MAKERERE UNIVERSITY
FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY
2010/2011 SEMESTER I
MTH 3105: DISCRETE MATHEMATICS
Test I Solutions

There are 5 (five) questions in this test, attempt all questions.
Time allowed: 1 (one) hour.

1. (a) Write contrapositive for each of the following statements: *6 marks*
   (i) The audience will go to sleep if the chairperson gives the lecture.

If the audience does not sleep, then the chairperson does not give the lecture

   (ii) You may inspect the aircraft only if you have the proper security clearance.

If you do not have the proper security clearance, then you cannot inspect the aircraft

   (iii) If x^2 is an even number, then x is an even number.

If x is an odd number, then x^2 is an odd number

   (b) Apply DeMorgan's law to $\neg(\exists x(Q(x) \longrightarrow \forall y P(y)))$ *4 marks*

$\equiv \forall x \neg(Q(x) \longrightarrow \forall y P(y))$
$\equiv \forall x \neg(\neg Q(x) \vee \forall y P(y))$
$\equiv \forall x(Q(x) \wedge \exists y \neg P(y))$

2. (a) What conclusion can you make, given the followings are true? *4 marks*

   (i) $\neg q \longrightarrow P(a)$ (iii) $s \vee \neg\exists x Q(x)$
   (ii) $q \longrightarrow Q(b)$ (iv) $\forall x \neg P(x)$

1. By (iv), for any a, ¬P(a) is true.
2. By (i), apply contrapositive and we get q is true.
3. By (ii), Q(b) is true.
4. By (iii), since ∃xQ(x) is false, s must be true.

   (b) A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:

   - If both Tom and Jesse are lying then Leo is telling the truth.
   - If Tom or Hackson is lying, then Jesse is also lying.
   - If Tom is telling the truth then John is lying.
   - John is a well respected teacher so he never lies.
   - Either Leo or Hackson is lying.

   What conclusion can you make? There may be more than one liar. Show your steps.
6 marks

Let A be "John is lying", J be "Jesse is lying", T be "Tom is lying" and etc. Then by the given facts, we have

i. $T \wedge J \longrightarrow \neg L$ iii. $\neg T \longrightarrow A$ v. $L \vee H$
ii. $T \vee H \longrightarrow J$ iv. $\neg A$

and we can decide that
1. By (iv), A is false.
2. By (iii), applying contrapositive we get T is true.
3. By (ii), we get J is also true.
4. By (i), L is false.
5. By (v), since L is false H must be true.

To conclude, Tom, Jesse and Hackson are lying

3. Let A, B, C be three arbitrary sets. Prove the following properties: *10 marks*
   (a) $A \cup B = B \cup A$ (d) $A \cup \emptyset = A$
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$ (e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (c) $A \backslash C \subseteq B \backslash C$, assuming A is a subset of B [for this part (c) only]

NB: Alternative solutions may use venn diagrams

a. $A \cup B ::= \{x \mid (x \in A) \vee (x \in B)\}$
$B \cup A ::= \{x \mid (x \in B) \vee (x \in A)\}$
$\equiv \{ x \mid (x \in A) \vee (x \in B)\}$ *commutative law,* ■

b. $(A \cap B) \cap C ::= \{x \mid ((x \in A) \wedge (x \in B)) \wedge (x \in C)\}$
$\equiv \{x \mid (x \in A) \wedge ((x \in B) \wedge (x \in C))\}$ *associative law*
$A \cap (B \cap C) ::= \{x \mid (x \in A) \wedge ((x \in B) \wedge (x \in C))\}$ ■

c. $A \mid C ::= \{(x \in A) \wedge (x \notin C)\}$
$B \mid C ::= \{(x \in B) \wedge (x \notin C)\}$
$A \subseteq B \longrightarrow \{\forall x, \text{ if } x \in A \text{ then } x \in B\}$
$\therefore A \mid C \subseteq B \mid C$ *by the same definition of subset* ■

d. $A \cup \emptyset ::= \{x \mid (x \in A) \vee (x \in \emptyset)\}$
$\equiv \{x \mid x \in A\}$
$\equiv A$ ■

e. $A \cap (B \cup C) ::= \{x \mid (x \in A) \wedge ((x \in B) \vee (x \in C))\}$
$\equiv \{x \mid ((x \in A) \wedge (x \in B)) \vee ((x \in A) \wedge (x \in C))\}$ *distributive law*
$(A \cap B) \cup (A \cap C) ::= \{x \mid ((x \in A) \wedge (x \in B)) \vee ((x \in A) \wedge (x \in C))\}$ ■

4. (a) Write a truth table for
   $p \oplus q \oplus r$
   where $\oplus \equiv$ exclusive OR. *5 marks*

   (b) Write a logical formula for the truth table besides and then simplify it. *5 marks*

p	q	r	Output
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	F

p	q	r	$p \oplus q \oplus r$
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F ≡
T	F	F	T
T	F	T	F
T	T	F	F
T	T	T	T

Either $\neg[(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge q \wedge r)]$
$\equiv (p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$
… factorizing p, opening brackets, & factorizing $(\neg q \vee \neg r)$ *…*
$\equiv (\neg q \vee \neg r) \wedge (p \vee q \vee r)$?

Or $(\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r)$
$\equiv \{\neg p \wedge [(\neg q \wedge r) \vee (q \wedge \neg r)]\} \vee \{p \wedge [(\neg q \wedge r) \vee (q \wedge \neg r)]\} \vee (p \wedge \neg q \wedge \neg r)$
$\equiv \{(\neg p \wedge p) \wedge [(\neg q \wedge r) \vee (q \wedge \neg r)]\} \vee (p \wedge \neg q \wedge \neg r)$
$\equiv (q \oplus r) \vee (p \wedge \neg q \wedge \neg r)$

5. (a) Simplify the following statement. $\neg (\neg q \wedge \neg (\neg q \vee s)) \vee (q \wedge (r \longrightarrow r))$ *4 marks*

$\equiv \neg (\neg q \wedge \neg (\neg q \vee s)) \vee q$ *identity laws*
$\equiv (q \vee (\neg q \vee s)) \vee q$
$\equiv (q \vee \neg q) \vee s \vee q$
$\equiv$ *true*

   (b) Assume that $\exists x \forall y P(x, y)$ is true and the domain of them is nonempty. Which of the following must be true? *6 marks*
   (i) $\forall x \forall y P(x, y)$ (ii) $\forall x \exists y P(x, y)$ (iii) $\exists x \exists y P(x, y)$

Only (iii) must be true