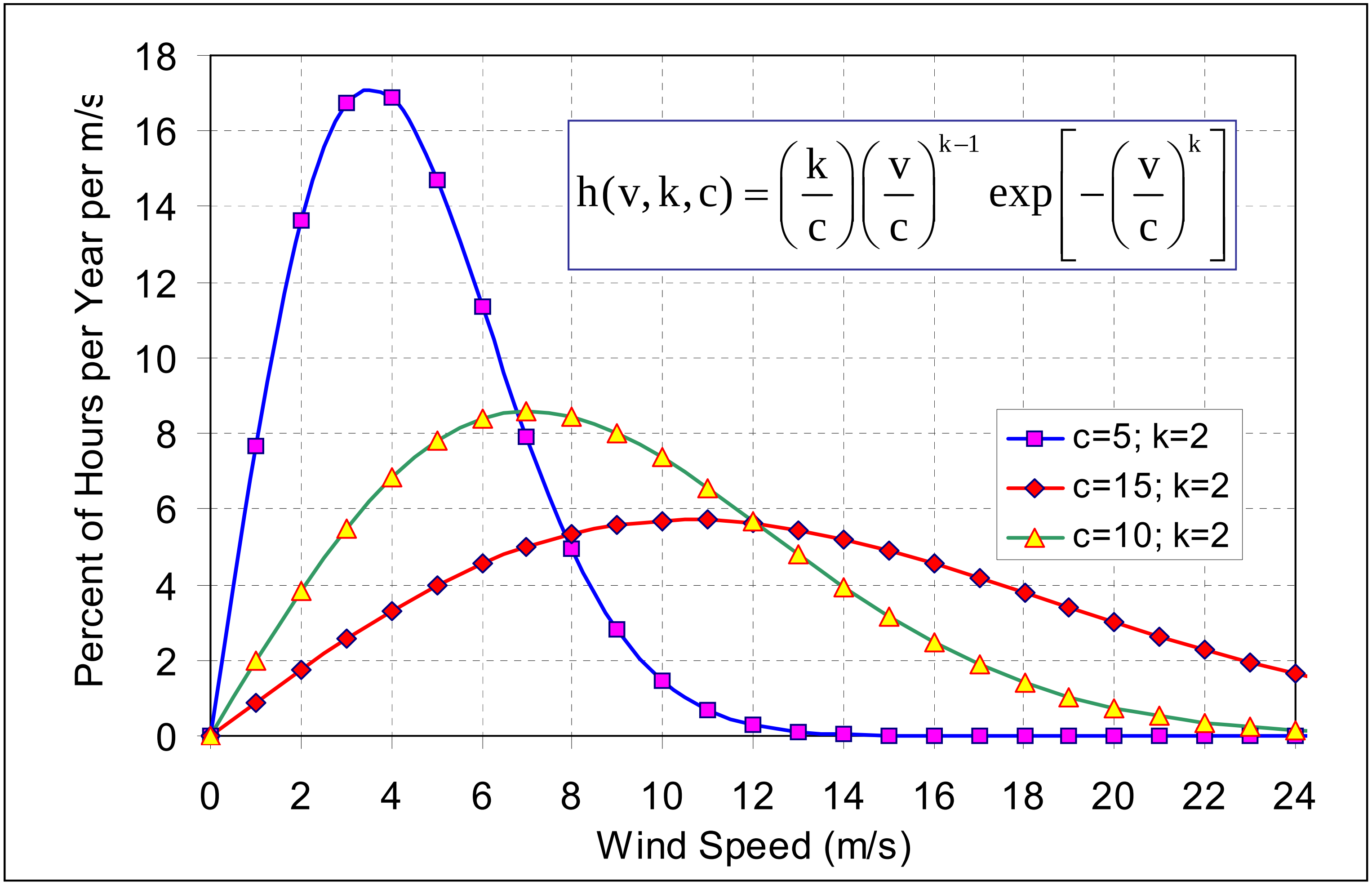
Wiebull Distribution

- For different values of the shape parameter



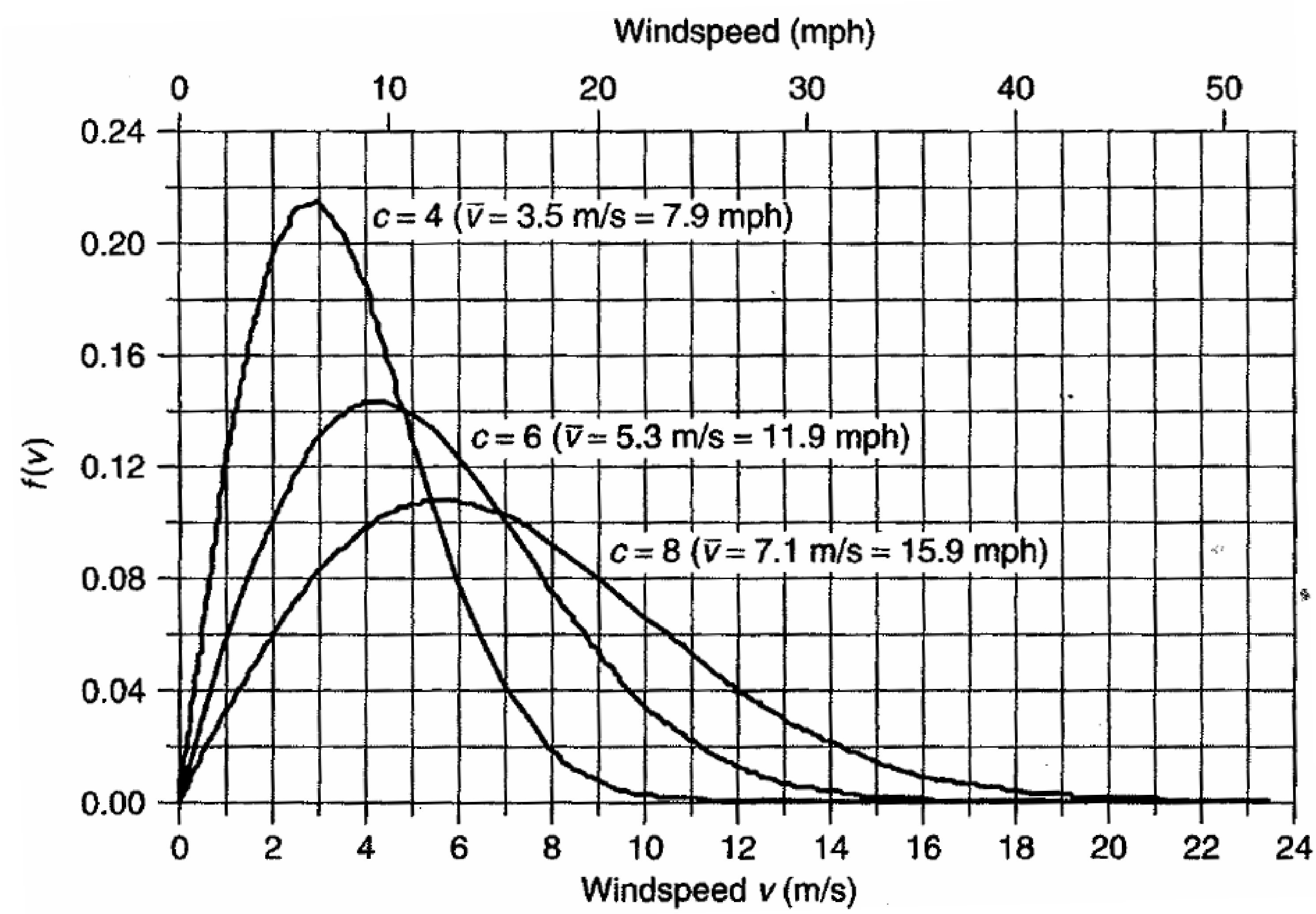
Wiebull Distribution

- For different values of the scale parameter



Rayleigh Disribution. (Weibull with k=2)

- Higher *c* implies higher average windspeeds



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Calculating Mean Wind Speed

- Let h(v) be the number of hours per year that the wind has speed v.
- The mean wind speed is then given by:

$$V_{mean} = \frac{1}{8760}\sum_{i=1}^{n} h(v_i)v_i$$

Exercise: Mean Wind Speed

- Determine the mean wind speed for the following speed frequency distribution:

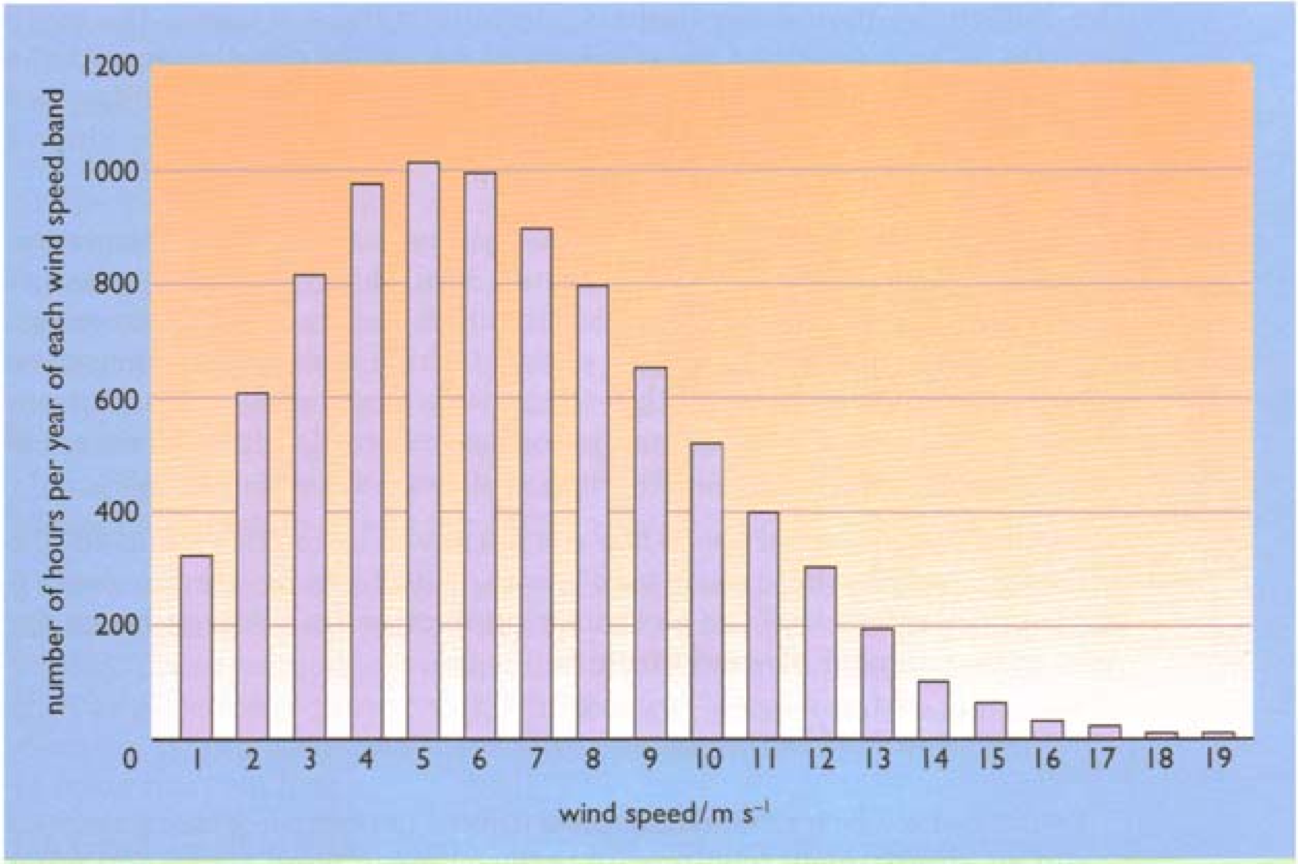
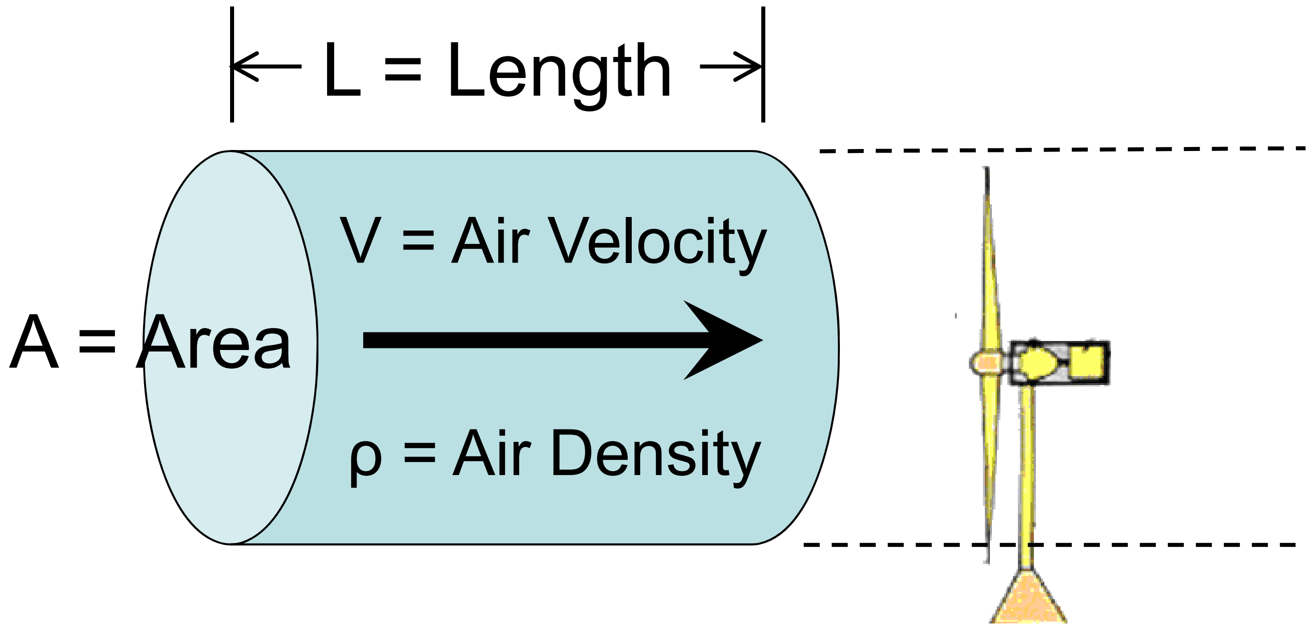


Figure 7.29 A wind speed frequency distribution for a typical site

speed	hours
1	320
2	610
3	818
4	980
5	1005
6	995
7	891
8	780
9	645
10	515
11	400
12	310
13	190
14	102
15	80
16	60
17	24
18	18
19	11
20	4
21	2

Kinetic Energy in Wind



Volume of Cylinder = AL

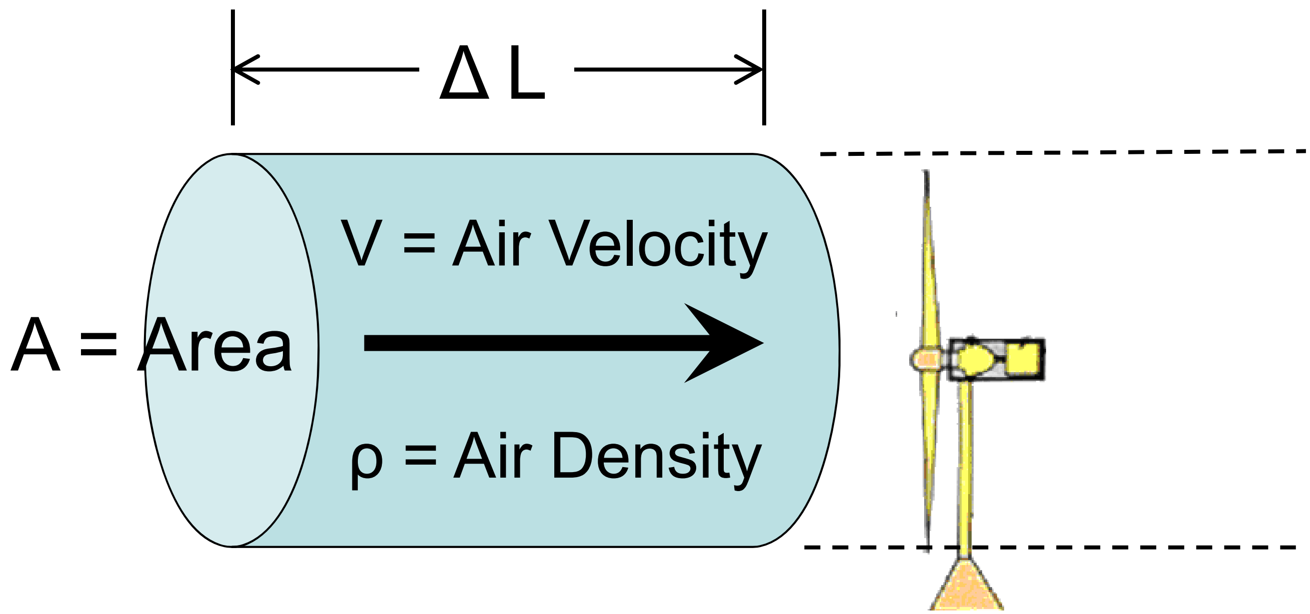
Mass of Air in Cylinder = ρAL

Kinetic Energy of cylinder moving at velocity V:

$$KE = \frac{1}{2} mV^2 = \frac{1}{2} (\rho AL) V^2$$

Wind Power

- ΔL = length of a cylinder of air that passes through the plane of the windmill during time Δt: $\Delta L = V \Delta t$



Δm = mass of air that passes the turbine during time Δt :

$$\Delta m = \rho A(\Delta L) = \rho A\,(V\,\Delta t)$$

Mass Flow Rate and Power

- And the mass flow rate of the air is

$$\Delta m = \rho A V \Delta t$$

$$\frac{\Delta m}{\Delta t} = \rho A V$$

- The energy in this cylinder of air is then

$$\Delta E = \frac{1}{2}(\Delta m)V^2$$

$$\Delta E = \frac{1}{2}(\rho A V \Delta t)V^2$$

- The power available in the wind is

$$\frac{\Delta E}{\Delta t} = \frac{1}{2}(\rho A V)V^2$$

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\rho A V^3$$

Power in the Wind

$$\text{Power} = \frac{1}{2}\rho A V^3$$

- Where:
  - ρ = air density (kg/m^3)
  - Area (m^2)
  - V = upstream (undisturbed) wind velocity (m/s)
  - Power (Watts)

Example – Power in the Wind

- Determine the power density (in W/m^2) in wind with a speed of 12 m/s. Assume an air density of 1.225 kg/m3.

$$\text{Power} = \frac{1}{2}\rho A V^3$$

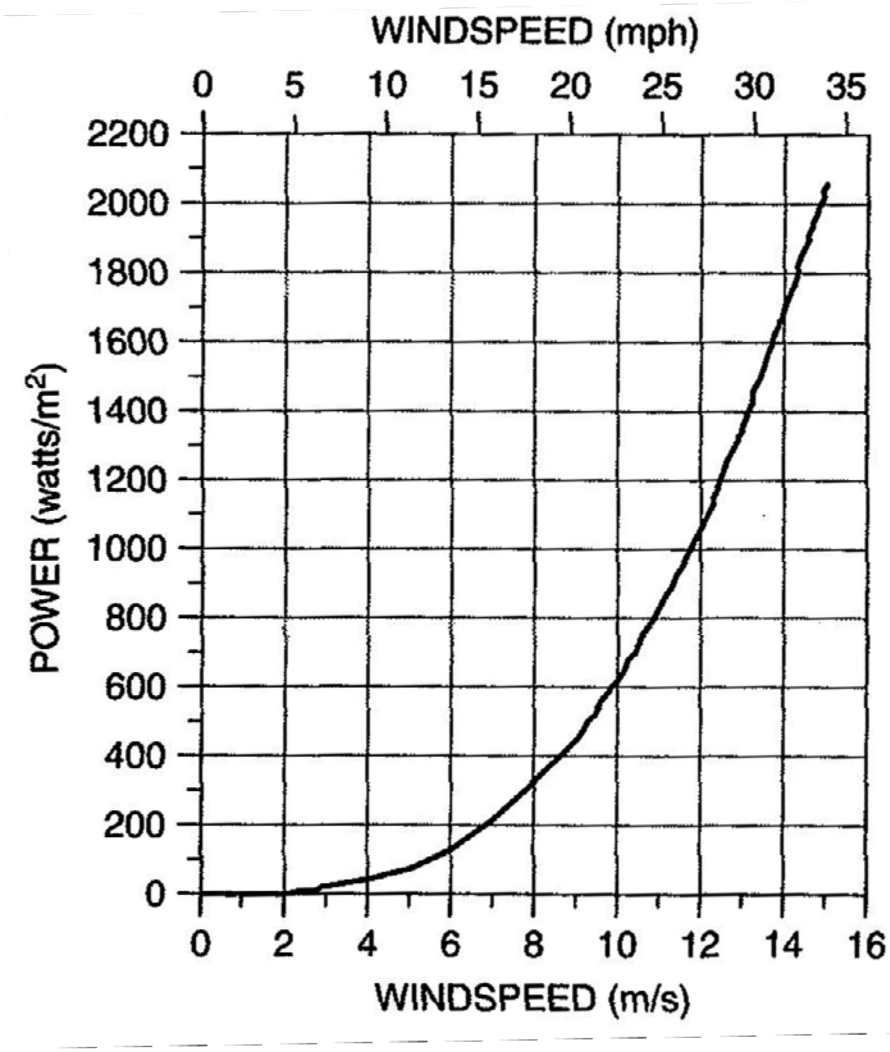
$$\text{Power Density} = \frac{P}{A} = \frac{1}{2}\rho V^3$$

$$\frac{P}{A} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(12\ \text{m/s}\right)^3 = 1058\ \frac{\text{W}}{\text{m}^2}$$

Power in the Wind

- Power increases like the cube of wind speed
- Doubling the wind speed increases the power by 8
- Energy in 1 hour of 20 mph winds is the same as energy in 8 hours of 10 mph winds

➢ **Nonlinear, so we cannot use average wind speed**



Example: Energy Density of Wind

$$\text{Energy} = (\text{Power})(\text{Time}) = \frac{1}{2}\rho A v^3 \Delta t$$

- 50 hours of 3 m/s winds and 50 hours of 9 m/s winds:
  - the "average" windspeed is 6 m/s

- 100 hours at an average of 6 m/s winds:

$$\frac{\text{Energy}}{(1\text{m}^2)} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(6\ \text{m/s}\right)^3 100\ \text{h} = 13.23\ \frac{\text{kWh}}{\text{m}^2}$$

- Calculate energy/m^2 of 50 hours of 3 m/s winds:

$$\frac{\text{Energy}}{(1\text{m}^2)} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(3\ \text{m/s}\right)^3 50\ \text{h}=0.827\ \frac{\text{kWh}}{\text{m}^2}$$

- Calculate energy/m^2 of 50 hours of 9 m/s winds:

$$\frac{\text{Energy}}{(1\text{m}^2)} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(9\ \text{m/s}\right)^3 50\ \text{h}=22.3\ \frac{\text{kWh}}{\text{m}^2}$$

- Now, add these two results to get the energy over 100 hours. Does this answer agree with the result on the previous slide?

$$.83\frac{\text{kWh}}{\text{m}^2} + 22.3\frac{\text{kWh}}{\text{m}^2} = 23.2\frac{\text{kWh}}{\text{m}^2}$$

Rayleigh Statistics – Average Power in the Wind

- To figure out average power in the wind, we need to know the average value of the ***cube*** of velocity:

$$P_{avg} = \left(\frac{1}{2}\rho A v^3\right)_{avg} = \frac{1}{2}\rho A \left(v^3\right)_{avg}$$

- With ***Rayleigh assumptions***, we can write the average power in the wind as follows:

$$\Rightarrow \quad P_{avg} = \left(\frac{6}{\pi}\right) \cdot \frac{1}{2}\rho A \left(v_{avg}\right)^3$$

Example: Average Power in the Wind

Estimate average power density in the wind at 50m when the windspeed at 10m has v_{avg} = 6m/s. Assume Rayleigh statistics, α = 1/7, and ρ = 1.225kg/m^3.

Estimate windspeed at 50 m:

$$\bar{v}_{50} = \bar{v}_{10}\left(\frac{H_{50}}{H_{10}}\right)^{\alpha} = 6\left(\frac{50}{10}\right)^{1/7} = 7.55 \text{ m/s}$$

Average power density in the wind at 50 m:

$$\frac{P_{avg}}{m^2} = \frac{6}{\pi}\cdot\frac{1}{2}(1.225)(7.55)^3 = 504 \text{ W/m}^2$$

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