

## MTH 3105: Discrete Mathematics

### Take Home Assignment I

1.
  - (a) Define the sequence  $c_0, c_1, \dots$  by the equations  $c_0 = 0$  and  $c_n = c_{\lfloor n/2 \rfloor} + 3$  for all  $n > 0$ . Prove that  $c_n \leq 2n$  for all  $n \geq 3$ . ( 5 Marks )
  - (b) Prove that any positive integer  $N$  is divisible by 11 if and only if the difference between the sum of odd digits and the sum of even digits is divisible by 11. ( 5 Marks )
2.
  - (a) Assume that  $\forall x \exists y P(x, y)$  is false and the domain of them is nonempty. Which of the following must be false? ( 2 Marks )
    - (i)  $\forall x \forall y P(x, y)$
    - (ii)  $\exists x \forall y P(x, y)$
    - (iii)  $\exists x \exists y P(x, y)$
  - (b) Let  $P(x)$  denote the statement “ $x$  is an accountant” and let  $Q(x)$  denote the statement “ $x$  owns a Porsche”. Write each statement below in first order logic. ( 2 Marks )
    - (i) All accountants own Porsches.
    - (ii) Some accountant owns a Porsche.
    - (iii) All owners of Porsches are accountants.
    - (iv) Someone who owns a Porsche is an accountant.
3.
  - (a) Prove that  $\sqrt{6}$  is irrational. ( 4 Marks )
  - (b) A detective has interviewed four witnesses to a crime. From their stories, the detective has concluded that:
    - (i) If the butler is telling the truth, then so is the cook.
    - (ii) The cook and the gardener cannot both be telling the truth.
    - (iii) The gardener and the handyman are not both lying.
    - (iv) If the handyman is telling the truth then the cook is lying.Who must be lying? There may be more than one liar. Show your steps. ( 6 Marks )
4.
  - (a) Simplify the following statement: ( 2 Marks )
$$\neg (\neg q \wedge \neg (\neg q \vee s)) \vee (q \wedge (r \rightarrow r))$$
  - (b) Determine whether or not the following arguments are valid:
    - (i) 
$$\frac{\neg p \rightarrow q, \neg q}{p}$$
 ( 4 Marks )
    - (ii) 
$$\frac{\neg p \rightarrow \neg q}{p \rightarrow q}$$
 ( 4 Marks )

This assignment is due 1 week from the 16<sup>th</sup> of October, 2023.