MTH 3105: Discrete Mathematics

Take Home Assignment I

1.
   (a) Prove that there is no positive integer solutions for:
   $4a^3 + 2b^3 = c^3$ (5 Marks)

   (b) Define $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$ for $n = 0, 1, 2, \ldots$ Prove that for any $n \geq 0$ we have:
   (5 Marks)

$$F_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^{n-1}$$

2.
   (a) Determine whether or not the following arguments are valid:

   a. $\dfrac{\neg p \to q, \neg q}{p}$ (4 Marks)

   b. $\dfrac{\neg p \to \neg q}{p \to q}$ (4 Marks)

   (b) If the tuple (a, b, c) satisfy the following conditions:
   - a, b and c are consecutive odd integers.
   - a, b and c are all primes.

   Then we call it a super prime tuple. Prove that (3, 5, 7) is the only super prime tuple.
   (4 Marks)

3.
   (a) Prove that $\sqrt{6}$ is irrational. (4 Marks)

   (b) A detective has interviewed five witnesses to a crime. From their stories, the detective has concluded that:
   - If both Tom and Jesse are lying, then Leo is telling the truth.
   - If Tom or Hackson is lying, then Jesse is also lying.
   - If Tom is telling the truth, then John is lying.
   - John is a well-respected teacher so he never lies.
   - Either Leo or Hackson is lying.

   What conclusion can you make? There may be more than one liar. Show your steps.
   (6 Marks)

4.
   (a) Prove that if p is prime and $p|ab$, then $p|a$ or $p|b$. (4 Marks)

   (b) Let X be the set $\{x \in \mathbb{Z} \mid x + 3 \text{ is a multiple of } 7\}$.

   Let Y be the set $\{y \in \mathbb{Z} \mid y - 4 \text{ is a multiple of } 7\}$.

   Prove that $X = Y$. (4 Marks)

END

This assignment is due 1 week from the 1st (SE) and 2nd (CS) of October, 2018.