MAKERERE UNIVERSITY

COLLEGE OF COMPUTING AND INFORMATION SCIENCES

SCHOOL OF COMPUTING AND INFORMATICS TECHNOLOGY

2016/2017 SEMESTER I

MTH 3105: DISCRETE MATHEMATICS

Test I

There are 4 (four) questions in this test (40 Marks), attempt all questions.
Time allowed: 1 (one) hour.

1.

a) Write contrapositive for each of the following statements: [6 Marks]

   i. The audience will go to sleep if the chairperson gives the lecture.
   ii. You may inspect the aircraft only if you have the proper security clearance.
   iii. If x^2 is an even number, then x is an even number.

b) Apply DeMorgan's law to $\neg(\exists x(Q(x) \rightarrow \forall y P(y)))$ [4 Marks]

2.

a) Prove that $\sqrt{10}$ is irrational. [5 Marks]

b) A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:

- If both Tom and Jesse are lying then Leo is telling the truth.
- If Tom or Hackson is lying, then Jesse is also lying.
- If Tom is telling the truth then John is lying.
- John is a well respected teacher so he never lies.
- Either Leo or Hackson is lying.

What conclusion can you make? There may be more than one liar. Show your steps. [5 Marks]

3. Using truth tables,

a) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent. [5 Marks]

b) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent. [5 Marks]

4. Show that if A and B are sets, then

a) $A - B = A \cap B^c$. [5 Marks]

b) $(A \cap B) \cup (A \cap B^c) = A$. [5 Marks]

END

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SECTION A

[*Answer all questions in this section. All questions carry equal marks (4)*]

1. Verify whether or not the following statement if valid. Give reasons to support your answer.

   $(\neg q \lor p) \leftrightarrow (q \rightarrow p)$

2. Letting the Universe of Discourse be the set of students at Makerere, let P(x) denote the statement "x likes CS2022". Express, in predicate logic, the sentence "At least two students like CS2022, though not everybody likes it".

3. Suppose that $|A| = m \geq 1$ and $|B| = n \geq 1$. What is the most that can be said about the relationship between m and n for each of the following to be true?

   a. There is an injection from A to B.

   b. There is a surjection from A to B.

4. Translate the following inference into propositional logic. If today is Thursday, then I have a test in CS or a test in Econ. If my Econ professor is sick, then I will not have a test in Econ. Today is Thursday and my Econ professor is sick. Therefore, I have a test in CS.

5. Prove by contradiction the following. For all rational number x and irrational number y, the sum of x and y is irrational.

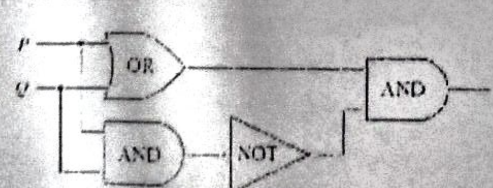
6. Use De Morgan's laws to write negation for the statement below.

   Hal is a Maths major and Hal's sister is a Computer Science major.

7. Use modus ponens or modus pollens to fill in the blank of the following argument so that they become valid inferences:

   If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole. There are more pigeons than there are pigeonholes. ∴ ....

8. Find the Boolean expressions that correspond to the circuit shown below. A dot indicates a soldering of two wires; wires that cross without a dot are assumed not to touch.



9. Write each of the following sentences symbolically, letting h = "It is hot" and s = "It is sunny."

   a. It is not hot but it is sunny.

   b. It is neither hot nor sunny.

10. Prove that for all integers n, if $n \bmod 5 = 3$ then $n^2 \bmod 5 = 4$

SECTION B

1.

   a. A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:

   - If both Tom and Jesse are lying, then Leo is telling the truth.
   - If Tom or Hackson is lying, then Jesse is also lying.
   - If Tom is telling the truth, then John is lying.
   - John is a well-respected teacher so he never lies.
   - Either Leo or Hackson is lying.

   What conclusion can you make? There may be more than one liar. Show your steps. *12 marks*

H and M lie
n, Jesse, Leo Hackson and John are lying.
$J \to L$
$H \to J$
$\to M$
M
$\vee H$

   b. Let $P(x)$ denote the statement "x is an accountant" and let $Q(x)$ denote the statement "x owns a Porsche". Write each statement below in first order logic. *8 marks*

      i. All accountants own Porsches. $\forall x\, P(x) \to Q(x)$

      ii. Some accountant owns a Porsche. $\exists x\, P(x) \to Q(x)$

      iii. All owners of Porsches are accountants $\forall x\, Q(x) \to P(x)$

      iv. Someone who owns a Porsche is an accountant $\exists x\, Q(x) \to P(x)$

2.

   a. Show that $2^n \geq n^2$, $n = 4, 5, \ldots$ *4 marks*

   b. Prove that if a and b are rational numbers, $b \neq 0$, and r is an irrational number, then $a+br$ is irrational. *4 marks*

   c. Show that for any integer n, $n^4 - 2n^3 - n^2 - 2n$ is divisible by 4. *6 marks*

   d. Prove that $\sqrt{10}$ is irrational. *6 marks*

3.

   a. Use De Morgan's laws to find the negation of each of the following statements:

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3.

   a. Use De Morgan's laws to find the negation of each of the following statements:

i. Jan is rich and happy. *2 marks*

ii. Carlos will bicycle or run tomorrow. *2 marks*

iii. Mei walks or takes the bus to class. *2 marks*

iv. Ibrahim is smart and hard working. *2 marks*

v. If you are doing MATH3105, then you are a year II student. *2 marks*

b. Using truth tables, show that: *5 marks @*

i. $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent

ii. $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent

4.

a. Show that if A and B are sets, then

i. $A-B = A \cap \overline{B}$ *5 marks*

ii. $(A \cap B) \cup (A \cap \overline{B}) = A$ *5 marks*

b. Determine whether each of these functions is a bijection from R to R.

i. $f(x) = 2x+1$ *2 marks*

ii. $f(x) = x^2 + 1$ *2 marks*

iii. $f(x) = x^3$ *2 marks*

iv. $f(x) = 2^x$ *2 marks*

v. $f(x) = \cos(x)$ *2 marks*

5.

a. Solve the followings, or give "unsolvable" if it has no solution.

i. $5x \equiv 11 \pmod{67}$ *4 marks*

ii. $99x \equiv 109 \pmod{81}$ *4 marks*

b. Show that $2! \cdot 4! \cdot 6! \cdots (2n)! \geq ((n+1)!)^n$ *5 marks*

c. Consider the famous Fibonacci sequence $\{x_n\}_{n=1}^{\infty}$, defined by the relations $x_1 = 1$, $x_2 = 1$, and $x_n = x_{n-1} + x_{n-2}$ for $n \geq 3$. Use an extended Principle of Mathematical Induction in order to show that for $n \geq 1$,

6 marks

$$x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

d. Use the results of part (c) to compute x_{20}. *1 mark*

END

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