CSC 2105: Discrete Mathematics

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Introduction to Discrete Mathematics

Overview

Motivation

Warmup Facts

1. If n is a positive integer, then there are n integers i such that $1 \leqslant i \leqslant n$
2. If m and n are positive integers with $m \leqslant n$, then the number of integers i such that $m \leqslant i \leqslant n$ is $n - (m-1) = n - m + 1$
3. if m and n are integers with $m \leqslant n$, then there are $n - m + 1$ integers i with $m \leqslant i \leqslant n$
4. Let k and n be positive integers. Then the number of multiples of k between 1 and n is $\lfloor n/k \rfloor$

If x is any real number, the **floor** of x, written $\lfloor x \rfloor$, is the largest integer less than or equal to x.

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Factors and Multiples

- If m and n are integers, then n is a **multiple of** m if $n = km$ for some integer k
- In otherwords, n is **divisible by** m, or m **divides** n, or m is a **divisor of** n, or m is a **factor of** n
- We write $m|n$ to mean "*m divides n*" and $m \nmid n$ in case $m|n$ is false
- if n is a multiple of m, then so is every multiple of n

Examples

1. We have $3|6$, $4|20$, $15|15$, $27|1998$, $4 \nmid 7$, $12 \nmid 11$, and $17 \nmid 1998$
2. For every nonzero integer n, we have $1|n$ and $n|n$, since $n = n \cdot 1 = 1 \cdot n$
3. Since $0 = 0 \cdot n$ whenever n is an integer, 0 is always a multiple of n (every integer is a divisor if 0). On the other hand, if n is a multiple of 0, then $n = k \cdot 0 = 0$, so the only multiple of 0 is 0 itself.

Factors and Multiples

Proposition

Proposition

If m and n are positive integers such that $m \mid n$, then $m \leqslant n$ and $\frac{n}{m} \leqslant n$

Proof

Let $k = \frac{n}{m}$. Then k is an integer and $n = km$. Since $n \neq 0$ and since n and m are both positive, k can't be 0 or negative. The smallest positive integer is 1, so $1 \leqslant k$. Hence $m = 1 \cdot m \leqslant k \cdot m = n$. Since $k \mid n$, the same argument shows that $k \leqslant n$.

Factors and Multiples

Theorems

1. An integer n greater than 1 is a prime if and only if its only positive divisors and 1 and n
2. Every integer greater than 1 can be written uniquely as a prime or as the product of two or more primes where the prime factors are written in order of nondecreasing size
   - $100 = 2 \cdot 2 \cdot 5 \cdot 5 = 2^2 \cdot 5^2$
   - $168 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 2^3 \cdot 3 \cdot 7$
   - $175 = 5 \cdot 5 \cdot 7 = 5^2 \cdot 7$
   - $192 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^6 \cdot 3$
   - $641 = 641$
   - $999 = 3 \cdot 3 \cdot 3 \cdot 37 = 3^3 \cdot 37$
   - $1024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{10}$
3. For positive integers m and n, we always have $gcd(m, n) \cdot lcm(m, n) = mn$
4. Let m and n be positive integers
   a) Every common divisor of m and n is a divisor of $gcd(m, n)$
   b) Every common multiple of m and n is a multiple of $lcm(m, n)$

Special Sets

Sets

- Natural numbers, $\mathbb{N} = 0, 1, 2, 3, 4, 5, 6, ...$
- Positive integers, $\mathbb{P} = 1, 2, 3, 4, 5, 6, 7, ...$
- The set of all integers, positive, zero, or negative, will be denoted by $\mathbb{Z}$ [for the German word "Zahl"]
- Rational numbers denoted by $\mathbb{Q}$
- Real numbers denoted by $\mathbb{R}$

Some notation

- We will consistently use braces { }, not brackets [ ] or parentheses ( ), to describe sets
- $73 \in \mathbb{Z}$, $73 \in \mathbb{N}$, $-73 \notin \mathbb{N}$, and $-73 \in \mathbb{Z}$
- $\{\ :\quad \}$
  E.g., $\{n : n \in \mathbb{N}$ and n is even$\}$, $\{n^2 : n \in \mathbb{N}\}$, $\{(-1)^n : n \in \mathbb{N}\} = \{$-1,1$\}$

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Special Sets

Examples

1. As with the familiar inequality $\leqslant$, we can run the assertions: $\mathbb{P} \subseteq \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
2. We have $\mathcal{P}(\varnothing) = \{\varnothing\}$ where $\mathcal{P}(S)$ is the set of all subsets of the set S, called the **power set** of S. Note that the one-element set $\{\varnothing\}$ is different from the empty set $\varnothing$.
3. Let $\Sigma = \{a, b, c, d, ..., z\}$ consist of the twenty-six letters of the English alphabet. Any string of letters (word) from Σ belongs to the infinite set Σ^* (e.g., *math, is, fun, aint, lieblich, amour, zzyzzoomph, etcetera, etc*)
    - The American language, a subset of Σ^*, consists of the words in the latest edition of *Webster's New World Dictionary of the American Language*
    - If $\Sigma = \{a, b\}$, then $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, aab, ...\}$ where λ is the *empty word, null word, or null string*.

Conclusion

- We have had some motivation for Discrete Mathematics with the help of warm up questions
- We have had a closer look at some properties of Factors and Multiples
- We have had a brief introduction to Sets and had examples of some special sets
- We shall revisit Sets (and Sequences) when we look at the Set Theory and Functions