MTH 3105: Discrete Mathematics

Take Home Assignment II

1. Define $F_0 = 0$, $F_1 = 1$ and $F_k = F_{k-1} + F_{k-2}$ for $k > 1$. Show that
   (a) $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$ (5 Marks)
   (b) $\sum_{i=0}^{n} F_i^2 = F_n F_{n+1}$ (5 Marks)
2.
   (a) Find the multiplicative inverse of 12345 modulo 211. (5 Marks)
   (b) Let $X = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$. Define the relation $\equiv$ on X by
   $(x, y) \equiv (z, t) \Leftrightarrow xt = yz$
   for every $(x, y), (z, t) \in X$.
      i. Show that this is an equivalence relation on *X*. (4 Marks)
      ii. Find the equivalence classes of (0, 1) and of (3, 3). (6 Marks)
3. Use the principle of mathematical induction to verify that: (5 Marks @)
   (a)
   $$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n}$$
   (b) for all $n \geq 1$, the sum of the squares of the first 2*n* positive integers is given by the formula
   $$1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$
4.
   (a) Show that a $6 \times n$ board ($n \geq 2$) can be tiled with L-shaped tiles, without gap and overlapping. Each L-shaped tile covers three squares. (5 Marks)
   (b) A particular random number generator is able to give a bunch of random integers from 0 to 65535. How many numbers do you need from this generator to guarantee that there exists two subsets of integers that sum to the same number? Show your steps. (5 Marks)
   (c) Prove the following equality using combinatorial proof. (Note: No marks will be given if it is not a combinatorial proof.) (5 Marks)
   $$\binom{n}{r} = \sum_{k=r-1}^{n-1} \binom{k}{r-1} \; for\ 0 < r \leq n$$

This assignment is due 1 week from the 13th of November, 2023.