MTH 3105: Discrete Mathematics, November 16, 2015

Take Home Assignment II

1.
   (a) What conclusion can you make, given the followings are true? (4 Marks)
   - $\neg q \rightarrow P(\mathrm{a})$
   - $q \rightarrow Q(b)$
   - $s \vee \neg\exists x Q(x)$
   - $\forall x\, \neg P(x)$

   (b) Show that for any integer n, $n^4 + 2n^3 - n^2 - 2n$ is divisible by 4. (6 Marks)

   (c) A number n is a sum of two squares if $n = a^2 + b^2$ for some integers a and b. If x and y are both sum of two squares, prove that xy is also a sum of two squares. (6 Marks)

2.
   (a) Simplify the following statement. $\neg(\neg q \wedge \neg(\neg q \vee s)) \vee (q \wedge (r \rightarrow r))$ (4 Marks)

   (b) Define $F_0 = 0$, $F_1 = 1$, and $F_k = F_{k-1} + F_{k-2}$ for $k > 1$. Show that:

      i. $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$ (5 Marks)

      ii. $\sum_{i=0}^{n} F_n^2 = F_n F_{n+1}$ (5 Marks)

3.
   (a) Determine whether there exists an inverse of 101 modulo 4620. If such an inverse exists, use the Euclidean Algorithm to determine the inverse. (4 Marks)

   (b) Use the inverse in (a) above to solve for x or give "Unsolvable" if there exists no solution in: (6 Marks)

   $101x \equiv 26 \pmod{4620}$

This assignment is due 1 week from date.