INSTRUCTIONS
(i) Work out in a group of four.

(ii) Deadline for submission is 7th /10/ 2017

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1. With $S = \{a,b,c\}$, let f,g: S→S be given by $f = \{(a,b), (b,c), (c,a)\}$ and $g = \{(a,b), (b,a), (c,c)\}$. Determine each of the following functions. Write your answers as a collection of ordered pairs. (a) $f \circ g$; (b) $g \circ f$; (c) f^{-1}; (d) g^{-1}.

2. Let m be a positive integer, and let p, q, and r be integers. Show that if $p \equiv q$ (mod m), then $p - r \equiv q - r$ (mod m).

3. Let $p,q \in \mathbf{N}$. Prove that if $p + q \geq 100$, then either $p \geq 50$ or $q \geq 50$. What type of proof did you use?

4. Prove that if z^3 is irrational, then x is irrational. State the method of proof used in this case.

5. Let $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

   be a matrix representing a relation R on a set A. Answer the following 3 true/false exercises

   (i) R is reflexive.

   ☐ True   ☐ False

   (ii) R is symmetric.

   ☐ True   ☐ False

   (iii)   R is antisymmetric.

   ☐ True   ☐ False

6. Let B = {e, f, g, h} and let R = {(e, f), (f, g), (g, h), (h, f)} be a relation on B.

Draw the directed graph representing **R** and deduce whether it is reflexive, symmetric and antisymmetric. Also represent the information using the matrix method.

7. Construct a truth table for the compound proposition $(m \land \neg n) \rightarrow (r \lor n)$ and deduce whether it is a tautology or not with an explanation

8. For all integers, p and q, show that if p – q is even, then $p^3 - q^3$.