MAKERERE UNIVERSITY
FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY
2010/2011 SEMESTER I
MTH 3105: DISCRETE MATHEMATICS
Test I

Patricia

There are 5 (five) questions in this test, attempt all questions.
Time allowed: 1 (one) hour.

1. (a) Write contrapositive for each of the following statements: *6 marks*
   (i) The audience will go to sleep if the chairperson gives the lecture.
   (ii) You may inspect the aircraft only if you have the proper security clearance.
   (iii) If x^2 is an even number, then x is an even number.
   (b) Apply DeMorgan's law to $\neg(\exists x(Q(x) \rightarrow \forall y P(y)))$ *4 marks*

2. (a) What conclusion can you make, given the followings are true? *4 marks*
   (i) $\neg q \rightarrow P(a)$ (iii) $s \vee \neg\exists x Q(x)$
   (ii) $q \rightarrow Q(b)$ (iv) $\forall x \neg P(x)$
   (b) A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:
   - If both Tom and Jesse are lying then Leo is telling the truth.
   - If Tom or Hackson is lying, then Jesse is also lying.
   - If Tom is telling the truth then John is lying.
   - John is a well respected teacher so he never lies.
   - Either Leo or Hackson is lying.

   What conclusion can you make? There may be more than one liar. Show your steps. *6 marks*

3. Let A, B, C be three arbitrary sets. Prove the following properties: *10 marks*
   (a) $A \cup B = B \cup A$ (d) $A \cup \emptyset = A$
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$ (e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (c) $A \backslash C \subseteq B \backslash C$, assuming A is a subset of B [for this part (c) only]

4. (a) Write a truth table for $p \oplus q \oplus r$ where $\oplus \equiv$ exclusive OR. *5 marks*

   (b) Write a logical formula for the truth table besides and then simplify it. *5 marks*

p	q	r	Output
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	F

5. (a) Simplify the following statement. $\neg(\neg q \wedge \neg(\neg q \vee s)) \vee (q \wedge (r \rightarrow r))$ *4 marks*
   (b) Assume that $\exists x \forall y P(x, y)$ is true and the domain of them is nonempty. Which of the following must be true? *6 marks*
   (i) $\forall x \forall y P(x, y)$
   (ii) $\forall x \exists y P(x, y)$
   (iii) $\exists x \exists y P(x, y)$

MAKERERE UNIVERSITY

FACULTY OF COMPUTING AND IT

DEPARTMENT OF COMPUTER SCIENCE
MTH 3105: DISCRETE MATHEMATICS
COURSEWORK

LECTURER. JOHNATHAN KIZITO

NAME	REGN NO	STUDENT NO.
KIBIRIGE TIMOTHY	09/U/13761/EVE	209005419

1a.

If the tuple (a, b, c) satisfy the following conditions:

- a, b and c are consecutive odd integers.
- a, b and c are all primes.

Then we call it a super prime tuple. Prove that (3, 5, 7) is the only super prime tuple.

Solution.

If am to list a sample grouping, we get,

(3,5,7),(9,11,13),(15,17,19),(21,25,27).

From the above you notice that the first integer is divisible by 3 for all the different groupings. But if we are to consider prime number groups, the first group is the only group with all primes i.e the only one satisfying condition 2. Thus from the definition of super prime tuple, (3,5,7) is the only super prime tuple.

1(b) Define F0 = 0, F1 = 1, Fn+2 = Fn+1 + Fn for n = 0, 1, 2, ... Prove that for any n ≥ 0 we have
Fn ≤ ((1 + √5)/2)^(n – 1).

Solution.

When n=0, F0=0, Substituting for n=0 in ((1 + √5)/2)^(n – 1), we get 0.618.

(F0=0) ≤=0.618 (0.618 is got as a result of substituting for n in given condition)

When n=1, F1=1. Substituting for n=1 in ((1 + √5)/2)^(n – 1), we get 1.

F1=1 and 1=1 (still in agreement with the given condition)

When n=2, F2=F1+F0= 0+1=1

Substituting for n=2 in ((1 + √5)/2)^(n – 1), we get 1.618.

(F2=1) ≤1.618 (still in agreement with the given condition)

When n=3, F3= F1+F2 =1+1=2

Substituting for n=3 in ((1 + √5)/2)^(n – 1), we get 2.618.

(F3=2) ≤2.618.(this is still in agreement with the condition given)

2.5

Therefore, Fn $((1+\sqrt{5})/2)$ ^ (n-1), for all n>=0

2a. Assume that $\forall x \exists y P(x, y)$ is false and the domain of them is nonempty. Which of the following must be false?

(i) $\forall x \forall y P(x, y)$

(ii) $\exists x \forall y P(x, y)$

(iii) $\exists x \exists y P(x, y)$

solution

$\forall x \forall y P(x,y)$ is false

2b. Let P(x) denote the statement "x is an accountant" and let Q(x) denote the statement "x
owns a Porsche". Write each statement below in first order logic.

(i) All accountants own Porsches.

(ii) Some accountant owns a Porsche.

(iii) All owners of Porsches are accountants

(iv) Someone who owns a Porsche is an accountant

Solution.

(i) $\forall x P(x) \rightarrow Q(x)$

(ii) $\exists x P(x) \rightarrow Q(x)$

(iii) $\forall x Q(x) \rightarrow P(x)$

(iv) $\exists x Q(x) \rightarrow P(x)$

$P(10, 4)$

$\frac{1}{4!}\left(\frac{10!}{6!}\right)^2$

No 3a. Prove $\lceil x \rceil = -\lfloor -x \rfloor$ for any real number x

Solution

using the notation [x] = greatest integer <= x, and (x) = smallest integer >=x, x = M +f where M is an integer and 0 =< f < 1 its fractional part, then [x] = M and (x) = M+1 when f > 0, and [x] = M = (x) when f = 0.

When f =0. Then [x] = M and - (- x) = - (- M) = M.

When f >0. Then - x = -M - f = - M -1 + 1-f, note that 1-f is -x 's fractional part since 0 < 1-f < 1. So [x] = M and (-x) = - M -1 + 1 = - M.

thus, $\lceil x \rceil = -\lfloor -x \rfloor$ as required.

3b. A detective has interviewed four witnesses to a crime. From their stories, the detective has concluded that:
(i) If the butler is telling the truth, then so is the cook.
(ii) The cook and the gardener cannot both be telling the truth.
(iii) The gardener and the handyman are not both lying.
(iv) If the handyman is telling the truth then the cook is lying.
(v) Who must be lying? There may be more than one liar. Show your steps

Solution.

Let B be "butler telling the truth", C be "cook telling the truth", G be "gardener telling the truth", H be "handyman telling the truth".

i. $B \rightarrow C$

ii. $\neg C \vee \neg G$

iii. $G \wedge H$

iv. $H \rightarrow \neg C$.

By (iii), G and H is true

By (iv) C is false, since H is true.

By (ii), since C is false, G must be true

By (i),apply contra positive, and we get B is false.

Conclusion.

Butler and cook are lying.

No.4

Define the sequence c0, c1, ... by the equations c0 = 0 and cn = $c\lfloor n/2 \rfloor + 3$ for all $n > 0$. Prove that cn ≤ 2n for all n ≥ 3.

Solution.

From the recursion equation , c0=0, c1=c0+3=3, c2=c1+3=6, c3=c1+3=6, c4=c2+3=9, c5=c2+3=9.

$cn = c\lfloor n/2 \rfloor + 3$
$cn = c\lfloor n/4 \rfloor + 3 + 3$
$cn = c\lfloor n/8 \rfloor + 3 + 3 + 3$
$cn = c\lfloor n/16 \rfloor + 3 + 3 + 3 + 3$
and so on

we can see from the continuity that
$cn = \lfloor lg(n) \rfloor + 3$
where lg is the logarithm to the base two.

now $\lfloor lg(n) \rfloor + 3 \leq lg(n) + 3$

lets determine the relation between $lg(n) + 3$ and $2n$ and lets represent the relation by a ? mark

so $lg(n) + 3 \ ? \ 2n$
subtracting 3 from both sides we get => $lg(n) \ ? \ 2n -3$
now taking 2 to the power of everything we get => $2^{(lg(n))} \ ? \ 2^{(2n -3)}$
$n \ ? \ 2^{(2n -3)}$
for $n = 1$, $1 \ ? \ 2^{(-1)}$ i.e $1 > 1/2$
for $n = 2$, $2 \ ? \ 2^{(1)}$ i.e $2 = 2$
for $n = 3$, $3 \ ? \ 2^{(3)}$ i.e $3 < 8$
for $n = 4$, $4 \ ? \ 2^{(5)}$ i.e $4 < 32$
and this is very obvious that for all higher n $n < 2^{(2n-3)}$ which implies that for all $n \geq 3$, $lg(n) + 3 \leq 2n$

Hence $\lfloor lg(n) \rfloor + 3 \leq 2n$ for all values of $n \geq 3$.

Patricia

MAKERERE UNIVERSITY

FACULTY OF COMPUTING & INFORMATICS TECHNOLOGY

END OF SEMESTER I EXAMINATION 2010/2011

PROGRAMME: BSC

YEAR OF STUDY: SE I, CS II

COURSE NAME: Discrete Mathematics

COURSE CODE: MTH 3105

DATE: Friday 10th December 2010 **TIME: 12:00 – 03:00**

EXAMINATION INSTRUCTIONS

1. ATTEMPT ALL QUESTIONS IN SECTION A (40 MARKS)
2. ATTEMPT THREE (03) QUESTIONS IN SECTIONS B (60 MARKS)
3. DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO
4. ALL ROUGH WORK SHOULD BE IN YOUR ANSWER BOOKLET

SECTION A

Each question in this section carries 4 marks

1. Show that $2^n \geq n^2$, $n = 4, 5, \ldots$
2. Prove that for all integers a and b, if $a \mid b$ then $a^2 \mid b^2$.
3. Write contrapositive for each of the following statements
   a. If you are a CS year 2 student, then you are taking MTH 3105.
   b. x^2 divides y^2 if x divides y.
4. Apply DeMorgan's law to $\neg(\exists x(Q(x) \rightarrow \forall y P(y)))$
5. Write a truth table for $\neg p \vee q$
6. What conclusion can you make, given the followings are true?
   - $\neg q \rightarrow P(\mathrm{a})$
   - $q \rightarrow Q(b)$
   - $s \vee \neg\exists x Q(x)$
   - $\forall x \neg P(x)$
7. A *Full House* is a hand with three cards of one value and two cards of another value. How many different hands contain a *Full House* from a deck of 52 cards?
8. Let R be the relation on $\{1, 2, 3, 4, 5\}$ defined by mRn if and only if $m - n$ is even. Draw a picture for this relation.
9. Let R_1 and R_2 be relations from a set S to a set T. Show that:
   $(R_1 \cup R_2)^{\leftarrow} = R_1^{\leftarrow} \cup R_2^{\leftarrow}$.
10. Given two functions $f: X \rightarrow Y'$ and $g: Y \rightarrow Z$ so that Y' is a subset of Y, we define the composition of f and g as the function $g \circ f: X \rightarrow Z$, where

$$g \circ f(x) = g(f(x)).$$

Complete this table that tells whether they are injective, surjective:

Function f	Function g	$g \circ f$ injective?	$g \circ f$ surjective?
$f: X \rightarrow Y$ surjective	$g: Y \rightarrow Z$ surjective		
$f: X \rightarrow Y$ injective	$g: Y \rightarrow Z$ surjective		

SECTION B

$n^4 - 2n^3 - n^2 + 2n$

1.

a. A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:

- If both Tom and Jesse are lying then Leo is telling the truth.
- If Tom or Hackson is lying, then Jesse is also lying.
- If Tom is telling the truth then John is lying.
- John is a well respected teacher so he never lies.
- Either Leo or Hackson is lying.

What conclusion can you make? There may be more than one liar. Show your steps. *10 marks*

b. A detective has interviewed four witnesses to a crime. From their stories, the detective has concluded that:

- If the butler is telling the truth, then so is the cook.
- The cook and the gardener cannot both be telling the truth.
- The gardener and the handyman are not both lying.
- If the handyman is telling the truth then the cook is lying.

Who must be lying? There may be more than one liar. *10 marks*

2.

a. Show that for any integer n, $n^4 + 2n^3 - n^2 - 2n$ is divisible by 4. *6 marks*

b. Prove that $\sqrt{10}$ is irrational. *8 marks*

$(ac+bd)^2$

c. A number n is a sum of two squares if $n = a^2 + b^2$ for some integers a and b. If x and y are both sum of two squares, prove that xy is also a sum of two squares. *6 marks*

3.

a. Write a truth table for $p \oplus q \oplus r$ *8 marks*

b. Write a logical formula for the truth table below and then simplify it (if possible). *4 marks*

P	Q	Output
T	T	T
T	F	F
F	T	T
F	F	T

c. Simplify the following statement. $\neg(\neg q \wedge \neg(\neg q \vee s)) \vee (q \wedge (r \rightarrow r))$ *8 marks*

4.

a. Assume that $\exists x \forall y P(x, y)$ is true and the domain of them is nonempty. Which of the following must be true?

i. $\forall x \forall y P(x, y)$

ii. $\forall x \exists y P(x, y)$

iii. $\exists x \exists y P(x, y)$

b. Assume that $\forall x \exists y P(x, y)$ is false and the domain of them is nonempty. Which of the following must be false?

i. $\forall x \forall y P(x, y)$

ii. $\exists x \forall y P(x, y)$

iii. $\exists x \exists y P(x, y)$

c. Let $P(x)$ denote the statement "x is an accountant" and let $Q(x)$ denote the statement "x owns a Porsche". Write each statement below in first order logic.

i. All accountants own Porsches.

ii. Some accountant owns a Porsche.

iii. All owners of Porsches are accountants

iv. Someone who owns a Porsche is an accountant

5.

a. Solve the followings, or give "unsolvable" if it has no solution.

i. $19x \equiv 20 \pmod{77}$

ii. $5x \equiv 11 \pmod{67}$

iii. $49x \equiv 98 \pmod{21}$

iv. $99x \equiv 109 \pmod{81}$

v. $105x \equiv 143 \pmod{100}$

b. There are k kinds of postcards, each with a limited amount. Let there be a_i copies of the i-th postcard. How many ways are there to send these postcards to n friends? (Each friend can receive zero cards, as well as multiple copies of the same card.)

Group Copy

MAKERERE UNIVERSITY EXAMINATIONS
FACULTY OF COMPUTING & INFORMATION TECHNOLOGY
SEMESTER II 2006

CSC 1201 : COMPUTATIONAL MATHEMATICS II

MONDAY 15TH MAY, 2006 8:00 AM - 11:00 AM

INSTRUCTIONS:

(i) Answer any **FOUR** *questions.*

(ii) Read all the instructions on the Answer book.

Question 1 [Propositional Logic]

(a) (i) Define what is meant by a language.

(ii) State the symbols of proposition logic $\mathcal{L}_p$.

(b) Construct truth tables for

(ii) $(\neg A \to \neg B) \to ((\neg A \to B) \to A)$

(ii) $Q \to P \leftrightarrow R \vee Q$

(c) Rewrite the following as statements in proposition logic:

(i) If the network server is on-line and the user has typed a query command then query log will be updated if and only if the user has typed a query command from a secure terminal else a security error is flagged.

(ii) If the moon is out and it is not snowing, then Noeline will go out for a walk if and only if Emilly accepts to stay home looking after their sick mother.

(d) Define the terms

(i) Formula.

(ii) Subformular in L_p. Hence Find the subformulas of

(ii1) $(\neg A_1 \vee A_2)$

(ii2) $A_1 \to A_2 \to A_3$

(e) Explain the terms below :

(i) A formular A in L_p is satisfiable.

(ii) A formular A in L_p is a tautology.

(f) Prove that $(p \wedge (p \to q)) \to q$ is a tautology in L_p

450
120
570
80

CSC 1201 SEM II 2005/2006

Question 2 [Sets and Numbers]

(a) (i) Define what is meant by

(i1) a set .

(i2) two sets A and B being equal.

(ii) State the two ways of describing a set.

$P_s = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{ \quad \{1,2,3\}\}.$

(b) (i) Which of the following sets are equal

no. of elements in a set

$$A = \{1,2,3\} \qquad B = \{3,2,1,3\}$$
$$C = \{1,2,2,3\} \qquad D = \{3,1,2,3\}$$

State the cardinality of set C. $= 4$

State the powerset of set A

(ii) Determine the elements in the set

(i1) $\{n + \frac{1}{n} \ : \ n \in \{1,2,3,5,7\}\}$

(i2) $\{n + (-1)^n \ : \ n \in \mathbb{N}\}$

$P = 2^n = 2^3 = 8$

(c) Let $A = \{a, b, c\}$, write a computer program (or develop an algorithm) that lists all the subsets B of A, where $|B| = 2$.

(d) During freshers' orientation at the main building, two showings of the latest Jennifer Lopez movies were presented. Among the 600 ICT freshers, 80 attended the first movie and 125 attended the second showing, while 450 did not make it to either showing.

(i) How many of the 600 freshers attended twice.

(ii) How many attended either shows.

(iii) What is the probability that a fresher chosen at random attended only one of the movie show.

(e) (i) One of the special sets of numbers declared in any programming language is $\mathbb{Q}$. With a relevant example, define $\mathbb{Q}$.

(ii) Prove that $\sqrt{2}$ is an irrational number ($\mathbb{I}$).

(iii) Solve the equation $3x - 5 = -x$.
According to the classification of numbers , to which group does the x above belong?

(iv) State which of the following statements are true, and which are false.

$\mathcal{E} = 600$

Q → Real No's

R → Rational

C = Complex

(a) $\mathbb{Z} \subseteq \mathbb{Q}$ (b) $\mathbb{Q} \subseteq \mathbb{R}$

(c) $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$ (d) $\mathbb{C} \cup \mathbb{R} = \mathbb{R}$

for $(n \in A = n \in B: \ |B| = 2)$

print ("$A \subseteq B$")

$655 - x = 600$

Question 3 [Methods of Proofs]

(a) (i) State any four methods of proofs.

(ii) Define what is meant for two formulas to be logically equivalent.

(b) Stating the method of proof to use, determine the validity of the statements

(i) All popes reside at the Vatican.
Benedict xvi resides at the Vatican.
Therefore, Benedict xvi is a pope.

(ii) My programming language is C++.
Therefore, it is not Java.

(iii)

$$\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$$

(iv)

$$p \rightarrow (q \vee r) \equiv \neg r \rightarrow (p \rightarrow q)$$

(v) All first year Computer Science students use LATEX

(c) Brown, Jones and Smith are suspected of a crime. They testify as follows:

- Brown : Jones is guilty and Smith is innocent.
- Jones : If Brown is guilty then so is Smith.
- Smith : Iam innocent but atleast one of the others is guilty.

Let b, j and s be statements
"Brown is innocent", "Jones is innocent", and "Smith is innocent".
Draw a truth table for the three testimonies to answer the following questions

(i) Are the three testimonies consistent?

(ii) The testimonies of one follows from the other, which from which?

(iii) Assuming everyone is innocent, who committed perjury?

(iv) Assuming all testimony is true, who is innocent and who is guilty ?

Question 4 [Discrete Probability]

(a) (i) State the axioms of probability.

(ii) Define (give its mathematical definition) the expectation $E(X)$

(b) A coin and a die are thrown together
Draw the sample space of the experiment, and from it determine the probability of obtaining

(i) a tail and a 4.

(ii) a head and an odd number.

(c) An insurance company offers a hospitalization policy to individuals in a certain group. For a 1-year period, the company will pay \$100 per day, to a maximum of 5-days, to each day the policy holder is hospitalized. The company estimates that, the probability that a person is hospitalized for exactly one day is 0.001; for exactly 2 days , 0.002; for exactly 3 days, 0.003; for exactly 4 days, 0.004; for 5 or more days 0.008. Find the expected gain per policy to the company if the annual premium is \$10.

(d) An urn contains ten marbles, each of which shows a number. Five marbles show 1, two show 2, three show 3. A marble is drawn at random, if X is the number that shows up, determine

(i) E(X) 0.018

(ii) Var(X) 0.982

(e) For

$$f(x) = \begin{cases} \frac{kx^2}{x!} & ; \ x = 1, 2, 3, 4 \\ 0 & ; \ \text{elsewhere} \end{cases}$$

Where k is a constant, determine

(i) the value of the constant k.

(ii) $P(x \geq 3)$

(iii) $P(x = 3/x \geq 2)$

(iv) $E(5x^2 - 3x)$

$(\Sigma x)^2 - \Sigma(x^2)$

Question 5 [Permutation and Combination]

(a) Define what is meant by

(i) a permutation of r elements from n objects

(ii) a combination of r elements from n objects

(b) At a restaurant a complete dinner consists of an appetizer, an entre, a dessert, and a beverage. For the appetizer, the choices are, soup or juice; for the entre, the choices are chicken, fish, steak or lamb, for the dessert the choices are cherries jubilee, fresh peach cobbler, chocolate truffle cake, a blueberry roly-poly; for the beverage, the choices are coffee, tea or milk. How many complete dinner are possible?

(c) Given the digits: 1, 2, 4, 6, 9, 0.

(i) How many three digit odd number arrangements are possible.

(ii) How many even digit number arrangements are possible.

(d) In how many ways can letters of the word MATHEMATICS be arranged?

(i) in a line.

(ii) if the word must start with S.

(iii) if the word must start and end with M.

1 → 2
2 → 4
3 → 4
4 → 4.

(e) From a group of 10 boys and 8 girls, 2 pupils are chosen at random. Find the probability that they are both girls.

(f) A committee of 8 members consists of one married couple together with 4 other men and 2 other women. From the committee, a working party of 4 persons is to be formed. Find the number of different working parties which can be formed if

(i) there is no restriction.

(ii) may not contain both the husband and his wife.

(iii) must contain 2 men and 2 women.

(iv) must contain at least one man and at least one woman.

The 8 committee members sit around an octagon table, their positions being decided by drawing lots. Find the probability of

(i) the man sitting next to his wife .

(ii) the 3 women sitting together.

(iii) the man sitting opposite his wife .

8 × 0.4 − 0.3

File :

Question 6 [Elementary Graph Theory]

a) (i) Define what is meant by a graph

(ii) Sketch the graphs $K_{3,2}$; and CL_8;

b) Seven towns a, b, c, d, e, f and g are net-linked (connected) by a system of highways as follows:

1) 1 – 22 goes from a to c passing through b.
2) 1 – 33 goes from c to d and then passes through b as it continues f.
3) 1 – 44 goes from d through e and a
4) 1 – 55 goes from f to b passing through g
5) 1 – 66 goes from g to d.

(i) Using vertices for towns and directed edges for segments of highways between towns, draw a directed graph that models this situation.

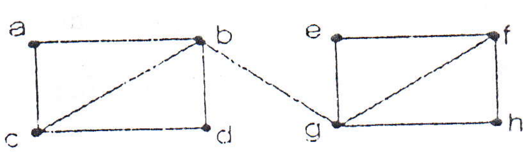
(ii) List the paths from g to a.

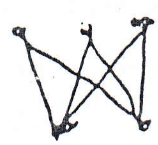
(iii) What is the smallest number of highways segments that would have to be closed down in order for travel from b to d to be disrupted.

(iv) Is it possible to leave town c and return there, visiting each of the other towns only once?

(v) What is the answer to part (iv) if we are not required to return to c. If Yes state the path(s).

(c) In the network below





(i) How many paths are there in the graph from a to h? State them.

(ii) How many of these are of length 5?

(d) Define what is meant by a graph to be complete.

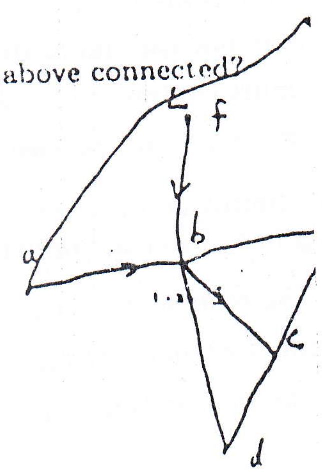
(i) Give an example of a complete graph with 3 vertices. Is the graph above connected? Defend your answer

(ii) Give an example of an uncomplete graph with 3 vertices.

(e) With relevant examples

(i) define what is meant by a pseudo graph

(ii) distinguish between an in-degree and out-degree of a vertex.



END

f e

1 — Jay.

Patrick A.

MAKERERE UNIVERSITY EXAMINATIONS
FACULTY OF COMPUTING & INFORMATION TECHNOLOGY
SEMESTER II 2006

Discrete Mathematics

CSC 1201 : COMPUTATIONAL MATHEMATICS II

MONDAY 15TH MAY, 2006 — 8:00 AM - 11:00 AM

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(i) Formula.

(ii) Subformular in L_p . Hence Find the subformulas of

(ii1) $(\neg A_1 \vee A_2)$

(ii2) $A_1 \rightarrow A_2 \rightarrow A_3$

(e) Explain the terms below :

(i) A formular A in L_p is satisfiable.

(ii) A formular A in L_p is a tautology.

(f) Prove that $(p \wedge (p \rightarrow q)) \rightarrow q$ is a tautology in L_p

Question 2 [Sets and Numbers]

(a) (i) Define what is meant by

(i1) a set .

(i2) two sets A and B being equal.

(ii) State the two ways of describing a set.

$\mathbb{P}S = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

(b) (i) Which of the following sets are equal

cardinality number of elements in a set

$$\begin{array}{llll} A & = \{1,2,3\} & B & = \{3,2,1,3\} \\ C & = \{1,2,2,3\} & D & = \{3,1,2,3\} \end{array}$$

powerset is a set of all subsets of A

State the cardinality of set C. # 4

State the powerset of set A.

$\mathbb{P}S = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2,3\}\}$.

$\{1,2\}$, 1

$P = 2^n = 2^3 = 8$

(ii) Determine the elements in the set

(i1) $\{n + \frac{1}{n} \; : \; n \in \{1, 2, 3, 5, 7\}\}$

(i2) $\{n + (-1)^n \; : \; n \in \mathbb{N}\}$

(c) Let $A = \{a, b, c\}$, write a computer program (or develop an algorithm) that lists all the subsets B of A, where $|B| = 2$.

(d) During freshers' orientation at the main building, two showings of the latest Jennifer Lopez movies were presented. Among the 600 ICT freshers, 80 attended the first movie and 125 attended the second showing, while 450 did not make it to either showing.

(i) How many of the 600 freshers attended twice.

(ii) How many attended either shows.

(iii) What is the probability that a fresher chosen at random attended only one of the movie show.

(e) (i) One of the special sets of numbers declared in any programming language is $\mathbb{Q}$. With a relevant example, define $\mathbb{Q}$.

(ii) Prove that $\sqrt{2}$ is an irrational number ($\mathbb{I}$).

(iii) Solve the equation $3x - 5 = -x$.
According to the classification of numbers , to which group does the x above belong?

(iv) State which of the following statements are true, and which are false.

(a)	$\mathbb{Z} \subseteq \mathbb{Q}$	(b)	$\mathbb{Q} \subseteq \mathbb{R}$
(c)	$\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$	(d)	$\mathbb{C} \cup \mathbb{R} = \mathbb{R}$

Q → Real No's

R → Rational

C = Complex

Z 4

4

for $(n \in A = n \in B)\ |B| = 2)$ $H \leq 2$

printf("A ⊆ B").

$655 - x = 600$

Question 3 [Methods of Proofs]

(a) (i) State any four methods of proofs.

(ii) Define what is meant for two formulas to be logically equivalent.

(b) Stating the method of proof to use, determine the validity of the statements

(i) All popes reside at the Vatican.
Benedict xvi resides at the Vatican.
Therefore, Benedict xvi is a pope.

(ii) My programming language is C++.
Therefore, it is not Java.

(iii)

$$\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$$

(iv)

$$p \rightarrow (q \vee r) \equiv \neg r \rightarrow (p \rightarrow q)$$

(v) All first year Computer Science students use LATEX

(c) Brown, Jones and Smith are suspected of a crime. They testify as follows:

- Brown : Jones is guilty and Smith is innocent.
- Jones : If Brown is guilty then so is Smith.
- Smith : Iam innocent but atleast one of the others is guilty.

Let b, j and s be statements
"Brown is innocent", "Jones is innocent", and "Smith is innocent".
Draw a truth table for the three testimonies to answer the following questions

(i) Are the three testimonies consistent?

(ii) The testimonies of one follows from the other. which from which?

(iii) Assuming everyone is innocent, who committed perjury?

(iv) Assuming all testimony is true, who is innocent and who is guilty ?

Subformula

$(\neg A_1 \vee A_2) \rightarrow \neg A_1$

A_2

$\neg A \vee A_2$

$A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow A_1$

Question 4 [Discrete Probability]

(a) (i) State the axioms of probability.

(ii) Define (give its mathematical definition) the expectation $E(X)$

(b) A coin and a die are thrown together
Draw the sample space of the experiment, and from it determine the probability of obtaining

(i) a tail and a 4.

(ii) a head and an odd number.

(c) An insurance company offers a hospitalization policy to individuals in a certain group. For a 1-year period, the company will pay \$100 per day, to a maximum of 5-days, to each day the policy holder is hospitalized. The company estimates that, the probability that a person is hospitalized for exactly one day is 0.001; for exactly 2 days , 0.002; for exactly 3 days, 0.003; for exactly 4 days, 0.004; for 5 or more days 0.008. Find the expected gain per policy to the company if the annual premium is \$10.

(d) An urn contains ten marbles, each of which shows a number. Five marbles show 1, two show 2, three show 3. A marble is drawn at random, if X is the number that shows up, determine

(i) E(X) 0.018 ·

(ii) Var(X) 0.982 ×10

1 | 2 |

H

T

X

(e) For

$$f(x) = \begin{cases} \frac{kx^2}{x\,!} & ; \quad x = 1, 2, 3, 4 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Where k is a constant, determine

(i) the value of the constant k.

(ii) $P(x \geq 3)$

(iii) $P(x = 3/x \geq 2)$

(iv) $E(5x^2 - 3x)$

$(\sum x)^2 - \sum(x^2)$

Question 5 [Permutation and Combination]

(a) Define what is meant by

(i) a permutation of r elements from n objects

(ii) a combination of r elements from n objects

(b) At a restaurant a complete dinner consists of an appetizer, an entre, a dessert, and a beverage. For the appetizer, the choices are, soup or juice; for the entre, the choices are chicken, fish, steak or lamb, for the dessert the choices are cherries jubilee, fresh peach cobbler chocolate, truffle cake, a blueberry roly-poly; for the beverage, the choices are coffee, tea or milk. How many complete dinner are possible?

(c) Given the digits; 1, 2, 4, 6, 9, 0.

(i) How many three digit odd number arrangements are possible.

(ii) How many even digit number arrangements are possible.

1 → 2
2 → 4
3 → 4
4 → 4.

(d) In how many ways can letters of the word MATHEMATICS be arranged?

(i) in a line.

(ii) if the word must start with S.

(iii) if the word must start and end with M.

(e) From a group of 10 boys and 8 girls, 2 pupils are chosen at random. Find the probability that they are both girls.

(f) A committee of 8 members consists of one married couple together with 4 other men and 2 other women. From the committee, a working party of 4 persons is to be formed. Find the number of different working parties which can be formed if

(i) there is no restriction.

(ii) may not contain both the husband and his wife.

(iii) must contain 2 men and 2 women.

(iv) must contain at least one man and at least one woman.

The 8 committee members sit around an octagon table, their positions being decided by drawing lots. Find the probability of

(i) the man sitting next to his wife .

(ii) the 3 women sitting together.

(iii) the man sitting opposite his wife .

8 + 0.4 - 0.3 + 0.2 + 0.1

file partmgr.sys

Question 6 [Elementary Graph Theory]

(a) (i) Define what is meant by a graph

(ii) Sketch the graphs $K_{3,2}$; and CL_5;

(b) Seven towns a, b, c, d, e, f and g are net-linked (connected) by a system of highways as follows:

1) 1 – 22 goes from a to c passing through b.
2) 1 – 33 goes from c to d and then passes through b as it continues f.
3) 1 – 44 goes from d through e and a
4) 1 – 55 goes from f to b passing through g
5) 1 – 66 goes from g to d.

(i) Using vertices for towns and directed edges for segments of highways between towns, draw a directed graph that models this situation

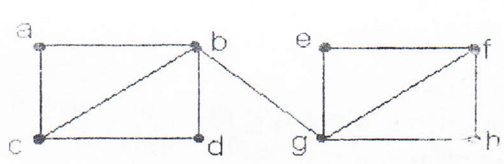
(ii) List the paths from g to a.

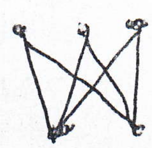
(iii) What is the smallest number of highways segments that would have to be closed down in order for travel from b to d to be disrupted.

(iv) Is it possible to leave town c and return there, visiting each of the other towns only once?

(v) What is the answer to part (iv) if we are not required to return to c. If Yes state the path(s).

(c) In the network below





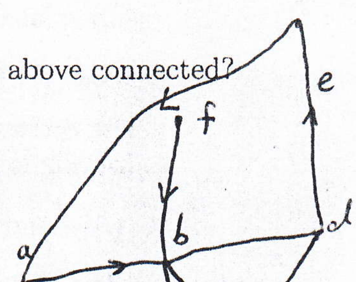
(i) How many paths are there in the graph from a to h? State them.

(ii) How many of these are of length 5?

(d) Define what is meant by a graph to be complete.

(i) Give an example of a complete graph with 3 vertices. Is the graph above connected? Defend your answer

(ii) Give an example of an uncomplete graph with 3 vertices.



(e) With relevant examples

(i) define what is meant by a pseudo graph

(ii) distinguish between an in-degree and out-degree of a vertex.

END

f e